Erratum

On the derivation of the Buckley–Leverett model from the two fluid Navier–Stokes equations in a thin domain

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(a) The corrected page 69 should read:

Now plug the formula (2.18) into (2.14³). Then after averaging we obtain an equation for $a(x_2, t)$. Note that by definition of η^0 , $\int_0^a \eta^0(z, x_2, t) dz = \eta_1 a(x_2, t)$. Then we have

$$\begin{aligned} \int_0^{h(x_2)} \frac{\partial \eta^0}{\partial t} dz &= \int_0^{h(x_2)} (\eta_1 - \eta_2) \frac{\partial a}{\partial t} \delta(z - a(x_2, t)) dz = (\eta_1 - \eta_2) \frac{\partial a}{\partial t}, \\ \int_0^{h(x_2)} \frac{\partial}{\partial x_2} (\eta^0 v_2^0) dz &= \int_0^{h(x_2)} \left((\eta_1 - \eta_2) \frac{\partial a}{\partial x_2} v_2^0(z, x_2, t) \delta(z - a(x_2, t)) + \eta^0 \frac{\partial v_2^0}{\partial x_2} \right) dz \\ &= \frac{\partial}{\partial x_2} \int_0^{h(x_2)} \eta^0 v_2^0 dz - h'(x_2) \eta_2 \underbrace{v_2^0(h, x_2, t)}_0 \\ &= \frac{\partial}{\partial x_2} \left\{ \eta_1 \int_0^a v_2^0 dz + \eta_2 \int_a^{h(x_2)} v_2^0 dz \right\}, \\ \int_0^{h(x_2)} \frac{\partial}{\partial z} (\eta^0 v_1^1) dz &= \int_0^{h(x_2)} \left((\eta_1 - \eta_2) \frac{\partial a}{\partial x_2} v_1^1(z, x_2, t) \delta(z - a(x_2, t)) + \eta^0 \frac{\partial v_1^1}{\partial x_2} \right) dz \\ &= 0. \end{aligned}$$

We note that equation (2.14¹) implies continuity of v_1^1 and at the interface $z = a(x_2, t)$ we have

$$\begin{aligned} v_1^1(a, x_2, t) &= - \int_0^{a(x_2, t)} \frac{\partial v_2^0}{\partial x_2}(z, x_2, t) dz \\ &= - \frac{\partial}{\partial x_2} \int_0^a v_2^0 dz + \frac{\partial a}{\partial x_2} v_2^0(a, x_2, t). \end{aligned} \quad (2.20)$$

Consequently,

$$(\eta_1 - \eta_2) \frac{\partial a}{\partial t} + \frac{\partial}{\partial x_2} \left\{ \eta_1 \int_0^a v_2^0 dz + \eta_2 \int_a^{h(x_2)} v_2^0 dz \right\} = 0. \quad (2.21)$$

The mass conservation and (2.21) give the following initial boundary value problem for $a(x_2, t)$:

$$\begin{cases} \frac{\partial a}{\partial t} + \frac{\partial}{\partial x_2} \left(\int_0^{a(x_2, t)} v_2^0(\xi, x_2, t) d\xi \right) = 0 & \text{in }]0, L[\times]0, T[, \\ a(x_2, 0) = a_0(x_2) & \text{in }]0, L[, \\ a(0, t) = a_1(t) & \text{on }]0, T[. \end{cases} \quad (2.22)$$

Obviously $s(x_2, t) = a(x_2, t)/h(x_2)$ represents the saturation of the first fluid. The natural question is whether the equation (2.22¹) relates to the well-known Buckley–Leverett equation.

Firstly, we are able to establish the solvability of (2.22) only if $v_2^0(a, x_2, t) \geq 0$. Formula (2.19) gives $v_2^0(a, x_2, t)$ and we see that $v_2^0(a, x_2, t) > 0$ for $a \in]0, h(x_2)[$ and $v_2^0(0, x_2, t) = v_2^0(h(x_2), x_2, t) = 0$. Using the classical theory (see, e.g., Bourgeat and Mikelić [21] for the particular case of the Buckley–Leverett system), we have a

(b) Analogously, equations (2.31) and (2.32) on page 72 should read:

$$\frac{\partial a}{\partial t} + \frac{\partial}{\partial x_2} \left(\int_0^{a(x_2, t)} v_2^0(\xi, x_2, t) d\xi \right) = 0 \quad (2.31)$$

and

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial x_2} \left(\int_0^{b(x_2, t)} v_2^0(\xi, x_2, t) d\xi \right) = 0. \quad (2.32)$$

(c) The text starting with equation (3.16), page 75, and ending with equation (3.18), should read:

After some calculations, we obtain the following system for $a(z, t)$:

$$\begin{cases} \frac{\partial a^2}{\partial t} + \frac{\partial}{\partial z} \left(2 \int_0^{a(z, t)} \xi v_3^0(\xi, x_2, t) d\xi \right) = 0 & \text{in }]0, L[\times]0, T[, \\ a(z, t = 0) = a_0(z) & \text{in }]0, L[, \\ a(z = 0, t) = a_1(t) & \text{in }]0, T[, \end{cases} \quad (3.16)$$

where $v_3^0(r, z, t)$ is the axial component of the leading order term in the asymptotic expansion of the velocity at the interface. It is given by

$$v_3^0(r, z, t) = 4Q(t) \frac{\eta_1 R^2(z) - \eta_2 r^2 + (\eta_2 - \eta_1) a^2(z, t)}{\eta_1 R^4(z) + (\eta_2 - \eta_1) a^4(z, t)} + g\varphi(r, a^2(z, t)), \quad (3.17)$$

with $Q(t)$, the inflow flux, defined by

$$Q(t) = \int_0^{R(0)} u_{00}(r, t) r dr = \int_0^{R(L)} u_{0L}(r, t) r dr,$$

and

$$\begin{aligned}
2\eta_1\eta_2\varphi(r, a^2(z, t)) = & -\frac{\eta_1 R^2(z) - \eta_2 r^2 + (\eta_2 - \eta_1)a^2(z, t)}{\eta_1 R^4(z) + (\eta_2 - \eta_1)a^4(z, t)} \\
& \times \left\{ \eta_1 \varrho_2 R^4(z) + a^4(z, t)(\varrho_1 \eta_2 - 2\eta_1(\varrho_1 - \varrho_2) - \varrho_2 \eta_1) \right. \\
& \quad \left. + 2\eta_1(\varrho_1 - \varrho_2)R^2(z)a^2(z, t) \right\} \\
& + \left\{ (\varrho_1 - \varrho_2)\eta_1 a^2 \ln \left(\frac{R^2(z)}{a^2(z, t)} \right) + \varrho_2 \eta_1 R^2(z) \right. \\
& \quad \left. + a^2(z, t)(\varrho_1 \eta_2 - \varrho_2 \eta_1) - \varrho_1 \eta_2 r^2 \right\}. \tag{3.18}
\end{aligned}$$