Perturbation Theory for the Asymptotic Decay Rates in the Queues with Markovian Arrival Process and/or Markovian Service Process – Correction *

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There was a minor error at the beginning of section 6.0 in [1]. This has now been corrected together with all the key associated results.

The matrices in (6.1) can be expressed as

$$A_0 = D_1 \otimes I, \quad A_1 = C_1 \oplus C_2 \quad \text{and} \quad A_2 = I \otimes D_2, \quad (6.1)$$

where I is a identity matrix and

$$B_0 = C_1 \otimes I, \quad B_1 = A_2.$$

It is proved in [2] that

$$\chi(z) = z[\varphi_1(z^{-1}) + \varphi_2(z)] \quad (6.2)$$

and

$$\chi'(\sigma) < 0. \quad (6.3)$$

So σ is the unique root to the equation

$$\varphi_1(z^{-1}) + \varphi_2(z) = 0, \quad 0 < z < 1,$$

* The authors had initially incorporated this correction in their paper [1], but an earlier version of the paper was published by mistake.

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i.e.,

$$z = pf[(-\varphi_2(z)I - C_1)^{-1}D_1] = \Delta(z), \quad 0 < z < 1.$$

From (6.2) and (6.3),

$$\varphi'_2(\sigma) - \sigma^{-2}\varphi'_1(\sigma^{-1}) < 0.$$

By calculating we have

$$\Delta'(\sigma) = \frac{\sigma^2\varphi'_2(\sigma)}{\varphi'_1(\sigma^{-1})} < 1.$$

Theorem 6.1.

$$\frac{|\log \sigma - \log \tilde{\sigma}|}{-\log \sigma} \leq \frac{8m^3\epsilon\epsilon}{1 - \Delta'(\sigma)} + O(\epsilon^2). \quad (6.4)$$

Remark. Once more, the fact that the sensitivity of σ or $\log \sigma$ depends on the value of $\Delta'(\sigma)$ is not surprising at all if one looks at the graph of $z = \Delta(z)$, $0 < z < 1$. The closer $\Delta'(\sigma)$ is to 1, the more sensitive is σ or $\log \sigma$.

References

- [1] A.S. Alfa, J. Xue and Q. Ye, Perturbation theory for the asymptotic decay rates in the queues with Markovian arrival process and/or Markovian service process, *Queueing Systems* 36 (2000) 287–301.
- [2] J. Abate, L. Choudhury and W. Whitt, Asymptotics for steady-state tail probabilities in structured Markov queueing models, *Commun. Statist. Stochastic Models* 1 (1994) 99–143.