Correction to Construction of Panoramic Image Mosaics with Global and Local Alignment*

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In Section 3 of the original paper (Shum and Szeliski, 2000), we introduce the following optimization, which underlies the efficiency of our algorithm:

An important element of this framework, which we exploit, is to perform the motion estimation between the current new input image and a *warped* (resampled) version of the mosaic. This allows us to estimate only *incremental* deformations of images (or equivalently, *instantaneous* motion), which greatly simplifies the computation of the gradients and Hessians required in our gradient descent algorithm (e.g., compare the Hessians computed below with those presented in Szeliski, 1996). Thus, to register two images $I_0(\mathbf{x})$ and $I_1(\mathbf{x}')$, where $\mathbf{x}'$ is computed using some parametric motion model $\mathbf{m}$, i.e., $\mathbf{x}' = \mathbf{f}(\mathbf{x}; \mathbf{m})$, we first compute the warped image

$$\tilde{I}_1(\mathbf{x}) = I_1(\mathbf{f}(\mathbf{x}; \mathbf{m})) \quad (9)$$

(in our current implementation, we use bilinear pixel resampling). The task is then to find a deformation of $\tilde{I}_1(\mathbf{x})$ which brings it into closer registration with $I_0(\mathbf{x})$ and which can also be used to update the parameter $\mathbf{m}$.

In Section 3.1, we then make the specialization of $\mathbf{x}' \sim \mathbf{M}\mathbf{x}$ where $\mathbf{x} = (x, y, 1)$ and $\mathbf{x}' = (x', y', 1)$ are homogeneous or projective coordinates, and $\sim$ indicates equality up to scale.

To recover the parameters, we first align the resampled image $\tilde{I}_1$ with the other image I_0 , i.e., we warp the resampled image $\tilde{I}_1$ by $\mathbf{x}'' \sim (\mathbf{I} + \mathbf{D})\mathbf{x}$. Ignoring errors

introduced by the double resampling operation, this is equivalent to resampling the original image $\tilde{I}_1$ by

$$\mathbf{x}' \sim \mathbf{M}(\mathbf{I} + \mathbf{D})\mathbf{x}.$$

The original update Eq. (13) should therefore be written as

$$\mathbf{M} \leftarrow \mathbf{M}(\mathbf{I} + \mathbf{D}) \quad (13)$$

instead of $\mathbf{M} \leftarrow (\mathbf{I} + \mathbf{D})\mathbf{M}$. This correction should be made in Eq. (13), the second line following Eq. (14), and in Fig. 3.

Similarly, Eqs. (25) and (27) should now read

$$\mathbf{R}_{kl} \leftarrow \mathbf{R}_{kl}\hat{\mathbf{R}}(\Omega) \quad \text{or} \quad \mathbf{M} \leftarrow \mathbf{V}\mathbf{R}_{kl}\hat{\mathbf{R}}(\Omega)\mathbf{V}^{-1} \quad (25)$$

and

$$\mathbf{M}' \approx \mathbf{V}\mathbf{R}_k\mathbf{R}_l^{-1}[\mathbf{I} + \mathbf{X}(\Omega)]\mathbf{V}^{-1} = \mathbf{M}(\mathbf{I} + \mathbf{D}_\Omega), \quad (27)$$

and Eqs. (29) and (30) should now read

$$\mathbf{M} \leftarrow \mathbf{M}(\mathbf{I} + e_k\mathbf{D}_{110}) \quad (29)$$

and

$$\begin{aligned} \mathbf{M} &\leftarrow (\mathbf{I} + e\mathbf{D}_{110})\mathbf{V}\mathbf{R}_k\mathbf{R}_l^{-1}\mathbf{V}^{-1}(\mathbf{I} - e\mathbf{D}_{110}) \\ &\approx \mathbf{M}(\mathbf{I} + e\mathbf{D}_e). \end{aligned} \quad (30)$$

Finally Eq. (32) should read

$$\mathbf{M} \leftarrow \mathbf{M}(p + \delta p) \approx \mathbf{M}\left(\mathbf{I} + \mathbf{M}^{-1}\frac{\partial \mathbf{M}}{\partial p}\delta p\right) \quad (32)$$

and the following line should use $(\partial \mathbf{M} / \partial p)\mathbf{M}^{-1}$.

In practice, using the old (erroneous) formulae may not make that much difference. In particular, the

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operations being interchanged (e.g., $\mathbf{M}$ and $(\mathbf{I} + \mathbf{D})$) are sometimes truly commutative, e.g., for translations or rotations around the same axis. However, for faster (and assured) convergence, the correct formulae should be used.

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References

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