Eventualities in Time

The localization of eventualities in the Platonist and the Reductionist picture of time

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Abstract. This paper addresses the general question of how eventualities should be localized in time. The answer given here is mainly inspired by the model theory of Discourse Representation Theory (DRT) as presented in Kamp and Reyle (1993). We show that there is a mathematical problem concerning the localization function used in that book. And we propose a remedy for this problem. The general picture that emerges should be interesting not only for DRT but for other theories using an eventuality-based semantics as well.

Key words: DRT, eventualities, formal semantics, localization function, model theory, Platonist picture of time, Reductionist picture of time, temporal semantics

1. Introduction

In an eventuality-based semantics sentences are seen in the first place as descriptions of *eventualities*. For example Shem kicked Shaun is read as the statement that an eventuality *ev* such that *kick(ev, Shem, Shaun)* held was localized in the past of the utterance time. The eventuality-based view goes back to Davidson (1966).

Our goal is to investigate the localizations of eventualities in time. The general strategy we use is the one chosen in Kamp and Reyle (1993), namely the following. We start from a structured set of eventualities so that we can talk about an eventuality preceding some eventuality or about it overlapping some eventuality. This structure we call a *pre-eventuality-structure*; it is nothing else than the *event structure* defined by Kamp and Reyle. Each eventuality is then localized at a certain *interval*; that is, at a *convex* set of instants. The localization function must at least be a homomorphism, as overlapping eventualities must be localized at overlapping intervals and preceding eventualities at preceding intervals.

Following Kamp and Reyle, we look at *instants* in two ways. First, instants are construed as maximal sets of pairwise overlapping eventualities. This position reduces instants in a certain sense to eventualities, so it is called the *Reductionist position* or the *Reductionist picture* in this paper. Second, instants are seen as existing independently of eventualities, and eventualities are localized in the interval structure constructed out of these instants. This position is called the *Platonist position* or the *Platonist picture* in this paper.¹

When looking at the Platonist picture, we consider first functions we call *Platonist pre-localization-functions*. These are homomorphisms with respect to precedence and overlap, and moreover the intersection of the localizations of all eventualities that form a maximal set of pairwise overlapping eventualities (or in other words: that form an instant in the Reductionist picture) is an interval.

We then identify two intervals if and only if they are the localizations of the same eventualities. If that procedure leads to a *homomorphic contraction* of the Platonist picture, it is natural to inquire whether we can embed the instant structure of the Reductionist picture into that homomorphic contraction.

We call such Platonist pre-localization-functions for which this homomorphic contraction is well defined *Platonist localization functions*. And we show that not every Platonist pre-localization-function is a Platonist localization function as well. Moreover we show that for Platonist localization functions there can be always defined an embedding in a natural way.

The idea of considering such an embedding goes back to Kamp and Reyle (1993). We show, however, that for the localization functions used in Kamp and Reyle (1993), which we call *Kampian localization functions*, and which are a special case of Platonist pre-localization-functions, the homomorphic contraction is not guaranteed to exist.

This text is based on a report of the Institute for Computational Linguistics of the University of the Saarland, see Dünge (1998a).²

2. Basic Structures

In this section, the basic structures needed for the localization of eventualities in time are introduced, namely *instant structures*, *interval structures* and *pre-eventuality-structures*.

2.1. INSTANT STRUCTURES

DEFINITION 1. Let T be a set, $T \neq \emptyset$ and let $<$ be a binary relation on T . Consider the following axioms:

$$t_1 < t_2 \rightarrow \neg t_2 < t_1 \quad (T1)$$

$$(t_1 < t_2 \wedge t_2 < t_3) \rightarrow t_1 < t_3 \quad (T2)$$

$$t_1 \neq t_2 \rightarrow (t_1 < t_2 \vee t_2 < t_1) \quad (T3)$$

(T1) is called the axiom of **asymmetry**, (T2) is called the axiom of **transitivity**. (T3) is called the axiom of **linearity**.

If $<$ fulfils (T1) together with (T2), it is called a **strict partial order** or a **precedence relation**.

If $<$ fulfils (T1) - (T3), it is called a **linear order**. In that case an element of T is called an **instant** or a **point of time** and $\mathcal{T} = (T, <)$ is called an **instant**

structure. The binary relation which is the union of the equality relation $=$ and the linear order $<$ is denoted by $\leq$.

A note on terminology: there is more than one definition of the term *instant structure* around. Definition 1 is the one used in Kamp and Reyle (1993). However there are authors who *require* of an instant structure just the fulfilment of (T1) and (T2) or these axioms together with other axioms. For example, van Benthem (1991) does not require (T3). For many applications in formal semantics of natural language, however, linearity seems to be a natural choice.

Sometimes it is desirable to investigate equivalence classes of instants rather than instants themselves. For instance, given a function that localizes eventualities at intervals of time, one might want to identify such points of time that are elements of the localizations of the same eventualities. In general, given an instant structure $\mathcal{T} = (\mathbb{T}, <)$ and the set $A = \{[t] \mid t \in \mathbb{T}\}$ of equivalence classes of instants relative to some given equivalence relation one wants $\mathcal{T}$ to induce a linear order $<'$ on A in the following way:

$$[t] <' [t'] \text{ iff } [t] \neq [t'] \text{ and } t < t' \quad (1)$$

Note, however, that the relation $<'$ on A is well defined if and only if $<$ is a *congruence relation* with respect to A . And $<$ is said to be a **congruence relation** with respect to A if and only if the following two conditions are fulfilled:

$$([t] \neq [t'], t < t', \tilde{t} \in [t]) \rightarrow \tilde{t} < t' \quad (\text{Contr1})$$

$$([t] \neq [t'], t < t', \tilde{t}' \in [t']) \rightarrow t < \tilde{t}' \quad (\text{Contr2})$$

If these conditions are fulfilled, then the function $t \mapsto [t]$ is a surjective homomorphism from $\mathbb{T}$ to A and the instant structure $\mathcal{A} = (A, <')$ is called the **homomorphic contraction** of $\mathcal{T}$ with respect to A . We shall write $\mathcal{A} = (A, <')$ instead of $\mathcal{A} = (A, <')$.

2.2. INTERVAL STRUCTURES

Given an instant structure $\mathcal{T}$, there is a natural way to define an interval structure related to $\mathcal{T}$:

DEFINITION 2. Let $\mathcal{T} = (\mathbb{T}, <)$ be an instant structure. Then an **interval** of $\mathcal{T}$ is a **convex** set of instants; that is a set $X \subseteq \mathbb{T}$, $X \neq \emptyset$, such that if $t_1, t_2 \in X$, $t_3 \in \mathbb{T}$ and $t_1 < t_3 < t_2$, then $t_3 \in X$. The set of intervals of $\mathcal{T}$ is denoted by $\text{Int}(\mathcal{T})$.

Let X_1 and X_2 be intervals of $\mathcal{T}$, then the relation of **precedence**, $<$, the relation of **overlap**, $\circ$, and the relation of **inclusion**, $\subseteq$, are defined as follows:

$$X_1 < X_2 \text{ iff for all } t_1 \in X_1 \text{ and } t_2 \in X_2 : t_1 < t_2 \quad (\text{I1})$$

$$X_1 \circ X_2 \text{ iff } X_1 \cap X_2 \neq \emptyset \quad (\text{I2})$$

$$X_1 \subseteq X_2 \text{ iff for every } t_1 \in X_1 : t_1 \in X_2 \quad (\text{I3})$$

If $X_1 \subseteq X_2$ then we say that X_1 is a **subinterval** of X_2 . The structure $\mathcal{INT}(\mathcal{T}) = (\text{Int}(\mathcal{T}), <, \bigcirc, \subseteq)$ is called the **interval structure** of $\mathcal{T}$.

Let us now mention just a few axioms fulfilled by an interval structure. We have the axiom of **Convexity**:

$$X_1 < X_2 < X_3 \rightarrow ((X_1 \subseteq X_4 \wedge X_3 \subseteq X_4) \rightarrow X_2 \subseteq X_4) \quad (\text{CONVEX})$$

The relation of inclusion is *transitive*, *reflexive* and fulfils the axiom of **Antisymmetry**:

$$(X_1 \subseteq X_2 \wedge X_2 \subseteq X_1) \rightarrow X_1 = X_2 \quad (\text{ANTIS})$$

We will come back to these axioms later on.

2.3. PRE-EVENTUALITY-STRUCTURES

In order to talk about *eventualities* we need concepts of temporal *precedence*, *overlap* and *inclusion*. One way to think about these concepts is illustrated here using *pre-eventuality-structures*.³

DEFINITION 3. A **pre-eventuality-structure** is a quadruple $\mathcal{EV} = (\text{EV}, <, \bigcirc, \subseteq)$ such that EV is a nonempty set (whose elements will be denoted by ev or ev_i where $i \in \mathbb{N}$), and $\bigcirc$, $<$ and $\subseteq$ are binary relations on EV such that for all $\text{ev}_i \in \text{EV}$:

$$\text{ev}_1 < \text{ev}_2 \rightarrow \neg \text{ev}_2 < \text{ev}_1 \quad (\text{P1})$$

$$(\text{ev}_1 < \text{ev}_2 \wedge \text{ev}_2 < \text{ev}_3) \rightarrow \text{ev}_1 < \text{ev}_3 \quad (\text{P2})$$

$$\text{ev}_1 \bigcirc \text{ev}_1 \quad (\text{P3})$$

$$\text{ev}_1 \bigcirc \text{ev}_2 \rightarrow \text{ev}_2 \bigcirc \text{ev}_1 \quad (\text{P4})$$

$$\text{ev}_1 < \text{ev}_2 \rightarrow \neg \text{ev}_2 \bigcirc \text{ev}_1 \quad (\text{P5})$$

$$(\text{ev}_1 < \text{ev}_2 \wedge \text{ev}_2 \bigcirc \text{ev}_3 \wedge \text{ev}_3 < \text{ev}_4) \rightarrow \text{ev}_1 < \text{ev}_4 \quad (\text{P6})$$

$$\text{ev}_1 < \text{ev}_2 \vee \text{ev}_1 \bigcirc \text{ev}_2 \vee \text{ev}_2 < \text{ev}_1 \quad (\text{P7})$$

EV is called the set of **eventualities**. $<$ is called the relation of **precedence**, $\bigcirc$ is called the relation of **overlap**, $\subseteq$ is called the relation of **inclusion** and defined as follows:

$$\text{ev}_1 \subseteq \text{ev}_2 \text{ iff } (\text{ev} < \text{ev}_2 \rightarrow \text{ev} < \text{ev}_1) \wedge (\text{ev}_2 < \text{ev} \rightarrow \text{ev}_1 < \text{ev})$$

The definition of a pre-eventuality-structure is essentially identical to the definition of an **event structure** as given in Kamp and Reyle (1993, p. 667), the only difference being that in Kamp and Reyle (1993) the inclusion relation is not seen *explicitly* as a part of the definition of an event structure.

EXAMPLE 1. Let $\mathcal{T}$ be an instant structure. Then the interval structure of $\mathcal{T}$, $\mathcal{INT}(\mathcal{T})$, is a pre-eventuality-structure.

LEMMA 1. Let $\mathcal{EV} = (\mathbf{EV}, <, \bigcirc, \subseteq)$ be a pre-eventuality-structure. Then $<$ and $\subseteq$ between them fulfil CONVEX.

In a pre-eventuality-structure the axiom of Antisymmetry does not have to hold; see the following example.

EXAMPLE 2. Consider a set $\mathbf{EV}$ with two elements, let $\mathbf{EV} = \{\text{ev}_1, \text{ev}_2\}$. Let $< = \emptyset$ and $\bigcirc = \mathbf{EV} \times \mathbf{EV}$.

Then $\mathcal{EV} = (\mathbf{EV}, <, \bigcirc, \subseteq)$ is a pre-eventuality-structure, but $\subseteq$ is not antisymmetric as $\text{ev}_1 \subseteq \text{ev}_2$ and $\text{ev}_2 \subseteq \text{ev}_1$ without ev_1 and ev_2 being identical.

So $\subseteq$ need not be *anti-symmetric*. That is as it should be, however: we are only interested in *temporal inclusion* here, not in the question of *identity* of eventualities. Furthermore, this example shows that there are other pre-eventuality-structures than just the instant structures. More examples and a discussion of several first-order properties of a pre-eventuality-structure are to be found in Dünges (1998a).

Note in passing that pre-eventuality-structures with a vacuous ordering relation and a universal overlap relation have the virtue of being relatively simple. That is why we sometimes use structures of this type as counterexamples for certain claims. In many cases we could have used structures without a vacuous ordering relation or without a universal overlap relation for these purposes as well, however.

3. The Localization of Eventualities in Time

The localization of eventualities in time can be done in two essentially different ways. In the *Reductionist* picture of time, instants are seen as maximal sets of pairwise overlapping eventualities. Hence, it is natural to say that an eventuality is going on at an instant iff it is an element of this instant. In the *Platonist* picture of time, instants are seen as existing independently of eventualities. Here we have more choice about how to localize an eventuality in time than in the Reductionist picture.

3.1. THE REDUCTIONIST PICTURE

DEFINITION 4. Let $\mathcal{EV} = (\mathbf{EV}, <, \bigcirc, \subseteq)$ be a pre-eventuality-structure. Then i is called an **instant** of $\mathcal{EV}$, if:

1. $i \subseteq \mathbf{EV}$ and $i \neq \emptyset$
2. $\text{ev}_1, \text{ev}_2 \in i \rightarrow \text{ev}_1 \bigcirc \text{ev}_2$
3. if $H \subseteq \mathbf{EV}, i \subseteq H$ and for all $\text{ev}_1, \text{ev}_2 \in H, \text{ev}_1 \bigcirc \text{ev}_2$ then $H \subseteq i$.

The set of instants of $\mathcal{EV}$ is denoted by $I(\mathcal{EV})$. If i is an instant, we say that an eventuality ev **occurs at** i , iff $ev \in i$. The **precedence relation** $<$ on $I(\mathcal{EV})$ is defined as follows:

$$i_1 < i_2 \leftrightarrow \text{there are } ev_1 \in i_1, ev_2 \in i_2 \text{ such that } ev_1 < ev_2$$

The pair $\mathcal{I}(\mathcal{EV}) = (I(\mathcal{EV}), <)$ is called the **instant structure of $\mathcal{EV}$** . The interval structure $\mathcal{INT}(\mathcal{I}(\mathcal{EV}))$ of the instant structure of $\mathcal{EV}$ is also called the **interval structure of $\mathcal{EV}$** .

Incidentally, Kamp and Reyle seem to allow in Kamp and Reyle (1993, p. 668) that an instant be the empty set. Presumably this is an oversight.

The instant structure of a pre-eventuality-structure, $(I(\mathcal{EV}), <)$, is an instant structure in the sense of definition 1, as the precedence relation on $I(\mathcal{EV})$ is asymmetric, transitive and linear.

The natural way to define a localization function in the Reductionist picture is the following.

DEFINITION 5. Let $\mathcal{EV}$ be a pre-eventuality-structure. The **Reductionist localization function** of $\mathcal{EV}$ is the function $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(\mathcal{I}(\mathcal{EV}))$, $ev \mapsto \{i \in I(\mathcal{EV}) \mid ev \in i\}$.

LEMMA 2. The Reductionist localization function is a homomorphism with respect to precedence and overlap.

At first sight, the Reductionist picture seems to be very attractive, because of the intimate connection between pre-eventuality-structures and instant structures it provides. But that feature is not unproblematic, as it can be difficult to state conditions on a pre-eventuality-structure that guarantee that its instant structure is in accordance with the intuitions one has about the structure of time. So, if the instant structure $\mathcal{T} = (T, <)$ is not allowed to be just any old linear order, it is a natural move to choose $\mathcal{T}$ freely and to allow $\mathcal{T}$ and $\mathcal{EV}$ to be ‘*irreducible primitives*’, see Kamp and Reyle (1993, p. 671). The localization of $\mathcal{EV}$ in $\mathcal{T}$ is then done in the *Platonist picture*.

From this, one could get the impression that the Reductionist picture is preferable, and that the usage of the Platonist picture is mainly motivated by the wish to avoid technical difficulties. But the following argument shows that the Platonist picture is in no sense a second best choice for the formalization of natural language.

In the Reductionist picture instants are constructed out of eventualities. In contrast, the Platonist picture allows us to see instants as existing independently of eventualities. Consequently, the Reductionist picture is ontologically more sparse than the Platonist picture. In view of Occam’s razor, you might see this as an advantage the Reductionist picture has over the Platonist picture, for a *philosophical* description of eventualities in time. But for *formal semantics of natural language*,

arguably Occam's razor does not cut here. To be more precise, the Platonist picture seems to be more appropriate for formal semantics just because of its greater ontological richness. The reason is, that there are phrases in natural language which seem to refer *directly* to temporal items, rather than to maximal sets of pairwise overlapping eventualities. Take for example at 10, which seems to refer to an instant or to an interval having just one instant as its member.

3.2. THE PLATONIST PICTURE

Here, the question is how to localize eventualities in an interval structure constructed from instants of a previously given instant structure. First we introduce *Platonist pre-localization-functions*. Then the wish to embed the *Reductionist* instant structure into a certain homomorphic contraction of the *Platonist* instant structure leads us to *Platonist localization functions*. Finally *Kampian localization functions* are discussed. These are a special case of Platonist pre-localization-functions and they represent the view of Kamp and Reyle as given in Kamp and Reyle (1993).

The concept of Platonist localization functions is heavily inspired by the work of Kamp and Reyle as presented in Kamp and Reyle (1993). But not every Platonist localization function is a Kampian localization function and not every Kampian localization function is a Platonist localization function. We will argue for the preferability of Platonist localization functions.

3.2.1. *Platonist pre-localization-functions*

First two requirements are discussed which any localization function must obviously fulfil. This leads us to the concept of *Platonist pre-localization-functions*.

Then we identify two instants of the Platonist instant structure if and only if they are elements of the localizations of the same eventualities. Intuitively, it would be nice to construct the homomorphic contraction of the Platonist instant structure with respect to the equivalence class resulting from this identification and to embed the *Reductionist* instant structure into this homomorphic contraction. Why should we be interested in this homomorphic contraction? Kamp and Reyle argue in Kamp and Reyle (1993, p.671) that this structure retains something of the spirit of the *Reductionist* picture. Perhaps a more down-to-earth motivation for it is the following: in this way, two instants are identified iff they can not be distinguished from each other by virtue of the eventualities going on at either of them. So this view has an operational flavour.

We will show, however, that the existence of this homomorphic contraction is not guaranteed for every Platonist pre-localization-function. So these functions do not capture all our intuitions about suitable localization functions, even though they are a first step in the right direction.

So, how should eventualities be localized in time? As we are committed to CONVEX in our pre-eventuality-structures, eventualities should be localized at *convex* sets of instants, that is: at intervals. Furthermore it is natural to posit that

the localization should be done via a homomorphism. Moreover, it is desirable that the intersection I of the localizations of all eventualities that, between them, yield one *instant* in the *Reductionist* picture is an interval, too. Note that intervals are nothing else than convex sets of instants. So the arbitrary intersection of intervals is an interval, or it is the empty set. Thus we only have to guarantee, that $I \neq \emptyset$. It turns out that this requirement is not fulfilled by every homomorphism, see the following example.

EXAMPLE 3. *Let us construct a pre-eventuality-structure $\mathcal{EV} = (\text{EV}, \circ, <, \subseteq)$ in the following way. Let $\text{EV} = \{\text{ev}_i \mid i \in \mathbb{N}\}$, let $< = \emptyset$, $\circ = \text{EV} \times \text{EV}$.*

*Consider the instant structure $\mathcal{T} = (\mathbb{N}, <)$, where $\mathbb{N}$ denotes the natural numbers and $<$ their usual order. Clearly, the function $h : \mathcal{EV} \rightarrow \text{INT}(\mathcal{T})$ defined as $h(\text{ev}_i) = \mathbb{N} \setminus \{0, 1, 2, \dots, i\}$ is a homomorphism with respect to precedence and overlap. But h is not a suitable localization function. Why is this so? As all eventualities overlap in our structure, we have only one instant in the *Reductionist* picture, namely EV . So let us look at $I := \bigcap \{h(\text{ev}_i) \mid \text{ev}_i \in \text{EV}\}$. For the sake of a contradiction, let us assume that there is an $i \in I$. But $i \notin h(\text{ev}_i)$. Consequently $i \notin I$. So $I = \emptyset$.*

So we arrive at the following definition:

DEFINITION 6. *Let $\mathcal{EV}$ be a pre-eventuality-structure and $\text{INT}(\mathcal{T})$ an interval structure. A **Platonist pre-localization-function** of $\mathcal{EV}$ with respect to $\text{INT}(\mathcal{T})$ is a function $\text{LOC} : \mathcal{EV} \rightarrow \text{INT}(\mathcal{T})$ such that:*

1. $\text{ev}_1 < \text{ev}_2 \rightarrow \text{LOC}(\text{ev}_1) < \text{LOC}(\text{ev}_2)$
2. $\text{ev}_1 \circ \text{ev}_2 \rightarrow \text{LOC}(\text{ev}_1) \circ \text{LOC}(\text{ev}_2)$
3. *for every $i \in \mathcal{I}(\mathcal{EV}) : \bigcap \{\text{LOC}(\text{ev}) \mid \text{ev} \in i\} \neq \emptyset$*

LEMMA 3. *Every Reductionist localization function is a Platonist pre-localization-function.*

Proof. Follows directly from the definitions. □

That not every Platonist pre-localization-function is a Reductionist localization function can be seen when looking at *empty instants*. By an **empty instant** we understand an instant $t \in \mathcal{T}$ such that there is no eventuality $\text{ev} \in \text{EV}$ such that $t \in \text{LOC}(\text{ev})$. Let us call the set of empty instants a Platonist pre-localization-function LOC provides $\text{EI}(\text{LOC})$. Trivially, $\text{EI}(\text{LOC}) = \emptyset$ for every Reductionist localization function. But there are Platonist pre-localization-functions which have $\text{EI}(\text{LOC}) \neq \emptyset$.

Now let us identify two instants of the Platonist picture if and only if they are elements of the localizations of the same eventualities:

DEFINITION 7. *Let $\mathcal{T} = (\mathcal{T}, <)$ be an instant structure and $\text{LOC} : \mathcal{EV} \rightarrow \text{INT}(\mathcal{T})$ a Platonist pre-localization-function.*

We define T' to be the set of all equivalence classes under the relation $\equiv$ which is defined as follows:

$$t \equiv t' \text{ iff } \forall \text{ev} (t \in \text{LOC}(\text{ev}) \leftrightarrow t' \in \text{LOC}(\text{ev}))$$

Intuitively, it is desirable to define the homomorphic contraction of the Platonist instant structure $\mathcal{T} = (T, <)$ with respect to T' and, moreover, to embed the Reductionist instant structure $\mathcal{I}(\mathcal{EV})$ into this homomorphic contraction of $\mathcal{T}$. A natural candidate for such an embedding is the function emb introduced in the following:

LEMMA 4. *Let $\mathcal{T} = (T, <)$ be an instant structure and $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(\mathcal{T})$ a Platonist pre-localization-function.*

Then LOC induces the function $\text{emb} : \mathcal{I}(\mathcal{EV}) \rightarrow T'$ defined as $\text{emb}(i) = \bigcap \{\text{LOC}(\text{ev}) \mid \text{ev} \in i\}$.

Proof. We have to show, that $\text{emb}(i) \in T'$. That means that we have to show that the following two conditions are satisfied:

1. $(t \in \text{emb}(i) \wedge t \equiv t') \rightarrow t' \in \text{emb}(i)$
2. $(t \in \text{emb}(i) \wedge t' \in \text{emb}(i)) \rightarrow t \equiv t'$

Trivially, (1) is fulfilled, as two instants are equivalent if and only if they are elements of the localizations of the same eventualities.

For (2), suppose, that $t \in \text{emb}(i)$ and $t' \in \text{emb}(i)$. Note, that if $\text{ev} \in i$, then $t \in \text{LOC}(\text{ev})$ if and only if $t' \in \text{LOC}(\text{ev})$. But if $t \in \text{emb}(i)$ and $t \in \text{LOC}(\tilde{\text{ev}})$, then $\tilde{\text{ev}} \in i$ as we will show now. Suppose, that $t \in \text{LOC}(\tilde{\text{ev}}) \cap \bigcap \{\text{LOC}(\text{ev}) \mid \text{ev} \in i\}$. Then we have $\text{LOC}(\tilde{\text{ev}}) \cap \text{LOC}(\text{ev})$ for all $\text{ev} \in i$. But that means that $\tilde{\text{ev}} \cap \text{ev}$ for all $\text{ev} \in i$. Thus, $\tilde{\text{ev}} \in i$, as i is an instant in the Reductionist picture. $\square$

So far, everything looks fine. We are not through, however, as two questions are still open. First, we have to ask whether the homomorphic contraction $\mathcal{T}' = (T', <)$ of $\mathcal{T} = (T, <)$ really exists. And, second, given $\mathcal{T}'$, we have to see, whether $\text{emb} : \mathcal{I}(\mathcal{EV}) \rightarrow \mathcal{T}'$ is a homomorphism. Let us turn to the first of these questions now.

Remember that the homomorphic contraction of $\mathcal{T} = (T, <)$ with respect to T' exists just in case $<$ is a congruence relation with respect to T' . That means, that the conditions (Contr1) and (Contr2) have to be fulfilled. The following example shows that these conditions are not fulfilled for every Platonist pre-localization-function.

EXAMPLE 4. *Let $\text{EV} = \{\text{ev}\}$, $\bigcirc = \text{EV} \times \text{EV}$, $< = \emptyset$. Then $\mathcal{EV} = (\text{EV}, <, \bigcirc, \subseteq)$ is a pre-eventuality-structure. Let the instant structure $\mathcal{T} = (T, <)$ be given as $T = \{t, t', t''\}$ and $< = \{(t', t), (t, t''), (t', t'')\}$.*

Now $\text{LOC}(\text{ev}) = \{t\} = [t]$ is a Platonist pre-localization-function. But we have $[t] = \{t\}$ and $[t'] = \{t', t''\}$. So $<$ is not a congruence relation with respect to T' .

Consequently, we have to be more restrictive in our choice of localization functions. This leads us to *Platonist localization functions*.

3.2.2. Platonist localization functions

Here, we are looking for a necessary and sufficient condition on Platonist pre-localization-functions $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(T)$ which allows us to define the homomorphic contraction $\mathcal{T}'$ of $\mathcal{T} = (\mathcal{T}, <)$ with respect to $\mathcal{T}'$. Functions which allow for this definition will be called *Platonist localization functions*. It will turn out, that these functions induce an embedding of the instant structure of the Reductionist picture, $\mathcal{I}(\mathcal{EV})$, into $\mathcal{T}'$ via the function emb .

So what restrictions do we have to pose in order for $\mathcal{T}' = (\mathcal{T}', <)$ to be well defined? Let us first define a convenient shorthand.

DEFINITION 8. *Let X be a nonempty set and $\mathbf{R}$ a binary relation on X . Let $A \neq \emptyset$ and $B \neq \emptyset$ be two subsets of X such that $a\mathbf{R}b$ for all $a \in A, b \in B$. In that case, we write $\mathbf{A}\mathbf{R}\mathbf{B}$.*

The concept of a localization function we are most interested in is the following.

DEFINITION 9. *Let $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(T)$ be a Platonist pre-localization-function such that the following two postulates are fulfilled:*

- (i) $\text{EI}(\text{LOC}) \neq \emptyset \rightarrow (\text{EI}(\text{LOC}) < \text{LOC}(\text{EV}) \vee \text{LOC}(\text{EV}) < \text{EI}(\text{LOC}))$
- (ii) $\forall t_1, t_2 ((\exists \text{ev}_1 (t_1 \in \text{LOC}(\text{ev}_1)) \wedge \exists \text{ev}_2 (t_2 \in \text{LOC}(\text{ev}_2)) \wedge \exists \text{ev} (\{t_1\} < \text{LOC}(\text{ev}) < \{t_2\})) \rightarrow (\exists \text{ev}_1 (t_1 \in \text{LOC}(\text{ev}_1) \wedge \neg t_2 \in \text{LOC}(\text{ev}_1)) \vee \exists \text{ev}_2 (t_2 \in \text{LOC}(\text{ev}_2) \wedge \neg t_1 \in \text{LOC}(\text{ev}_2))))$

*Then LOC is called a **Platonist localization function** of $\mathcal{EV}$ with respect to $\mathcal{INT}(T)$.*

The interest in these functions is based on the following two results:

PROPOSITION 1. *Let $\mathcal{EV}$ be a pre-eventuality-structure and $\mathcal{T} = (\mathcal{T}, <)$ an instant structure. Let $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(T)$ be a Platonist pre-localization-function.*

Then the homomorphic contraction of $\mathcal{T}$ with respect to $\mathcal{T}'$ is well defined if and only if LOC is a Platonist localization function.

Proof. Let LOC be some Platonist pre-localization-function.

Remember that the homomorphic contraction of $\mathcal{T}$ with respect to $\mathcal{T}'$ is well defined if and only if the conditions (Contr1) and (Contr2) are fulfilled. But we do not need to look at both of these conditions, as they are essentially symmetric. So let us only investigate (Contr1), which is repeated here:

$$([t] \neq [t'], t < t', \tilde{t} \in [t]) \rightarrow \tilde{t} < t'$$

Let us assume the left hand side of (Contr1) in the following. Trivially, we only have to look at $\tilde{t} \neq t$, so let us assume this in the following, too.

Linearity of $<$ leads to $\tilde{t} < t$ or $t < \tilde{t}$. By transitivity of $<$, $\tilde{t} < t$ immediately gives $\tilde{t} < t'$ and we are done. So we assume in the following $t < \tilde{t}$.

There are exactly two cases which cause the equivalence classes $[t]$ and $[t']$ to be different:

1. there is an eventuality ev such that $t \in \text{LOC}(ev)$ and $t' \notin \text{LOC}(ev)$
2. there is an eventuality ev' such that $t \notin \text{LOC}(ev')$ and $t' \in \text{LOC}(ev')$

Let us look at (1) first.

Linearity of $<$ together with $t' \notin \text{LOC}(ev)$ leads to $\{t'\} < \text{LOC}(ev)$ or $\text{LOC}(ev) < \{t'\}$. As $t < t'$ by our first assumption, we have $\text{LOC}(ev) < \{t'\}$. This gives $\tilde{t} < t'$. So this case is unproblematic.

Now let us look at (2).

From our first assumption we have $t < t'$ and, as we are only looking at possibly problematic cases for the well-definedness of $<$ on $\mathcal{T}'$, we always assume that $t < \tilde{t}$. So we only have to investigate two cases.

The case $\{t\} < \{\tilde{t}\} < \text{LOC}(ev')$ gives the unproblematic result $\tilde{t} < t'$. The other case to consider is:

$$\{t\} < \text{LOC}(ev') < \{\tilde{t}\} \quad (*)$$

As this implies $t' < \tilde{t}$, this case yields a criterion for our problem. Moreover, the exclusion of a case like $(*)$ is not only necessary but also sufficient for the well definedness of the precedence relation on $\mathcal{T}'$. So all we have to do is to exclude that the localization of an eventuality has to its left and to its right an instant such that these instants are equivalent. Note that two instants can only be equivalent if *both* of them are empty instants or *neither* of them is an empty instant.

Let us look at empty instants first. Now, *any* two empty instants are equivalent. So we have to exclude that the localization of an eventuality has both on its left and on its right an empty instant. Empty instants, however, do not have to be excluded altogether. There are exactly two cases where no harm can be done by them: first, if all of them lie *before* the localization time of any eventuality and, second, if all of them lie *after* the localization time of any eventuality. This leads us to condition (i) of definition 9.

Now let us look at cases like $\{t_1\} < \text{LOC}(ev) < \{t_2\}$ where neither t_1 nor t_2 is an empty instant. We have to make sure, that t_1 and t_2 are not equivalent. Clearly, $[t_1] \neq [t_2]$ if and only if there is an eventuality ev_1 such that $t_1 \in \text{LOC}(ev_1)$ and $t_2 \notin \text{LOC}(ev_1)$ or there is an eventuality ev_2 such that $t_2 \in \text{LOC}(ev_2)$ and $t_1 \notin \text{LOC}(ev_2)$. This leads us to condition (ii) of definition 9.

So we have shown that the Platonist localization functions are exactly such Platonist pre-localization-functions $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(\mathcal{T})$ which allow for the definition of the homomorphic contraction $\mathcal{T}'$ of $\mathcal{T}$. $\square$

PROPOSITION 2. Let $\mathcal{EV}$ be a pre-eventuality-structure and $\mathcal{T} = (\mathcal{T}, <)$ an instant structure. Let $\text{LOC} : \mathcal{EV} \rightarrow \text{INT}(\mathcal{T})$ be a Platonist localization function. Let $\mathcal{T}'$ be the homomorphic contraction of $\mathcal{T}$ with respect to $\mathcal{T}'$.

Then $\text{emb} : \mathcal{I}(\mathcal{EV}) \rightarrow \mathcal{T}'$ is an embedding of $\mathcal{I}(\mathcal{EV})$ into $\mathcal{T}'$.

Proof. For a Platonist pre-localization-function we have already considered the function $\text{emb} : \mathcal{I}(\mathcal{EV}) \rightarrow \mathcal{T}'$ which is defined as $\text{emb}(i) = \bigcap \{\text{LOC}(\text{ev}) \mid \text{ev} \in i\}$. We now have to show that emb is a homomorphism if LOC is a Platonist localization function.

So let $i_1 < i_2$. Then $\text{ev}_1 < \text{ev}_2$ for all $\text{ev}_1 \in i_1, \text{ev}_2 \in i_2$. Consequently, $\text{LOC}(\text{ev}_1) < \text{LOC}(\text{ev}_2)$ for all $\text{ev}_1 \in i_1, \text{ev}_2 \in i_2$. Thus $\text{emb}(i_1) \neq \text{emb}(i_2)$. Furthermore, there is a $t_1 \in \text{emb}(i_1)$ and a $t_2 \in \text{emb}(i_2)$ such that $t_1 < t_2$. So we conclude that $\text{emb}(i_1) < \text{emb}(i_2)$. Thus emb is a homomorphism. $\square$

In example 4 we have already seen a Platonist pre-localization-function which is not a Platonist localization function because condition (i) in definition 9 is violated. For a Platonist pre-localization-function which violates condition (ii) in definition 9, consider the following example.

EXAMPLE 5. Let the set of eventualities be $\mathcal{EV} = \{\text{ev}, \text{ev}'\}$. Let the precedence relation be the empty set while the overlap relation is the reflexive symmetric closure of $\{(\text{ev}, \text{ev}')\}$.

Let the set of instants be $\mathcal{T} = \{t, t', \tilde{t}\}$ while precedence is defined as $\leq = \{(t, t'), (t', \tilde{t}), (t, \tilde{t})\}$. Then the following Platonist pre-localization-function is not a Platonist localization function: $\text{LOC}(\text{ev}) = \{t'\}$, $\text{LOC}(\text{ev}') = [t, \tilde{t}]$.

The following result is immediate.

LEMMA 5. Every Reductionist localization function is a Platonist localization function.

Finally, note that there is a price to be paid if you prefer Platonist localization functions to ‘pure’ Platonist pre-localization-functions: typically, you will need a ‘bigger’ pre-eventuality-structure if you insist in the former functions. The reason is that with Platonist localization functions you might need more eventualities than are talked about in a given discourse, whereas with Platonist pre-localization-functions this does not seem to be the case.

3.2.3. Kampian localization functions

For a function to count as *localization function* in the sense of Kamp and Reyle (1993, p.671) it has to be a Platonist pre-localization-function, but two further requirements are mentioned by Kamp and Reyle. We call Platonist pre-localization-functions that meet these requirements *Kampian localization functions*.

DEFINITION 10. Let $\mathcal{EV}$ be a pre-eventuality-structure and $\text{LOC} : \mathcal{EV} \rightarrow \text{INT}(\mathcal{T})$ a Platonist pre-localization-function such that the following conditions are satisfied:

1. $\mathcal{T}$ has the structural properties of $\mathbb{R}$.
2. The localization of an event(uality) is a closed interval which would be thought of as the smallest closed interval of $\mathcal{T}$ which temporally includes the event(uality).

Then LOC is called a **Kampian localization function**.

What do conditions 1 and 2 of the above definition mean? Let us first look at condition 1. Kamp and Reyle choose the instant structure of the reals for their model theory, because $\mathbb{R}$ is the standard time structure for physics. We would not like to follow them here, however, as we think that a time structure suitable for formal semantics of natural language does not have to be suitable for natural science as well.

Now let us look at condition 2 of definition 10. No justification is given in Kamp and Reyle (1993) for this condition. So what could it mean? It is not stated explicitly in Kamp and Reyle (1993) whether this condition is to be taken to pertain to *all* eventualities, or only to certain eventualities, namely to *events*.⁴ Even if we knew whether condition 2 referred to eventualities or just to events, we still had to guess at its mathematical content, as it is not evident to us what a ‘*smallest closed interval which temporally includes the event(uality)*’ should be. We discuss this problem in detail in Dünge (1998a).

But even though the concept of a Kampian localization function seems not to be clearly defined, there are functions that should be counted as Kampian localization functions in any event. Looking at one of these we can show the following result.

LEMMA 6. *Not every Kampian localization function is a Platonist localization function.*

Proof. Consider the following variant of example 4: Let the set of eventualities be $\mathcal{EV} = \{\text{ev}\}$. Let precedence and overlap be given as $\leq = \emptyset$ and $\circ = \mathcal{EV} \times \mathcal{EV}$, respectively. Let the instant structure be $\mathcal{T} = (\mathbb{R}, <)$ and $\text{LOC}(\text{ev}) = \{13\} = [13, 13]$.

Trivially, LOC is a Platonist pre-localization-function. And $[13, 13]$ simply has to be a smallest closed interval including anything... So LOC should really be a Kampian localization function. But LOC is not a Platonist localization function as condition (i) of definition 9 is violated. $\square$

4. Conclusion

In this paper we have looked at the localization of eventualities in the Reductionist and in the Platonist picture of time. We started from a Platonist pre-localization-function $\text{LOC} : \mathcal{EV} \rightarrow \mathcal{INT}(\mathcal{T})$ and considered the set of equivalence classes $\mathcal{T}'$ given by identifying instants that are members of the localizations of the same eventualities. We looked for the homomorphic contraction $\mathcal{T}'$ of $\mathcal{T}$ with respect to $\mathcal{T}'$.

We arrived at the following results. The instant structure of the Reductionist picture, $\mathcal{I}(\mathcal{EV})$, can be embedded into $\mathcal{T}'$, if $\mathcal{T}'$ exists. We found a criterion for the existence of $\mathcal{T}'$, namely, $\mathcal{T}'$ exists if and only if LOC is a Platonist localization function. Every Reductionist localization function is a Platonist localization function. Not every Platonist pre-localization-function and not every Kampian localization function is a Platonist localization function.

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Notes

¹ The terms *Platonist position* and *Reductionist position* are borrowed from Newton-Smith, see Newton-Smith (1980). The meaning of these words here, however, bears only a rough resemblance to their meaning in Newton-Smith (1980). By the *Reductionist* position and the *Platonist* position Newton-Smith means two positions with respect to the nature of time that usually are attributed to Leibniz and Isaac Newton, respectively. Both positions are discussed extensively in Newton-Smith (1980). An early discussion of these positions can be found in the correspondence which Leibniz had with Clarke about the physics of Newton, see Leibniz and Clarke (1989).

² The present text is a small part of the report. In the report, in addition several first-order properties of pre-eventuality-structures are discussed. Furthermore we discuss the theory of tense and aspect of DRT and the localizations of states and events there. A small part of the present text, without any proofs, has appeared in the Proceedings of the MFCS98 Workshop on Mathematical Linguistics, Dönges (1998b).

³ In the model theory of DRT, finer structures than *pre-eventuality-structures* are needed. For example, eventualities are distinguished into *states* and *events*. Furthermore, each eventuality has a beginning and an end. We reserve the name *eventuality structure* for such finer structures. We will not discuss them here. See Dönges (1998a).

⁴ This confusion is due to the fact that what we have introduced as a pre-eventuality-structure is termed in Kamp and Reyle (1993) first an '*event structure*' and only some pages later this mathematical structure is transformed into a so-called '*eventuality structure*', just by calling that what was formerly called the set of events now the set of eventualities and by introducing a partition of the set of eventualities into a set of states and a set of events. See Kamp and Reyle (1993, p.667) and Kamp and Reyle (1993, p.673).

References

- Benthem, J. van. *The Logic of Time – A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse*, Synthese Library, Kluwer Academic Publishers, Dordrecht/Boston/London: second edition, 1991 (first edition 1983).
- Davidson, D. The logical form of action sentences. In N. Rescher, editor, *The Logic of Decision and Action*, 81–95, University of Pittsburgh Press, Pittsburgh 1966.
- Dönges, P. (1998a). Eventualities in time. CLAUS report 104, Universität des Saarlandes, Computer-linguistik, Saarbrücken, <http://www.coli.uni-sb.de/>

- Düngen, P. (1998b). Eventualities in time. In M. Kudlek editor, *Proceedings of the MFCS98 Workshop on Mathematical Linguistics*, 51–60, Universität Hamburg, Fachbereich Informatik; This workshop was held at Brno on August 27 and 28, 1998.
- Kamp, H. and U. Reyle. *From Discourse to Logic*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1993.
- Leibniz, G.W. and S. Clarke. Briefwechsel zwischen G.W. Leibniz und S. Clarke. In W. Winter, editor, *Gottfried Wilhelm Leibniz – Philosophische Schriften*, 357–455, Vol. 5. Wissenschaftliche Buchgesellschaft, Darmstadt, 1989. Edited and translated by Werner Winter.
- Newton-Smith, W.H. *The Structure of Time*, Routledge and Kegan Paul, London/Boston/Hanley, 1980.