LOVE NUMBERS AND ELASTIC ENERGY OF DEFORMATION FOR VENUS

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Abstract. Love numbers of second order of Venus are calculated with resolving fundamental differential equations of elastic body according to the parameters of the density and the elasticity of material by means of the PVM94-01 Venus model. Meanwhile, the elastic energy of deformation of second order due to the tidal perturbation of the Sun and that due to rotational centrifugal potential are also calculated. The values of Love number provide a basis for model of internal structure of Venus. The numerical calculation of the elastic energy of deformation gives a magnitude evaluation of the perturbation terms to the Hamiltonian expressions in the study of dynamics of the elastic Venus.

Keywords: Elastic energy of deformation, Love number, Venus

1. Introduction

Venus has been the object of space exploration for more than 30 years. A wealth of data enriches man's knowledge on Venus geodesy. At present, important geodetic parameters of Venus are provided with high accuracy (e.g., Bills and Synnott, 1987). The geodetic parameters for both Venus and the Earth adopted in this study are shown in Table I.

In Table I a is average radius of a celestial body, ρ the average density, m and $m_{\oplus}$ the masses of a planet and the Earth, and I/ma^2 the dimensionless moment of inertia (I is the mean moment of inertia). By the comparison of the geodetic parameters between Venus and the Earth, Venus is reminiscent of a twin planet of Earth. Therefore, it is quite natural to construct an internal model of Venus on the basis of an Earth model. A comprehensive review on the construction of Venus' model may be found from a survey by Zharkov (1983). A sequence of models for

TABLE I

Geodetic parameters of Venus and the Earth

	a (km)	ρ (g cm^{-3})	$m/m_{\oplus}$	I/ma^2	Ω (rad s^{-1})
Venus	6051.53	5.2446	0.8150	0.329–0.341	2.99236×10^{-7}
Earth	6378.136	5.51483	1.0	0.3307	7.292115×10^{-5}

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the interior of Venus is constructed under the assumption of hydrostatic equilibrium by Zhang and Zhang (1995). The basic conclusion of these works is that the interior structure of Venus is similar to that of Earth. Apparently, we can study Venus, one of the terrestrial planets, with the method by which the Earth is studied.

The deformation of the Earth by the tidal attractions of the Sun and Moon has been dealt with by various authors (Jeffreys and Vicente, 1957; Munk and MacDonald, 1960; Moritz, 1982). They regarded the Earth as an elastic body in theoretical studies of the rotational motion of a deformable Earth. On the analogy of the Earth, in the first approximation Venus may also be thought as an elastic body. When Venus is considered as an elastic body, Venus is deformed by external forces (such as the tide-generating force of the Sun, rotational centrifugal force, etc.). Perturbing potentials of the external forces and various deformational effects can be expanded with spherical harmonic functions, and functional relations can be definitively set up between their corresponding terms of same order through a dimensionless number or a partial derivative, these relations are Love numbers. The Love numbers of Venus are the quantities that are related to the internal structure and the distributions of the elasticity and the density of material of Venus. Szeto (1983) estimated the Love numbers by means of a certain simplified model of Venus. Bodri (1987) computed second degree Love numbers for radially heterogeneous compressible models of Venus. Zhang (1992) computed the static Love numbers using the latest geodetic parameters for the parametric models of Venus. When we consider the tidal response feature of Venus, it is necessary to estimate its Love numbers.

In this paper, the displacement vectors of deformation are obtained by resolving fundamental differential equations of elastic body, which is based on a Hamiltonian theory for the rotation motion of an elastic body (Getino and Ferrandiz, 1990, 1991). By means of the PVM94-01 Venus model given by Zhang and Zhang (1995), the Love numbers of second order of Venus are acquired. Meanwhile, the elastic energy of deformation of second order of Venus due to the tidal perturbation of the Sun and that due to rotational centrifugal potential are also calculated.

2. Fundamental Equations of Elastic Body

It is assumed that the basic state of an elastically deformable body is in hydrostatic equilibrium; the stress on a perturbed body is the sum of an basic stress and an additional stress, and the additional stress depends upon the strain and can be calculated with Hooke's law; the density and rheological parameters are considered as function of the distance from the center of mass of an elastic body; and the additional stress and strain both are so small, that their effects can be regarded as perturbations.

The fundamental differential equation of an elastic body is

$$\rho \partial^2 \mathbf{u} / \partial t^2 = \rho \nabla V + \nabla \cdot \sigma, \tag{1}$$

where ρ, V, σ are the density, potential of the forces and tensor of stress of a deformed elastic body, respectively; $\mathbf{u} = (u, v, w)$ is the displacement vector of deformation of a body element.

Because of the deformation of mantle layer is small in comparison with the basic state, merely the term of first order may be remained. The density ρ can be obtained from the law of mass conservation as follows

$$\rho = \rho_0 - U\,\mathrm{d}\rho_0/\mathrm{d}r - \rho_0\Delta, \tag{2}$$

where ρ_0 is the density in the basic state. $U = (xu + yv + zw)/r$ is the radical displacement, $\Delta = \partial u/\partial x + \partial v/\partial y + \partial w/\partial z$ is the volume divergence, r is the distance from the center of mass of an elastic body.

Potential V is the sum of the basic potential V_0 and the perturbing potential H, namely

$$\begin{aligned} V &= V_0 + H, \\ H &= W' + W, \end{aligned} \tag{3}$$

where W' is the additional deformational potential, W is tide-generating potential. H admits Poisson's formula

$$\nabla^2 H = 4\pi G(U\,\mathrm{d}\rho_0/\mathrm{d}r + \rho_0\Delta), \tag{4}$$

where G is the gravitational constant.

The stress tensor consists of the basic stress and the additional stress caused by strain

$$\begin{aligned} \sigma_{xx} &= -(P_0 - U\,\mathrm{d}P_0/\mathrm{d}r) + \lambda\Delta + 2\mu\partial u/\partial x, \\ \sigma_{xy} &= \mu(\partial v/\partial x + \partial u/\partial y), \\ \sigma_{xz} &= \mu(\partial w/\partial x + \partial u/\partial z), \qquad \text{etc.} \end{aligned} \tag{5}$$

where λ, μ are Lame's elastic parameters, P_0 is the hydrostatic pressure in the basic state.

If the perturbing potential, H, is expressed in an expansion of the form

$$H_n = K_n(r)W_n, \tag{6}$$

where W_n is the spherical harmonic function of the perturbing potential, $K_n(r)$ is solely a function of r, the expressions of the displacement vector can be written as (Getino, 1993)

$$u_n = F_n(r)\partial W_n/\partial x + G_n(r)xW_n,$$

$$v_n = F_n(r)\partial W_n/\partial y + G_n(r) y W_n, \tag{7}$$

$$w_n = F_n(r) W_n/\partial z + G_n(r) z W_n,$$

where $F_n(r)$, $G_n(r)$ are only functions of r.

From Equations (3) and (6), we obtained

$$W'_n = (K_n(r) - 1) W_n. \tag{8}$$

Inserting Equations (2), (3), (5), (6) and (7) into Equations (1) and (4), respectively, and letting

$$s = r/a, \quad F_n(s) = F_n(r)/a^2,\ G_n(s) = G_n(r),\ K_n(s) = K_n(r)/4\pi G a^2, \tag{9}$$

where a is the mean radius of an elastic body, as for the case where $n = 2$, the differential equations needed to find the functions $F_2(s)$, $G_2(s)$ and $K_2(s)$ are obtained as follows

$$\begin{aligned}
&\mu F''_2 + [\mathrm{d}\mu/\mathrm{d}s + (2\lambda + 6\mu)/s]F'_2 + (\lambda + \mu)sG'_2 + [(2/s)\,\mathrm{d}(P_0 + \mu)/\mathrm{d}s]F_2 \\
&\quad + [s\,\mathrm{d}(P_0 + \mu)/\mathrm{d}s + 5\lambda + 7\mu]G_2 + 4\pi G a_2 \rho_0 K_2 = 0, \\
&(\lambda + 2\mu)s^2 G''_2 + 2(\lambda + \mu)F''_2 + 2[\mathrm{d}(\lambda + \mu)\mathrm{d}s - (\lambda + \mu)/s]F'_2 \\
&\quad + [s^2 \mathrm{d}(\lambda + 2\mu)/\mathrm{d}s + (6\lambda + 14\mu)/s]G'_2 + 4\pi G a^2 \rho_0 s K'_2 \\
&\quad + 2[(-1/s)\,\mathrm{d}P_0/\mathrm{d}s + \rho_0\,\mathrm{d}^2 V_0/\mathrm{d}s^2]F_2 + [s^2 \rho_0\,\mathrm{d}^2 V_0/\mathrm{d}s^2 \\
&\quad + s\,\mathrm{d}(5\lambda + 4\mu - 4P_0)/\mathrm{d}s]G_2 = 0, \\
&K''_2 - (2\rho_0/s)F'_2 - \rho_0 s G'_2 + (6/s)K'_2 - (2/s)(\mathrm{d}\rho_0/\mathrm{d}s)F_2 \\
&\quad - (s\,\mathrm{d}\rho_0/\mathrm{d}s + 5\rho_0)G_2 = 0,
\end{aligned} \tag{10}$$

where $F''_2 = \mathrm{d}^2 F_2(s)/\mathrm{d}s^2$, $F'_2 = \mathrm{d}F_2(s)/\mathrm{d}s, \ldots$. To find the displacement vectors will, therefore, be turned to do the numerical integration of the functions $F_2(s)$, $G_2(s)$ and $K_2(s)$ in Equation (10).

Being integrated Equations (10) must satisfy corresponding boundary conditions. By making $\mathbf{u}$, V, σ, $\nabla(V_0 + K)$ continuous at the discontinuous surface in the mantle of an elastic body, the boundary conditions are obtained as

$$\begin{aligned}
&(F_2)_{\text{above}} = (F_2)_{\text{below}}, \quad (G_2)_{\text{above}} = (G_2)_{\text{below}}, \quad (K_2)_{\text{above}} = (K_2)_{\text{below}}, \\
&[\mu(F'_2 + 2F_2/s + sG_2)]_{\text{above}} = [\mu(F'_2 + 2F_2/s + sG_2)]_{\text{below}}, \\
&[\lambda(2F'_2/s + sG'_2 + 5G_2) + \mu(2F'_2/s + 2sG'_2 + 4G_2)]_{\text{above}} \\
&\quad = [\lambda(2F'_2/s + sG'_2 + 5G_2) + \mu(2F'_2/s + 2sG'_2 + 4G_2)]_{\text{below}},
\end{aligned} \tag{11}$$

$$(K_2')_{\text{above}} = (K_2')_{\text{below}} + (1/s)(2F_2 + s^2 G_2)[(\rho_0)_{\text{above}} - (\rho_0)_{\text{below}}],$$

where the suffixes 'above' and 'below' refer to the values above and below the surface of boundary surface, respectively.

3. Elastic Energy of Deformation

When an elastic body is deformed by the attraction of outside body, an elastic energy of deformation is produced.

If Equation (7) are expressed in spherical coordinates instead of Cartesian coordinates, they can be written as

$$\begin{aligned} u_{rn} &= (nF_n/r + rG_n)W_n, \\ u_{\theta n} &= (F_n/r)(\partial W_n/\partial\theta), \\ u_{\phi n} &= (F_n/r\sin\theta)(\partial W_n/\partial\phi). \end{aligned} \tag{12}$$

Then the elastic energy of deformation per volume unit can be expressed as (Getino and Ferrandiz, 1991)

$$E_d = (\lambda/2)(e_{rr} + e_{\theta\theta} + e_{\phi\phi})^2 + \mu[e_{rr}^2 + E_{\theta\theta}^2 + e_{\phi\phi}^2 + 2(e_{r\theta}^2 + e_{r\phi}^2 + e_{\theta\phi}^2)], \tag{13}$$

where e_{ij} are the components of the deformation, they relate to the components of the displacement vector (u_r, u_θ, u_ϕ) with following expressions

$$\begin{aligned} e_{rr} &= \partial u_{rn}/\partial r, \\ e_{\theta\theta} &= (1/r)(\partial u_{\theta n}/\partial\theta) + u_{rn}/r, \\ e_{\phi\phi} &= (1/r\sin\theta)(\partial u_{\phi n}/\partial\phi) + (1/r)(u_{rn} + \cot\theta\, u_{\phi n}), \\ e_{r\theta} &= [(1/r)(\partial u_{rn}/\partial\theta) - u_{\theta n}/r + \partial u_{\theta n}/\partial r]/2, \\ e_{r\phi} &= [(1/r\sin\theta)(\partial u_{rn}/\partial\phi) - u_{\phi n}/r + \partial u_{\phi n}/\partial r]/2, \\ e_{\theta\phi} &= [(1/r)(\partial u_\phi/\partial\theta)) - (1/r)\cot\theta\, u_{\phi n} + (1/r\sin\theta)(\partial u_{\theta n}/\partial\phi)/2. \end{aligned} \tag{14}$$

Inserting Equations (12) and (14) into Equation (13), and taking into account the property of the spherical harmonics, for $n = 2$, Equation (13) becomes

$$\begin{aligned} E_d = {} & (\lambda/2) f_2^2 W_2^2 + \mu\{(Q_1^2 + 2Q_2^2 - 12Q_2Q_3)W_2^2 \\ & + (Q_4^2/2)[(\partial W_2/\partial\theta)^2 + (1/\sin^2\theta)(\partial W_2/\partial\phi)^2] \\ & + Q_3^2[(6W_2 + \partial^2 W_2/\partial\theta^2)^2 + (2/\sin^2\theta)(\partial^2 W_2/\partial\theta\partial\phi \\ & - \cot\theta\,\partial W_2/\partial\phi)^2 + (\partial^2 W_2/\partial\theta^2)^2], \end{aligned} \tag{15}$$

where

$$
\begin{aligned}
f_2 &= (2/r)F_2' + rG_2' + 5G_2,\\
Q_1 &= (2/r)F_2' + rG_2' + (2/r)F_2 + 3G_2,\\
Q_2 &= (2/r)F_2 + G_2,\\
Q_3 &= (1/r)F_2,\\
Q4 &= (1/r)F_2' + (2/r)F_2 + G_2.
\end{aligned}
\tag{16}
$$

The tidal perturbing potential of the second order, W_2, can be showed as

$$W_2 = (GM/R^3)r^2 P_2(\cos z), \tag{17}$$

where M is the mass of the perturbing body, R the distance from the elastic body to the perturbing body, z the zenith distance of the perturbing body, and $P_2(\cos z)$ the Legendre polynomial of second order.

The total elastic energy of deformation, E, is obtained by integrating E_d in the entire mantle as follows

$$E = \int E_d \, \mathrm{d}V = \int_r^a \int_0^\pi \int_0^{2\pi} E_d r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\phi. \tag{18}$$

With the following results of integration taken into account

$$
\begin{aligned}
&\int_0^\pi \int_0^{2\pi} W_2^2 r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi = (GM/R^3)^2 (12\pi/15) r^6,\\
&\int_0^\pi \int_0^{2\pi} [(\partial W_2/\partial\theta)^2 + (1/sin^2\theta)(\partial W_2/\partial\phi)^2] r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi\\
&\qquad = (GM/R^3)^2 (24\pi/15) r^6,\\
&\int_0^\pi \int_0^{2\pi} [(6W_2 + \partial^2 W_2/\partial\theta^2)^2 + (2/\sin^2\theta)(\partial^2 W_2/\partial\theta\,\partial\phi - \cot\theta\,\partial W_2/\partial\phi)^2\\
&\qquad + (\partial^2 W_2/\partial\theta^2)^2] r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi = (GM/R^3)^2 24\pi r^6.
\end{aligned}
\tag{19}
$$

Finally, the expression of the total elastic energy of deformation, E, is obtained as

$$E = (GM/R^3)^2 (2\pi/5)(I_\lambda + 2I_\mu), \tag{20}$$

where

$$I_\lambda = (a^3/16\pi^2 G^2) \int_s \lambda (2s^2 F_2' + s^4 G_2' + 5s^3 G_2)^2 \, \mathrm{d}s,$$

$$I_\mu = (a^3/16\pi^2 G^2) \int_s \mu [7s^4 F_2'^2 + 30s^2 F_2^2 + 14s^6 G_2^2 + S^8 G_2'^2 + 20s^3 F_2 \times (F_2' + sG_2) + 4s^5 G_2'(sF_2' + F_2) + 18s^5 F_2' G_2 + 6s^7 G_2' G_2] \, \mathrm{d}s. \tag{21}$$

4. Numerical Results

The PVM94-01 Venus model given by Zhang and Zhang (1995) is adopted, the parameters of this model are listed in the Table II, in which r is the distance from the mass center of Venus, ρ_0 the density, V_p the velocity of longitudinal wave, V_s velocity of transverse wave, $s = r/a$, $a = 6051.53 \times 10^5$ cm (cf. Table I) the mean radius of Venus.

The analytical expressions of the rheological constants and their derivatives may be calculated with the following equations

$$\begin{aligned}
&\mu = \rho_0 V_s^2, \qquad \lambda = \rho_0 V_p^2 - 2\mu, \\
&\mathrm{d}\mu/\mathrm{d}s = V_s^2 \, \mathrm{d}\rho_0/\mathrm{d}s + 2V_s\rho_0 \, \mathrm{d}V_s/\mathrm{d}s, \\
&\mathrm{d}\lambda/\mathrm{d}s = V_p^2 \, \mathrm{d}\rho_0/\mathrm{d}s + 2V_p\rho_0 \, \mathrm{d}V_p/\mathrm{d}s - 2\mathrm{d}\mu/\mathrm{d}s, \\
&g(s) = (4\pi Ga/s^2) \int \rho_0(s) s^2 \, \mathrm{d}s, \qquad \mathrm{d}g/\mathrm{d}s = -(2g/s) + 4\pi Ga\rho_0, \\
&\mathrm{dP}_0/\mathrm{d}s = -a\rho_0 g = \rho_0 \, \mathrm{d}V_0/\mathrm{d}s, \qquad \mathrm{d}V_0/\mathrm{d}s = -ag, \\
&\mathrm{d}^2 V_0/\mathrm{d}s^2 = -a \, \mathrm{d}g/\mathrm{d}s,
\end{aligned} \tag{22}$$

where g is gravitational acceleration.

By means of the analytical expressions computed from Equation (22), the coefficients of functions, F_2'', F_2', F, G_2'', $G_2' \ldots$, in Equation (10) from Venusian crust to the lower mantle may be obtained with s as their independent variable, for example, for $s = 0.55045$, we have

$$F_2'' = -14.15326F_2' - 1.28011G_2' + 9.25788F_2 - 12.22529G_2 - 0.54590K_2,$$

$$\begin{aligned}
G_2'' = \; & 80.93151F_2' - 32.10854G_2' - 0.29822K_2' - 47.84808F_2 \\
& + 63.11774G_2 + 2.51983K_2,
\end{aligned} \tag{23}$$

TABLE II
PVM94-1 Venus model

Zone	r (km)	ρ_0 (g cm^{-3})	V_p (km s^{-1})	V_s (km s^{-1})
Core	0.00–3331.06	$11.0031 - 0.1088s$ $-5.2654s^2 - 2.0290s^3$	$10.0856 - 0.0891s$ $-5.9955s^2 - 1.6658s^3$	0.0
Lower mantle	3331.06–5585.61	$5.9274 - 0.6953s$ $-1.4302s^2$	$17.7173 - 3.3930s$ $-3.8698s^2$	$9.1671 - 1.8605s$ $-2.1221s^2$
Upper mantle	5585.61–5971.53	$7.4900 - 3.7420s$	$20.6294 - 11.0292s$	$11.4033 - 6.0966s$
Crust	5971.53–6051.53	2.85	6.00	3.50

$$K_2'' = 18.57171F_2' + 2.81357G_2' - 10.90017K_2' - 8.24707F_2 + 24.30759G_2.$$

Integrating numerically Equations (10), we must consider the boundary conditions between the discontinuous surface, including the crust and the upper mantle (s = 0.98676), the upper mantle and the lower mantle (s = 0.92301), the lower mantle and the core (s = 0.55045), and those at the surface of Venus (s = 1.0). The results thus obtained are

$$\begin{aligned} F_2 &= -0.07939 + 5.03454s - 15.27656s^2 + 16.70031s^3 - 6.33066s^4, \\ G_2 &= -1.80606 + 8.20289s - 8.96405s^2 + 0.94574s^3 + 1.82763s^4, \\ K_2 &= 8.13738 - 17.50264s + 8.49681s^2 + 9.53843s^3 - 7.40817s^4. \end{aligned} \tag{24}$$

These results are rounded off to five decimal places and it would be necessary to multiply them by the factor $1/(4\pi Ga^2)$. The distributions of F_2, G_2 and K_2 are shown in Figures 1 and 2.

5. Results and Discussion

5.1. Love numbers

As dimensionless numbers that manifest the elastic nature of an elastic body, Love numbers are ratios between the deformational effects and the corresponding perturbing potential or those between the former and the partial derivatives over a coordinate to the later. The Love numbers of Venus, therefore, can be calculated from the displacement vectors of deformation obtained in the preceding section.

Love number k_2 is ratio between the additional deformational potential and the tide-generating potential on the surface, namely

$$W_2' = k_2 W_2. \tag{25}$$

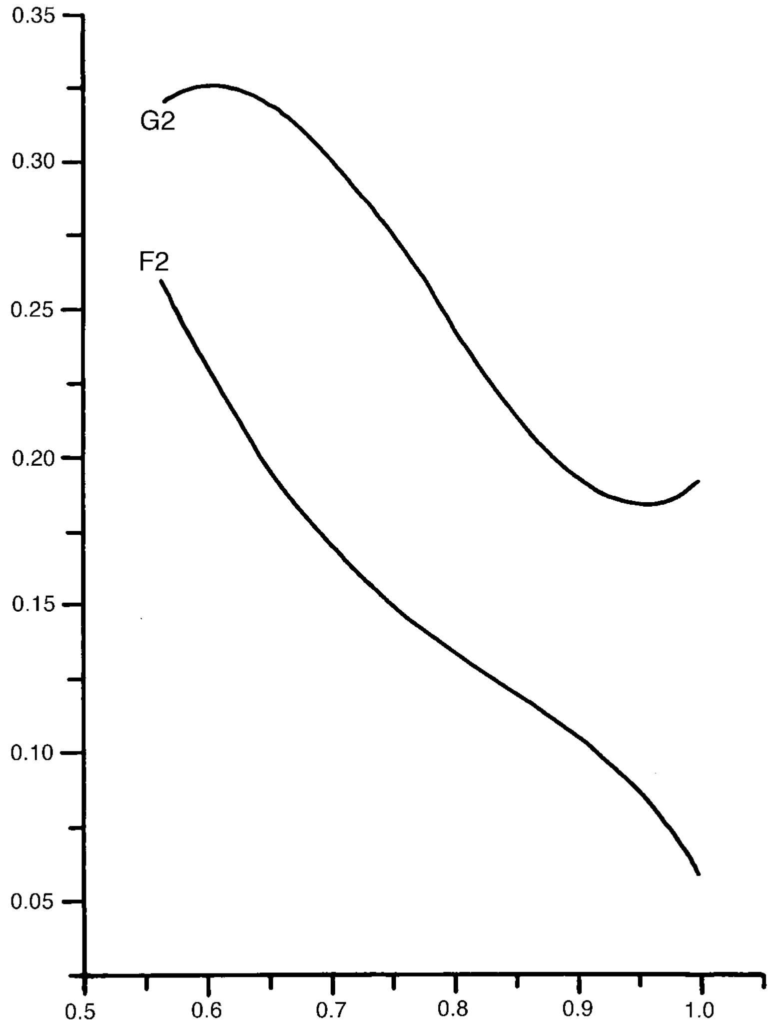

Figure 1. The distribution of F_2 and G_2 for s from 0.55045 to 1.

Love numbers, h_2 and l_2, are ratio between the heights of the body tide and the equilibrium tide, and that between the horizontal displacements of the body tide and the equilibrium tide on the surface, namely

$$u_r = (h_2/g)W_2, \quad u_\theta = (l_2/g)(\partial W_2/\partial\theta), \quad u_\phi = (l_2/g\,\sin\theta)(\partial W_2/\partial\phi). \tag{26}$$

When s = 1.0, from Equation (24) we have

$$K_2 = 1.26182/(4\pi Ga^2), \quad F_2 = 0.04823/(4\pi Ga^2),$$

$$G_2 = 0.20613/(4\pi Ga^2). \tag{27}$$

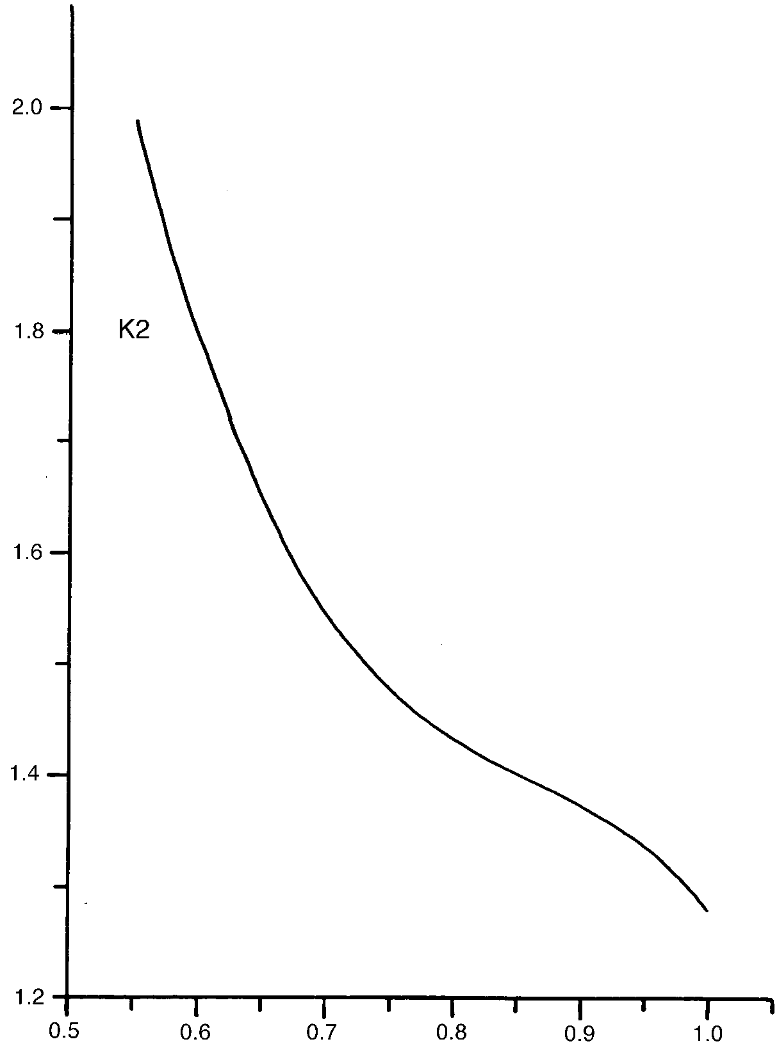

Figure 2. The distribution of K_2 for s from 0.55045 to 1.

From Equations (8) and (25), we obtain

$$k_2 = K_2 - 1 = 0.26182.$$

As for $n = 2$, Equation (12) turn into

$$u_r = (2F_2/r + rG_2)W_2,$$

$$u_\theta = (F_2/r)(W_2/\partial\theta), \tag{28}$$

$$u_\phi = (F_2/r\sin\theta)(\partial W_2/\partial\phi).$$

Then, from Equations (26) and (28) h_2 and l_2 are obtained

$$h_2 = (ga/s)[2F_2(s) + s^2 G_2(s)],$$

$$l_2 = (ga/s)F_2(s). \tag{29}$$

Inserting F_2 and G_2 of Equation (27) into above formulae, as $s = 1.0$, and taking account of the gravitation on the surface of Venus, $g = (4/3)\pi Ga\rho$, where $\rho = 5.2446$ g cm^{-3} (cf. Table I) being the mean density of Venus, we obtain

$$h_2 = 0.52900, \qquad l_2 = 0.08432.$$

The numerical limits of the Love numbers of Venus have been given by Zhang (1991) as $k_2 = 0.18 \sim 0.26$, $h_2 = 0.36 \sim 0.52$, $l_2 \approx 0.08$. The results obtained in this paper are in accordance with his upper limits, because in PVM94-01 Venus model the rigidity coefficient μ of the core layer has been taken as 0. This consequence also shows that the algorithm carried out in this paper is practicable. The Love numbers relate the deformational effects of Venus to the corresponding perturbing potential. The values of Love numbers are quite sensitive to the internal structure of Venus. It is possible to improve the internal model of Venus when the future observation on the Love number of Venus becomes available.

5.2. Elastic energy of deformation

Using Equation (20) we can compute the total elastic energy of deformation, with the heliocentric gravitational constant GM and the distance R taken as, respectively,

$$GM = 1.327124 \times 10^{26} \text{ cm}^3 \text{ s}^{-1}, \qquad R = 1.081592610 \times 10^{13} \text{ cm},$$

and with the respective integral limits in Equations (21) taken from $s = 0.55045$ to $s = 1.0$, thus, we obtain

$$I_\lambda + 2I_\mu = 2.70582 \times 10^{50} \text{ g cm}^2 \text{ s}^{-2}.$$

Finally, we obtain the elastic energy of deformation of second order of the Venus, E, due to the tidal perturbation of the solar gravitation as

$$E = 3.78205 \times 10^{24} \text{ g cm}^2 \text{ s}^{-2}.$$

The elastic energy of deformation due to the rotational centrifugal potential may be obtained by means of similar procedure, as $\Omega = 2.99236 \times 10^{-7}$ rad/s (cf. Table I) being put to use, it is obtained that

$$E_r = (\Omega^2/3)^2(2\pi/5)(I_\lambda + 2I_\mu) = 3.03003 \times 10^{23} \text{ g cm}^2 \text{ s}^{-2}.$$

Taking the principle momentum of inertia of Venus as $C = 0.6062 \times 10^{45}$ g cm^2 (Habibullin,1995), we obtain the principal term of the Hamiltonian, the kinetic energy of zero-order of Venus, $T_0 = 2.7147 \times 10^{31}$ g cm^2 s^{-2}, hence

$$E/T_0 = 1.3932 \times 10^{-7}, \qquad E_r/T_0 = 1.1162 \times 10^{-8}.$$

The elastic energy of deformation of the Earth is caused by the tidal perturbation of the lunar and solar gravitation. According to the Earth's PREM model, it is obtained that $E = 1.5378 \times 10^{24}$ g cm^2 s^{-2}, $E_r = 5.3454 \times 10^{32}$ g cm^2 s^{-2}, $T_0 = 2.1376 \times 10^{36}$ g cm^2 s^{-2} (Xia and Xiao, 2002). Evidently, the Venusian elastic energy of deformation caused by the tidal perturbation potential is greater than that of the earth. The cause is that the sun finds itself more distant to the earth than to Venus, and although the distance between the moon and the earth is greatly smaller than that between the sun and Venus, the mass of the moon is mostly less than that of the sun. The Earth's elastic energy of deformation caused by the rotational centrifugal potential and the kinetic energy of zero-order both are mostly greater than those of Venus because the Earth rotates more rapidly than Venus. We study the elastic energy of deformation which is produced in the Venus' elastic mantle on being deformed by the solar tidal force and the rotational centrifugal force. The results provide a magnitude evaluation of the perturbation terms to the Hamiltonian expressions. The study of elastic energy of deformation is favorable to the research of the rotational movement of Venus with Hamiltonian.

The calculations of Venus' Love numbers and its elastic energy of deformation with Venus' model may inspire the directions of future Venus' exploration or make to suggest proposals of actual effect. These directions or proposals would be aimed to examine and to improve the existing Venus' models.

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