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Explaining Insurance Policy Provisions via Adverse Selection

VIRGINIA R. YOUNG AND MARK J. BROWNE
School of Business, University of Wisconsin–Madison, Madison, WI 53706

Abstract

In this article, we show that common insurance policy provisions—namely, deductibles, coinsurance, and maximum limits—can arise as a result of adverse selection in a competitive insurance market. Research on adverse selection typically builds on the assumption that different risk types suffer the same size loss and differ only in their probability of loss. In this study, we allow the severity of the insurance loss to be random and, thus, generalize the results of Rothschild and Stiglitz [1976] and Wilson [1977]. We characterize the separating equilibrium contracts in a Rothschild-Stiglitz competitive market. By further assuming a Wilson competitive market, we show that an anticipatory equilibrium might be achieved by pooling, and we characterize the optimal pooling contract.

1. Introduction

In an insurance market with perfect information, risk-averse individuals prefer policies that offer full coverage when insurance is priced actuarially fair (Mossin [1968]). However, most insurance policies do not offer full coverage. In the literature, several explanations have been provided for this occurrence. One of the primary results of this type is due to Arrow [1971]: if the insurance premium depends only on the policy's actuarial value, then insureds prefer policies that provide full coverage above a deductible. The optimality of deductibles to consumers when insurance entails transactions costs has been recognized by Townsend [1979], Brennan and Solanki [1981], Mayers and Smith [1981], Huberman, Mayers, and Smith [1983], Schlesinger [1981], and Turnbull [1983], among others.

Raviv [1979] demonstrates that, in a perfect information market, coinsurance is either a result of risk aversion of insurers or of nonlinearity in insurance costs. Raviv also shows that policy limits are part of an optimal insurance contract when rates are regulated. Huberman, Mayers, and Smith [1983] argue that policy limits are a consequence of the limit on liability individuals receive through bankruptcy laws.

Information asymmetry between insurer and policyholder can also explain incomplete coverage. For example, under conditions of moral hazard, Shavell [1979] shows that optimal insurance will be less than full coverage. Adverse selection also provides rationale for incomplete coverage. Rothschild and Stiglitz [1976] show that equilibrium is either nonexistent or separating, with low risks buying a policy that provides partial coverage and high risks buying a policy that provides full coverage. In a competitive market, Wilson [1977] shows that an anticipatory equilibrium will either be separating like that of Rothschild-Stiglitz or pooling with both high and low risks buying the same policy at a common price. Because there is only one loss size in the models of Rothschild-Stiglitz and

Wilson, it is impossible to know whether the specific policy provisions include a deductible, coinsurance, or a maximum limit. Fluet and Pannequin [1977] study the design of insurance contracts under adverse selection when the severity of the loss is random and when first-best solutions are not realizable, as do we. They focus on competitive insurance markets, as in Rothschild and Stiglitz [1976], and extend the results of Rothschild and Stiglitz to the continuous loss case.[1] Specifically, if one defines low versus high risks according to their expected losses (that is, a low risk has an expected loss less than or equal to a high risk), then in a separating equilibrium, high risks buy full coverage, while low risks buy only partial coverage.

In this article, we extend the literature on the design of optimal insurance contracts under adverse selection in two ways. First, we give the necessary and sufficient conditions satisfied by the optimal separating insurance policy for low risks. Second, and more important, we extend the results of Wilson to the continuous loss case. We characterize both the optimal separating and pooling contracts in a Wilson competitive market and describe how one can determine whether the equilibrium is separating or pooling. If the equilibrium is separating, then the equilibrium contracts are the same as in the (extended) Rothschild-Stiglitz model. Under assumptions of information asymmetry, we, thereby, explain common provisions found in many insurance policies, such as deductibles, coinsurance, and maximum limits.

In Section 2, we state our basic assumptions, and in Section 3, we formulate our problem concerning equilibrium separating and pooling insurance policies in a competitive market. We analyze the equilibrium separating and pooling policies in Section 4. In Section 5, we include a constraint on the policies that will protect the insurer against underreporting of losses and determine necessary conditions for the resulting equilibrium insurance policies. In Section 6, we present an illustrative example in which we determine the equilibrium insurance policies and show that the common insurance provisions of deductibles, coinsurance, and maximum limits can be realized in our model.

2. Insurance risks and feasible contracts

Assume that insurers compete for the business of L low risks and H high risks in the market. A risk is defined to be *low* versus *high* if the expected loss of a low risk is not greater than the expected loss of a high risk. N is the total number of insureds in the market—that is, $N = L + H$. Denote the initial wealth of an individual risk by w. Let u denote the utility of wealth, a twice-differentiable function. Individuals are risk averse—that is, $u' > 0$ and $u'' < 0$. Low and high risks are identical except for their loss distributions, which are a mixture of a point mass at zero and a continuous severity distribution above that. Denote the cumulative loss distributions of low and high risks by F_L and F_H, respectively. Let $p_i, i = L$ or H, be the probability that a risk suffers a loss; $p_i = 1 - F_i(0) \leq 1$. Let $f_i, i = L$ or H, denote the probability density function of the severity of loss. We can, therefore, write $F_i, i = L$ or H, as follows:

$$F_i(x) = \begin{cases} 0, & x < 0; \\ 1 - p_i, & x = 0; \\ (1 - p_i) + p_i \int_0^x f_i(t)\,dt, & x > 0. \end{cases}$$

Throughout this article, we assume that f_L and f_H have the same support—that is, they are positive on the same set of nonnegative real numbers—and we assume that $E_L[X] \leq E_H[X]$.

Let $I : \mathbf{R}^+ \to \mathbf{R}^+$ denote an insurance policy that pays $I(x)$ to an insured if the insured suffers a loss of size x. Assume that the indemnity benefit an insured receives, following a loss x, lies between zero and x—that is, $0 \leq I(x) \leq x$. Such an insurance policy I is called *feasible*. Later, in Section 5, we further restrict feasible policies to be nondecreasing in order to guard against underreporting of losses. We assume that insureds can buy at most one insurance policy to cover their potential losses. The expected utility of an individual of type i, $i = L$ or H, who buys an insurance policy I for a premium P is

$$\begin{aligned} U_i(I, P) &= E_i[u(w - P + I(x) - x)] \\ &= \int_0^\infty u(w - P + I(x) - x)\, dF_i(x) \\ &= (1 - p_i)u(w - P) + p_i \int_0^\infty u(w - P + I(x) - x) f_i(x)\, dx. \end{aligned}$$

3. Separating and pooling equilibria

Assume that insurers behave as if they are risk neutral and that administrative expenses and investment income are zero. Assume that the insurance market is a competitive market, as in Rothschild and Stiglitz [1976], and that each insurance policy earns nonnegative profits. A set of contracts is in *equilibrium* if there does not exist an additional contract that, if also offered, would make a positive profit. Due to long-run competition, each policy will earn zero expected profits in equilibrium. Information asymmetries exist in the market because of regulatory prohibitions on underwriting or the inability of insurers to acquire relevant information.

Fluet and Pannequin [1977] show that full insurance for both low and high risks constitutes a separating equilibrium at an actuarially fair price if and only if $E_L[X] = E_H[X]$. They also show that if equilibrium is achieved by a pair of separating policies and $E_L[X] < E_H[X]$, then the equilibrium coverage for high risks is full insurance at the actuarially fair price of $E_H[X]$ and the equilibrium coverage for low risks I_S solves the following optimization problem:

$$\max_{I,P} U_L(I, P) = \max_{I,P} \left[(1 - p_L)u(w - P) + p_L \int_0^\infty u(w - P + I(x) - x) f_L(x)\, dx \right], \tag{1}$$

subject to

$$P \geq p_L \int_0^\infty I(x) f_L(x)\, dx, \tag{2}$$

$$0 \leq I(x) \leq x, \tag{3}$$

and

$$u(w - E_H[x]) \geq U_H(I)$$
$$= (1 - p_H)u(w - P) + p_H \int_0^\infty u(w - P + I(x) - x) f_H(x)\, dx. \tag{4}$$

The last constraint (4) ensures high risks will not prefer the separating policy of low risks to full coverage.

Now, further assume that insurers are nonmyopic, as in Wilson [1977]—that is, they will not offer policies that will become unprofitable if other policies are removed from the market in response to the introduction of a new policy. It may occur that both risks will prefer a pooling policy to their optimal separating policies. In that case, any pooling policy that is priced actuarially fair and that does not maximize the expected utility of the low risks can be improved upon for the low risks. The old policy will be removed from the market because it will eventually lose money, due to the following straightforward result, and because insurers are nonmyopic.

Lemma 1: *Suppose that $E_L[X] < E_H[X]$. For any pooling policy I with an actuarially fair premium P, such that the high risks prefer I to full coverage with an actuarially fair premium of $E_H[X]$, we have that $P < E_H[I(X)]$.*

Therefore, if the equilibrium is a pooling equilibrium, then the optimal pooling policy I_P solves the following problem:

$$\max_{I,P} U_L(I, P) = \max_{I,P} \left[(1 - p_L)u(w - P) + p_L \int_0^\infty u(w - P + I(x) - x) f_L(x)\, dx \right], \tag{5}$$

subject to

$$P \geq \int_0^\infty I(x) \frac{L p_L f_L(x) + H p_H f_H(x)}{N}\, dx, \tag{6}$$

and

$$0 \leq I(x) \leq x. \tag{7}$$

Note that competition will force both premium constraints (2) and (6), to hold at the optimal contracts. For the optimal separating policies to constitute an equilibrium, we also have the self-selection constraints that at least one risk class will prefer its optimal separating policy to the optimal pooling policy. Similarly, for the optimal pooling policy to constitute an equilibrium, we have the self-selection constraints that both risk classes will prefer the optimal pooling policy to their optimal separating policies. In the next section, we uniquely determine the optimal separating and pooling insurance policies.

4. Design of the optimal insurance contracts

In this section, we characterize the optimal separating and pooling insurance contracts, as given in Section 3, (1) to (4) and (5) to (7). Recall that $E_L[X] \leq E_H[X]$ and that f_L and f_H have the same support. We first state necessary and sufficient conditions for the optimal separating insurance contracts, then for the optimal pooling contract. The optimal separating insurance contract for high risks is full coverage at an actuarially fair premium, $E_H[X]$, as shown in Fluet and Pannequin [1997]. The optimal separating contract for low risks is characterized in the following proposition.

Proposition 1: *The optimal separating insurance policy I_S for the low risks is characterized by the following necessary and sufficient conditions: There exists a nonnegative constant λ such that*

a. $$I_S(x_0) = 0 \text{ iff } MUC_L - \lambda MUC_H \geq u'(w - P - x_0)\left\{1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right\}.$$

b. $$I_S(x_0) = x_0 \text{ iff } MUC_L - \lambda MUC_H \leq u'(w - P)\left\{1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right\}.$$

c. $$0 < I_S(x_0) < x_0 \text{ iff } MUC_L - \lambda MUC_H = u'(w - P + I_S(x_0) - x_0)\left\{1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right\}.$$

$MUC_L = E_L[u'(w - P + I_S(X) - X)]$ and $MUC_H = E_H[u'(w - P + I_S(X) - X)]$ are the marginal utility costs under the insurance policy I_S of the low and high risks, respectively. The premium P equals the expected indemnity benefit $P = p_L \int_0^\infty I_S(x) f_L(x)\, dx$. Also, the self-selection constraint (4) holds at the optimum, and one can use this constraint to determine the value of λ.

Proof. See the appendix. □

One can interpret the conditions in Proposition 1 economically. Indeed, in each condition one compares the marginal utility cost to the low risks of paying additional premium, MUC_L, with the marginal utility benefit of receiving the corresponding additional indemnity benefit, adjusted for the net marginal benefit to the high risks. For example, $I_S(x_0) = 0$ if and only if the marginal utility cost to the low risks is higher than the adjusted benefit. We have the following corollaries of Proposition 1, which we state without proof. The first corollary generalizes a result of Mossin [1968].

Corollary 1.1: *The optimal separating policy for the low risks I_S is full coverage iff $E_L[X] = E_H[X]$.*

Corollary 1.2: *If the ratio of severity density functions of the high and low risks, $\frac{f_H(x)}{f_L(x)}$, is increasing for all losses $x > 0$, then the optimal policy I_S has full coverage only between 0 and some loss amount l (possibly zero) and coinsurance above that amount.*

Therefore, if the severity densities satisfy a monotone likelihood ratio (MLR) ordering—that is, $\frac{f_H(x)}{f_L(x)}$ increases with respect to x—then the optimal separating contract for the low risks is not necessarily deductible insurance. In this case, low risks are willing to give up coverage at higher loss amounts in order to receive full coverage at lower amounts, where they are more likely to incur losses. In Section 5, we present an example that illustrates this corollary. See Ormiston and Schlee [1993] for further insight into risk preferences when distributions satisfy MLR dominance.

Corollary 1.3: *If* $\frac{f_H(x)}{f_L(x)}$ *is increasing for all losses* $x > 0$, *and if* $0 < I_S(x) < x$, *for the optimal policy* I_S, *then* $I_S'(x) < 1$.

Corollary 1.3 is similar to one in Raviv [1979]: our corollary states that if the severity densities satisfy an MLR ordering, then I_S grows more slowly than losses increase. In this case, a policyholder has no incentive to create incremental damage when a loss has occurred. The next corollary gives an instance in which a deductible policy is optimal.

Corollary 1.4: *If the low and high risks have the same severity distribution and differ only in their probabilities of having a positive loss, with* $p_L < p_H$, *then the optimal policy* I_S *has a deductible* d. *For losses,* x, *above the deductible,* $I_S(x) = x - d$.

The next corollary shows that when the severity density for the low risks grows relatively more quickly than the one for the high risks, a type of (possibly nonlinear) disappearing deductible policy is optimal.

Corollary 1.5: *If* $\frac{f_H(x)}{f_L(x)}$ *is decreasing for all losses* $x > 0$, *then the optimal policy* I_S *has a deductible* d *(possibly zero) with* $I_S'(x) > 1$ *for losses above the deductive.*

Now we turn our attention to the optimal pooling contract. In the next proposition, we state necessary and sufficient conditions satisfied by the optimal pooling insurance contract. The proof parallels the one of Proposition 1, so we omit it.

Proposition 2: *The optimal pooling insurance policy* I_P *is characterized by the following necessary and sufficient conditions*:

a. $$I_P(x_0) = 0 \textit{ iff } MUC_L \geq u'(w - P - x_0)\frac{Np_L f_L(x_0)}{Lp_L f_L(x_0) + Hp_H f_H(x_0)}.$$

b. $$I_p(x_0) = 0 \textit{ iff } MUC_L \leq u'(w - P)\frac{Np_L f_L(x_0)}{Lp_L f_L(x_0) + Hp_H f_H(x_0)}.$$

c. $$0 < I_p(x_0) < x_0 \textit{ iff } MUC_L = u'(w - P + I_P(x_0) - x_0)\frac{Np_L f_L(x_0)}{Lp_L f_L(x_0) + Hp_H f_H(x_0)}.$$

MUC_L represents the marginal utility cost of the low risks, $E_L[u'(w - P + I_P(X) - X)]$, and x_0 is a given loss amount in $(0, \infty)$. The premium P equals the expected indemnity benefit

$$P = \int_0^\infty I_P(x) \frac{Lp_L f_L(x) + Hp_H f_H(x)}{N} dx.$$

The left side of each expression in (a), (b), and (c) in Proposition 2 is the marginal utility cost of paying additional premium. The corresponding right sides are the marginal utility benefits of receiving the corresponding additional indemnity benefit at x_0. Therefore, $0 < I_P(x_0) < x_0$ if and only if the marginal utility cost equals the marginal utility benefit. Corollaries 1.1 to 1.5 also hold for the optimal pooling insurance I_P, but in the interest of conserving space, we do not repeat them.

Propositions 1 and 2 determine the optimal separating and pooling policies. These propositions, together with self-selection constraints, completely determine the equilibrium in a given market. Specifically, the equilibrium will be separating unless both risks prefer the optimal pooling policy to their optimal separating policies.

5. Maximum limits

The conditions of Propositions 1 and 2 do not preclude the optimal policy from decreasing with increasing loss. If a policy exhibits this behavior, then policyholders will be motivated to represent their losses downward. However, the price of the insurance policy does not anticipate this reporting hazard, and if policyholders misrepresent their losses downward, then the insurers will lose money on average. In order to prevent this behavior, we restrict the class of feasible insurance policies by requiring that a policy be nondecreasing with increasing loss. That is, to the optimization problems stated in Section 3, we add the constraint $I'(x) \geq 0$, for all $x > 0$. This constraint leads to possible maximum limits on coverage.

In the first proposition, we give necessary conditions for the optimal separating policy of the low risks when feasible insurance policies are required to be nondecreasing. The proof parallels the one of Proposition 1, so we omit it.

Proposition 3: *The optimal separating policy I_S for the low risks that solves the optimization problem given in* (1) *to* (4) *subject to the additional constraint that $I'(x) \geq 0$, for all $x > 0$, satisfies the following necessary conditions: There exists a nonnegative constant λ, such that*

a. *If the policy I_S has a deductible $d > 0$—that is, if $I_S(x_0) = 0$, for all $x_0 \leq d$, and $I_S(x_0) > 0$, for all $x_0 > d$—then*

$$MUC_L - \lambda MUC_H \geq \frac{\int_{x_0}^{d} [u'(w - P - x)(p_L f_L(x) - \lambda\, p_H f_H(x))]\, dx}{\int_{x_0}^{d} p_L f_L(x)\, dx}, \quad \forall\, x_0 < d.$$

b. *If* $I_S(x_0) = x_0$, *then*

$$MUC_L - \lambda MUC_H \le u'(w - P)\left(1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right).$$

c. *If* $0 < I_S(x_0) < x_0$ *and if* $I_S'(x_0) > 0$, *then*

$$MUC_L - \lambda MUC_H = u'(w - P + I_S(x_0) - x_0)\left(1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right).$$

d. *If* $I_S > 0$ *and if* $I_S' \equiv 0$, *on the interval* (u_1, u_2), *then*

$$\frac{\int_{x_0}^{u_2} [u'(w - P + I_S(u_1) - x)(p_L f_L(x) - \lambda p_H f_H(x))]\, dx}{\int_{x_0}^{u_2} p_L f_L(x)\, dx} \le MUC_L - \lambda MUC_H$$
$$\le \frac{\int_{u_1}^{y_0} [u'(w - P + I_S(u_1) - x)(p_L f_L(x) - \lambda p_H f_H(x))]\, dx}{\int_{u_I}^{y_0} p_L f_L(x)\, dx},$$

for all x_0, y_0 *in* (u_1, u_2). MUC_L *and* MUC_H *are the marginal utility costs under the insurance policy* I_S *of the low and high risks, respectively. The premium* P *equals the expected indemnity benefit* $P = p_L \int_0^\infty I_S(x) f_L(x) dx$, *and the self-selection constraint* (4) *holds at the optimum.*

We have the following corollary to Proposition 3 that gives one case for which the optimal separating insurance has a maximum limit.

Corollary 3.1: *If* $0 < I_S(x) < x$ *and*

$$-\frac{u''(w - P + I_S(x) - x)}{u'(w - P + I_S(x) - x)} < -\frac{d}{dx} \ln\left(1 - \lambda \frac{p_H f_H(x)}{p_L f_L(x)}\right)$$
$$= \frac{\lambda}{1 - \lambda \frac{p_H f_H(x)}{p_L f_L(x)}} \frac{p_H}{p_L} \frac{d}{dx}\left(\frac{f_H(x)}{f_L(x)}\right),$$

for all x *greater than some loss amount* u, *then the optimal policy* I_S *has a maximum limit. Specifically, there exists a loss amount* $m \le u$ *such that* $I_S(x) = I_S(m)$, *for all* $x \ge m$.

Proof. See the appendix. □

In Corollary 3.1, the left side of the inequality is the absolute risk aversion evaluated at the outcome under the optimal separating policy. If the risk aversion of the low risks is bounded by the right side, which depends on the relative growth of the severity densities, then the low risks are willing to give up coverage at large losses. In fact, they prefer an indemnity benefit that is decreasing with respect to the loss amount for large losses. Because that policy is not feasible, the optimal policy has a maximum limit.

In the next proposition, we give necessary conditions for the optimal pooling policy when feasible insurance policies are required to be nondecreasing.

Proposition 4: *The optimal pooling policy I_P that solves the optimization problem given in* (5) *to* (7) *subject to the additional constraint that $I'(x) \geq 0$, for all $x > 0$, satisfies the following necessary conditions*:

a. *If the policy I_P has a deductible $d > 0$, then*

$$MUC_L \geq \frac{p_L \int_{x_0}^{d} u'(w - P - x) f_L(x)\,dx}{\int_{x_0}^{d} \frac{Lp_L f_L(x) + Hp_H f_H(x)}{N}\,dx}, \quad \forall x_0 < d.$$

b. *If $I_P(x_0) = x_0$, then*

$$MUC_L \leq u'(w - P) \frac{Np_L f_L(x_0)}{Lp_L f_L(x_0) + Hp_H f_H(x_0)}.$$

c. *If $0 < I_P(x_0) = x_0$ and if $I'_P(x_0) > 0$, then*

$$MUC_L = u'(w - p + I_p(x_0) - x_0) \frac{Np_L f_L(x_0)}{Lp_L f_L(x_0) + Hp_H f_H(x_0)}.$$

d. *If $I_P > 0$ and if $I'_P \equiv 0$, on the interval (u_1, u_2), then*

$$\frac{p_L \int_{x_0}^{u_2} u'(w - P + I_P(u_1) - x) f_L(x)\,dx}{\int_{x_0}^{u_2} \frac{Lp_L f_L(x) + Hp_H f_H(x)}{N} dx} \leq MUC_L \leq \frac{p_L \int_{u_1}^{y_0} u'(w - P + I_P(u_1) - x) f_L(x) dx}{\int_{u_1}^{y_0} \frac{Lp_L f_L(x) + Hp_H f_H(x)}{N}\,dx},$$

for all x_0, y_0 in (u_1, u_2). MUC_L is the marginal utility cost of the low risks under the policy I_P, and the premium P equals the expected indemnity benefit

$$P = \int_0^{\infty} I_P(x) \frac{Lp_L f_L(x) + Hp_H f_H(x)}{N}\,dx.$$

One can interpret the conditions in Propositions 3 and 4, as in Propositions 1 and 2. Indeed, in Proposition 4, MUC_L is the marginal utility cost to the low risks of paying additional premium. The right side in (a), for example, is the marginal utility benefit of receiving the corresponding additional indemnity benefit between x_0 and the deductible d. Because this marginal utility benefit is lower than the marginal utility cost, the benefit is zero. One can interpret the remaining conditions of this proposition and of Proposition 3 similarly.

The following corollary to Proposition 4 parallels Corollary 3.1.

Corollary 4.1: *If* $0 < I_P(x) < x$ *and*

$$-\frac{u''(w - P + I_P(x) - x)}{u'(w - P + I_P(x) - x)} < \frac{d}{dx} \ln\left(L + H\frac{p_H f_H(x)}{p_L f_L(x)}\right),$$

for all x *greater than some loss amount* u, *then the optimal policy* I_P *has a maximum limit. Specifically, there exists a loss amount* $m \leq u$ *such that* $I_P(x) = I_P(m)$, *for all* $x \geq m$.

The conditions of Corollaries 3.1 and 4.1 are satisfied, for example, when risks have constant risk aversion, as measured by $-\frac{u''(w)}{u'(w)}$, when there exists at least one loss x that satisfies the given inequality, and when the severity distributions for the low and high risks come from a linear exponential family with $\theta_L < \theta_H$: A family of distributions is called a *linear exponential family of probability distributions* if the distributions have densities of the form (Lehmann [1991])

$$f(x \mid \theta) = e^{\theta x} h(\theta)\, q(x),$$

in which h is a nonnegative function on a parameter space, and q is a nonnegative function on a subset of $\mathbf{R}$ ($\mathbf{R}^+$ in this case). For example, the gamma family of distributions, $G(\alpha, \beta)$, with fixed shape parameter $\alpha > 0$ and variable scale parameter $\beta > 0$, is a linear exponential family. Indeed,

$$f(x \mid \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0.$$

If we let $\theta = -\beta$, then $h(\theta) = \frac{(-\theta)^\alpha}{\Gamma(\alpha)}$, $q(x) = x^{\alpha-1}$, and θ ranges over the negative real numbers.

6. Illustrative example

In this section, we provide an illustrative example with which we show how the insurance policy provisions of deductibles, coinsurance, and maximum limits can be realized through our model. Let $L = 9$ and $H = 1$—that is, the high risks constitute 10 percent of the market. Assume that low risks have $p_L = 0.20$ and have loss severities distributed according to the exponential distribution with mean \$1,000. Similarly, high risks have $p_H = 0.80$ and have loss severities distributed according to the exponential distribution with mean \$2,000. Assume that both the low and high risks have an exponential utility with constant risk aversion $\alpha = 0.0003$—that is, $u(w) = -e^{-\alpha w}$. It follows that low and high risks are willing to pay up to \$274 and \$2,628 for full coverage, respectively.

First, consider the case of optimal insurance without imposing the nondecreasing constraint. The expected utility of the low risks under the optimal pooling insurance, I_{P1}, is $U_L(I_{P1}) = -1.0840$, and the expected utility of the high risks under the optimal pooling insurance is $U_H(I_{P1}) = -2.006$. We ignore the factor of $e^{-\alpha w}$ in each expected utility.

The expected utility of the high risks under full coverage is $U_H(I_{\text{Full}}) = -1.616$. Thus, the high risks are better off if they separate from the low risks, and it follows that the equilibrium

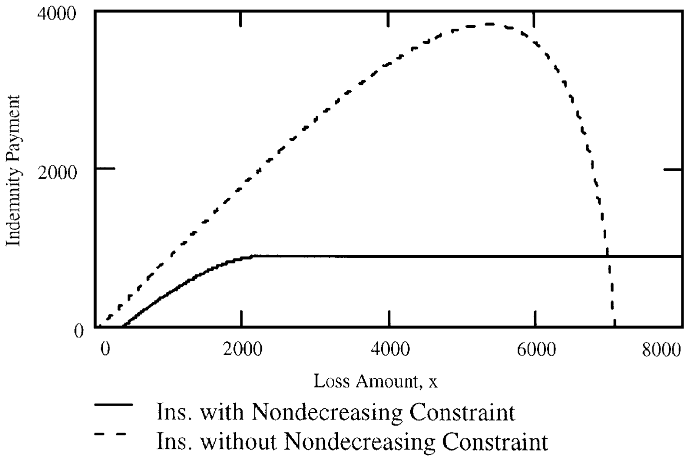

Figure 1. Indemnity benefit for the low risks under equilibrium separating contracts with and without the nondecreasing constraint.

will be a separating one. The expected utility of the low risks, under the optimal separating insurance policy, I_{S1}, is $U_L(I_{S1}) = -1.064$. Thus, the expected utility of the low risks is also higher under the optimal separating insurance policy than under pooling insurance but slightly less than under full coverage at an actuarially fair premium; $U_L(I_{\text{Full}}) = -1.062$.

See figure 1 for a graph (dotted line) of the optimal separating policy of the low risks for loses ranging from \$0 to \$8,000. The optimal insurance has a deductible of about \$61 and coinsurance above that amount. Note that the optimal insurance is decreasing for losses greater than approximately \$5,350. This is unrealistic, as pointed out by Huberman, Mayers, and Smith [1983] because the insured is, thus, motivated to misrepresent his or her loss downward if it is greater than \$5,350.

Now, consider the case of optimal insurance in which we impose the nondecreasing constraint. The expected utility of the low risks under the optimal pooling insurance, I_{P2}, is $U_L(I_{P2}) = -1.0842$, and the expected utility of the high risks under the optimal pooling insurance is $U_H(I_{P2}) = -1.919$. Note that the low risks lose a small amount of expected utility—namely, 0.0002—due to the nondecreasing constraint. One can think of the difference as the utility cost of the reporting hazard. However, the high risks gain expected utility because they get better coverage at high losses under I_{P2} than under I_{P1}.

Similarly, the expected utility of the low risks under the optimal separating insurance, I_{S2}, is 0.009 less than without the nondecreasing constraint; it is $U_L(I_{S2}) = -1.073$. Again, the risks buy separating insurance policies. See figure 1 for a graph (solid line) of the optimal nondecreasing separating policy of the low risks for losses ranging from \$0 to \$8,000. This policy has a deductible of about \$350, coinsurance above that amount, and a maximum limit at the loss amount \$2,400. For losses above \$2,400, the indemnity benefit is about \$900. In figure 1, the indemnity benefit graphed with the dotted line is preferred by the low risks to the one graphed with the solid line, but the former is not likely to be seen in the market because of the relative ease of underreporting losses.

7. Summary

In this work, we characterize the equilibrium insurance contract in a market with asymmetric information. We show that if the risk classes separate, then the equilibrium is a Rothschild-Stiglitz separating equilibrium, determined by Proposition 1 and described in Fluet and Pannequin [1997]. We show that if the market is a Wilson market and the risk classes pool, then the optimal insurance contract is given by Proposition 2. In Propositions 3 and 4, we describe the optimal separating and pooling policies when a nondecreasing constraint is imposed. By varying the relationship between the loss distributions of the low and high risks, one can explain insurance policy provisions commonly encountered in insurance. These provisions include deductibles, coinsurance, and maximum limits.

We give an example that shows that low risks are willing to give up coverage for large losses when they are very unlikely to have large losses, relative to the high risks In that example, we also show that the low risks lose utility due to the underreporting hazard. We suggest future research into models that allow for subsidies between policies, as in Spence [1978], and that incorporate transaction costs.

Appendix: Proofs

Proof of Proposition 1

Write I_s for the optimal separating insurance for the low risks. We can rewrite the optimization problem in (1) to (4) as follows: For a fixed $\lambda \geq 0$, find $I_S(\lambda)$ to solve the following optimization problem:

$$\max_{I,P} [U_L(I, P) - \lambda U_H(I, P)],$$

subject to $P = p_L \int_0^\infty I(x) f_L(x)\, dx$, and $0 \leq I(x) \leq x$. One can find the value of λ that determines the optimal separating policy I_S by the self-selection constraint:

$$u(w - E_H[x]) = U_H(I_S(\lambda), E_L[I_S(\lambda)]).$$

Suppose that I_S is zero at $x_0 > 0$, and consider an insurance policy I given by

$$I(x) = I_S(x) + \varepsilon[H(x - (x_0 - \delta)) - H(x - (x_0 + \delta))],$$

in which $\varepsilon > 0$, $\delta > 0$; that is, I is I_S increased by a small amount ε in a small δ-neighborhood of x_0. Here H is the Heaviside function defined by

$$H(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Competition forces the premium constraint to hold at the optimum, so without loss of generality, we can write the premium P of the policy I as follows:

$$P(I) = P(I_S) + \varepsilon \int_{x_0-\delta}^{x_0+\delta} p_L f_L(x)\,dx$$
$$= P(I_S) + \varepsilon(2\delta) p_L f_L(x_0^*),$$

in which x_0^* is some number between $x_0 - \delta$ and $x_0 + \delta$.

Now,

$$U_L(I_S, P(I_S)) - \lambda U_H(I_S, P(I_S)) \geq U_L(I, P(I)) - \lambda U_H(I, P(I))$$
$$= U_L(I_S, P(I_S)) - \lambda U_H(I_S, P(I_S)) - 2\varepsilon\delta(\mathrm{MUC}_L - \lambda \mathrm{MUC}_H) p_L f_L(x_0^*)$$
$$+ \varepsilon \int_{x_0-\delta}^{x_0+\delta} u'(w - P(I_S) + I_S(x) - x)(p_L f_L(x) - \lambda p_H f_H(x))\,dx + o(\varepsilon).$$

After simplifying this expression and letting ε and δ approach zero, we obtain the following necessary condition for $I_S(x_0) = 0$:

$$\mathrm{MUC}_L - \lambda \mathrm{MUC}_H \geq u'(w - P - x_0)\left\{1 - \lambda \frac{p_H f_H(x_0)}{p_L f_L(x_0)}\right\}.$$

Similarly, we obtain necessary conditions for $I_S(x_0) = x_0$ and for $0 < I_S(x_0) < x_0$, as stated in parts (b) and (c) of Proposition 1. To see that the conditions are also sufficient to determine the optimal insurance, note that becuase u is concave, one and only one of the conditions holds at a specific x_0, except possibly for equality at the boundaries. □

Proof of Corollary 3.1

Take the natural logarithm of both sides of condition (c); then, differentiate with respect to x to obtain the following:

$$I_S'(x)\frac{u''(w - P + I_S(x) - x)}{u'(w - P + I_S(x) - x)} = -\frac{u''(w - P + I_S(x) - x)}{u'(w - P + I_S(x) - x)} + \frac{d}{dx}\ln\left(1 - \lambda \frac{p_H f_H(x)}{p_L f_L(x)}\right).$$

If $-\frac{u''(w-P+I_S(x)-x)}{u'(w-P+I_S(x)-x)} < -\frac{d}{dx}\ln\left(1 - \lambda \frac{p_H f_H(x)}{p_L f_L(x)}\right)$, then $I_S'(x) < 0$, for all $x > u$, contradicting a requirement of condition (c). Thus, the policy has a maximum limit. □

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Note

1. Other recent research also considers the case of random loss severity under conditions of information asymmetry. Doherty and Jung [1993] allow random loss severity but constrain the form of the insurance contract. They focus on finding when first-best equilibrium occurs and thereby eliminate the problem of adverse selection. Such situations occur when the supports of the loss distributions of different risks are nonoverlapping; in that case, the size of an insured's loss may reveal whether an insured is a high risk. Doherty and Schlesinger [1995] assume that individuals have different probabilities of loss but have the same loss severity distributions. They also restrict the form of the insurance policies to be coinsurance with a fixed proportion of coinsurance for all losses. (In this work, we show that in this case, the equilibrium insurance contract is deductible insurance.) Landsberger and Meilijson [1994, 1996], study adverse selection in a monopolistic insurance market and determine conditions under which a (quasi) first-best outcome exists.

References

ARROW, K.J. [1971]: *Essays in the Theory of Risk Bearing*. Chicago: Markham.

BRENNAN, M.J., and SOLANKI, R. [1981]: "Optimal Portfolio Insurance," *Journal of Financial and Quantitative Analysis*, 16, 279–290.

DOHERTY, N.A., and JUNG, H.J. [1993]: "Adverse Selection When Loss Severities Differ: First-Best and Costly Equilibria," *Geneva Papers on Risk and Insurance Theory*, 18, 173–182.

DOHERTY, N.A., and SCHLESINGER, H. [1995]: "Severity Risk and the Adverse Selection of Frequency Risk," *Journal of Risk and Insurance*, 62, 649–665.

FLUET, C., and PANNEQUIN, F. [1997]: "Insurance Contracts Under Adverse Selection with Random Loss Severity," Working Paper, University of Quebec at Montreal.

HUBERMAN, G., MAYERS, D., and SMITH, C.W. [1983]: "Optimal Insurance Policy Indemnity Schedules," *Bell Journal of Economics*, 14, 415–426.

LANDSBERGER, M., and MEILIJSON, I. [1994]: "Monopoly Insurance Under Adverse Selection When Agents Differ in Risk Aversion," *Journal of Economic Theory*, 63, 392–407.

LANDSBERGER, M., and MEILIJSON, I. [1996]: "Extraction of Surplus Under Adverse Selection: The Case of Insurance Markets," *Journal of Economic Theory*, 69, 234–239.

LEHMANN, E.L. [1991]: *Theory of Point Estimation*. Belmont, California: Wadsworth.

MAYERS, D., and SMITH, C.W. [1981]: "Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets," *Journal of Business*, 14, 407–434.

MOSSIN, J. [1968]: "Aspects of Rational Insurance Purchasing," *Journal of Political Economy*, 79, 553–568.

ORMISTON, M.B., and SCHLEE, E.E. [1993]: "Comparative Statics Under Uncertainty for a Class of Economic Agents," *Journal of Economic Theory*, 61, 412–422.

RAVIV, A. [1979]: "The Design of an Optimal Insurance Policy," *American Economic Review*, 69, 84–96.

ROTHSCHILD, M., and STIGLITZ, J. [1976]: "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629–649.

SCHLESINGER, H. [1981]: "The Optimal Level of Deductible in Insurance Contracts," *Journal of Risk and Insurance*, 48, 465–481.

SHAVELL, S. [1979]: "On Moral Hazard and Insurance," *Quarterly Journal of Economics*, 10, 541–562.

SPENCE, M. [1978]: "Product Differentiation and Performance in Insurance Markets," *Journal of Public Economics*, 10, 427–447.

TOWNSEND, R. [1979]: "Optimal Contracts and Competitive Contracts with Costly State Verification," *Journal of Economic Theory*, 22, 265–293.

TURNBULL, S.M. [1983]: "Additional Aspects of Rational Insurance Purchasing," *Journal of Business*, 56, 217–229.

WILSON, C. [1977]: "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, 97, 167–207.