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Exclusions and the Demand for Property Insurance

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Abstract

The paper examines property insurance contracts in which consumers choose the upper limit on coverage. Exclusions are of two types, and both reduce the demand for insurance of the included perils. A practical implication is that an insurer can raise the demand for fire insurance by offering an earthquake rider, and profit from the rider even when the premia are ceded in such a way that the rider does not raise profit directly. The results do not require assumptions about correlations between included and excluded losses, which is interesting because correlations are decisive in most of the other literature on background risk.

Key words: background risk, property insurance, upper limit, exclusion

1. Introduction

Exclusions are part of property insurance. A simple contract covers losses from a single peril but excludes losses from all other perils and from the insured peril in some instances. Alternatively, in a complex contract like homeowners insurance that covers a wide range of perils, the exclusions are fewer but more acutely recognized, typically catastrophe risks such as flood and earthquake.

Demand for insurance is impacted by excluded risks. The lack of earthquake coverage can be a significant factor in the demand for homeowners insurance, and limitations on coverage of automobile liability can influence the decision to purchase coverage for collision. There are two contrary tendencies here: Exclusions might raise demand by heightening risk aversion or lower it by degrading the quality of the insurance offered. The balance between these tendencies depends upon the form of the insurance contract, as this paper shows.

Historically, the growth of coverages often consists of adding new perils to contracts for previously covered perils. Thus current homeowners policies arise from coverage first of fire and then from successive accumulation of theft coverage, liability coverage, and so on. There are numerous incentives for this type of accumulation, and the possibility examined here is that the addition of a new coverage might increase demand for the old ones.

2. Precursors

The classic theory of demand for insurance assumes that all risks to wealth are insurable. The literature on background risk as developed in Doherty and Schlesinger [1983], Mayers and Smith [1983], Kimball [1990], Kimball and Eeckhoudt [1992], Gollier [1996], and

Garratt and Marshall [1999] amends the classic theory by recognizing the incompleteness of insurance markets. In a typical application of background risk, wealth consists of risky real property, which is insurable, and a risky financial or human capital portfolio, which is not, and the presence of the uninsurable risk influences the demand for insurance of the insurable one.

The assumptions of the classic and background-risk models are summarized in the table below. Risky wealth is w in the classic case, it is insurable, and consumers select a vector of parameters θ such as premium, deductibles and coinsurance, leading to insurance $I(w;\theta)$. In the case of background risk, the components of wealth are random variables x and y. The risks are additive and insurance is available only for the x component.

	utility	insurance
classic	$U(w)$	$I(w;\theta)$
background risk	$U(x+y)$	$I(x;\theta)$

Existing results on background risk are similar to each other and typically depend upon the correlation of x and y. Positive correlation or a small negative correlation both imply that demand for insurance of x at unfair prices will rise in the presence of variability in y. Sufficient negative correlation reduces demand for insurance of x because the consumer possesses in y a kind of self-insurance. These results are usually obtained by supposing that the consumer demand is expressed through choice of the rate of coinsurance. That approach is taken in Mayers and Smith [1983], Doherty and Schlesinger [1983], and some of the results in Kimball and Eeckhoudt [1992]. Other results of Kimball and Eeckhoudt and those in Gollier [1996] concern the deductible, but correlations are still central. Garratt and Marshall [1999] examine a different model of insurance—the same one used below—in the presence of a particular type of background risk, and again find that correlations matter.

3. Damage and conversion

The coinsurance model is natural in the classic theory, where markets insure variations in wealth. In that context it is as good as any other model and more convenient than most. Its attractions are less compelling when markets insure damage to property. The problem is that damage—which is insurable—is not always equivalent to the reduction of wealth that consumers want to insure. The difference arises because property ownership entails the right to convert the property by pulling down existing improvements and replacing them with something different. Consideration of whether to convert is continual because typically the current building on a site is in some measure depreciated and obsolete, and it may be entirely different from the building that would have the highest value on the site by today's standards. Eventually the opportunity to convert is profitably exercised. Conversion opportunities are important for insurance because damage, whether insured or not, can provoke early conversion. After slight damage the building is restored to its previous condition, but when damage passes a critical level the building is razed and replaced by something different.

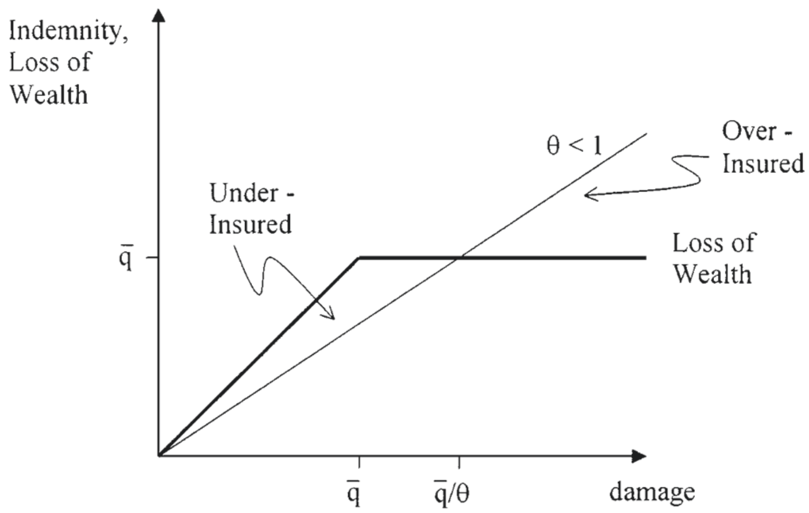

Figure 1. Coinsurance of damage with bounded loss of wealth.

Conversion governs the relation between damage and loss of wealth. So long as damage is not too great, the property is restored and loss of wealth is the same as damage. In that domain, each increment of damage is a corresponding reduction in wealth. Above the critical level of damage, further damage does not further reduce wealth. Damage can increase beyond, perhaps far beyond, the point at which the loss of wealth ceases. Thus the amount of damage that is barely sufficient to induce conversion is the upper bound on loss of wealth, as elaborated in Garratt and Marshall [1996, 1999].

The relationship between damage and loss of wealth is shown in figure 1. For levels of damage below a critical level $\bar{q}$, damage coincides with loss of wealth. For damage beyond $\bar{q}$, loss of wealth is constant at $\bar{q}$. The figure shows why coinsurance policies are not well suited to property insurance. A risk averse consumer wants full insurance of wealth, but the contract is written in terms of damage. A contract in damage having a coinsurance rate of $\theta = 1$ is full insurance for slight damage and more-than-full insurance for damage above the upper bound. The optimum coinsurance policy would have $\theta < 1$ chosen so that the marginal disutility of becoming more under-insured for damages below $\bar{q}/\theta$ is just offset by the marginal utility of becoming less over-insured for damages above $\bar{q}/\theta$. The situation is illustrated in figure 1. Clearly, this is not a good insurance policy.

A better policy has payoffs that mimic the piecewise linear relation between damage and wealth. It supplies full insurance up to an upper limit, which is the choice variable for the consumer. The optimum choice of upper limit is the critical point at which loss of wealth ceases. Damage may grow beyond that point, but the indemnity, like the reduction in wealth, does not. The theoretical considerations are borne out in reality. Coinsurance policies are uncommon in property insurance, but policies featuring the upper limit as the choice variable are prevalent. Thus on descriptive and a priori grounds it is proper to examine background risk in property insurance using a model that centers on choice of the upper limit.

4. Exclusions in property insurance

The exclusion risks studied here are of two types. One is exclusion by peril—a homeowner's policy covers the peril of fire but excludes the peril of rising waters. The second type of exclusion is by event—the policy excludes losses even from the insured perils if the cause of loss is, for instance, an act of war or a civil insurrection. Exclusion by event is parallel to ideas studied by Chen [1997] in his work on insolvency of insurance companies.

4.1. Exclusion by peril

The structure of exclusion by peril is naturally described as follows: The consumer has some nonrandom components of wealth that are ignored. In addition, she has a property with undamaged value $\bar{v}$ that is subject to two perils. Damages from the separate perils are represented by the random variables t and r, and total damage is f which satisfies $f \geq \max[t, r]$. Insurance is available for t only.

Let the value of the land and improvements in the best use be v^*, and let the cost of building the optimum improvements be c. Among the conversion options, the greatest attainable net value is $v^* - c$. When the owner of damaged property restores it, she attains wealth $\bar{v} - f$. Hence, the decision either to restore or convert is the decision to select the greater of $\bar{v} - f$ and $v^* - c$. At the critical level of damage the options are equally valuable. Thus the critical level of damage that corresponds to the bound on loss of wealth is $\bar{q} = \bar{v} - (v^* - c)$, and pre-insurance wealth is given by

$$\bar{v} - \min[f, \bar{q}].$$

The risk of this wealth is of course dependent upon the distribution of the insurable risk of t (probability density function $h(t)$) and upon that of the total risk f (conditional probability density function $g(f \mid t)$), which may or may not be dependent on t.

In choosing an insurance contract, the consumer selects an upper limit, which is denoted by b and thought of as "bound." There is no coinsurance or deductible. Insurance is full up to the upper limit and thus for any realization of insurable damage t, the insurance payment is $\min[b, t]$. Its expected value is the fair premium

$$P(b) = \int_0^b th(t)\, dt + b \int_b^\infty h(t)\, dt. \tag{1}$$

Combining the pre-insurance wealth with the insurance variables, insured wealth is

$$w = \bar{v} - \min[f,\ \bar{q}] + \min[b, t] - P(b). \tag{2}$$

The option to convert appears in the second term, and the indemnity with upper limit is in the third term. The exclusion is seen in the difference between the actual damage f and the insurable damage t. Other features are standard.

The consumer of insurance is a risk averse maximizer of expected utility with utility function u. The consumer's problem is to choose the upper limit on insurance, b, so as to maximize

$$E_{t,f}u(\bar{v} - \min[f, \bar{q}] + \min[b, t] - P(b))$$
$$= \int_{t=0}^{\infty}\left[\int_{f=t}^{\infty} u(\bar{v} - \min[f, \bar{q}] + \min[b, t] - P(b))g(f \mid t)\,df\right]h(t)\,dt \quad (3)$$

Damage from the uninsured peril is conveniently implicit. Less conveniently, the function being optimized is not globally concave in b. This difficulty is described in detail in Garratt and Marshall 1999 and circumvented in the proof below.

It is natural to focus attention on the point $b = \bar{q}$, at which the insurable peril is fully insured. It is the optimum if the uninsurable peril is absent. The surprising result is that adding the uninsurable peril *reduces* the optimum upper limit to a level below $\bar{q}$. The general intuition is as follows. Total damage f is distributed over values that are always above t. Assuming that $b = \bar{q}$, for $t > \bar{q}$ the cap on loss is attained and loss of wealth is fully indemnified. For insurable damage in the range $(0, \bar{q})$, total damage is above t and therefore expected marginal utility of wealth is higher than in the states $t > \bar{q}$. Reducing the upper limit shifts wealth toward the high marginal utility states. The proof below is structured like the intuition. The interesting situations are ones in which the whole loss f sometimes exceeds the insurable loss t, and sometimes exceeds the cap on damage, $\bar{q}$. Otherwise the situation reduces trivially to a single insurable loss. The following result holds true if either of the non-triviality conditions does.

Proposition 1: *Suppose that u is risk averse and insurance is fairly priced. If* $f = t$, *the optimum upper bound is* $b = \bar{q}$. *If there is positive probability mass on at least one of the sets,* $\{(t, f) \mid t \in (0, \bar{q}),\ f \in (t, \bar{q})\}$ *and* $\{(t, f) \mid t \in (0, \bar{q}),\ f \in (\bar{q}, \infty)\}$, *then the optimum upper bound on insurance of insurable risk t is* $b < \bar{q}$.

Proof. Since concavity is not global, the proof consists of checking that for all $b \geq \bar{q}$, the derivative of the objective is negative. Let the objective in Eq. (3) be denoted by $K(b)$.

$$K(b) = E[u(w) \mid t < b]\Pr(t < b) + E[u(w) \mid t \geq b]\Pr(t \geq b) \quad (4)$$

The derivative is

$$K'(b) = E[u'(w) \mid t < b]\left.\frac{\partial w}{\partial b}\right|_{t<b}\Pr(t < b) + E[u'(w) \mid t \geq b]\left.\frac{\partial w}{\partial b}\right|_{t\geq b}\Pr(t \geq b) \quad (5)$$

From Eqs. (1) and (2), it can be seen that

$$\left.\frac{\partial w}{\partial b}\right|_{t<b} = -\Pr(t \geq b) \quad \text{and} \quad \left.\frac{\partial w}{\partial b}\right|_{t\geq b} = \Pr(t < b) \quad (6)$$

Therefore

$$\text{sign } K'(b) = \text{sign}\{-E[u'(w) \mid t < b] + E[u'(w) \mid t \geq b]\}$$

For $t \geq b$, wealth is $\bar{v} - P(b) + b - \bar{q}$, which bears a constant marginal utility. For $t < b$, wealth is either $\bar{v} - P(b) + t - f$ or $\bar{v} - P(b) + t - \bar{q}$, depending on whether $f > \bar{q}$. In either case wealth is lower than in $t \geq b$ states and marginal utility is higher. It follows that

$$E[u'(w) \mid t < b] > E[u'(w) \mid t \geq b]$$

and hence that $K'(b) < 0$. If $f = t$, insurance is full at $b = \bar{q}$, and by well-known arguments, no higher utility is attainable for the risk averse consumer at fair prices. □

The intuition behind the result is straightforward. Suppose that fairly-priced fire insurance is available but earthquake coverage is not available. Suppose that a property owner insures against fire up to the critical point. Then in case of no loss or in case of fire, the owner has the same wealth, but his wealth is lower if the uninsured earthquake strikes. The only way to transfer wealth to the earthquake states is to reduce the premium paid for fire insurance. Such transfers reduce fire insurance, but at first this effect is unimportant because, near the full-insurance point, the consumer is risk neutral. Eventually the loss of fire coverage weighs equally with the increase of earthquake-state wealth, and that equation determines the optimum. Thus demand for fire insurance is less in the presence of an uninsurable earthquake risk.

The intuition exposes another interesting aspect of exclusions. Suppose that both earthquake and fire occur together, as is typical in reality. Suppose that the uninsurable earthquake produces damage beyond the critical point, so that conversion is certain and loss of wealth is at its upper bound. Suppose in addition that the earthquake starts a fire. The fire does not increase the loss of wealth already suffered, but the fire insurance indemnities compensate part or all of the wealth lost in the earthquake. Although an incentive to increase the insurable loss exists here, the law is firm in maintaining that such indemnities are collectible. Thus when both perils strike at once, insured damage may be welcome because it fills the gap in coverage left by the exclusion.

Exclusion by peril is a form of background risk. This is seen by writing wealth as the sum of an insurable risk $x = \bar{v} - \min[t, \bar{q}]$, and an uninsurable risk $y = \min[t, \bar{q}] - \min[f, \bar{q}]$. Previous results on background risk depend on the correlation between the two risks. Specifically, a reduction of insurance requires a negative correlation. This is not the case here. In fact, different densities for damage t lead to different correlations. Think of the case that $f = 2t$, and suppose that $\bar{v} = 3, \bar{q} = 2$, and $h(t)$ is uniform on [0, 3]. Then the covariance between x and y as defined above is $cov(x, y) = -\frac{1}{9}$. Switching to a density $h(t) = \frac{e^{-t}}{1-e^{-3}}$, so that more probability is stacked up on small losses, the covariance is $cov(x, y) = .0129807$. Thus the correlation of x and y can be either positive or negative, without upsetting the result. The examples show that results on background risk derived for coinsurance policies are not applicable to all settings.

In some settings it is assumed that the background risk is either independent of the insurable risk or, at least, growing as the insurable loss grows [e.g., Gollier, 1996]. Neither assumption is satisfied in the model of property insurance being studied here. The distribution of the background risk is naturally dependent on the insurable risk, it tends to fall as the insurable loss rises, and it certainly reaches zero when the insurable loss reaches the critical point for conversion.

4.2. *Exclusion by event*

The other type of exclusion is triggered by an event such as an act of war or civil insurrection that negates all indemnities. Let e be a random variable that equals 1 in the event of war or civil insurrection, and 0 otherwise.

Consider an all-peril homeowners insurance contract with an exclusion for war damage. Damage is denoted by t, and it is excluded from the insurance contract when $e = 1$. In determining the conversion option and the bound on loss of wealth, t has the same role as f had in the previous section. War happens with probability π. In that case, the probability distribution function of damage is $g(t)$. With probability $1 - \pi$ there is no war, and the house suffers damage t according to the probability density function $h(t)$. The fair premium is given by

$$P(b) = (1-\pi)\left[\int_0^b th(t)\,dt + b\int_b^\infty h(t)\,dt\right] \tag{7}$$

The goal of the consumer is to choose the upper limit b to maximize

$$\begin{aligned} J(b) = {} & (1-\pi)E[u(\bar{v} - \min[t,\bar{q}] + \min[t,b] - P(b)) \mid e = 0] \\ & + \pi E[u(\bar{v} - \min[t,\bar{q}] - P(b)) \mid e = 1] \end{aligned} \tag{8}$$

Exclusion by event unambiguously lowers demand for fairly priced insurance. Suppose the contrary. Suppose for purposes of later contradiction that the optimum is at $b = \bar{q}$. Then in peacetime the consumer has a deterministic wealth of $\bar{v} - P(\bar{q})$. In wartime the highest level of wealth is $\bar{v} - P(\bar{q})$, which is reached only if damage is zero. Otherwise wealth is lower, and that happens with positive probability. It follows that the expected marginal utility of wealth in wartime is above that in peacetime. Since insurance is fairly priced, the consumer benefits by transferring wealth to the war states, and the wealth transfer is accomplished by reducing b. Thus optimum insurance is less than full.

The formal proof parallels the intuition.

Proposition 2: *Suppose that u is risk averse and that there is positive probability mass on the set* $\{(t,e) \mid t \in (0,\bar{q}), e = 1\}$. *Insurance is fairly priced. Then the optimum upper bound on insurance of insurable risk t is* $b < \bar{q}$.

Proof. The objective can be rewritten from Eq. (8) as

$$\begin{aligned} J(b) = {} & \pi u(\bar{v} - P(b)) + (1-\pi)E[u(\bar{v} - \min[t,\bar{q}] + \min[t,b] - P(b)) \mid e = 0] \\ & + \pi E[u(\bar{v} - \min[t,\bar{q}] - P(b)) \mid e = 1] - \pi u(\bar{v} - P(b)). \end{aligned} \tag{9}$$

Consider the first two terms as

$$J_1(b) = \pi u(\bar{v} - P(b)) + (1 - \pi)E[u(\bar{v} - \min[t, \bar{q}] + \min[t, b] - P(b)) \mid e = 0]$$

This is the objective of a consumer with a probability π of no loss and a conditional probability density of $h(t)$ in case of loss. This consumer can purchase fairly priced insurance at the price $P(b)$ given in Eq. (7). The case is covered by Proposition 1, from which it is known that $J_1'(\bar{q}) = 0$ and for all $b > \bar{q}$, $J_1'(b) < 0$. Consider the last two terms as

$$J_2(b) = \pi E[u(\bar{v} - \min[t, \bar{q}] - P(b)) \mid e = 1] - \pi u(\bar{v} - P(b))$$

and consider any $b \geq \bar{q}$. The derivative is

$$J_2'(b) = -P'(b)\pi\{E[u'(\bar{v} - \min[t, \bar{q}] - P(b)) \mid e = 1] - u'(\bar{v} - P(b))\}.$$

Clearly the first term dominates because marginal utility is higher in poor states. Thus $J_2'(b) < 0$ for all b on $[\bar{q}, \infty)$. Combining results, the conclusion is that $J'(b) < 0$ for all b on $[\bar{q}, \infty)$. Thus optimum b is less than $\bar{q}$. □

Proposition 2 shows how degradation in the quality of insurance leads to reduced demand for it. The result makes sense: instead of fully insuring for the included perils, the consumer accepts some risk from them in order to transfer wealth into states associated with the excluded loss.

Exclusion by event is a form of background risk. Let $x = \bar{v} - t$ if $e = 0$ and $x = 0$ otherwise; and $y = \bar{v} - t$ if $e = 1$ and $y = 0$ otherwise. Then wealth is $x + y$ and the x component is insurable while the y component is not. The correlation between the two risks is always negative, regardless of the choice of the conditional densities $h(t)$ and $g(t)$. Thus the result is consistent with the literature on background risk, in which a negative correlation is a necessary condition for a reduction in demand for insurance.

5. Concluding remarks

Conditions on correlation are not needed to get results for the contracts studied in this paper. The effect of background risk is unambiguous. It reduces demand for insurance of the insurable risk.

The results on demand for insurance reveal profit opportunities for insurers. They imply that insurers might gain in two ways from extending coverages, a primary profit from the new coverage, and a secondary profit from increased demand for existing coverage. In some cases the secondary effect can be bigger than the primary and can be, in fact, the whole motivation for extending coverages. Thus when coverage of theft joins with coverage of fire in a homeowner's insurance policy, profits from the increased demand for fire coverage may be more important than the profits arising directly from theft insurance. Similarly, earthquake coverage may be offered under a reinsurance pact that absorbs all profits from it, but the supplier of homeowners insurance can still gain from the heightened demand for the underlying coverage.

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