The Geneva Papers on Risk and Insurance Theory, 23: 29–40 (1998)

Changes in Background Risk and the Demand for Insurance

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Abstract

The demand for insurance against loss from a particular risky asset is likely to depend on other risks the decision-maker faces. For independently distributed other risks, referred to as background risk, Eeckhoudt and Kimball [1992] determine the effect on insurance demand of introducing background risk. Recently, Eeckhoudt, Gollier, and Schlesinger [1996] determine conditions on preferences such that first- and second-degree stochastic deteriorations in background risk lead to a decrease in the decision-maker's willingness to accept other risks. These results, although formulated in a general decision model, also apply to insurance demand. This article continues analysis of this question by determining the effect on insurance demand of several other general changes in background risk.

Key words: background risk, stochastic dominance, coinsurance, deductibles

1. Introduction

Much of the work concerning the demand for insurance assumes that the insurable risky asset is the only source of risk in the decision-maker's portfolio. During the past fifteen years, however, a number of articles have considered the effect of additional sources of risk on the decision to insure.[1] This analysis has been carried out in a mean-variance decision model and in an expected-utility maximization context. The additional sources of risk have usually been assumed to be distributed independently from the insurable risk, although a limited amount of research has been carried out when the different sources of risk are correlated in a known and specific way.

In the model analyzed here, *background risk* is the term used to describe an exogenous and independently distributed source of risk that is part of the portfolio containing the insurable risk. The main question considers the effect of changes in the distribution function for this background risk on the decision to insure. The analysis is conducted assuming that expected utility is maximized.

Two recently published articles address this particular question. Eeckhoudt and Kimball [1992] (EK) determine the effect of introducing background risk into an insurance demand model. In their work, insurance demand is measured by either the coinsurance or the deductible level. They show that under a straightforward, intuitive, and previously assumed restriction on preferences, the introduction of background risk increases the demand for insurance. For this result it does not matter whether insurance demand is measured by the coinsurance or the deductible level.

Very recently, Eeckhoudt, Gollier, and Schlesinger [1996] (EGS) investigate the effect on the risk-taking attitude of the decision-maker of a change in, rather than the introduction of, background risk. EGS determine the effect on risk-taking preferences of first- or second-degree stochastic dominant deteriorations in background risk. Not surprisingly, to unambiguously determine the effect of these two broad categories of changes, EGS find that "the necessary and sufficient conditions that are derived are fairly restrictive upon preferences." Although EGS do not discuss insurance demand explicitly, their results indirectly give information concerning the impact of changes in background risk on the demand for insurance because the willingness to insure depends on the risk-taking attitude of the insured.

This research continues and extends the work begun in these two articles. Using expected-utility maximization as the objective, and assuming independently distributed, additive, and exogenous background risk, the effect on the demand for insurance of three well-known but less broadly defined changes in background risk is determined. This is accomplished without imposing the strong conditions on preferences identified by EGS. In fact, the results presented here do not require restrictions on preferences that go beyond those imposed by EK in determining the effect of an introduction of background risk. The three main results determine the effect on the demand for insurance of a strong increase in risk, a simple first-degree stochastic dominant deterioration in background risk, and a simple increase in risk. The method that is used to prove these results can also be employed when examining other changes in background risk as well. As in EK, insurance demand can be measured by either the coinsurance or the deductible level.

The article is organized as follows. First, the most commonly analyzed model of insurance demand in the literature, the coinsurance demand model, is used to determine the effects of various changes in background risk on the demand for coinsurance. Most of the notation and assumptions are introduced in this section. In Section 3, deductible insurance is analyzed, and results similar to those for coinsurance concerning the effect of changes in background risk on the demand for deductible insurance are stated. Because of the similarities, the results in this section are given with only a brief sketch of a proof.

2. Background risk and coinsurance

Assume a decision-maker is endowed with a risky asset whose value is M when no loss occurs but that this asset is subject to random loss x ranging from 0 to M. Formally, the loss x is represented as a random variable with cumulative distribution function (CDF) $F(x)$ whose support is in $[0, M]$. Further assume that this risky asset can be insured. One unit of insurance sells for price p and provides indemnification $I(x)$ when the loss is x. The decision-maker can choose θ between 0 and 1, covering fraction θ of the loss.[2] When a fraction θ of loss is covered, both the indemnification and the price are scaled by θ.

In addition to this insurable risky asset and its insurance, the decision-maker also holds another risky asset referred to as background risk.[3] That is, the decision-maker's portfolio contains an additional component that is treated as fixed when making the decision to insure, and this component is distributed independently from the insurable risk x. This background risk is denoted y, and $G(y)$ is its CDF. The support of y is assumed to be some interval $[c, d]$.

The decision-maker's preferences are represented by utility function $u(Z)$, which is assumed continuously differentiable at least four times. $r(Z) = -u''(Z)/u'(Z)$ denotes the measure of absolute risk aversion, $\rho(Z) = -u'''(Z)/u''(Z)$, the measure of absolute prudence, and $\tau(Z) = -u''''(Z)/u'''(Z)$, the measure of absolute temperance. $u'(Z) \geq 0$ and $u''(Z) < 0$ are assumed throughout, and various assumptions concerning $r(Z)$, $\rho(Z)$, and $\tau(Z)$, are imposed as needed.

Formally, the decision-maker is assumed to choose θ in [0, 1] to maximize expected utility from Z where $Z = M - x + \theta[I(x) - p] + y$. The indemnification function $I(x)$ is assumed to be nondecreasing and continuous, and to satisfy $0 \leq I(x) \leq x$ and $I'(x) \leq 1$, when that derivative exists. It is assumed that the price of insurance, relative to the indemnification provided, is such that there is an interior solution to this optimization.[4] That is, it is assumed that the expected-utility maximizing level for θ is determined by the equation $E_y E_x u'(Z)[I(x) - p] = 0$. Under strict risk aversion, the second-order condition for the maximization is satisfied.

The question of concern—how does the optimal level for θ change as the distribution function for background risk y is altered?—is answered by determining whether $E_y E_x u'(Z)$ $[I(x) - p]$ is increased or decreased from its zero value as $G(y)$, the CDF for background risk, is changed. If $E_y E_x u'(Z)[I(x) - p]$ increases (decreases) with the change in y, then the optimal θ is larger (smaller) as a result of the change. The results of EK and EGS concerning this question are reviewed in the next two paragraphs.

As indicated earlier, Eeckhoudt and Kimball have determined the impact on θ of the introduction of background risk. They determine the effect on θ of changing y from a nonrandom value μ to a random variable with mean equal to μ. This introduction of background risk is represented as a change from an initial CDF $G(y)$, which is degenerate at μ, to a new CDF $H(y)$, which has mean μ. EK show that it is sufficient that $u(Z)$ display positive and decreasing risk aversion and decreasing prudence ($r(Z) \geq 0$, $r'(Z) \leq 0$ and $\rho'(Z) \leq 0$), for the introduction of background risk to lead to an increase in the optimal θ.[5] These assumptions on $u(Z)$ have a simple interpretation that is provided by EK and also by Kimball [1993].

Eeckhoudt, Gollier, and Schlesinger have given necessary and sufficient conditions on $u(Z)$ for determining when *any* first-degree stochastic deterioration (FSD) or second-degree stochastic deterioration (SSD) in background risk from any initial distribution of this risk would cause the decision-maker to act in a more risk-averse manner toward the insurable primary risk. These two categories of changes for y are much broader than the introduction of risk considered by EK. As a result, the conditions on $u(Z)$ needed to determine the effect of the background risk change are stronger. EGS indicate that in order to determine the effect on risk taking of an arbitrary FSD deterioration in y, preferences must be such that the lower bound for $\rho(Z)$ is larger than the upper bound for $r(Z)$ over all possible values for Z. Similarly, to determine the effect of an arbitrary SSD change, this same condition must hold, and in addition, the temperance measure $t(Z)$ must have a lower bound that is larger than the upper bound of $r(Z)$. These are very strong restrictions on preferences and as yet do not have an intuitive interpretation.

The results presented here begin to fill the gap between the findings of EK and those of EGS. This is accomplished by considering less general changes in background risk than those of EGS. By imposing more restrictions on the change in background risk that occurs, less restrictive assumptions are required on preferences. In fact, we show that without imposing any restriction on $u(Z)$ beyond those employed by EK, the effect on the optimal θ of a change for y from any one of three broad categories of change can be determined. Two of these categories contain the introduction of background risk as a special case, and hence those results extend the EK finding.

Before presenting the three main theorems, two preliminary lemmas are useful. These lemmas are stated and discussed below, and the proofs are provided in the appendix. The lemmas demonstrate properties of the function $m(y) = E_x u'(Z)[I(x) - p]$ and its derivatives. Note that the $m(y)$ function is obtained by taking the expectation of $u'(Z)[I(x) - p]$ only with respect to random variable x.[6] The function $m(y)$ is of interest because the first-order condition defining the optimal θ takes the form $E_y m(y) = 0$. Hence, the effect of a change in the distribution function for y on this expectation and θ, depends on the properties of $m(y)$. Of special importance are the increasing/decreasing and concavity/convexity characteristics of $m(y)$. For example, if one determines that $m(y)$ is nonincreasing in y for all y, then any first-degree stochastic deterioration in y leads to a higher value for $E_y E_x u'(Z)[I(x) - p]$ and hence a larger optimal value for θ. In fact, it is the case that the conditions identified by EGS for determining the effect of an FSD change in y are sufficient to show that $m(y)$ is nonincreasing.

Lemma 1: *If preferences satisfy $r(Z) \geq 0$ and $r'(Z) \leq 0$, then at the optimal θ there exists a y^* such that $m(y) \geq 0$ for all $y \leq y^*$ and $m(y) \leq 0$ for all $y \geq y^*$. Furthermore, $m'(y) \leq 0$ whenever $m(y) \geq 0$. Also $E_y m'(y) \leq 0$.*

This lemma indicates that risk aversion and decreasing absolute risk aversion (DARA) are sufficient to yield a $m(y)$ function, which although not necessarily nonincreasing in y, does possess some of the important properties of such a function. These properties include (1) $m(y)$ changes sign once from positive to negative as y increases, (2) $m(y)$ has negative slope whenever its value is positive, and (3) on average, $m(y)$ has a negative slope. These properties, of course, are not the same as $m(y)$ nonincreasing and are *not* enough to determine the effect on θ of all first-degree stochastic dominant (FSD) deteriorations of y. They are sufficient, however, to determine the effect of a simple FSD change, as well as other changes.[7] Furthermore, these determinations are accomplished without imposing the strong assumption on preferences identified by EGS or even the assumption of decreasing prudence used by EK. In fact, the imposed assumption, DARA, is shown by Pratt [1964] to be necessary to determine the effect of even a change in *nonrandom* background wealth. DARA is clearly the minimum restriction that one would expect to impose in order to determine the effect of a change in background risk on the decision to insure.

Lemma 2: *If preferences satisfy $\rho(Z) \geq 0$ and $\rho'(Z) \leq 0$, then at the optimal θ there exists a y^{**} such that $m'(y) \leq 0$ for all $y \leq y^{**}$ and $m'(y) \geq 0$ for all $y \geq y^{**}$. Furthermore, $m''(y) \geq 0$ whenever $m'(y) \leq 0$. Also $E_y m''(y) \geq 0$.*

This lemma indicates that positive and decreasing absolute prudence (DAP) is sufficient to yield a $m(y)$ function that has several properties implied by convexity of $m(y)$—that is, (1) the first derivative of $m(y)$ changes sign once from negative to positive as y increases, (2) $m'(y)$ is positively sloped whenever it is negative, and (3) $m(y)$ has an average second derivative that is positive. These properties are *not* equivalent to convexity of $m(y)$ and are not enough to determine the effect on θ of all Rothschild and Stiglitz changes in risk. These properties are sufficient, however, to determine the effect on θ of two special types of risk increases. These restrictions on $u(Z)$ are those used by EK in determining the effect of an introduction of background risk and are less restrictive than those of EGS for determining the effect of general SSD changes in y.

The two lemmas can be combined and summarized as follows. If preferences display risk aversion, DARA, and DAP, then the function $m(y)$ is positive, decreasing, and convex for y in $[c, y^*]$, negative, decreasing and convex for y in $(y^*, y^{**}]$, and negative and increasing on $(y^{**}, d]$. It is possible that y^{**} is coincident with d—that is, $m(y)$ is nonincreasing and convex for all y. The conditions of EGS do imply this property of $m(y)$. Loosely speaking, under DARA and DAP, the function $m(y)$ is well behaved with regard to determining the effects of FSD or SSD changes in background risk for the lower values for y (y in $[c, y^{**}]$) but may not be so well behaved for higher values. In addition, on average, $m(y)$ is well behaved implying that changes that are uniformly applied to y throughout its domain have determinate effects on θ. With these preliminaries taken care of, it is now possible to use these properties of $m(y)$ to demonstrate three results concerning the effect of changes in background risk on the optimal level of coinsurance.

Meyer and Ormiston [1985] define a generalization of an introduction of risk called a strong increase in risk. For background risk y, a strong increase in risk is represented as a Rothschild and Stiglitz change from CDF $G(y)$ whose initial support is some interval $[y_1, y_2]$ to $H(y)$ with support $[c, d]$ containing $[y_1, y_2]$. In addition, the difference in the cumulative distribution functions, $[H(y) - G(y)]$, is required to be nonincreasing on (y_1, y_2). It is this later condition that restricts the change to a subset of Rothschild and Stiglitz increases in risk.

Strong increases in risk are represented by $[H(y) - G(y)]$, which are nonnegative and nondecreasing on $[c, y_1]$, nonincreasing on (y_1, y_2), and nonpositive and nondecreasing on $[y_2, d]$. $[H(y) - G(y)]$ changes sign one time from positive to negative at some y_0 in $[y_1, y_2]$. Strong increases in risk take probability mass from its current distribution and redistribute the mass at or outside the endpoints of the initial support. The definition of a strong increase in risk ensures that "enough" of the probability mass transfer occurs at the left-hand end of the support where the $m(y)$ function is well behaved. Also, because the introduction of risk is a strong increase in risk, the following theorem is a generalization of the EK result to this broader category of risk increases.

Theorem 1: *Strong increases in background risk cause an increase in* θ *when* $r(Z) \geq 0$*,* $r'(Z) \leq 0$*, and* $\rho'(Z) \leq 0$.

Proof. Because the proof of this theorem uses partial expectations, it is more convenient to use integral rather than expectation notation. Under the new background risk with CDF $H(y)$, the expression to be signed is $\int_c^d m(y)\,dH(y) = \int_c^d m(y)\,d[H(y) - G(y)]\,dy$. The second expression follows because θ satisfies the first-order condition for CDF $G(y)$ and hence has value 0. Integration by parts yields $\int_c^d m'(y)[G(y) - H(y)]\,dy$ as the expression to be signed. Two cases are to be considered. First, assume that $y^{**} \geq y_0$—that is, $[G(y) - H(y)]$ changes sign from negative to positive to the left of where $m'(y)$ changes sign from negative to positive. When this is the case, $\int_c^d m'(y)(G(y) - H(y))\,dy \geq \int_c^{y^{**}} m'(y)[G(y) - H(y)]\,dy$. Now, for y in $[c, y^{**}]$, $m'(y)$ is nonpositive and nondecreasing and $[G(y) - H(y)]$ changes sign from negative to positive and has negative expectation. Thus, $\int_c^{y^{**}} m'(y)[G(y) - H(y)]\,dy \geq 0$, and $\int_c^d m'(y)[G(y) - H(y)]\,dy \geq 0$ for this case.

The second case to consider is where $y^{**} < y_0$, which implies that $y^* < y_0$ also. For this case, $\int_c^d m'(y)[G(y) - H(y)]\,dy \geq \int_c^{y_0} m'(y)[G(y) - H(y)]\,dy = \int_c^{y^*} m'(y)[G(y) - H(y)]\,dy + \int_{y^*}^{y_0} m'(y)[G(y) - H(y)]\,dy$. The first term in this expression is positive because $m'(y) \leq 0$ and $[G(y) - H(y)] \leq 0$ for y in $[c, y^*]$. The second term is also positive. To see this, integrate by parts to obtain $\int_{y^*}^{y_0} m'(y)[G(y) - H(y)]\,dy = \int_{y^*}^{y_0} m(y)d[H(y) - G(y)]$. Now on $[y^*, y_0]$, $m(y) \leq 0$, and for strong increases in risk, $[H(y) - G(y)]$ is nonincreasing. Thus, $\int_c^d m'(y)[G(y) - H(y)]\,dy \geq 0$ for this case as well and we can conclude that the decision-maker chooses a higher θ when y undergoes a strong increase in risk. □

Answers to the comparative statics question posed here have been successfully determined in two different ways. Under the CDF approach that was just employed, one determines how $E_y E_x u'(Z)[I(x) - p]$ changes as the CDF for y changes from $G(y)$ to $H(y)$. A second approach calculates the effect on $E_y E_x u'(Z)[I(x) - p]$ of transforming random variable y—that is, replacing y with transformed value $t(y)$. The next two results concern the effect of changing background risk using a transformation of the background risk variable. This approach is described more completely by Meyer and Ormiston [1989].

Under the transformation approach, random variable y is transformed and hence replaced in the model by function $t(y)$. $t(y)$ is assumed to be a nondecreasing function so that the ranking of values for y is not altered. To determine the effect of replacing y by $t(y)$, it is convenient to augment the decision model so that it includes background risk variable $[y + \delta k(y)]$ rather than y, where $k(y)$ is defined by $k(y) = t(y) - y$. The purpose of this step is that when $\delta = 0$ the original background risk specification prevails, and when $\delta = 1$, the new background risk is $t(y)$, the transformed value. Hence, the effect on θ of changing δ from 0 to 1 is the effect of transforming background risk y by $t(y)$. Differential calculus methods can be used to determine the effect of this change.

With $[y + \delta k(y)]$ representing background risk, the new outcome variable Z is given by $Z = M - x + \theta[I(x) - p] + [y + \delta k(y)]$. The first-order expression for the optimal θ is $E_y E_x u'(Z)[(I(x) - p]$, with δ only appearing in the $u'(Z)$ term. As before, define $m(y)$ as the expectation of this expression only with respect to x; $m(y) = E_x u'(Z)[(I(x) - p]$. At $\delta = 0$, θ is chosen so that $E_y m(y) = 0$. What must be determined is how this expectation changes as δ is increased to $\delta = 1$. Taking the derivative with respect to δ yields $E_y E_x u''(Z)k(y)[I(x) - p] = E_y k(y) E_x u''(z)[I(x) - p]$. Signing this derivative for all δ in $[0, 1]$, determines the direction of the effect on θ of changing δ from 0 to 1.

Meyer [1989] identifies transformation $t(y)$ as first-degree stochastic dominant improving whenever $k(y) = t(y) - y \geq 0$—that is, whenever the transformation does not decrease any value for y. Ormiston [1992] focuses on a subset of these FSD changes by requiring that $k'(y) \leq 0$. These changes are called simple FSD transformations of y. Simple FSD transformations are FSD improvements that require that the improvement be larger when y is small than when y is large.

Recalling the earlier discussion concerning the well behavedness of $m(y)$, it is important to recognize that simple FSD changes, by requiring the change in y to be larger when y is small than when y is large, imply that more change occurs where $m(y)$ is well behaved rather than where it need not be. In this way, simple FSD changes and also simple increases in risk are similar to strong increases in risk. As a consequence of this concentration of the change in the left tail, one can show that a simple FSD deterioration is sufficient to determine the sign of the effect on insurance demand. The following theorem presents this result for simple FSD deteriorations rather than improvements ($k(y) \leq 0$, $k'(y) \geq 0$) to maintain consistency with the EK and EGS discussion.

Theorem 2: *At the optimal* θ, $d\theta/d\delta \geq 0$ *for all* δ *in* [0, 1] *if* $k(y) \leq 0$, $k'(y) \geq 0$ *and* $r(Z) \geq 0$ *and* $r'(Z) \leq 0$.

Proof. We must determine the sign of $E_y k(y) E_x u''(Z)[I(x) - p]$ for all δ when $k(y)$ is nonpositive and nondecreasing. Consider $n(y) = E_x u''(Z)[I(x) - p]$ so the expression to be signed is $E_y k(y)n(y)$. Using the argument in the proof of Lemma 1, DARA implies that $n(y) \leq -r(Z_0)E_x u'(Z)[I(x) - p] = -r(Z_0)m(y)$. Consequently, $E_y k(y)n(y) \geq -E_y r(Z_0)k(y)m(y)$. Now $m(y)$ changes sign from positive to negative and $E_y m(y) = 0$. With $r(Z_0)k(y)$ nonpositive and nondecreasing, this implies that $E_y r(Z_0)k(y)m(y) \leq 0$ and hence $E_y k(y)n(y) = E_y k(y)E_x u''(Z)[I(x) - p] \geq 0$. Therefore, $d\theta/d\delta \geq 0$. This proof uses the fact that θ is optimal for each δ but holds for all δ in [0, 1]. □

The transformation approach has also been used to examine changes in y that are either second-degree stochastic dominant changes or Rothschild and Stiglitz changes in risk. Transformation $t(y)$ is a second-degree stochastic dominant deterioration in background risk if $k(y)$ satisfies $\int_c^t k(y)\,dG(y) \leq 0$ for all t in $[c, d]$. If in addition, $\int_c^d k(y)\,dG(y) = 0$, then the change is a Rothschild and Stiglitz increase in risk.[8] Meyer and Ormiston [1989] discuss a subcategory of these changes where $k(y)$ is monotonically increasing calling those changes simple increases in risk. A change from any distribution of background risk with mean μ to one where the background risk variable is degenerate at μ is a simple decrease in risk. Hence, the category simple changes in risk contains the introduction of risk as a special case. The following theorem then is also a generalization of the result of Eeckhoudt and Kimball.

Theorem 3: *At the optimal* θ, $d\theta/d\delta \geq 0$ *for all* δ *in* [0, 1] *if* $E_y k(y) = 0$, $k'(y) \geq 0$ *and* $r(Z) \geq 0$, $r'(Z) \leq 0$ *and* $\rho'(Z) \leq 0$.

Proof. As in Theorem 2, we must sign $E_y k(y) E_x u''(Z)[I(x) - p]$. Now $E_y k(y)$ is zero and $k(y)$ changes sign once from negative to positive at some point s in $[c, d]$. As in Theorem 2, let $n(y) = E_x u''(Z)[I(x) - p]$ so $E_y k(y)n(y)$ is to be signed. Recall that DARA and DAP together imply that $n(y)$ changes sign at most one time from negative to positive as y increases and $E_y n(y) \leq 0$. Let y^{**} denote the point where this sign change occurs. It can be shown that $n'(y) \geq -(1 + \delta k'(y))\rho(Z_0)n(y)$. Thus $n(y)$ is also nondecreasing on $[c, y^{**}]$ where $n(y)$ is negative.

Consider three cases. First, if $s = y^{**}$, the sign changes of $n(y)$ and $k(y)$ occur at the same point and $E_y k(y)n(y) \geq 0$, and we are done. Next consider the case where $s < y^{**}$. $E_y k(y)n(y) = \int_c^d k(y)n(y)\, dG(y) \geq \int_c^{y^{**}} k(y)n(y)\, dG(y)$. Over the interval $[c, y^{**}]$, $k(y)$ changes sign once from negative to positive and $\int_c^{y^{**}} k(y)\, dG(y) \leq 0$. In addition, $n(y)$ is nonpositive and nondecreasing over this interval and therefore, $\int_c^{y^{**}} k(y)n(y)\, dG(y) \geq 0$.

The final case to consider is where $s > y^{**} \geq y^*$. Now, $\int_c^d k(y)n(y)\, dG(y) \geq \int_c^s k(y) n(y)\, dG(y)$. Furthermore, on the interval $[c, s]$, $\int_c^s n(y)\, dG(y) \leq 0$, $n(y)$ changes sign once from negative to positive, and $k(y)$ is nonpositive and nondecreasing. Hence, $\int_c^s k(y) n(y)\, dG(y) \geq 0$ implying that $\int_c^d k(y)n(y)\, dG(y) \geq 0$ as well. □

This proof is also valid when $E_y k(y) \leq 0$ rather than $E_y k(y) = 0$ is imposed and hence deals with simple SSD deteriorations in background risk as well as simple increases in risk. It should also be mentioned that these specialized changes in background risk are only sufficient for the determinate comparative static result. Kimball [1993] discusses the issue of necessity using the concept of patently more risky.

3. Background risk and deductible insurance

Assume now that the decision-maker can choose a level D for the deductible in an insurance policy where the quantity of insurance selected is adjusted using the deductible level. That is, the indemnification function is

$$I(x) = \begin{cases} 0 & \text{if } x \leq D \\ (x - D) & \text{if } x > D, \end{cases}$$

where D is the level of the deductible. Let $P = \phi(D)$, which is assumed to be continuous, twice differentiable, and decreasing in D, denote the premium function for this insurance. Final wealth when insurance is of the deductible form is given by

$$Z = \begin{cases} M - x - \phi(D) + y & \text{if } x \leq D \\ M - D - \phi(D) + y & \text{if } x > D. \end{cases}$$

Let $U(D)$ denote expected utility from final wealth—that is, $U(D) \equiv E[u(Z)]$. When D is selected to maximize $U(D)$, the first-order condition is $U_D(D^*) = 0$, where

$$U_D = \int_c^d \left[-\int_0^D u'(M - x - \phi + y)\phi'\, dF(x) \right.$$
$$\left. - \int_D^M u'(M - D - \phi + y)(\phi' + 1)\, dF(x) \right] dG(y)$$

and D^* is the optimal level for the deductible. Meyer and Ormiston [1998] have shown that if the price of this insurance is convex in the level of expected indemnification, then the second-order condition for this maximization is satisfied. It is assumed that an interior solution prevails—that is, the optimal D is defined by $U_D(D) = 0$.

To determine the effect on the optimal D of a change in background risk y, the same steps as in the previous section are taken. The first step is to define $m(y)$ as the expectation of the first-order expression taken only with respect to variable x. That is,

$$m(y) = \left[-\int_0^D u'(M - x - \phi + y)\phi' \, dF(x) - \int_D^M u'(M - D - \phi + y)(\phi' + 1) \, dF(x) \right].$$

While this expression is more complicated to write down than for coinsurance analysis, the expression does exhibit one very important and simplifying attribute that is the same as for coinsurance. As with coinsurance, it is the case that background risk y only appears in $u'(Z)$. Hence, all of the derivatives of $m(y)$ are related to $m(y)$ in that they differ only in the order of differentiation for the utility function. For instance,

$$m'(y) = \left[-\int_0^D u''(M - x - \phi + y)\phi' \, dF(x) - \int_D^M u''(M - D - \phi + y)(\phi' + 1) \, dF(x) \right],$$

which is identical to the expression for $m(y)$ except for the order of the differentiation applied to $u(Z)$. This implies that using the same methods as for coinsurance, one can show that $E_y m'(y) = -E_y r(Z) m(y)$, as well as other similar relationships among higher-order derivatives of $m(y)$. In short, one can show that this $m(y)$ derived in the deductible insurance model satisfies the properties listed in Lemma 1 and Lemma 2. Hence, we state without formal proof the following theorem.

Theorem 4: *If preferences satisfy $r(Z) \geq 0$ and $r'(Z) \leq 0$, then simple FSD deteriorations in y lead to decreases in D. If preferences also satisfy $\rho'(Z) \leq 0$, then strong and simple increases in background risk lead to a decrease in D.*

4. Conclusion

This work has shown that if one considers changes in background risk that lie in any one of three broad categories of change that have been identified in the literature, the conditions imposed by Eeckhoudt and Kimball to determine the effect of an introduction of background risk are sufficient to determine the effect of these changes as well. Because these conditions are less restrictive than those identified by Eeckhoudt, Gollier, and Schlesinger as necessary for determining the effect of all FSD or SSD changes in y, we have illustrated again the potential to tradeoff assumptions concerning utility for those applying to the change in risk under consideration.

Appendix

Lemma 1: *If preferences satisfy $r(Z) \geq 0$ and $r'(Z) \leq 0$, then at the optimal θ there exists a y^* such that $m(y) \geq 0$ for all $y \leq y^*$ and $m(y) \leq 0$ for all $y \geq y^*$. Furthermore, $m'(y) \leq 0$ whenever $m(y) \geq 0$. Also $E_y m'(y) \leq 0$.*

Proof. First, we show that there exists a Z_0 that does not depend on x, such that $m'(y) \leq -r(Z_0)m(y)$ for all y. Recall that $m'(y) = E_x u''(Z)[I(x) - p] = -E_x r(Z)u'(Z)[(I(x) - p]$. Because $I(x)$ is nondecreasing and continuous, there exists a x_0 in $[0, M]$ such that $[I(x_0) - p] = 0$, and hence, $[I(x) - p] \leq 0$ for $x \leq x_0$ and $[I(x) - p] \geq 0$ all $x > x_0$. Define $Z_0 = M - x_0 + y$. Assuming DARA and recalling that $z_x \leq 0$, it is the case that $r(Z_0) \geq r(Z)$ for all $x \leq x_0$ and $r(Z_0) \leq r(Z)$ for all $x > x_0$. Consequently, $E_x[r(Z_0) - r(Z)]u'(Z)[I(x) - p] \leq 0$ or equivalently, $r(Z_0)E_x u'(Z)[I(x) - p] \leq E_x r(Z)u'(Z)[I(x) - p]$. The left side of this inequality is $r(Z_0)m(y)$ and the right side is $-m'(y)$. Hence, $m'(y) \leq -r(Z_0)m(y)$. This implies that $m'(y) \leq 0$ whenever $m(y) \geq 0$.

At the optimal θ, $E_y m(y) = 0$. Therefore, $m(y)$ is always zero, or it must change sign. At any y^* in $[c, d]$ such that $m(y^*) = 0$, $m'(y^*) \leq 0$—that is, $m(y)$ is nonincreasing whenever it is zero. Therefore, it is impossible for $m(y)$ to change from negative to positive as y increases. When $m(y)$ is identically zero, $m'(y) \leq 0$ for all y holds trivially. For the case where $m(y)$ does change sign as y increases, the sign change must be from positive to negative and can only occur one time. Finally, to see that not only does $m(y)$ change sign from positive to negative, on average it is decreasing, observe that $E_y m'(y) \leq -E_y r(Z_0)m(y) \leq 0$. This follows from the combination of two facts. First, at the optimal θ, $E_y m(y) = 0$ and $m(y)$ changes sign from positive to negative, and second, under DARA, $r(Z_0)$ is decreasing in y. This allows one to conclude that $E_y r(Z_0)m(y) \geq 0$, and therefore $E_y m'(y) \leq 0$. □

Lemma 2: *If preferences satisfy $\rho(Z) \geq 0$ and $\rho'(Z) \leq 0$, then at the optimal θ there exists a y^{**} such that $m'(y) \leq 0$ for all $y \leq y^{**}$ and $m'(y) \geq 0$ for all $y \geq y^{**}$. Furthermore, $m''(y) \geq 0$ whenever $m'(y) \leq 0$. Also $E_y m''(y) \geq 0$.*

Proof. The proof follows the steps in the proof of Lemma 1. Recall that $m''(y) = E_x u'''(Z)[I(x) - p] = -E_x \rho(Z)u''(Z)[I(x) - p]$. For the Z_0 identified in the proof of Lemma 1, it is the case that $\rho(Z_0) \geq \rho(Z)$ for all $x \leq x_0$ and $\rho(Z_0) \leq \rho(Z)$ for all $x > x_0$. Consequently, $E_x[\rho(Z_0) - \rho(Z)]u''(Z)[I(x) - p] \geq 0$ or equivalently, $\rho(Z_0)E_x u''(Z)[I(x) - p] \geq E_x \rho(Z)u''(Z)[I(x) - p]$. The left side of this inequality is $\rho(Z_0)m'(y)$, and the right side is $-m''(y)$. Hence, $m''(y) \geq -\rho(Z_0)m'(y)$. This implies that $m''(y) \geq 0$ whenever $m'(y) \leq 0$. $m'(y)$ is always negative, or it must change sign from negative to positive. To see this, observe that at any y^{**} in $[c, d]$ such that $m'(y^{**}) = 0$, $m''(y^{**}) \geq 0$—that is, $m'(y)$ is nondecreasing whenever it is zero. Therefore, it is impossible for $m'(y)$ to change from positive to negative as y increases. When $m'(y)$ is always zero or negative, $m''(y) \geq 0$ for all y. For the case where $m'(y)$ does change sign as y increases, the sign change must be from negative to positive and can only occur one time.

Finally, to see that not only does $m'(y)$ change sign from negative to positive, on average it is increasing, observe that $E_y m''(y) \geq -E_y \rho(Z_0) m'(y) \geq 0$. This follows from the fact that at the optimal θ, $E_y m'(y) \leq 0$ and $m'(y)$ changes sign from negative to positive. Hence, with $\rho(Z_0)$ decreasing in y, one can conclude that $E_y \rho(Z_0) m'(y) \leq 0$ and therefore that $E_y m''(y) \geq 0$. □

Acknowledgments

The authors wish to thank two reviewers and also the participants of the 1997 Risk Theory Seminar, Tuscaloosa, Alabama, for helpful comments.

Notes

1. Mayers and Smith [1983], Doherty and Schlesinger [1983a, 1983b], Turnbull [1983], Briys, Kahane, and Kroll [1988], and Eeckhoudt and Kimball [1992] are among those who do consider insurance demand when other risks are present.
2. The term *coinsurance* is often used to describe only the case where $I(x) = x$. General forms for $I(x)$ are allowed here.
3. Background risk can be composed of several components as long as the sum of these random variables is exogenous and independently distributed from the insurable risk.
4. The price for insurance must be more than expected indemnification or $\theta = 1$ is the optimal solution for all risk-averse decision-makers. For prices greater than expected indemnification, strictly positive risk aversion is needed to obtain solutions where $\theta > 0$.
5. They actually do this for the case where the indemnification function is $I(x) = x$, but our generalization shows that their result holds for general forms for $I(x)$ as well. Also, Gollier and Pratt [1996] weaken the assumptions on utility needed to determine the effect of an introduction of risk using the concept of risk vulnerability.
6. The function $E_x u'(Z)[I(x) - p]$ depends on M, p, and θ as well as y, but since the properties with respect to y are the main concern, for simplicity these variables are suppressed in the notation.
7. Another specialized FSD change for $G(y)$ is the monotone probability ratio (MPR) change identified by Eeckhoudt and Gollier [1995]. The effect of this change on the demand for insurance can be determined using an additional property of $m(y)$. The demonstration of this property—that the expectation of $m'(y)$ taken for all values of y less than a specified value is always negative—is available from the authors. Since MPR changes include all monotone likelihood ratio changes, that category of FSD change is taken care of as well.
8. Transformations that are increases in risk can be decomposed into an FSD deterioration and an FSD improvement where the improvement results from adding a constant to y to restore its mean to the original value. Assuming only DARA, these two changes individually have the opposite effect on θ. The total effect is signable only under stronger conditions on utility such as those assumed in Theorem 3.

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