CORRIGENDUM TO “ON THE DISTRIBUTION OF PRIMES IN INTERVALS OF LENGTH $\log^\theta N$ ”

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After further investigations on the subject we found that our method of proving Theorem 2 in [1] actually works for intervals of length $\log^\theta N$ with a sufficiently large exponent θ (we give an explicit value below), in contrast to $\theta > 1$ as was mistakenly stated.

First we give the correct statements of Theorem 2 and Corollary 2 of [1].

THEOREM 2. *Let $\varepsilon > 0$, $H \geq L^{311+\varepsilon}$ and $1 \leq K \leq L^{1/2}$. Then*

$$J(N, H) \ll H^2 \sum_{j=1}^J H_j^{-2} I(N, T_j) \\ + NH^2 \left(\frac{1}{K^2} + \left(\frac{KL^{1/2}}{H} \right)^{\frac{6}{155}} L^{12} + \frac{HL^2}{N \log^2 H} \right).$$

COROLLARY 2. *Let $\varepsilon > 0$, $0 < \delta < 1/8$; then for $H \geq L^{311+\varepsilon}$ and $H = o(N)$ we have*

$$I(N, T) = o\left(\frac{N^3}{T^2 L}\right) \quad \forall T : \frac{N^\delta}{50} \leq T \leq \frac{NL^{1/2}K}{100H} \Rightarrow J(N, H) = o(NH^2),$$

where $K \rightarrow \infty$ and $K = o(L^\varepsilon)$ (as $N \rightarrow \infty$).

Corollary 2 is an immediate consequence of Theorem 2. The proof of Theorem 2 follows the steps in [1] up to the point where zero density estimates are used (p. 165, line 9).

However, we failed to see that Theorem 9.19(A) of [3], i.e.

$$N(\sigma, T) \ll T^{3/2-\sigma} \log^5 T,$$

is too weak near $\sigma = 1$. Indeed, we use this bound only in the range $\frac{1}{2} \leq \sigma \leq \frac{152}{155}$, while we use Theorem 11.3 of [2], i.e.

$$N(\sigma, T) \ll T^{35(1-\sigma)/36} \log^{16} T,$$

in the range $152/155 \leq \sigma \leq 1$.

Equation (26) of [1] thus modifies to

$$(26) \quad \sum_{j=1}^J F_1(j) \ll N L^{12} \sum_{j=1}^J H_j^{-6/155} + N e^{-cL^{1/4}} \ll N L^{12} \xi^{\frac{6}{155}} + N e^{-cL^{1/4}}.$$

Finally, gathering the estimates (11)–(14), (27) of [1] and (26) above we get

$$\begin{aligned} J(N, H) &\ll \frac{NL}{\xi^2} + H^2 \sum_{j=1}^J H_j^{-2} I(N, T_j) + NH^2 \left(\frac{HL^2}{N \log^2 H} + \exp(-cL^{1/4}) \right) \\ &\quad + NH^2 \left(L^{12} \xi^{\frac{6}{155}} + L\xi^2 + \frac{L^{18}}{N^{2\delta/5}} \right), \end{aligned}$$

whence, choosing $\xi = \frac{KL^{1/2}}{H}$ with $1 \leq K \leq L^{1/2}$ we have

$$\begin{aligned} J(N, H) &\leq H^2 \sum_{j=1}^J H_j^{-2} I(N, T_j) \\ &\quad + NH^2 \left(\frac{1}{K^2} + \left(\frac{KL^{1/2}}{H} \right)^{\frac{6}{155}} L^{12} + \frac{HL^2}{N \log^2 H} \right) \end{aligned}$$

and Theorem 2 follows.

References

- [1] G. Coppola and A. Vitolo, On the distribution of primes in intervals of length $\log^{\theta} N$, *Acta Math. Hungar.*, **70** (1996), 151–166.
- [2] A. Ivic, *The Riemann Zeta Function*, Wiley Interscience (New York, 1985).
- [3] E. C. Titchmarsh, *The Theory of the Riemann Zeta Function*, second edition, University Press (Oxford, 1986).

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