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Coalition-Proof Allocations in Adverse-Selection Economies

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Abstract

We reexamine the canonical adverse selection insurance economy first studied by Rothschild and Stiglitz [1976]. We define *blocking* in a way that takes private information into account and define a *coalition-proof correspondence* as a mapping from coalitions to allocations with the property that allocations are in the correspondence, if and only if, they are not blocked by any other allocations in the correspondence for any subcoalition. We prove that the *Miyazaki allocation*—the Pareto-optimal allocation (possibly cross-subsidized) most preferred by low-risk agents—is coalition-proof.

Key words: adverse selection, coalition proof, insurance

Adverse-selection economies have been particularly troublesome for economists. As Rothschild and Stiglitz [1976] showed, equilibrium, the way one might naturally define it, often does not exist. While modifications of their definition of equilibrium have been proposed that solve the existence problem, these take the form of specifying particular institutional arrangements through which agents interact, with different specifications yielding different predictions (Wilson [1979] and Hellwig [1987]). It would be desirable to have a predictive notion that is not heavily dependent on a particular extensive form.

An alternative to specifying an explicit game form is to examine solutions that are more cooperative in nature. Miyazaki [1977] represents an early example of this approach, applied to an adverse-selection labor-market environment. The Miyazaki solution is built on the observation that in some cases Pareto-efficiency requires cross-subsidization between types. In the insurance setting, the Miyazaki allocation is the Pareto-optimal allocation most favored by the low-risk agents. Since low-risk agents pay the subsidy, Miyazaki argues that this equilibrium is supported by their threat to abandon the arrangement and self-insure.

Several authors have presented corelike notions of equilibrium in adverse-selection environments that deliver the Miyazaki allocation (Boyd and Prescott [1986], Boyd, Prescott, and Smith [1988], and Marimon [1988]). The extension of the core to adverse-selection environments, however, is not straightforward. The most direct approach is to impose incentive compatibility on agents' decisions about joining blocking coalitions, but this results in a core that is empty in exactly those cases in which the Rothschild-Stiglitz equilibrium fails to exist. The authors cited above, address this problem by further restricting the notion of a successful blocking coalition. They require that a blocking coalition survive when the agents left behind are allowed to react to the deviation.

In an adverse-selection credit-market environment with a continuum of types (Lacker and Weinberg [1993]), we explored a different approach to finding a cooperative equilibrium, utilizing the notion of coalition-proofness introduced by Bernheim, Peleg, and Whinston [1987] (hereafter BPW). Rather than considering reactions by the complement of the deviating coalition, coalition-proofness considers reactions by the deviating coalition itself. A "credible" blocking coalition must itself be immune from further deviations by "credible" subcoalitions. BPW introduced coalition-proofness as a refinement of Nash equilibrium in finite normal-form games and defined it recursively by considering all possible sequences of subcoalitions of agents. Greenberg [1989] showed that coalition-proofness could be defined equivalently as a stable set (see also Greenberg [1991] and Kahn and Mookherjee [1992]).

In a recent paper, Kahn and Mookherjee [1995] (hereafter KM) present a coalition-proof equilibrium for the standard adverse-selection insurance environment. Following BPW, they impose coalition-proofness as a refinement of Nash equilibrium in an incomplete-information game of contract proposal and acceptance. Their approach might be viewed as a blend of cooperative and noncooperative approaches to equilibrium in adverse-selection economies. The unique result under their approach is the Rothschild-Stiglitz allocation—the Pareto-optimum of the type-wise break-even, separating allocations.

The purpose of this note is to examine the implications of coalition-proofness in isolation for the adverse-selection insurance environment. Given our definition of blocking, we define coalition-proof allocations using Greenberg's stable-set approach, as in Lacker and Weinberg [1993] and Lacker [1994]. We show that the Miyazaki allocation is coalition-proof; specifically, the correspondence consisting of the Miyazaki allocation for any arbitrary coalition constitutes a stable set. Therefore, it appears that it is not coalition-proofness per se that rules out cross-subsidization in the KM analysis.

In closely related work, Asheim and Nilssen [1994] present an extensive form-contracting game with renegotiation in which the unique perfect Bayesian equilibrium is the Miyazaki allocation. Their extensive form thus implements the solution suggested by the cooperative literature. Their result also underscores the point made by Wilson [1979] and Hellwig [1987]; the noncooperative equilibrium in an adverse-selection economy depends critically on the extensive form.

The next two sections describe the environment and feasible allocations. In Section 3 we define the coalition-proof correspondence. In Section 4 we define and characterize the Miyazaki allocations. In Section 5 we prove that they are the coalition-proof correspondence.

1. The economy

We study the simple insurance environment described in Rothschild and Stiglitz [1976]. The economy is populated by a continuum of risk-averse agents who each receive a random endowment of a single consumption good. All agents have identical utility functions $U(c)$, where $U'(c) > 0$ and $U''(c) < 0$ for all $c \geq 0$. Each agent receives a random endowment e drawn from the two-element set $\{e_g, e_b\}$, where $e_g > e_b > 0$. There are two types of agents; type H (high-risk) agents, and type L (low-risk) agents. Type $H(L)$ agents receive the bad endowment with probability $p^H(p^L)$ and the good endowment with probability

$1 - p^H(1 - p^L)$. We assume that $1 > p^H > p^L > 0$. Let $\Pi^H(\Pi^L)$ be the measures of type $H(L)$ agents, $\Pi^H, \Pi^L > 0$, and let $\Pi = (\Pi^H, \Pi^L)$.

Although all agents know the distribution of types, Π, each individual's type is private information. There is no technology available for verifying or evaluating agents' types. Agents know their own types before interacting with other agents. The endowment realization of each agent is publicly observed, as are any contracts into which an agent may enter.

2. Allocations

An allocation for this economy must describe the consumption of each agent in every possible state of the world, but since there are a continuum of agents, there is no uncertainty in the aggregate endowment. We will consider only allocations in which an agent's consumption depends on their own endowment. We also restrict attention to allocations that treat all agents of a given type identically, since agents are only different in a meaningful way through their types.

Formally, an allocation consists of four nonnegative scalars: c_i^T, $T = H, L$, $i = g, b$, the consumption of a type T agent with endowment e_i. Let $c^L = (c_g^L, c_b^L)$ and $c^H = (c_g^H, c_b^H)$ be the allocations of type L and type H agents respectively, and let $c = (c^L, c^H)$.

We will want to define allocations and feasibility for arbitrary subsets of the population. For any arbitrary set of agents let ϕ^L and ϕ^H be the measures of type L and type H agents, respectively, and let $\phi = (\phi^L, \phi^H)$. Since a set of agents is a subset of the entire set of agents in the economy, we have $\phi^L \le \Pi^L$ and $\phi^H \le \Pi^H$. We will define feasibility for an allocation-coalition pair to encompass resource and incentive feasibility. An allocation-coalition pair (c, ϕ) is *resource feasible* if

$$\begin{aligned}\phi^H\big[p^H c_b^H + (1 - p^H)\, c_g^H\big] + \phi^L\big[p^L c_b^L + (1 - p^L)\, c_g^L\big]\\ \le \phi^H[p^H e_b + (1 - p^H)\, e_g] + \phi^L[p^L e_b + (1 - p^L)\, e_g]. \qquad (1)\end{aligned}$$

This definition of resource feasibility makes use of the law of large numbers, which here implies that the fraction of agents realizing any particular outcome is equal to the probability of that outcome. An allocation is *incentive feasible* if

$$p^H u\big(c_b^H\big) + (1 - p^H) u\big(c_g^H\big) \ge p^H u\big(c_b^L\big) + (1 - p^H) u\big(c_g^L\big), \qquad (2)$$

and

$$p^L u\big(c_b^L\big) + (1 - p^L) u\big(c_g^L\big) \ge p^L u\big(c_b^H\big) + (1 - p^L) u\big(c_g^H\big). \qquad (3)$$

Define the set of feasible allocations $C(\phi)$ for coalition ϕ as the set of all allocations satisfying (1) through (3).[1]

3. The coalition-proof allocations

Our definition of coalition-proof allocations for this economy differs from the notion of coalition-proof Nash equilibrium (CPNE) proposed by BPW and KM in that we do not explicitly define a noncooperative game to which the full definition of the CPNE can be applied. Instead, we simply seek allocations that are "coalition proof" in the same sense that strategy profiles are coalition proof in the CPNE. Coalition-proofness is based on the notion of a blocking coalition and so closely parallels the definition of the core of an economy. The notion of a blocking coalition must be adjusted, however, to take account of the ex ante private information feature of our environment.

First, define the set of possible subcoalitions to a coalition ϕ as

$$\Phi(\phi) \equiv \{\hat{\phi} \in \mathbb{R}^2 \mid 0 \leq \hat{\phi}^L \leq \phi^L, \text{ and } 0 \leq \hat{\phi}^H \leq \phi^H\}.$$

We will denote the set of all possible subcoalitions as $\Phi = \Phi(\Pi)$. We will denote the set of allocations that are feasible for some coalition as C, the union of $C(\phi)$ for $\phi \in \Phi$.

We adopt the following notion of a blocking coalition.

Definition 1: *An allocation $c \in C(\phi)$ for coalition ϕ is* blocked *by a coalition $\hat{\phi} \in \Phi(\phi)$ and allocation $\hat{c}$ if*

(a) $\hat{c} \in C(\hat{\phi})$;

(b) *For $T = H, L$, if $\hat{\phi}^T > 0$ then $p^T u(\hat{c}_b^T) + (1-p^T)u(\hat{c}_g^T) \geq p^T u(c_b^T) + (1-p^T)u(c_g^T)$;*

(c) *The inequality in (2) is strict for at least one T for which $\hat{\phi}^T > 0$;*

(d) *For $T = H, L$, if $\hat{\phi}^T < \phi^T$ (some type T agents are left behind) then*

$$p^T u\big(c_b^T\big) + (1 - p^T)u\big(c_g^T\big) \geq p^T u\big(\hat{c}_b^H\big) + (1 - p^T)u\big(\hat{c}_g^H\big) \quad \text{if } \hat{\phi}^H > 0,$$

and

$$p^T u\big(c_b^T\big) + (1 - p^T)u\big(c_g^T\big) \geq p^T u\big(\hat{c}_b^L\big) + (1 - p^T)u\big(\hat{c}_g^L\big) \quad \text{if } \hat{\phi}^L > 0.$$

The first condition in the above definition states that the blocking allocation is resource and incentive feasible for that blocking coalition. Conditions (b) and (c) state the usual requirement that members of a blocking coalition are at least as well off and some members are strictly better off. The fourth condition is added to respect the constraints of private information. It states that if a deviating coalition intends to leave behind any agents of type T, then those left behind cannot strictly prefer to join the coalition (including by claiming to be another type).[2]

Some definitions of blocking in similar private information environments (notably Boyd, Prescott and Smith [1988] and Marimon [1988]) require the deviating coalition to anticipate what will become of the complement coalition after the deviation. Our condition (d) requires only that those in the complement coalition compare the status quo allocation to the proposed deviation. However, our "coalition-proof" requirement plays the role of considering what might happen after a deviation occurs. KM require blocking allocations to be proper subsets of the original coalition, thus ruling out a priori blocking by the coalition of the whole. Our approach allows deviations by the coalition of the whole. KM also require strict inequality in (b), so that all members of a deviating coalition must be made strictly better off. Strict inequality in (b), in conjunction with (d), would also imply that deviating coalitions must take either all or none of any given type. Our approach allows deviating coalitions that strictly dominate only for a subset of their members.

The *core* of this economy—defined as the set of allocations that are unblocked according to the above definition—may be empty. The core is empty in the cases in which the Rothschild-Stiglitz equilibrium fails to exist or is Pareto dominated. The Rothschild-Stiglitz equilibrium exists only when the ratio of high- to low-risk agents exceeds a threshold below which there exists a feasible pooling allocation that gives the low-risk agents the same expected utility as in the Rothschild-Stiglitz equilibrium. When the ratio lies in a range just above this threshold, equilibrium exists but is Pareto dominated.

Figure 1 provides some intuition on the case when the core is empty. The Rothschild-Stiglitz allocation consists of c^{HO} and c^{LO} for the high- and low-risk types, respectively: a precise definition is given in Section 4 below. The dashed line in figure 1 represents the pooling allocations that breaks even. The allocation (c^{H^*}, c^{L^*}) is a feasible alternative that Pareto-dominates the Rothschild-Stiglitz allocation. Thus (c^{HO}, c^{LO}) is blocked by a coalition of the whole and does not lie in the core. Allocations like (c^{H^*}, c^{L^*}) are blocked in turn by allocations like c', with a coalition that includes all of the low-risk types and

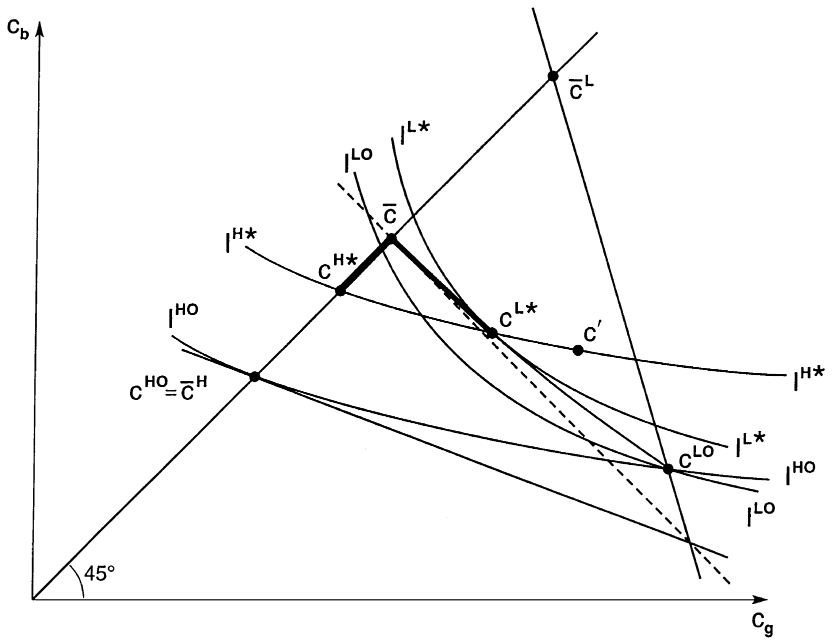

Figure 1. The core may be empty.

not too many high risk-types. Note that high-risk types are indifferent between c^{H^*} and c', so the blocking coalition can consist of any fraction of the high-risk types. In cases like that illustrated in figure 1, any Pareto-optimal allocation can be blocked by a coalition that includes only some of the high-risk agents. However, the corresponding blocking allocations are not feasible for the coalition of the whole.

Coalition-proofness places more stringent conditions on blocking coalitions. We require allocations to be immune only to deviations by coalitions that are themselves immune to further deviations by similarly immune subcoalitions.

Definition 2: *A* coalition-proof correspondence *(CPC) is a mapping* $\sigma : \Phi \to C$, *with the following properties*:
(i) $\sigma(\phi) \subseteq C(\phi)$ *for all* $\phi \in \Phi$; *and*
(ii) $c \in \sigma(\phi)$ *if and only if there does not exist a* $\hat{\phi} \in \Phi(\phi)$ *and* $\hat{c} \in \sigma(\hat{\phi})$ *that block* c.

This definition states that any allocation-coalition pair that is not coalition proof can be blocked by a subcoalition with an allocation that is coalition proof. On the other hand, if an allocation is coalition proof, then any proposed deviation is deterred by a credible threat of further deviation from the deviating coalition. The requirement of coalition-proofness is weaker than the requirement that an allocation be unblocked. As a result, any core allocation is also coalition-proof.[3]

Our definition follows Greenberg's [1989] "von Neumann and Morgenstern abstract stable set," defined as a partition of a stable system. A stable system is comprised of a set D, which in our case consists of allocation-coalition pairs (c, ϕ), and a dominance relation, which in our case is blocking. In Greenberg's application a typical element of D is a coalition together with a strategy profile. He shows that, given a notion of blocking for strategy profiles, the definition of a CPNE in terms of the coalition-proof correspondence is fully equivalent to the recursive definition given in Bernheim, Peleg, and Whinston [1987], allowing extension of the coalition-proof concept to environments with an infinite number of agents.[4]

In our definition of the coalition-proof correspondence the conditions required for a subcoalition to block a coalition are symmetric across coalition-subcoalition pairs in the sense that they depend only on the preferences, endowments, and allocations within the initial coalition. In particular, blocking does not depend on whether the coalition being blocked is the coalition of the whole or is itself a deviating coalition. As a result, the coalition-proof correspondence for a given coalition yields the coalition-proof allocation for an economy consisting of just that coalition. This incorporates the notion that deviations are private in the sense that only those within a deviation can react to it (Kahn and Mookherjee [1995], p. 118).

4. The Miyazaki allocation

Miyazaki [1977], in a labor-market version of this economy, proposed that equilibrium allocations can be defined as the solution to the following programming problem.

The Miyazaki problem

$$\begin{aligned} \text{Max}\quad & p^L u\big(c_b^L\big) + (1 - p^L) u\big(c_g^L\big) \\ \text{s.t.}\quad & (1), (2), (3), \text{ and} \\ & p^H u\big(c_b^H\big) + (1 - p^H) u\big(c_g^H\big) \geq U^H, \end{aligned} \tag{4}$$

where

$$\begin{aligned} U^H \equiv \underset{\tilde{c}_b^H, \tilde{c}_g^H}{\text{Max}}\quad & p^H u\big(\tilde{c}_b^H\big) + (1 - p^H) u\big(\tilde{c}_g^H\big) \\ \text{s.t.}\quad & p^H \tilde{c}_b^H + (1 - p^H)\tilde{c}_g^H \leq p^H e_b + (1 - p^H)\, e_g. \end{aligned}$$

The Miyazaki problem maximizes the expected utility of the low-risk agents subject to resource and incentive feasibility. The additional constraint, (4), states that the high-risk agents receive no less expected utility than they would receive were they to band together and self-insure, U^H. Note that to achieve U^H the high-risk agents receive full insure at fair odds. Define $c^*(\phi)$ as the solution to the Miyazaki problem, and call this the *Miyazaki allocation* for the coalition ϕ.[5] In figure 1, (c^{H^*}, c^{L^*}) is the Miyazaki allocation. The curve connecting $\bar{c}$ with c^{L^*} and c^{LO} is the boundary of the set of allocations that are feasible for low-risk types; raising the (full insurance) high-risk allocation from c^{HO} toward $\bar{c}$ relaxes the incentive constraint and allows more insurance for the low-risk types along the aforementioned curve. The Miyazaki allocation is the low-risk agents' most preferred point along this curve. Note that the bold portion of the curve represents the low-risk agents' consumption in the Pareto-optimal allocations; the corresponding high-risk agents' consumption lies along the bold portion of the 45 degree line.

Define full-insurance consumption at fair odds as $\bar{c}^T$ for type T agents, so that $\bar{c}^T = p^T e_b + (1 - p^T)\, e_g$. The *Rothschild-Stiglitz allocation*, c^O, is the type-wise break-even, separating allocation that provides full insurance at fair odds for the high-risk agents ($c_b^{HO} = c_g^{HO} = \bar{c}^H$). Low-risk agents receive fair odds but only partial insurance because of the incentive constraint (2) that high-risk agents do not prefer c^{LO} over c^{HO}. Thus c_b^{LO} and c_g^{LO} are defined jointly by Eq. (2) and

$$p^L c_b^{LO} + (1 - p^L) c_g^{LO} = p^L e_b + (1 - p^L)\, e_g.$$

Rothschild and Stiglitz [1976] identify this allocation as the Nash equilibrium. If the ratio of high-risk to low-risk agents is too small, then a feasible pooling allocation exists that gives the low-risk agents as much expected utility as c^{LO}, and their equilibrium does not exist. Finally, define $V^S(c^T) = p^S u(c_b^T) + (1 - p^S) u(c_g^T)$ and $E^S[c^T] = p^S c_b^T + (1 - p^S)\, c_g^T$ for $S, T = L, H$.

We collect together here some of the important properties of the Miyazaki allocations, stated without proof (see Crocker and Snow [1985]).

Proposition 1: (*Properties of* $c^*(\phi)$)

1. $c^{L^*}(\phi^L, 0) = \bar{c}^L$ *for all* $\phi^L \in (0, \Pi^L)$, *and* $c^{H^*}(0, \phi^H) = \bar{c}^H$ *for all* $\phi^H \in (0, \Pi^H)$ (*full insurance at fair odds for single-type coalitions*).
2. *For all* ϕ *s.t.* $\phi^H, \phi^L > 0$:
   - (a) $c^*(\phi)$ *is unique and continuous in* ϕ;
   - (b) $c_b^{H^*}(\phi) = c_g^{H^*}(\phi) \geq \bar{c}^H$ (*full insurance at fair odds or better for high-risk agents*);
   - (c) $c_b^{L^*}(\phi) < c_g^{L^*}(\phi)$ (*incomplete insurance for low-risk agents*);
   - (d) $c^*(\phi)$ *satisfies* (1) *and* (2) *with equality* (*the resource constraint and the high-risk incentive constraint bind*);
   - (e) $c^*(\phi)$ *satisfies* (3) *as a strict inequality* (*the low-risk incentive constraint does not bind*);
   - (f) *There exists a* $\rho_0 \in (0, \infty)$ *s.t. if* $\phi^H/\phi^L \geq \rho_0$ *then* $c^*(\phi) = c^O$, *and if* $\phi^H/\phi^L < \rho_0$ *then* $c^*(\phi) \neq c^O$ (*type-wise break-even, separating allocation if and only if the ratio of high- to low-risk agents is larger than* ρ_0).
3. *As* $\phi^H/\phi^L \to 0$, $c^{L^*}(\phi) \to \bar{c}^L$ *and* $c^{H^*}(\phi) \to \bar{c}^L$ (*full insurance for all at low-risk fair odds as the high-risk agents disappear*).
4. $V^L(c^{L^*}(\phi))$ *and* $V^H(c^{H^*}(\phi))$ *are strictly decreasing in* ϕ^H/ϕ^L *for* $\phi^H/\phi^L < \rho_0$.

Note that the Miyazaki allocation is scale-invariant, in the sense that it depends only on the ratio of high-risk to low-risk populations.[6] When this ratio is greater than ρ_0, the Miyazaki allocation is identical to the type-wise break-even, separating allocation, the original Rothschild-Stiglitz equilibrium. When there are relatively few high-risk agents ($\rho < \rho_0$), low-risk agents are willing to subsidize high-risk agents ($c^{H^*}(\phi) > \bar{c}^H$) to loosen incentive constraints. In the limit, as the ratio of high- to low-risk agents goes to zero the low-risk Miyazaki allocation approaches full insurance at fair odds. Contrast this with equilibrium notions that predict the type-wise break-even separating allocation, which is invariant to the number of high-risk agents. In such equilibria, introducing a tiny number of high-risk agents in an economy with just low-risk agents results in a discrete jump in allocations.[7]

5. The Miyazaki allocation is coalition-proof

Our candidate CPC is just $\sigma^*(\phi) \equiv \{c^*(\phi)\}$, for each coalition a set consisting of the unique Miyazaki allocation. To prove it we must select an arbitrary coalition and an arbitrary feasible allocation and show that (1) if it is not the Miyazaki allocation for that coalition, then it can be blocked by a Miyazaki allocation for some coalition, and (2) if it is the Miyazaki allocation, then it cannot be blocked by a Miyazaki allocation for any subcoalition. We briefly sketch the argument before formally stating the result and the proof.

Two cases are handled quite easily. First, if the expected utility of the high-risk agents is lower in the arbitrary allocation than in the Miyazaki allocation, then the coalition of the whole blocks the arbitrary allocation; the Miyazaki allocation yields higher expected utility for both types of agents. Second, if the expected utility of the high-risk agents is greater in the arbitrary allocation than they would receive with *low-risk* full insurance, then the low-risk agents are doing quite poorly, and a coalition of just low-risk agents blocks the arbitrary allocation.

The difficult case lies between these two. The trick is to note that the expected utility of the high-risk agents in the Miyazaki allocation is strictly decreasing in ρ. A blocking coalition can be formed with all of the low-risk agents and some of the high-risk agents such that the Miyazaki allocation for that coalition gives the high-risk agents exactly the same expected utility as they receive in the arbitrary allocation. The high-risk agents are indifferent between the status quo and the blocking allocation, while the low-risk agents are strictly better off (unless the arbitrary allocation is the Miyazaki allocation for the original coalition).

A Miyazaki allocation cannot be blocked by a Miyazaki allocation for any deviating coalition. If the deviating coalition has a lower ratio of high- to low-risk agents, then either the allocations are identical (when $\rho > \rho_0$ in both allocations) or both types are strictly better off (when $\rho < \rho_0$ in the deviating coalition). In the latter case, the deviating coalition would attract all of the low-risk agents and *all*, not some, of the high-risk agents, and thus would fail to reduce the ratio of high- to low-risk agents. Similarly, any deviating coalition with a higher ratio of high- to low-risk agents would make both types strictly worse off if the allocation was at all different.

We can now state our central result.

Proposition 2: σ^* *is a CPC.*

Proof. Choose an arbitrary coalition ϕ, with $\rho = \phi^H/\phi^L$. Since only the ratio ϕ^H to ϕ^L matters, we will use ϕ and ρ interchangeably to refer to a given coalition.[8] Choose an arbitrary $c \in C(\phi)$. We will prove that if $c \neq c^*$ then there exists a subcoalition that blocks c with the Miyazaki allocation for that subcoalition, and if $c = c^*$ then no such subcoalition exists. If either $\phi^L = 0$ or $\phi^H = 0$ the proof is trivial, so assume from now on that $\phi^L, \phi^H > 0$.

First, suppose that $V^H(c^H) < V^H(c^{H^*}(\phi))$. The Miyazaki problem implies that $V^L(c^{L^*}(\phi)) \geq V^L(c^L)$, so conditions (b) and (c) of Definition 1 are satisfied. If we form a blocking coalition consisting of the entire coalition ϕ, no agents are left behind, so we need not check blocking condition (d). Therefore, the entire coalition with allocation $c^*(\phi)$ blocks c.

Next suppose that $V^H(c^H) \geq V^H(\bar{c}^L)$. This implies $E^H[c^H] \geq \bar{c}^L > E^H[e]$, which implies $E^L[c^L] < E^L[e]$, which implies $V^L(c^L) < V^L(\bar{c}^L)$. Consider a blocking coalition consisting of just the low-risk agents, $\phi' = (\phi^L, 0)$. The Miyazaki allocation for that coalition gives low-risk agents the full-insurance allocation $\bar{c}^L$. By construction $\bar{c}^L$ is feasible for ϕ', so blocking condition (a) is satisfied. Since $V^L(\bar{c}^L) > V^L(c^L)$, conditions (b) and (c) are satisfied. Since only high-risk agents are left behind, condition (d) requires $V^H(c^H) \geq V^H(\bar{c}^L)$, which was assumed. Therefore, coalition $\phi' = (\phi^L, 0)$ with Miyazaki allocation $\bar{c}^L$ blocks c.

Now suppose $V^H(c^{H^*}(\phi)) \leq V^H(c^H) < V^H(\bar{c}^L)$. Choose ρ' s.t. $V^H(c^{H^*}(\rho')) = V^H(c^H)$ and $\rho' \leq \rho$. We know such a ρ' exists from Proposition 1, properties (2a), (3), and (4). The definition of the Miyazaki allocation implies that $V^L(c^{L^*}(\phi)) \geq V^L(c^L)$, and Property 4 implies that $V^L(c^{L^*}(\rho')) \geq V^L(c^{L^*}(\rho))$. Therefore, $V^L(c^{L^*}(\rho')) \geq V^L(c^L)$ and $V^H(c^{H^*}(\rho')) = V^H(c^H)$, satisfying blocking condition (b). Since only high-risk agents are left behind, blocking condition (d) just requires $V^H(c^H) \geq V^H(c^{L^*}(\rho'))$. But this can be deduced from the incentive compatibility of $c^*(\rho')$, which implies $V^H(c^{H^*}(\rho')) \geq V^H(c^{L^*}(\rho'))$, and the fact that $V^H(c^{H^*}(\rho')) = V^H(c^H)$ by construction. Obviously $c^*(\rho')$ is feasible, so Definition 1 is satisfied except for condition (c). Coalition ρ' with Miyazaki allocation $c^*(\rho')$ blocks allocation c, therefore, if and only if $V^L(c^{L^*}(\rho')) > V^L(c^L)$.

Suppose $c^H \neq c^{H^*}(\rho')$. Since $V^H(c^H) \geq V^H(c^{H^*}(\phi))$ and u is strictly concave, we know that $E^H[c^H] > E^H[c^{H^*}(\phi)]$, and thus $E^L[c^L] < E^L[c^{L^*}(\phi)]$. Since $c^{L^*}(\phi)$ is the unique solution to the Miyazaki problem and $c^L \neq c^{L^*}(\phi)$, we have $V^L(c^{L^*}(\phi)) > V^L(c^L)$. Therefore, $V^L(c^{L^*}(\rho')) \geq V^L(c^{L^*}(\phi)) > V^L(c^L)$ and $(c^*(\rho'), \rho')$ blocks c.

Now suppose $c^H \neq c^{H^*}(\rho')$ but $c^L \neq c^{L^*}(\rho')$. Since c is feasible for coalition ρ' and $c^{L^*}(\rho')$ is the unique solution to the Miyazaki problem, we have directly that $V^L(c^{L^*}(\rho')) > V^L(c^L)$ and thus $(c^*(\rho'), \rho')$ blocks c.

If $c^H = c^{H^*}(\rho')$ and $c^L = c^{L^*}(\rho')$, then c is the Miyazaki allocation for ϕ, $\rho' = \rho$, and $(c^*(\rho'), \rho')$ fails to block c. We need to prove that no subcoalition blocks c with the Miyazaki allocation for that subcoalition. If $\rho \geq \rho_0$, then any coalition with $\rho'' \geq \rho_0$ has $V^T(c^{T^*}(\rho'')) = V^T(c^{T^*}(\rho))$, $T = H, L$, and will thus fail to block. Any coalition with $\rho'' < \rho_0$ has $V^T(c^{T^*}(\rho'')) > V^T(c^{T^*}(\rho))$, $T = H, L$, attracts both types, and thus fails to obtain the proportion ρ''. Suppose $\rho < \rho_0$; then any coalition with $\rho'' > \rho$ has $V^L(c^{L^*}(\rho'')) < V^L(c^{L^*}(\rho))$ and attracts no low-risk agents. The Miyazaki allocation for such a coalition of all high-risk agents gives them $V^H(\bar{c}^H) \leq V^H(c^{H^*}(\phi))$. Therefore, no coalition with $\rho'' > \rho$ is capable of blocking c^* in this case. Any coalition with $\rho'' < \rho$ has $V^L(c^{L^*}(\rho'')) > V^L(c^{L^*}(\rho))$ (since $\rho'' < \rho_0$) and will attract all of the low-risk agents, but because $V^H(c^{H^*}(\rho))$ is strictly decreasing for $\rho < \rho_0$, all of the high-risk agents are also attracted, so the allocation fails to attract the correct proportions of high- and low-risk agents. Therefore, there is no coalition whose Miyazaki allocation blocks c. □

Figure 2 illustrates Proposition 2. The allocation $(c^{H^*}(\Pi), c^{L^*}(\Pi))$ is the Miyazaki allocation for the coalition of the whole, Π. The bold wishbone-shaped locus represents the Miyazaki allocations for coalitions with $\rho = \phi^H/\phi^L < \Pi^H/\Pi^L$. The allocation c' blocks $(c^{H^*}(\Pi), c^{L^*}(\Pi))$ for a particular coalition ϕ. Since all of the low-risk types but only a fraction of the high-risk types join the blocking coalition, we have that $\phi^H/\phi^L < \Pi^H/\Pi^L$. The dashed line through c' is the pooling break-even line for the coalition ϕ. The allocation c' is blocked in turn by $(c^{H^*}(\phi), c^{L^*}(\phi))$, the Miyazaki allocation for the coalition ϕ. In this way, any non-Miyazaki allocation can be blocked by a Miyazaki allocation for some subcoalition. But a Miyazaki allocation cannot be blocked by any other Miyazaki allocation.

One difference between our setup and KM is the absence here of a separate class of agents acting as risk-neutral insurance firms. By itself, the presence of firms does not affect our results since risk-averse agents can always split off from a firm making positive profits. Under the additional assumption that risk-averse agents are incapable of joining together to self-insure, such a deviation is not allowed and allocations with positive firm profits can be sustained. In this case our CPC must be expanded; for any given collection of risk-averse agents, the set consists of the family of Miyazaki-type allocations indexed by the nonnegative level of firm profits (see note 6). Positive profits can be sustained because a set of agents insured by a single firm cannot, by assumption, self-insure in any subcoalition that does not include the firm. However, the requirement that insurance be obtained through

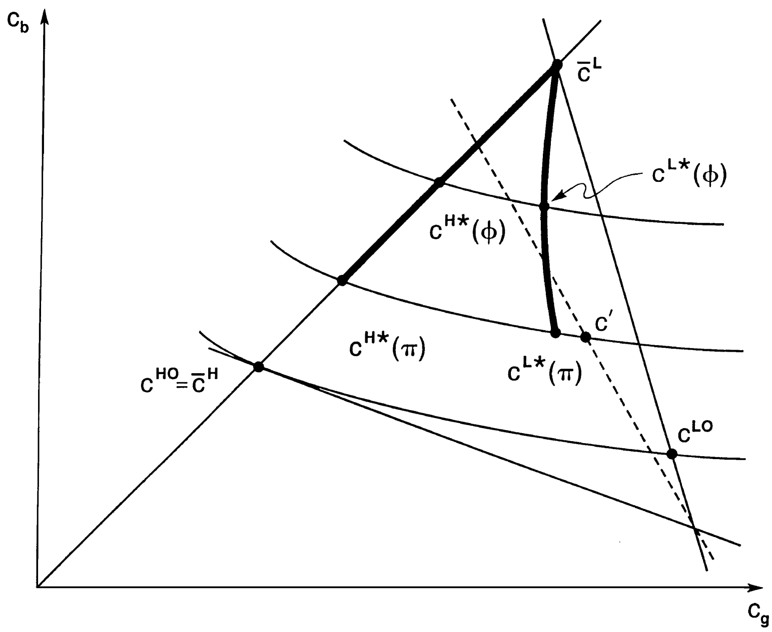

Figure 2. The Miyazaki allocation is coalition proof.

firms does not change the basic characteristics of coalition-proof allocations. Miyazaki-like allocations are still coalition-proof.

The other key differences between our results and those of KM stem from our different definitions of coalition-proofness. KM require that deviating coalitions be proper subsets, whereas we allow deviations by an entire coalition. KM motivate strict set inclusion by the need to guarantee the existence of a CPC (a "stable partition" in their terminology) in the presence of infinite strategy spaces. We have constructed a CPC using only weak set inclusion, and thus strict set inclusion does not seem essential for existence here. Imposing strict set inclusion in our framework, the Miyazaki allocation would not be coalition proof, and thus strict set inclusion does not seem to be innocuous.

Finally, we have proven our results only for the case of two types. The existence problem identified by KM could well be more serious with n types. This problem necessitates care in the details of how coalition-proofness is defined. Our results imply that in the two, type case predictions can be sensitive to seemingly technical details. This suggests that other approaches would be worth examining in the n type case as well. We leave this for future research.

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Notes

1. It would be possible to include in our economy another category of agents capable of serving as insurance firms, as in KM. Such risk-neutral agents would be willing to provide insurance to coalitions of risk-averse agents only if their portfolio of insurance contracts earns nonnegative expected profits. In our economy nonnegative expected profits for such a firm is equivalent to the resource feasibility constraint (1). In addition, the firm would face the same incentive feasibility conditions, (2) and (3), which we impose on coalitions. If coalitions of risk-averse agents can self-insure, then the presence of insurance firms does not expand the set of attainable allocations. If, on the other hand, coalitions of agents cannot self-insure, the presence of insurance firms raises the possibility of allocations in which such firms earn strictly positive profits.
2. This definition of blocking was adopted in Lacker and Weinberg [1993].
3. There is a similarity to the Riley [1979] reactive equilibrium, in which viable deviations are constrained by the anticipation that they would be undone by further profitable deviations. The difference is that the Riley equilibrium looks forward only two steps; further deviations are not themselves constrained by the prospect of subsequent deviations.
4. See also Greenberg [1991] and Kahn and Mookherjee [1992]. KM show that with an infinite set of agents and infinite strategy sets the stable set and the CPNE may fail to coincide and a stable set may fail to exist.
5. It is straightforward to generalize the Miyazaki allocation for n types. For type 1, the highest risk, define U^1 exactly the way U^H is defined, and define U^2 exactly the way U^L is defined. Assuming U^k has been defined for $k < j$, U^j is the maximum expected utility of type j consumers subject to resource and incentive feasibility and the constraints that all type $k < j$ consumers each receive at least U^k in expected utility.
6. A CPC is not required to be scale-invariant however.
7. If the economy includes insurance firms, then one could define a family of Miyazaki-type allocations for a given coalition, indexed by the expected profits of the insurance firms. Each allocation in the family satisfies Proposition 1, with the full-insurance allocations defined net of expected firm profits (e.g., $\bar{c}^L = p^L e_b + (1 - p^L)\, e_g - x$, where expected firm profits per insured agents are x). In such a setting, this entire family of Miyazaki-like allocations would be coalition-proof.
8. Note that our definition of coalition-proof correspondence allows for mappings that are not scale-invariant.

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