International Journal of Theoretical Physics, Vol. 41, No. 2, February 2002 (© 2002)

Erratum: Correction to Phase Operator on a Deformed Hilbert Space

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Received April 30, 2001

A modification of phase operator is given.

I studied a phase operator $P = (q^n + T^*T)^{-1/2}T$ in five of my papers (Das, 1999, 2000a,b, 2001a,b) to describe the phase vector and studied its various applications. It appears that the phase operator is to be modified to an appropriate one. By analogy I propose the modified phase operator to be

$$P = (q^{N+1} + T^*T)^{-1/2}T$$

where

$$Nf_n = nf_n$$

and $\{f_n\}$ is given in Das (1998).

Now the phase vector is obtained by solving the eigenvalue equation

$$Pf_\beta = \beta f_\beta \tag{1}$$

where $f_\beta(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n(z)$. That is,

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n. \tag{2}$$

Then

$$\begin{aligned} Pf_\beta &= \sum_{n=0}^{\infty} a_n \sqrt{[n]!}(q^{N+1} + T^*T)^{-1/2}Tf_n \\ &= \sum_{n=1}^{\infty} a_n \sqrt{[n]!}(q^{N+1} + T^*T)^{-1/2}\sqrt{[n]}\, f_{n-1} \end{aligned}$$

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0020-7748/02/0200-0371/0 © 2002 Plenum Publishing Corporation

$$= \sum_{n=1}^{\infty} a_n \sqrt{[n]!}\sqrt{[n]}(q^n + [n-1])^{-1/2} f_{n-1}$$

$$= \sum_{n=0}^{\infty} a_{n+1} \sqrt{[n+1]!}\sqrt{[n+1]}(q^{n+1} + [n])^{-1/2} f_n \tag{3}$$

and

$$\beta f_\beta = \beta \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n. \tag{4}$$

From (1)–(4) we observe that a_n satisfies the following difference equation:

$$a_{n+1}\sqrt{[n+1]!}\sqrt{[n+1]}(q^{n+1} + [n])^{-1/2} = \beta a_n \sqrt{[n]!}. \tag{5}$$

That is,

$$a_{n+1} = \frac{\beta a_n (q^{n+1} + [n])^{1/2}}{[n+1]}. \tag{6}$$

Hence,

$$a_1 = \frac{\beta (q + [0])^{1/2} a_0}{[1]}.$$

$$a_2 = \frac{\beta a_1 (q^2 + [1])^{1/2}}{[2]} = \frac{\beta^2 a_0 \sqrt{(q+[0])(q^2+[1])}}{[2]!}.$$

$$a_3 = \frac{\beta a_2 (q^3 + [2])^{1/2}}{[3]} = \frac{\beta^3 a_0 \sqrt{(q+[0])(q^2+[1])(q^3+[2])}}{[3]!}.$$

and so on. Thus,

$$a_n = \frac{\beta^n a_0 \sqrt{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}}{[n]!}.$$

Hence,

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n$$

$$= a_0 \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n.$$

where $\beta = |\beta|\, e^{i\theta}$ is a complex number. These vectors are normalizable in a strict sense only for $|\beta| < 1$.

Now, if we take $a_0 = 1$ and $|\beta| = 1$ we have

$$f_\beta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n. \tag{7}$$

Henceforth, we shall denote this vector as

$$f_\theta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n, \tag{8}$$

$0 \le \theta \le 2\pi$ and call f_θ a *phase vector* in H_q.

REFERENCES

Das, P. K. (1998). Eigenvectors of backwardshift on a deformed Hilbert space. *International Journal of Theoretical Physics* **37**(9), 2363.

Das, P. K. (1999). Phase distribution of Kerr vectors in a deformed Hilbert space. *International Journal of Theoretical Physics* **38**(6), 1807.

Das, P. K. (2000a). Erratum: Phase distribution of Kerr vectors in a deformed Hilbert space. *International Journal of Theoretical Physics* **39**(4), 1171.

Das, P. K. (2000b). Probability operator measure and phase measurement in a deformed Hilbert space. *International Journal of Theoretical Physics* **39**(4), 1037.

Das, P. K. (2001a). Squeezed vector and its phase distribution in a deformed Hilbert space. *International Journal of Theoretical Physics* **40**(4), 205.

Das, P. K. (2001b). Nonlinear phase changes in a deformed Hilbert space. *International Journal of Theoretical Physics* **40**(4), 217.