The Geneva Papers on Risk and Insurance Theory 22: 43–58 (1997)

An Option-Pricing Approach to the Costs of Export Credit Insurance

SEBASTIAN T. SCHICH
Money, Credit, and Capital Markets Section, Economics Department, Deutsche Bundesbank, Wilhelm-Epstein-Str. 14, 60431 Frankfurt a.M., Germany

Abstract

This article investigates the relationship between a debtor country's external financial indicators and the costs associated with the insurance of export credits to that country. For this purpose a stylized model of export credit insurance (ECI) is developed, the central idea being that ECI is similar to a contingent claim such as a European put option. Thus, tools from option pricing theory were used to calculate the price of ECI, implying that not only the current financial position but also the volatility of the changes in that position determine such costs. The empirical results of a statistical analysis of the premium rates for ECI, applied by a private export credit insurer to seventy-seven developing countries during 1993, provide some support for these hypotheses. In particular, the reserves-over-imports ratio of a debtor country and the volatility of the rates of change of this ratio appear to contribute significantly to the premium rates that apply to that country. Thus, the article provides evidence that option pricing parameters do play role in practical insurance pricing, even if this pricing is not explicitly based on these parameters. Premium rates are set as if an underlying option market operated. Thus, the trade of countries with volatile external financial positions is saddled with higher costs than that of countries with more stable positions.

Key words: trade financing costs, export credit insurance, credit risk, insurance premia, option pricing, non-parametric regressions

1. Introduction

Guaranteed export credits represent an important source of external financing for developing countries. For example, at the beginning of the 1990s the long-term guaranteed export credits of developing countries amounted to almost a fifth of their total disbursed long-term debt, and the short-term guaranteed credits to almost half of the total short-term external financing of some regional groups of developing countries. This article identifies how a country's external financial indicators and their (historical) development may affect the costs associated with this form of trade financing.

The direct way to address this question would be simply to ask the country risk experts in the export credit agencies (ECAs) how the economic and financial indicators characterizing a debtor country affect the premium charged for cover of export credits to that country. This, however, is unlikely to be a helpful approach because these premia are not always set according to a structured method involving quantitative techniques, or else information about their determination is confidential. Leaving the problem of confidentiality of the country risk assessment aside, even a detailed knowledge of the premium-setting policy at these agencies,

JEL classification: F34, G13, G22

including the one on which we have data, does not allow us to identify exactly how the development of a country's external financial indicators will influence the actual premium applied to it. Therefore, to gain an insight into the relationship between the external financial position of a country and its trade financing costs an indirect route was chosen. A theoretical model of international export credit was developed that allows us to derive an explicit formula indicating on what the insurance premia *should* depend. It is based on the idea that export credit insurance, viewed as a security, is analogous to a European put option. Such a put option gives the owner the right to sell an asset to the writer of the option, at a specified date and price. Similarly, export credit insurance (ECI) gives the exporter (or his bank) the right to sell his claim on the debtor to the insurance agency at a specified date. The analogy suggests that tools from option pricing theory might help to explain the value of such an insurance and to identify determinants of its costs. For example, the Black-Scholes portfolio arbitrage concept applied to our particular case implies that the premia for credit insurance should depend not only on the debtor's current external financial indicators but also on the volatility of the rate of changes therein.

These implications were also tested with observations using the example of the premium rates applied by the private Nederlandsche Creditverzekering Maatschappij, UK (NCM, UK) during 1993, to seventy-seven developing countries. These premium rates are interesting because they seem to be relatively close to a "market view" of credit risk, the British market for short-term export credit insurance—unlike those in several other European countries—being highly competitive. To the author's knowledge no similar data set has been collected and subjected to an empirical analysis before. These rates were compared with the level and the development of two of their external financial indicators, the reserves-over-imports ratio, describing a country's liquidity situation and the debt-service ratio, describing its solvency situation. A central finding from the empirical analysis was that the premium rates applying to a debtor country were negatively related to its reserves-over-imports ratio and positively to the volatility of the rates of changes of that ratio. There was only little evidence for the hypothesis that these rates were related to the debt-service ratio and the volatility of its rate of change. This points to the hypothesis that a country's liquidity is more relevant for its trade financing costs than its solvency situation.

The article is structured as follows. Section 2 explains the concept used for our theoretical model, the model itself, and the valuation formula for export credit insurance obtained from it. Then, Section 3 investigates whether the implications of the theoretical model are reflected in the observable data. Section 4 concludes the article.

2. An option-pricing approach to the premium for export credit insurance

The valuation concept is based on the idea of a fair premium rate, where *fair* denotes the premium rate that makes the export credit insurance contract a transaction with zero net-present value. In general the value of such an insurance contract depends on whether the debtor defaults and on what the value of recovery is in the case of such a default, where by *default* we mean any failure of the debtor to pay the full amount stated in the (explicit) debt contract. The uncertainty about whether the insurance contingency will occur and what the extent of the compensation payable will be are the most important single aspects of the valuation of such insurance contracts. In order to focus on that aspect it is assumed that such an uncertainty is shared equally by the insurer and the insured. This means that informational asymmetries, leading to problems such as moral hazard and adverse selection, are ignored.

The approach adopted here is to interpret export credit insurance as another contingent claim, which is similar to a *European put option*. Accordingly, an export credit insurance contract corresponds to a European put option, which gives the insured the right to sell his (insured) credit claim to the ECA at its maturity date. The price at which he can sell the claim (assuming, for simplicity, a complete insurance) is equivalent to the nominal value of the claim insured. The correspondence between such a put option and an insurance contract is reflected in their payoffs. For example, ignoring the existence of a claims-waiting period the pay-off from a (complete) credit insurance at the date of maturity of the credit is as follows:

$$\text{Payoff from insurance} = \max\{0, D - A\}, \tag{1}$$

where D denotes the value of the (insured) contractual debt service and A denotes the debt service that is actually made. The payoff is equal to the difference between the amount insured D and the actual payment by the debtor A, in the case of a default on that credit, and zero if there is no default. Thus, as the value of a common European put option depends on the stochastic price of the underlying stock, the value of the insurance contract depends on the (stochastic) actual debt service made by the debtor. And as the value of the put option depends on the prespecified exercise price, the value of the insurance contract depends on the nominal value of the contractual debt service that is insured.

Consider that the insurer is risk-neutral. This is a common assumption for an insurer, and it is justified on the grounds that the insurer can eliminate risk through the pooling of a large number of risks, provided they are independent. Under these circumstances, the "fair" premium rate depends on two aspects. First, it depends on the probability that the debtor's debt-servicing capacity falls short of his contractual debt service, meaning that he does not meet his debt-service demands in full. Second, it depends on the level of debt service that the debtor actually meets. The first event is generally understood as default of the debtor, and the amount that the debtor actually transfers in such a situation is often denoted by recovery. Thus, an insurer must estimate the probability of default and the extent of recovery in order to determine the price for the credit insurance.

Abstracting from reporting problems, it is fair to say that a country's future contractual debt service due is known. On the other hand, the country's debt-servicing capacity is not known. This stylized scenario is captured in the present model by assuming that the contractual debt service, denoted by D_t, is a deterministic variable and the debt-servicing capacity, denoted by K_t, a stochastic one. In particular we follow the conventional assumption (e.g., Bartolini and Dixit [1991], Claessens and van Wijnbergen [1990], and Klein [1994]) that the debt-servicing capacity K_t can be described by a Brownian motion. Thus, $dK_t/K_t = \mu dt + \sigma dW_t$, where μ is the trend parameter and σ the diffusion parameter of the Brownian motion, and W_t is the standard Wiener process, with $E(dW_t) = 0$ and the variance $E(dW_t^2) = dt$. In general, the debt-servicing capacity changes continuously, while debt is generally issued and becomes due at discrete intervals. For simplification it is assumed here that the debt instruments have a maturity of one period—that is, all debt issued at time t becomes due at time $t+1$. At that date of maturity, the debtor country repays its debt if its debt-servicing capacity exceeds its debt-servicing demands. Otherwise, it is in default and enters into debt renegotiation with its creditors:

$$\begin{aligned} \text{No default} \quad & \text{if } K_{t+1} \succ D_{t+1}; \\ \text{Default} \quad & \text{if } K_{t+1} \prec D_{t+1}. \end{aligned} \tag{2}$$

In practice, the agreements of such renegotiations include a variety of measures, all of which are designed to bring the contractual debt service more into line with the debtor's capacity to service it. For this purpose, debt and debt-service reductions are included as a common feature of debt restructurings. This scenario is reflected in the stylized rescheduling model of Claessens and van Wijnbergen [1990], which we adopt for convenience. It considers that the contractual debt service of the insolvent country is reduced to its actual debt-servicing capacity, so that the actual debt service, denoted by A_{t+1}, is given as follows:

$$A_{t+1} = \min\{D_{t+1}, K_{t+1}\}. \tag{3}$$

Thus the aggregate loss of creditors at time $t+1$, L_{t+1}, is given as

$$L_{t+1} = D_{t+1} - A_{t+1} = \max\{D_{t+1} - K_{t+1}, 0\}, \tag{4}$$

where $A_{t+1} = K_{t+1} \prec D_{t+1}$ in the case of default. In the absence of explicit and implicit seniority provisions the aggregate losses are shared proportionally by all creditors, so that the loss per claim is

$$\frac{L_{t+1}}{D_{t+1}} = \max\left\{0, \left(\frac{D_{t+1} - K_{t+1}}{D_{t+1}}\right)\right\}. \tag{5}$$

Consider that there exist competitive and risk-neutral ECAs that insure the holder of a defaultable claim against a loss arising from default of the debtor on that claim. Then the price of the insurance per dollar insured—that is, the premium rate, denoted by p_t—is given as its discounted expected payoff:

$$p_t = e^{-r} E_t\left[\max\left\{0, \left(\frac{D_{t+1} - K_{t+1}}{D_{t+1}}\right)\right\}\right], \tag{6}$$

where r is the risk-free interest rate and $E_t[\cdot]$ denotes the expected value at time t of the insurance contract at $t+1$, the time of the maturity of the credit. The above equation contains, on the left side, the safe return for the insurer from the insurance contract at time t and, on the right side, the present value of the expected (negative) payoffs from that contract. Competition ensures the equality of both sides.

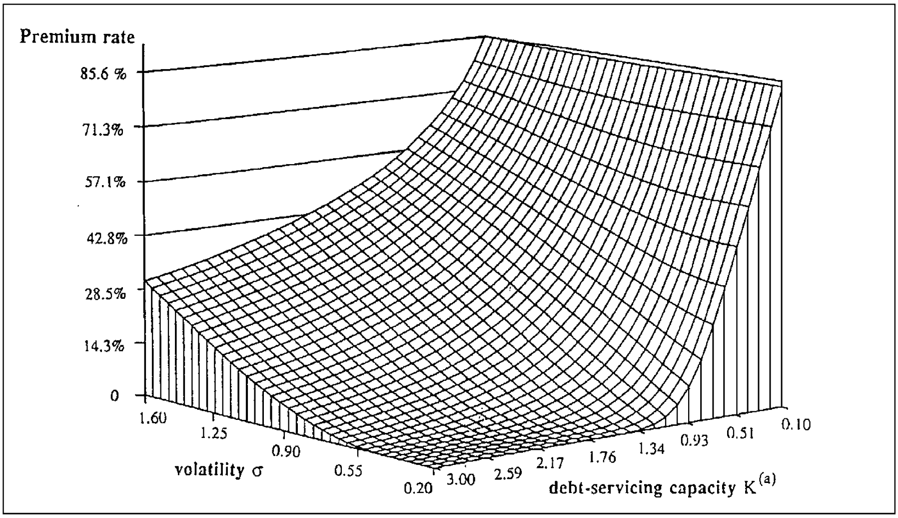
Consider that export credits are in the form of one-period zero-coupon bonds that are continuously compounded and have a nominal value of one at the date of maturity. Furthermore, let there be also an alternative debt instrument that is default-free but has otherwise exactly the same characteristics as these export credits. Let B_t and R denote the value at time t of the defaultable and the default-free debt instrument, respectively, with $B_t = \exp\{-b_t\}$ and $R = \exp\{-r\}$, where b_t is the interest rate of the risky claim and r the riskless interest rate. Thus, the debtor country's aggregate debt-service demands in period $t+1$, D_{t+1}, are given as the nominal value of the claims at the time of maturity B_{t+1} times the number of claims that have been issued in the previous period N_t. Since by assumption the debt instruments are in the form of zero-coupon bonds with a face value of one (that is, $B_{t+1} = 1$), it follows that $D_{t+1} = N_t$. Denoting by π_t the probability of default viewed from period t, Eq. (6) can be written as follows (see the appendix):

$$p_t = e^{-r}\pi_t E_t\left[\left(\frac{N_t - K_{t+1}}{N_t}\right) \middle| K_{t+1} \prec N_t\right], \tag{7}$$

where $E_t[\cdot|\cdot]$ denotes the expectation at time t about the debtors payment conditional on the occurrence of default. In other words, the price of the insurance is given as the discounted present value of the insurance in the case of default $E_t[\cdot|\cdot]$, weighted by the probability of default π_t. Given the assumptions about the stochastic dynamics of K_t, the premium for ECI at time t is given as follows:

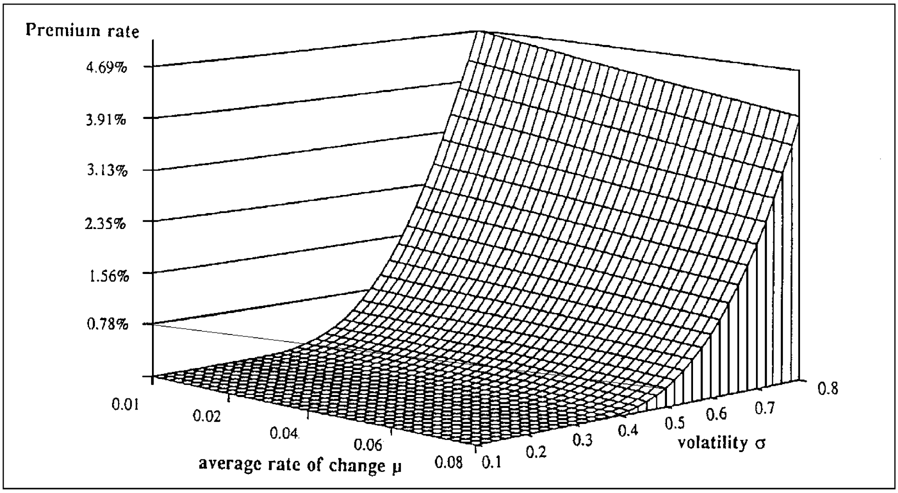
$$\begin{aligned} p_t = e^{-r}\Bigg[&\Phi\left(-\frac{\ln(K_t/N_t) + \mu - \sigma^2/2}{\sigma}\right) \\ &- e^{\mu}\frac{K_t}{N_t}\Phi\left(-\frac{\ln(K_t/N_t) + \mu + \sigma^2/2}{\sigma}\right)\Bigg], \end{aligned} \tag{8}$$

with $\Phi(\cdot)$ denoting the distribution function of the standard normal probability. The above equation expresses the price of export credit insurance at time t as a function of a limited number of variables that are all known at that time. For example, if the debt-servicing capacity K_t is small relative to the number of debt instruments N_t, then $\Phi(\cdot)$ is close to one and thus the price of the insurance is close to the price of the riskless asset, $R = \exp\{-r\}$. The impact of the debtor country-specific variables on the price of credit insurance is illustrated by a simulation analysis, shown in figures 1 and 2. Figure 1 depicts the relation between the debt-servicing capacity K_t and the volatility of the rate of change of that capacity σ on the one hand and the insurance premium rate p_t on the other (graph drawn on the basis of $r = \mu = 0.06$, $N_t = 1$, K_t and σ ranging from 0.1 to 3.0 and 0.2 to 1.6, respectively). It illustrates that the premium rate is very high for a very low K_t and a high σ. By contrast, it is very low for a high K_t and a low σ; however, it is always positive as long as σ is positive. Figure 2 depicts the sensitivity of the premium rate with respect to different values of μ and σ (graph drawn on the basis of $K_t = 1.5, N_t = 1, r = 0.06$, μ and σ ranging from 0.08 to 0.01 and 0.1 to 0.8, respectively). It demonstrates the crucial importance of the volatility in the concept used here.



Note: Debt-servicing capacity relative to the contractual debt service.

Figure 1. The premium rate, the debt-servicing capacity, and the volatility of the rate of changes of that capacity.



Note: Debt-servicing capacity relative to contractual debt service beign equal to 1.5.

Figure 2. The premium rate, the rate of changes of the debt-servicing capacity, and the volatility of the rate of changes of that capacity.

There is a sharp increase in the surface at $\sigma = 0.5$, the premium being only marginal if σ is lower than 0.5 but increasing exponentially with σ increasing above 0.5. On the other hand, the effect of the growth rate is limited, an increase in that rate reducing the premium rate only linearly.

The analysis of this section allows the formulation of a hypothesis in terms of the relationship between the level and the development of a debtor country's external financial position, on the one hand, and the export credit insurance costs applying to it, on the other. The less favorable the relation between a debtor country's current debt-servicing capacity and its contractual debt service and the higher the volatility of changes in that capacity, the higher are the costs associated with the insurance of export credits to that country. Furthermore, the less favorable the rate of change of that capacity, the higher are these costs. These hypotheses are tested with observations, and the results are reported in the next section.

3. Empirical analysis

The premium rates, in the following denoted by P, applied by the Nederlandsche Creditverzekering Maatschapij, UK during 1993 to seventy-seven developing countries were used as proxies for the premium rates of the theoretical model. They apply when the exporter—who is already insured under a general comprehensive policy—has credit exposure to any of the countries for which such premium surcharges are specified. Thus they represent the specific premium rate for insurance cover of the risk associated with that country and correspond to the concept of the export credit insurance premium rate, for which Eq. (8) has been derived. For the parameters on the right side, the following proxies were included:

DSR = the (inverse of the) debt-service ratio as a proxy for K/N,
DDSR = the average rate of change of the above ratio as a proxy for μ,
VDSR = the volatility of the rate of change of the above ratio as a proxy for σ, and, alternatively,
RES = the ratio of reserves over three months of imports as a proxy for K/N,
DRES = the average rate of change of the above ratio as a proxy for μ,
VRES = the volatility of the rate of change of the above ratio as a proxy of σ.

First, the relations between the premium rate surcharges and the country-specific variables were tested using linear OLS estimates. When regressing the variables directly on the premium rate surcharges, the hypothesis that the residuals are not normally distributed could not be rejected. A further analysis of the premium surcharges data suggested a classification of the debtor countries into one of four groups: those with zero premium rate surcharges, those with surcharges greater than zero but smaller (or equal) to 1 percent, those with surcharges greater than 1 but smaller (or equal) to 2 percent, and those with surcharges exceeding 2 percent. Thus, a premium rate surcharge class variable was created assigning each country a value between 1 and 4, where a 1 depicted countries with the most favorable (zero premium rate surcharges) and a 4 the countries with the least favorable terms (surcharges exceeding 2 percent). The results of the regressions on this variable are shown in Table 1. For example, the estimated coefficient of DSR was not significant. The coefficient estimate for VDSR was positive, as expected, but significant only when DDSR was eliminated from the regression and then only at the 10 percent level. The coefficient estimates for RES and VRES were negative and positive, respectively, as expected. Both

Table 1. Regression of export credit insurance premium rates on financial indicators.

	Equation 1	Equation 2		Equation 3	Equation 4
Constant	2.86	2.74	Constant	2.35	2.35
	(20.16)	(20.52)		(14.09)	(14.25)
	[0.00]	[0.00]		[0.00]	[0.00]
RES	−1.90	−1.64	DSR	0.53	0.85
	(−4.77)	(−4.30)		(0.90)	(1.23)
	[0.00]	[0.00]		[0.24]	[0.22]
DRES	2.87		DDSR	−0.69	
	(2.12)			−0.19	
	[0.00]			[0.85]	
VRES	4.57	5.74	VDSR	2.64	3.28
	(3.19)	(3.95)		(0.63)	(1.73)
	[0.00]	[0.00]		[0.53]	[0.09]
Adj. R^2	0.25	0.25	Adj. R^2	0.06	0.07
F-statistic	7.25	7.25	F-statistic	2.14	2.87
	[0.00]	[0.00]		[0.08]	[0.04]
DW	1.68	1.68	DW	1.96	1.97
JB	0.85	0.85	JB	1.30	1.29
	[0.65]	[0.65]		[0.52]	[0.52]

Note: OLS regressions of the premium rate category (1, 2, 3, and 4) on financial indicator variables, using a heteroscedasticity-consistent estimator [White 1980]. The numbers in parentheses are t-statistics and the numbers in square brackets are probability values. DW is the Durbin-Watson and JB the Jarque-Bera statistic (null hypothesis: normally distributed residuals). A dummy variable is included to account for the especially high premium surcharges applying to Argentina. It is always significantly positive at the 1 percent level and is not reported in the table for ease of exposition.

were highly significant—that is, at the 1 percent level. When DRES was eliminated from the regression, the test statistics improved marginally and the coefficient estimate for VRES increased. In all four equations the null hypothesis of normally distributed residuals (JB statistics) could not be rejected, and functional misspecification could not be detected. In sum, the results could be interpreted as support for the hypothesis that a country's current liquidity position and the volatility of that position are important determinants of the export credit insurance premia applying to it.

Second, the relations between the (noncategorized) premium rate surcharges and the country-specific financial variables were analyzed using nonparametric kernel regressions. The attractive aspect of this approach is that it requires less rigid assumptions about the distribution of the observed data and that the explanation and illustration of conclusions are fairly easily comprehensible to the reader. The estimates are shown in figures 3 and 5, and the associated density estimates in figures 4 and 6. The latter are an essential tool for interpreting the estimates. Specifically, the interpretation of the surfaces in figures 3

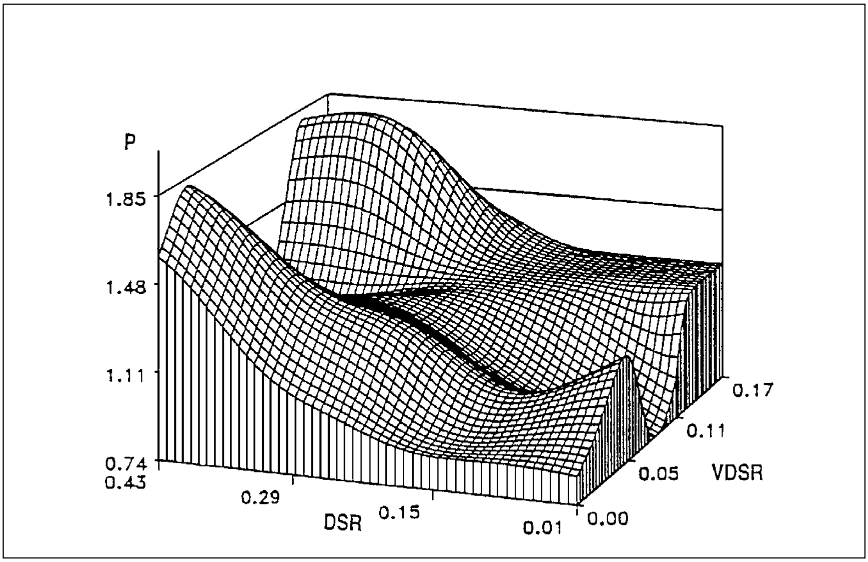


Figure 3. Nonparametric regression of the premium rates (P) on the debt-service ratios (DSR) and the volatilities of these ratios (VDSR).

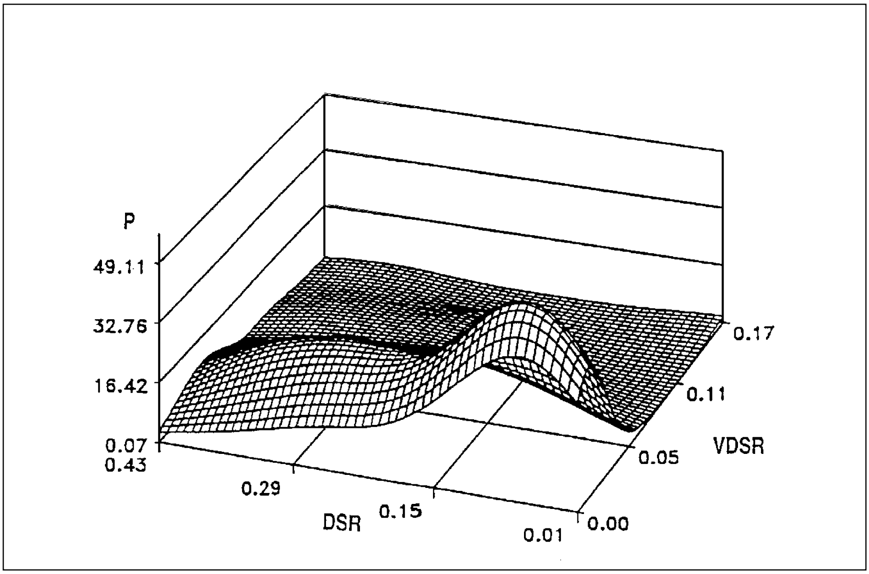


Figure 4. Density estimates.

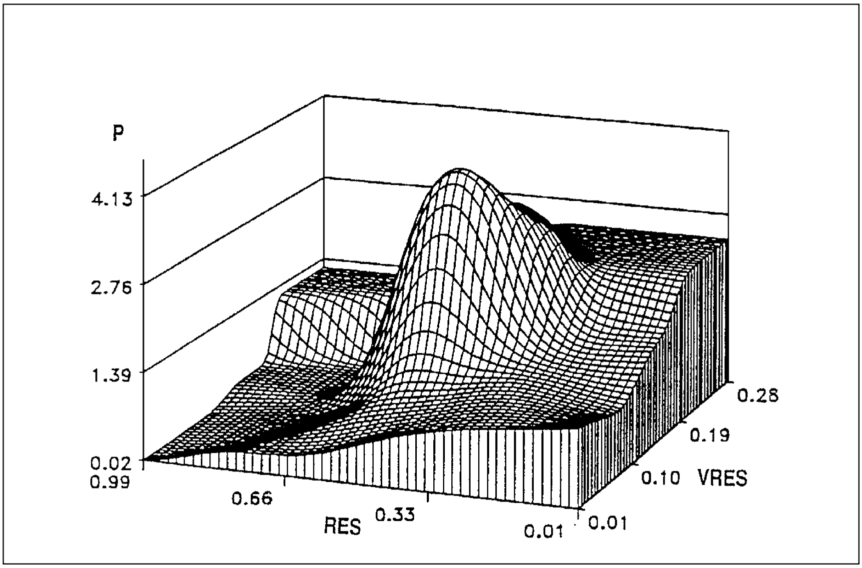


Figure 5. Nonparametric regression of the premium rates (P) on the reserves-over-imports ratios (RES) and the volatilities of these ratios (VRES).

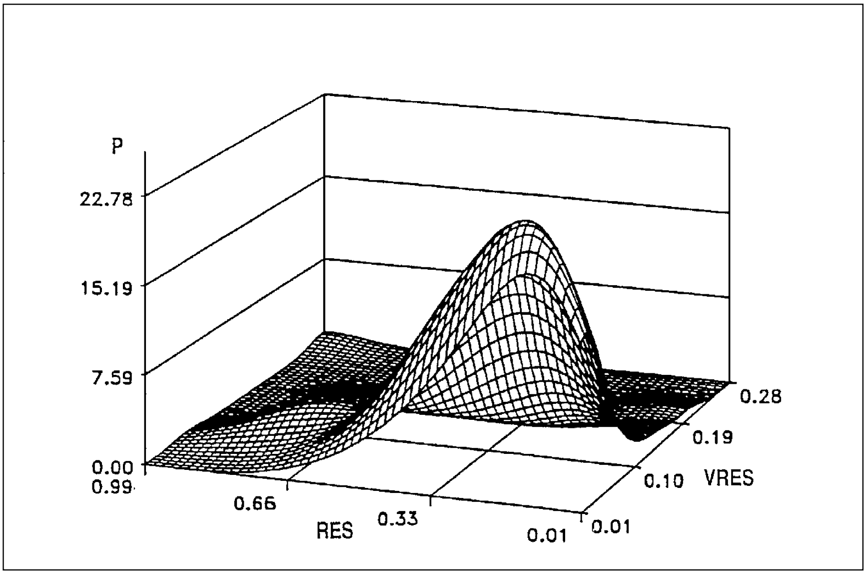


Figure 6. Density estimates.

and 5 should concentrate on those areas where most of the density is concentrated. Figure 3 illustrates the relation between the premium rates (denoted by P), on the one hand, and the debt-service ratio (denoted by DSR) and the volatility of its rate of change (denoted by VDSR), on the other. Figure 4 shows that the area with high density is the one in the south of the surface (viewed from the front axis—that is, the DSR-axis), with most density being concentrated in the southeast—that is, between DSR-values of 0.01 and 0.15. The estimates in this high-density area are not unequivocal. In the area with the highest density the premium surcharges appeared to be decreasing in DSR, while they appeared to be increasing in that variable in the other part of the high-density area. As far as it concerns the volatility of the rate of change of the debt-service ratio VDSR, the premium rates appeared to be always increasing in that variable throughout the high-density region. Figure 5 illustrates the relation between the premium surcharges and the reserves-over-imports ratio (denoted by RES) and the volatility of its rate of change (denoted by VRES). The area in the southeast of the surface plot is characterized by a high density. Starting from the origin in the southeast corner, the high-density area extends roughly up to the west to a RES-value of 0.66 and to the north to a VRES-value slightly larger than 0.10. In this area, P is decreasing in RES and increasing in VRES. To sum up, the estimates obtained from nonparametric regressions provide some support for the hypothesis that the premium surcharges are negatively related to the reserves-over-imports ratio and positively to the volatility of its rate of change but are not so much influenced by the debt-service ratio.

4. Conclusion

The parametric and nonparametric regression exercises convey an idea of the relation between the premium rates for the insurance of export credits to a country on the one hand and its level of, and volatility of the rate change of, its external financial indicators on the other. Since the sample size (seventy-seven observations) is small, the estimates should be treated with caution. However, if one wants to summarize the empirical results, the following can be said. They do not provide support for the hypothesis that the average rate of change—that is, the trend in the financial indicators of a country—affects the export credit insurance rates applying to it. Similarly, they do not provide support in favor of the hypothesis that indicators related to the solvency situation of a country affect such premium rates. However, they provide support for the hypothesis that the level and the volatility of the rate of changes of the liquidity indicator affect such rates. The trade of countries with unfavorable and volatile external financial positions is thus saddled with higher costs than that of countries with more stable positions. This result is in accordance with the qualitative implications of the Black-Scholes-based premium valuation concept developed in this article, which implies that the premium rates for export credit insurance to a country should depend not only on its current external financial position but also on the volatility of the rates of change therein. Thus, the article provides evidence that option pricing parameters do play role in practical insurance pricing, even if this pricing is not explicitly based on these parameters. Premium rates are set as if an underlying option market operated.

While the focus of the present article has been on analyzing insurance premiums that were observed in a competitive environment, the theoretical framwork developed therein may well be applied to the setting of similar premiums in public or publicly supported export credit insurance agencies where there is no or only little private competition. Or the concept may be used for the measuring of subsidies in the export credit insurance offered by such agencies, such subsidies being an issue of concern in various international forums (WTO, OECD, EU).

Appendix

Algebraic derivation of the premium rate formula

In order to obtain the formula for the fair premium for export credit insurance, two important assumptions are made—namely, that the insurer is risk-neutral and that the debtor country's debt-servicing capacity follows a geometric Brownian motion. As a result the fair premium—the one that makes the contract a zero-expected profit transaction—can be expressed in terms of its discounted expected (negative) payoffs as follows:

$$\begin{aligned} p_t &= e^{-r} E_t\left[\max\left\{0, \left(\frac{D_{t+1} - K_{t+1}}{D_{t+1}}\right)\right\}\right] \\ &= e^{-r} \pi_t E_t\left[\left(\frac{D_{t+1} - K_{t+1}}{D_{t+1}}\right) \middle| K_{t+1} \prec D_{t+1}\right], \end{aligned} \tag{A1}$$

where π_t is the probability of default in period $t + 1$ viewed from period t. Considering the stochastic dynamics of $\ln K_t$—that is, $\ln K_t = \ln K_{t-1} + (\mu - \sigma^2/2) + \sigma\omega_t$—this probability is given as follows:

$$\begin{aligned} \pi_t &= \text{prob}\{K_{t+1} \prec D_{t+1}\} \\ &= \text{prob}\{\ln K_{t+1} \prec \ln D_{t+1}\} \\ &= \text{prob}\{\ln K_{t+1} \prec \ln N_t\} \\ &= \text{prob}\{\ln K_t + \mu - \sigma^2/2 + \sigma\omega_{t+1} \prec \ln N_t\} \\ &= \text{prob}\left\{\omega_{t+1} \prec \frac{\ln N_t - \ln K_t - \mu + \sigma^2/2}{\sigma}\right\} \\ &= \Phi\left(-\frac{\ln(K_t/N_t) + \mu - \sigma^2/2}{\sigma}\right), \end{aligned} \tag{A2}$$

where $\Phi(\cdot)$ denotes the standard normal distribution function

$$\Phi(\omega) = \int_{-\infty}^{\omega} \phi(v)\, dv, \quad \text{with density } \phi(\omega) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}}. \tag{A3}$$

The expected loss, conditional on the event of default, is as follows:

$$\begin{aligned}
&E_t\left[\left(\frac{D_{t+1}-K_{t+1}}{D_{t+1}}\right)\,\middle|\,K_{t+1}\prec D_{t+1}\right]\\
&\quad=E_t\left[\left(\frac{N_t-K_{t+1}}{N_t}\right)\,\middle|\,K_{t+1}\prec N_t\right]\\
&\quad=1-E_t\left[\frac{K_{t+1}}{N_t}\,\middle|\,K_{t+1}\prec N_t\right]\\
&\quad=1-\frac{K_t}{N_t}E_t\left[\frac{K_{t+1}}{K_t}\,\middle|\,\frac{K_{t+1}}{K_t}\prec\frac{N_t}{K_t}\right]\\
&\quad=1-\frac{K_t}{N_t}E_t[e^{\ln K_{t+1}-\ln K_t}\mid \ln K_{t+1}-\ln K_t\prec \ln N_t-\ln K_t]\\
&\quad=1-\frac{K_t}{N_t}E_t\big[e^{\mu-\sigma^2/2+\sigma\omega_{t+1}}\mid \mu-\sigma^2/2+\sigma\omega_{t+1}\prec\ln(N_t/K_t)\big]\\
&\quad=1-\frac{K_t}{N_t}e^{\eta}E_t\left[e^{\sigma\omega_{t+1}}\mid\omega_{t+1}\prec-\frac{\ln(K_t/N_t)+\eta}{\sigma}\right],
\end{aligned}\tag{A4}$$

with $\eta = \mu - \sigma^2/2$. The expected value of $e^{\sigma\omega_{t+1}}$ on the condition that $\omega_{t+1} \prec -(\ln(K_t/N_t)+\eta)/\sigma$ can be calculated using standard algebra.

Thus,

$$E_t\left[e^{\sigma\omega_{t+1}}\mid\omega_{t+1}\prec-\frac{\ln(K_t/N_t)+\eta}{\sigma}\right]=e^{\sigma^2/2}\frac{\Phi\left(-\frac{\ln(K_t/N_t)+\mu+\sigma^2/2}{\sigma}\right)}{\Phi\left(-\frac{\ln(K_t/N_t)+\mu-\sigma^2/2}{\sigma}\right)}.\tag{A5}$$

$$E_t\left[\left(\frac{D_{t+1}-K_{t+1}}{D_{t+1}}\right)\,\middle|\,K_{t+1}\prec D_{t+1}\right]=1-e^{\mu}\frac{K_t}{N_t}\frac{\Phi\left(-\frac{\ln(K_t/N_t)+\mu+\sigma^2/2}{\sigma}\right)}{\Phi\left(-\frac{\ln(K_t/N_t)+\mu-\sigma^2/2}{\sigma}\right)}\tag{A6}$$

and, combining (A6) with (A2), the following equation is obtained:

$$\begin{aligned}
p_t=e^{-r}&\left[\Phi\left(-\frac{\ln(K_t/N_t)+\mu-\sigma^2/2}{\sigma}\right)\right.\\
&\left.\quad-e^{\mu}\frac{K_t}{N_t}\Phi\left(-\frac{\ln(K_t/N_t)+\mu+\sigma^2/2}{\sigma}\right)\right],
\end{aligned}\tag{A8}$$

which is contained in the text as Eq. (8).

The premium rate data

Algeria	3.15	Egypt	1.24	Nigeria	3.15
Argentina	6.00	Equat. Guinea	0.37	Pakistan	1.24
Bangladesh	1.24	Ethiopia	2.10	Panama	2.35
Barbados	0.90	Fiji	1.60	Papua N. Guinea	1.05
Belize	0.37	Gabon	1.60	Paraguay	1.24
Benin	1.24	Ghana	0.37	Peru	1.42
Bhutan	0.37	Grenada	1.24	Philippines	1.24
Bolivia	1.24	Guatemala	1.24	Poland	3.50
Brazil	1.24	Honduras	2.55	Portugal	0.00
Burkina Faso	1.24	Hungary	1.05	Romania	1.24
Burundi	0.37	India	1.24	Rwanda	0.37
Cameroon	1.60	Indonesia	0.37	Seychelles	0.90
Cape Verde	1.24	Jamaica	1.24	Solomon Isld	0.37
Ctl. Africa	1.24	Kenya	0.90	Sri Lanka	1.60
Chad	1.24	Lesotho	0.37	St. Kitts Nevis	1.05
Chile	0.37	Madagascar	1.60	St. Vincent	1.05
China	0.25	Malawi	1.24	Swaziland	1.42
Colombia	0.37	Malaysia	0.00	Thailand	0.37
Comoros	0.37	Maldives	0.37	Togo	1.42
Congo	1.60	Mali	1.60	Tonga	0.37
Costa Rica	1.24	Malta	0.00	Tunisia	0.70
Côte d'Ivoire	1.60	Mauritania	1.24	Turkey	1.42
Cyprus	0.00	Mauritius	0.37	Uruguay	0.37
Czechosl.	0.00	Mexico	1.24	Vanuatu	0.37
Dominica	1.24	Nepal	0.37	Venezuela	1.24
Ecuador	1.24	Niger	1.24		

Note: Indications of the premium surcharges in percent of amount insured, as of 1993.
Source: Nederlandsche Creditverzekering Maatschappij, UK.

Acknowledgments

I am grateful to Andrew Cornford, Francisco Ferreira, Henri Loubergé, Yaffa Machnes, Nick Stern, and the participants of a seminar at the London School of Economics and at the Twenty-Second Seminar of the European Group of Risk and Insurance Economists at the University of Geneva in 1995 for their helpful comments. All remaining errors are mine. The opinions expressed in this paper are solely those of the author and do not necessarily reflect those of the Deutsche Bundesbank.

Notes

1. Export credit insurance is given in the form of an insurance or a guarantee. Although differing from each other with respect to the contractual relations involved, they are economically equivalent in that the guarantor or insurer, respectively, provides the creditor with an insurance in exchange for the payment of a premium where the insurance is contingent on the extent of default.
2. In the practice of export credit insurance (ECI) the term *premium surcharges* is used. They are charged in addition to a basic insurance fee if credits are extended to those countries for which such surcharges are specified. Thus the term *premium surcharges* corresponds to the concept of *premium rates* that is often used in the theoretical insurance literature.
3. NCM, a private company, took over the responsibility of short-term credit insurance cover from ECGD and thus this short-term branch is now called NCM, UK. It continued to have access to political risk reinsurance from the public ECGD at preferential rates. However, we believe that such support, if there was any, was very limited. For example, this reinsurance was available in 1993 for less than ten country destinations altogether.
4. The debt-service ratio is defined as annual exports over total annual debt service and thus measures the proportion of foreign earnings absorbed by debt service. The reserves-over-imports ratio is defined as (the stock of) international reserves over the equivalent of three months of imports. It can be shown that the former is related to a country's solvency and the latter to its liquidity situation (Schich [1994], pp. 59–66). Empirical studies have often shown these indicators to be statistically significantly related to the event of rescheduling (see, e.g., Feder and Just [1977], Cline [1984, App. A], and Solberg [1988]).
5. In fact, there is a difference in that the put option gives the right to sell an asset *at* a specific date, whereas the insurance gives the right to sell the claim at *any time after* a specific date. However, the fairly general assumption of optimization on the part of the insured implies that the right to sell the claim is always exerted *at* the first possible date. Under these circumstances the two concepts are identical.
6. Thus some important technical assumptions are made. First, the changes in the debt-servicing capacity are intertemporally uncorrelated. This does not necessarily mean that, for example, the foreign-exchange earnings are intertemporally uncorrelated, but it does mean that, while anticipated changes in these earnings already form part of the country's debt-servicing capacity, unexpected ones (shocks) are intertemporally uncorrelated. Thus shocks are not forecastable from past expected changes. Second, the variance of the change in value is constant through time. Third, changes in the debt-servicing capacity can be either positive or negative, but the value of such capacity can never fall below zero when its starting value is positive. Such restriction is essential for a net debtor country because otherwise the intertemporal budget constraint will be violated.
7. Note that for μ equals r, Eq. (8) reduces to the Black-Scholes formula.
8. To be included in our sample, complete data had to be available in all dependent and independent variables. Data on the two financial indicators—the debt-service ratio and the reserves-over-ratio—were obtained from the United Nations data bank. The standard debt-service ratio was inverted, so that the expected signs of the two indicators were comparable. The average rates of change and the volatilities of these rates were calculated applying standard equations to historical data over three years. The results were similar when these rates were calculated over a five-year horizon.
9. In another test, the specific structural form of the equations was considered. The external financial indicator, its average rate of change, and the volatility of that rate were substituted into (8) to obtain hypothetical premium rates. These where then compared with the actual premium rates by means of correlation coefficients and regressions. The results were mixed. Using the debt-service ratio, the two series appeared to differ significantly. On the other hand, they appeared to be very similar when the reserves-over-imports ratio was used.
10. The kernel density estimator used is the standard bivariate one with the optimal window width, minimizing the mean square error as described in Silverman [1986, p. 86]. The estimates for the variables DRES and DDSR are not reported here because they did not show clear patterns.

References

BARTOLINI, L., and DIXIT, A. [1991]: "Market Valuation of Illiquid Debt and Implications for Conflicts Among Creditors," *IMF Staff Papers*, 38(4), 828–849.

BLACK, F. [1976]: "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 24–28.

BLACK, F., and SCHOLES, M. [1973]: "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637–654.

CLAESSENS, S., and VAN WIJNBERGEN, S. [1990]: "Secondary Market Prices Under Alternative Debt Reduction Strategies: An Option Pricing Approach with an Application to Mexico," *CEPR Discussion Paper*, 415.

CLARK, E.A. [1991]: *Cross-Border Investment Risk*. London: Euromoney Books.

CLINE, W.R. [1984]: *International Debt: Systemic Risk and Policy Response*. Cambridge, MA: MIT Press.

COHEN, D. [1990]: "A Valuation Formula for LDC Debt," *CEPR Discussion Paper*, 460.

DOOLEY, M.P., FOLKERTS-LANDAU, D., SYMANSKY, S.A., and TYRON, R.W. [1990]: "Debt Reduction and Economic Activity," *IMF Occasional Paper*, 68.

FEDER, G., and JUST, R. [1977]: "A Study of Debt-Servicing Capacity Applying Logit Analysis," *Journal of Development Economics*, 4, 25–38.

GENOTTE, G., KHARAS, H., and SADEQ, S. [1987]: "A Valuation Model for Developing Country Debt," *World Bank Economic Review*, 1(2), 169–176.

KLEIN, M. [1991]: "Bewertung von Länderrisiken durch Optionspreismethoden," *Kredit und Kapital*, 24(4), 484–507.

KLEIN, M. [1994]: *Bewertung von Länderrisiken*. Berlin: Duncker & Humblot.

MERTON, R.C. [1977]: "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantee," *Journal of Banking and Finance*, 1, 3–11.

NOCERA, S. [1989]: "Pricing an Interest Payment Guarantee: A Contribution to Debt Reduction Techniques," *IMF Working Paper*, WP/89/65.

SCHICH, SEBASTIAN T. [1994]: "Export Credits and the Cost of Trade Financing," Ph.D. dissertation, London School of Economics.

SCOTT, L. [1990]: "Pricing Floating-Rate Debt and Related Interest Rate Options," *IMF Working Paper*, WP/90/7.

SILVERMAN, B.W. [1986]: *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

SOLBERG, R.L. [1988]: *Sovereign Rescheduling: Risk and Portfolio Management*. London: Unwin Hyman.

WHITE, H. [1980]: "A Heteroscedastic-Consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity," *Econometrica*, 48, 817–838.

WINKLER, M. [1933]: *Foreign Bonds, An Autopsy: A Study of Defaults and Repudiations of Government Obligations*. Philadelphia: Swain.