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Erratum: Phase Distribution of Kerr Vectors in a Deformed Hilbert Space

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After the publication of my paper [1] I found a serious mistake in the proof of the completeness relation of phase vectors appearing on pp. 1811–1812. The correct version is as follows.

The completeness relation

$$I = \frac{1}{2\pi} \int_X \int_0^{2\pi} d\nu(x, \theta) |f_\theta\rangle\langle f_\theta| \tag{1}$$

where

$$d\nu(x, \theta) = d\mu(x)\, d\theta \tag{2}$$

may be proved as follows:

Here we consider the set X consisting of the points $x = 0,1,2, \ldots$, and $\mu(x)$ is the measure on X, which equals

$$\mu_n \equiv \frac{[n]!}{(q + [0])(q^2 + [1]) \ldots (q^n + [n - 1])}$$

at the point $x = n$, and θ is the Lebesgue measure on the circle.

Define the operator

$$|f_\theta\rangle\langle f_\theta|: \quad H_q \to H_q \tag{3}$$

by

$$|f_\theta\rangle\langle f_\theta| f = (f_\theta, f) f_\theta \tag{4}$$

with $f(z) = \Sigma_{n=0}^{\infty} a_n z^n$.

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Now,

$$(f_\theta, f) = \sum_{n=0}^{\infty} [n]! \, \frac{e^{-in\theta}}{\sqrt{[n]!}} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\ldots(q^n+[n-1])}{[n]!}} \, a_n$$

$$= \sum_{n=0}^{\infty} e^{-in\theta} \sqrt{(q+[0])(q^2+[1])(q^3+[2])\ldots(q^n+[n-1])} a_n \qquad (5)$$

Then,

$$(f_\theta, f)f_\theta = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n \, e^{i(m-n)\theta} \sqrt{\frac{(q+[0])(q^2+[1])\ldots(q^m+[m-1])}{[m]!}}$$
$$\times \sqrt{(q+[0])(q^2+[1])\ldots(q^n+[n-1])} f_m \qquad (6)$$

Using

$$\int_0^{2\pi} d\theta \; e^{i(m-n)\theta} = 2\pi\delta_{mn} \qquad (7)$$

we have

$$\frac{1}{2\pi} \int_X \int_0^{2\pi} d\nu(x, \theta) |f_\theta\rangle\langle f_\theta| f$$
$$= \int_X d\mu(x) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n f_m \sqrt{\frac{(q+[0])(q^2+[1])\ldots(q^m+[m-1])}{[m]!}}$$
$$\times \sqrt{(q+[0])(q^2+[1])(q^n+[n-1])} \, \frac{1}{2\pi} \int_0^{2\pi} e^{i(m-n)\theta} \, d\theta$$
$$= \sum_{n=0}^{\infty} a_n f_n \int_X \frac{(q+[0])(q^2+[1])\ldots(q^n+[n-1])}{\sqrt{[n]!}} \, d\mu\,(x)$$
$$= \sum_{n=0}^{\infty} a_n f_n \, \frac{(q+[0])(q^2+[1])\ldots(q^n+[n-1])}{\sqrt{[n]!}}$$
$$\times \frac{[n]!}{(q+[0])(q^2+[1])\ldots(q^n+[n-1])}$$
$$= \sum_{n=0}^{\infty} \sqrt{[n]!} \, a_n f_n = f \qquad (8)$$

Thus, (1) follows.

REFERENCES

1. P. K. Das (1999). Phase distribution of Kerr vectors in a deformed Hilbert space, *Int. J. Theor. Phys.* **38**, 1807–1815.
2. I. M. Gelfand and N. Ya. Vilenkin (1964). *Generalized functions*, Vol. 4, Academic Press.