Errata

Michio Jimbo and Jun'ichi Shiraishi: A Coset-Type Construction for the Deformed Virasoro Algebra (*Lett. Math. Phys.* **43**(2) (1998), 173–185) (Received: 10 February 1998)

In the Letter [1], Subsection 4.3, the proof of (c3) contains a gap concerning the commutability of the operations of taking residues. We give here a corrected proof.

Let

$$G^{(\lambda_0, \dots, \lambda_n)}(z_1, \dots, z_n) = \prod_{i < j} \eta(z_j/z_i) \psi^{(\lambda_0, \lambda_1)}(z_1) \dots \psi^{(\lambda_{n-1}, \lambda_n)}(z_n). \quad (1)$$

The following are direct consequences of Proposition 3.1:

LEMMA 1. (1) $G^{(\lambda_0, \dots, \lambda_n)}(z_1, \dots, z_n)$ is holomorphic except for simple poles at $z_j/z_i = p^{*-s}x^2$ ($i < j$, $s \geq 0$).

(2) We have

$$\begin{aligned} & G^{(\dots, \lambda_{k-1}, \lambda_k, \lambda_{k+1}, \dots)}(\dots, z_k, z_{k+1}, \dots) \\ &= \left(\frac{z_k}{z_{k+1}} \right)^{\frac{r}{2(r-1)}} \sum_{\mu} G^{(\dots, \lambda_{k-1}, \mu, \lambda_{k+1}, \dots)}(\dots, z_{k+1}, z_k, \dots) \times \\ & \quad \times W \left(\begin{array}{cc|c} \lambda_{k+1} & \lambda_k & z_k \\ \mu & \lambda_{k-1} & z_{k+1} \end{array} \right). \end{aligned}$$

(3) As $z_k \rightarrow x^{-2}z_{k+1}$ we have

$$\begin{aligned} & G^{(\lambda_0, \dots, \lambda_n)}(z_1, \dots, z_n) \\ &= \frac{\delta_{\lambda_{k-1}\lambda_{k+1}} c_{\lambda_k\lambda_{k+1}}}{1 - x^{-2}z_{k+1}/z_k} \prod_{i < k} \frac{(p^* \frac{z_{k+1}}{z_i}; p^*)_{\infty}}{(x^{-2} \frac{z_{k+1}}{z_i}; p^*)_{\infty}} \prod_{j > k+1} \frac{(p^* x^2 \frac{z_j}{z_{k+1}}; p^*)_{\infty}}{(\frac{z_j}{z_{k+1}}; p^*)_{\infty}} \times \\ & \quad \times \eta(x^2) G^{(\lambda_0, \dots, \lambda_{k-1}, \lambda_{k+2}, \dots, \lambda_n)}(z_1, \dots, z_{k-1}, z_{k+2}, \dots, z_n) + O(1). \end{aligned}$$

Our aim is to calculate

$$X = \text{res}_{z_1=x^{-2}z_2} f \left(\frac{z_2}{z_1} \right) T_{\pm}^{(\lambda)}(z_1) T_{\pm}^{(\lambda)}(z_2) \frac{dz_1}{z_1}. \quad (2)$$

In terms of the function (1), this can be written as follows:

LEMMA 2. *We have*

$$X = \operatorname{res}_{z_1=x^{-2}z_2} \varphi(z_2/z_1) \frac{dz_1}{z_1} \left(\operatorname{res}_{z'_1=z_1} H(z'_1, z_1, z_2) \frac{dz'_1}{z'_1} \right),$$

where

$$H(z'_1, z_1, z_2) = \operatorname{res}_{z'_2=z_2} G^{(\lambda, \lambda_{\pm}, \lambda, \lambda_{\pm}, \lambda)}(x^{r-2}z'_1, x^{-r+2}z_1, x^{r-2}z'_2, x^{-r+2}z_2) \frac{dz'_2}{z'_2},$$

$$\varphi(z) = \frac{x^{-2r+\frac{2r}{r-1}} (p^*x^{-2}z; p^*)_{\infty}}{\eta(p^*x^{-2})^2 (x^2z; p^*)_{\infty}} (1 - p^{*-1}z)(1 - z)(1 - p^*z).$$

LEMMA 3. *In the neighborhood of $z'_1 = z_1 = x^{-2}z_2$, $H(z'_1, z_1, z_2)$ is holomorphic except for simple poles at $z'_1 = z_1$, $z_1 = x^{-2}z_2$. As $z_1 \rightarrow x^{-2}z_2$, we have*

$$\begin{aligned} H(z'_1, z_1, z_2) = & -x^{r-2} \frac{c_{\lambda_{\pm}, \lambda} \eta(x^2)}{1 - x^{-2}z_2/z_1} G^{(\lambda, \lambda_{\pm}, \lambda)}(x^{r-2}z'_1, x^{r-2}z_2) \times \\ & \times \frac{1}{1 - p^*} \frac{(x^2 \frac{z_2}{z'_1}; p^*)_{\infty}}{(p^{*-1} \frac{z_2}{z'_1}; p^*)_{\infty}} \frac{(x^2; p^*)_{\infty}}{(p^*; p^*)_{\infty}}. \end{aligned} \quad (3)$$

Proof. Consider

$$G = G^{(\lambda, \lambda_{\pm}, \lambda, \lambda_{\pm}, \lambda)}(x^{r-2}z'_1, x^{-r+2}z_1, x^{r-2}z'_2, x^{-r+2}z_2).$$

From Lemma 1, the only poles of G in the neighborhood of $z'_1 = z_1 = x^{-2}z_2$, $z'_2 = z_2$ are

$$z'_1 = z_1, \quad z'_2 = z_2, \quad z_1 = x^{-2}z_2, \quad z'_1 = x^{-2}z'_2.$$

As $z_1 \rightarrow x^{-2}z_2$, we have

$$\begin{aligned} G = & \left(p^{*-1} \frac{z_2}{z'_2} \right)^{\frac{r}{2(r-1)}} \frac{c_{\lambda_{\pm}, \lambda}}{1 - x^{-2}z_2/z_1} G^{(\lambda, \lambda_{\pm}, \lambda)}(x^{r-2}z'_1, x^{r-2}z'_2) \times \\ & \times W \left(\begin{array}{cc|c} \lambda_{\pm} & \lambda & \frac{z_2}{z'_2} \\ \lambda & \lambda_{\pm} & \frac{z'_2}{z'_2} \end{array} \right) \times \\ & \times \eta(x^2) \frac{(x^2 \frac{z_2}{z'_1}; p^*)_{\infty}}{(p^{*-1} \frac{z_2}{z'_1}; p^*)_{\infty}} \frac{(x^2 \frac{z_2}{z'_2}; p^*)_{\infty}}{(p^{*-1} \frac{z_2}{z'_2}; p^*)_{\infty}} + O(1). \end{aligned} \quad (4)$$

Likewise, as $z'_1 \rightarrow x^{-2}z'_2$, we have

$$\begin{aligned} G &= \left(p^{*-1} x^2 \frac{z'_1}{z'_2} \right)^{\frac{r}{2(r-1)}} \frac{c_{\lambda_{\pm}, \lambda}}{1 - x^{-2} z'_2 / z'_1} G^{(\lambda, \lambda_{\pm}, \lambda)}(x^{-r+2} z_1, x^{-r+2} z_2) \times \\ &\quad \times W \left(\begin{array}{cc|c} \lambda_{\pm} & \lambda & x^2 \frac{z_1}{z'_2} \\ \lambda & \lambda_{\pm} & \end{array} \right) \times \\ &\quad \times \eta(x^2) \frac{(x^4 \frac{z_1}{z'_2}; p^*)_{\infty}}{(p^{*-1} x^2 \frac{z_1}{z'_2}; p^*)_{\infty}} \frac{(x^4 \frac{z_2}{z'_2}; p^*)_{\infty}}{(p^{*-1} x^2 \frac{z_2}{z'_2}; p^*)_{\infty}} + O(1). \end{aligned} \quad (5)$$

Taking the residue of (5) at $z'_2 = z_2$, we find that $H(z'_1, z_1, z_2)$ is regular at $z'_1 = x^{-2}z_2$. The behavior (3) is a consequence of (4). $\square$

Proof of (c3). From Lemmas 2 and 3, we can change the order of the residues as

$$X = \text{res}_{z'_1 = x^{-2}z_2} \frac{dz'_1}{z'_1} \left(\text{res}_{z_1 = x^{-2}z_2} \varphi(z_2/z_1) H(z'_1, z_1, z_2) \frac{dz_1}{z_1} \right).$$

Using (3) we find

$$\begin{aligned} X &= -x^{r-2} \varphi(x^2) \text{res}_{z'_1 = x^{-2}z_2} \frac{dz'_1}{z'_1} c_{\lambda_{\pm}, \lambda} \eta(x^2) \times \\ &\quad \times G^{(\lambda, \lambda_{\pm}, \lambda)}(x^{r-2} z'_1, x^{r-2} z_2) \frac{(x^2 \frac{z_2}{z'_1}; p^*)_{\infty}}{(p^{*-1} \frac{z_2}{z'_1}; p^*)_{\infty}} \frac{(x^2; p^*)_{\infty}}{(p^*; p^*)_{\infty}} \\ &= -x^{-r+\frac{2}{r-1}} c_{\lambda_{\pm}, \lambda}^2 \left(\frac{\eta(x^2)}{\eta(p^* x^{-2})} \right)^2 (1 - x^2)(1 - p^* x^2). \end{aligned}$$

After simplification, we obtain the result stated in Subsection 4.3. $\square$

There are also the following corrections.

(i) The right-hand side of Equation (13) should read

$$x^{2(\Delta_{\mu} - \Delta_{\lambda}) \mp 1/2} g_{\lambda}^{\mu} \delta_{v_{\lambda}} \times \text{id}, \quad \text{for } \mu = \lambda_{\pm}.$$

(ii) Line next to Equation (23), the formula for $c_{\mu\lambda}$ should read

$$c_{\mu\lambda} = x^{2(\Delta_{\mu} - \Delta_{\lambda}) \mp 1/2} g g_{\lambda}^{\mu}, \quad \text{for } \mu = \lambda_{\pm}.$$

(iii) Third line above Subsection 4.3: this equation should read

$$\frac{1}{(1-x^{-2}z)^2} \sum_{\mu, \nu} b_{\mu, \lambda} b_{\nu, \lambda} c_{\mu, \lambda} c_{\nu, \lambda} W \left(\begin{matrix} \nu & \lambda \\ \lambda & \mu \end{matrix} \middle| x^{-4} \right).$$

Acknowledgement

We thank T. Miwa for critical comments on the Letter [1].

References

1. Jimbo, M. and Shiraishi, J.: A coset-type construction for the deformed Virasoro algebra, *Lett. Math. Phys.* **43**(2) (1998), 173–185.