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Methodological issues: gender related indices

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Abstract
Gender related Index (GRI) is a singled valued function combining selected dimensions containing different number of components. Measurement of multidimensional GRIs involves measurement issues for meaningful aggregations and comparisons over time and space. The paper proposes gender inequality through multiplicative aggregation (GRI_{GM}) and gender similarity index (*GSI*) by cosine similarity, avoiding scaling, finding weights and satisfying desired properties. Inverse relationship derived between GRI_{GM} and *GSI*. Each proposed index reduces level of substitutability, effect of outliers. Each facilitates aggregation of country-wise scores to find world average, statistical testing of equality of mean of two countries or mean of a country at two time-periods using *Y*-scores obtained from each measure by linear transformations where $Y \in [0, 100]$ and $Y \sim N(\mu, \sigma^2)$. Each index can be used for ranking and classification of countries. GSI can indicate current distance of a country from the goal of zero gender inequality. GRI_{GM} can assess progress/decline in successive periods for women and separately for men. GRI_{GM} is preferred for satisfaction of time-reversal test, easy identification of critical areas and contribution of the components or dimensions to the index. Empirical verification of the proposed indices and their properties including robustness could be taken as future studies.

Keywords Measurement · Composite index · Gender inequality · Multiplicative aggregation · Cosine similarity

JEL Classification D63 · J16 · O15

1 Introduction

Gender equality is a basic human right (Article 1 of the 1948 Universal Declaration of Human Rights of United Nations, http://www.un.org/en/universal-declaration-human-rights). Need to achieve gender equality is well acknowledged at national and international levels. Elimination of All Forms of Discrimination against Women was adopted in 1979 by United Nations.

(http://www.ohchr.org/Documents/ProfessionalInterest/cedaw.pdf). Promoting equality and empowering women are included in goals of the Millennium Development Goals (MDGs, http://www.un.org/millenniumgoals), and also addressed in other international forum like Addis Ababa Action Agenda [1], 2030 Agenda for Sustainable Development [2], which used broader view of gender equality than the MDGs, and aimed at ending all discriminations and violence against women everywhere. Aims of the SDGs (http://www.un.org/sustainable development/sustainable-development-goals/) include among others total elimination of harmful practices on women like forced marriages, genital mutilation, universal access to sexual and reproductive health services, recognition of contributions of unpaid

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and domestic work, full access and participation of women in economic, political, social and public life at all levels, etc. (https://unstats.un.org/sdgs/metadata/).

UNCTAD [3] Found causal relationships between gender equality and economic growth. Improved gender equality benefits growth of education, economy, employment and access to finance [4]. Denying accessibility of half of the population i.e. women in the labour equity, financial equity and governance result in significant loss to a country, which was estimated at the level of 42–47 billion USD per annum for Asian and Pacific Ocean countries [5]. Reduced gender inequality helps better utilization of human capital and augments economic growth [6]. However, based on economic growth and changes in gender inequality (weighted by population size), [7] found small and inconsistent correlations.

Gender effect exists also on pattern of investment of income. While men spend more on entertain, women make more investments in fields like children`s education and health, which result in higher number and higher proportion of healthy and educated people for the country in future periods and thus contributing positively to the economy of the nation [8].

Equality in opportunities and potential to participate in the economy to women and girls can act as a catalyst for a strong engine of growth for more resilient, sustainable, and inclusive economies marching forward. Gender equality can boost private and public sector performance, and reduce income inequality. Major dimensions of gender equality to foster global prosperity are opportunities, participation in decision-making, access to resources, education and employment [9]. However, Global Gender Gap Report 2020 [10] indicated gender gaps in health, education and policy areas, over all forms of economic participations. Through multiple regression analysis [11], observed that gender does influence quality of life (QoL) and existence of gender inequality in five countries (China, India, Russia, South Africa, and Ghana), primarily due to influencing factors like lower social status, limited income, more barriers concerning access to health care and higher responsibilities in household chores, etc. Providing opportunities to women in public and private sectors does not mean fewer opportunities for men since *rising tide lifts all boats* and improved gender equality can significantly improve life satisfaction for all people of a country, both men and women [12].

Impact of economic crisis resulting from the COVID-19 pandemic on women and men were different in the European Union (EU) especially in access to the economy which resulted in higher female unemployment rates and likelihood of poverty for women in the EU [12]. Women have been redundant due to the impossibility of transposing the female-dominated sectors to a teleworking modality. Frequent interruptions faced by mothers (from children and partner) while "Working from Home", created difficulties to maintain their performances which affected their career progression and pay, implying increased gender gap in pay and pensions. In addition, increased numbers of incidences of violence, particularly from intimate-partner have given rise to increased number of women victims [13].

For a single dimension *X*, gender inequality may be given by Gini coefficient $|S_F - S_M|$ where S_F and S_M denote share of females and males respectively in *X* assuming same population sizes of male and female [14]. However, problem increases when instead of a single dimension, vectors of dimensions with varying order for different gender related indices (GRIs) are considered along with uneven population growths across countries. It is difficult to derive direct relationship between per capita GDP and $|S_F - S_M|$ since development of an economy depends on a host of factors other than gender equality.

The human development report of United Nations used two indices namely Gender and Development Index (GDI) and Gender Empowerment Measure (GEM) till 2010. Now, Gender Inequality Index (GII) is reported in lieu of GDI and GEM. While calculation of GDI were based on the women`s data only, GEM reflected inequality between men and women in three directions: (i) participation in political processes and the opportunity to make decision, calculated by proportion of men and women parliamentarians; (ii) participation in economy and the opportunity to make decision, calculated by proportion of (a) man and woman legislations, (b) senior officials and managers, (c) technical/qualified labour force; (iii) economic resources, calculated by the compensation, earned by the women and men in US Dollars with the parity of purchasing power.

A number of GRIs have been developed for comparing countries and orienting policy-decisions, like Gender Inequality Index (GII) by UNCTAD, Gender Gap Index (GGI) by WEF, Social Institution and Gender Index (SIGI) by OECD, New Gender Equality Index (GEI) by the African Development Bank (AfDB), etc. However, such GRIs differ in dimensions covered, components/variables, procedure of normalizing the input variables, and aggregation methods. Measurement and evaluation of multidimensional GRIs involve number of methodological issues for meaningful comparisons over time and space and ranking the countries [15]. Country-wise GRI scores and associated rankings are sensitive to methodological changes and may explain different proportion of variation of economic aspects [16]. Significant variation exists in country scores and ranks of countries for GII, GGI and SIGI even for the same time period [17].

A GRI is composite index (CI) which is a single valued function *f* combining *n*-number of selected components $X_1, X_2, \ldots\ldots, X_n$ where dimensions have different number of component variables in different units. CI maps the vector $(X_1, X_2, \ldots\ldots, X_n)^T$ on the real line, to find the position of a country. However, $\mathrm{GRI} = f(X_1, X_2, \ldots\ldots, X_n)$ involves several stages of construction where input variables could be pure numbers (say Maternal mortality ratio) or in percentages (like Share of parliamentary seats by women) or have different units; discrete or continuous, cardinal or ordinal. The chosen components are aggregated to obtain dimension scores which may be independent or correlated with different degrees. Such dimension scores are aggregated suitably to obtain GRI for countries. Variations in each stage result in different values of GRI with different properties. Possibilities of misinterpretation and manipulation of CI cannot be ruled out and questions of accuracy, reliability, appropriateness, etc. need to be addressed [18].

Country-wise GRI scores are combined to find the world average to assess extent by which women and men are converging or diverging in terms of the chosen dimensions. It is necessary to ensure meaningful aggregation at each stage and inequalities in one dimension (or a country) do not offset another. Possibility of offsetting effects is more for arithmetic aggregation of variables or dimensions following different (and unknown) distributions. For example, if M_L and F_L denote literacy rate of male and female respectively, $M_L > F_L$ for 50% of the countries under observations and for the rest, $F_L > M_L$, then the world average of gender inequality with respect to literacy rate could be zero.

For two random variables *X* and *Y* with probability density function f_X and f_Y respectively, the convolution of the two marginal pdfs $Z = X + Y$ is $f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z - x)dx$. Similar convolution for discrete random variables can be derived.

Un-weighted arithmetic mean (AM) of countries may be unreliable; since small countries get undue weights and contribution of populous countries get deflated [7]. In addition, parametric analysis and statistical inferences require meaningful use of the operations (say addition) with known distribution of the variables [19]. Found that country-level measures of gender equality are inconsistent and empirical findings are confounded by outliers and existing GRIs fail to test statistical significance of two countries.

Shortcomings of the existing GRIs limit meaningful comparisons over time and space, assessing responsiveness or tracking of the path of fluctuations of the index for a country across time, effect of initiatives taken or policy measures adopted. Construction of meaningful and sound GRI satisfying desired properties is the focus area.

The paper discusses the major methodological issues related to the various measures of GRIs, which is followed by proposed approaches of construction of better GRI and gender similarity index satisfying desirable properties and finally, the salient points of observations and benefits of the proposed method are discussed.

2 Current GRIs (Illustrative)

GII is a composite index (CI) which estimates women's unfavorable conditions in three dimensions and reflects the extent of inequality in achievements between men and women. It attempts to estimate the gender-based disadvantage in the following dimensions and components:

Reproductive Health Adolescent birth rate (ABR), Maternal mortality ratio (MMR)

Female Empowerment Share of parliamentary seats (PR) by each gender and proportion of adults with at least secondary education (SE).

Labour force participation rate (LFPR) For female and male

The dimensions are aggregated as follows:

$$\text{For women group}: G_F = \sqrt[3]{\left[\left(\frac{10}{MMR}.\frac{1}{ABR}\right)^{\frac{1}{2}}.\left(PR_F.SE_F\right)^{\frac{1}{2}}.LFPR_F\right]} \tag{1}$$

$$\textit{and for men group}: G_M = \sqrt[3]{\left[\left(PR_M \cdot SE_M\right)^{\frac{1}{2}} \cdot LFPR_M\right]} \tag{2}$$

Here, *MMR* is rescaled to account for the truncation of MMR at 10. The GII score of a country is obtained first by aggregating Eq. (1) and (2) (treating the genders equally) followed by aggregating the indices across dimensions as

$$G_{F,M} = \sqrt[3]{[\left(\overline{Health}.\overline{EmpowermentLFPR}\right]}$$

where $\overline{Health} = \frac{\sqrt{\frac{10}{MMR}.\frac{1}{ABR}}+1}{2}$; $\overline{Empowerment} = \frac{(PR_F.SE_F)^{\frac{1}{2}}+(PR_M.SE_M)^{\frac{1}{2}}}{2}$ and $\overline{LFPR} = \frac{LFPR_F+LFPR_M}{2}$.

GII score of a country ranges between 0 and 1. GII = 0 implies absence of gender equality and men and women are equally valued and favoured. GII = 1 indicates maximum inequality between men and women in all dimensions.

Higher GII implies greater gender disparities and more losses to human development. Average GII score of the world is 0.492 may be interpreted as loss in achievement across the chosen dimensions is 49.2% due to gender inequality. Empirically, countries with lower GII tend to show higher or increasing GDP. However, it is difficult to prove that additional GDP is due to lower GII only.

Calculation of GII involves data for each gender, finding geometric mean for each dimension separately for men and women, which is followed to unite average direction of men and women by harmonic mean. Harmonic mean reflects lower GII values for greater difference between men and women.

3 Limitations of GII

- Considers women-specific variables (MMR, AFR) with no counterpart for men
- Increase in MMR or AFR widen gender gap
- Does not consider wage differences and dimensions like unpaid works, asset ownership, childcare support, gender-based violence and participation in community decision-making by women etc.
- Maximum value of MMR and AFR are taken as 1 which is rather unrealistic.
- Does not consider direction of the gender gaps, so it is not possible to find relative position of men vis-à-vis women from the GII values [20].
- Parliamentary seats by women at national level exclude participations of women at local governments, community and public life, etc.
- Labour market dimension excludes information on unpaid works by women, like housekeeping, taking care of children and other family members, etc. at the cost of cutting leisure time and increased stress and physical exhaustions.
- Excludes the level of development of the country and thus, may not facilitate international comparisons.
- Method of combining the dimension scores may fail if score of a dimension is zero (say, no representation of women in parliamentary seats)

Gender Gap Index (GGI) estimates difference between women and men with respect to participation in economy, involvement in education, health/life safety and political involvement. The measure targets outcome variables and ranks countries accordingly rather than women's empowerment. GGI considers quantitative and qualitative data and focuses on measurement of gaps in outcome variables and not on input and policy variables or levels [6]. Thus, GGI ranks countries with gender equality and not with women's empowerment. It also helps in tracking progress over time. GGI reflects extent of gender disparities in the following four dimensions:

- *Economic participation & opportunity*: Labour force participation, Wage inequality, estimated earned income, legislators, senior officials and managers, professionals and technical workers of female over male.
- *Educational attainment*: Literacy rate, net enrolment rate of primary, secondary enrolment rate, gross tertiary enrolment rate of female over male.
- *Health and survival*: Sex ratio at birth (No. of female births ÷ No. of male births), Ratio of life expectancy of female over male.
- *Political empowerment*: Shares of seats in parliament, ministerial level by female and the same for male, Head of state position occupied by female over male during last 50 years.

4 Limitations of GGI

Component variables like labour force participation in percentages, economic participation as percentage of professionals; technical workers, etc. are in ratio scales. Results of survey on wage equality for similar work are in ordinal scales. Earned incomes are estimates.

Ratings reflect current situation of a country with respect to another country and are relative. Comparison of such relative ratings and points of GGI may lead to inaccurate conclusions. Equality in salary (only indicator of economic involvement and opportunities direction) is considered as qualitative, assessed by expert interviews, unlike other indicators. Combining cardinal and ordinal variables (with different values of reliability and validity) are problematic. Goodness of estimation may be questioned. Arithmetic mean (AM) of cardinal and ordinal variables may not be meaningful since higher rating of equality in salary may compensate lower values of cardinal components.

Summative scores of ordinal data with unknown distance between successive categories are not additive [21, 22] and hence, AM of ordinal scores may not be meaningful [23]. *K*-point scales ($K = 3, 4, 5, \ldots$ and so on) differ in mean, SD and influence item/test parameters more by value of *K* than the underlying variable [24]. Mean and variance get increased as *K* increases [25]. Moreover, *K*-point scales result in different distributions of test scores and different values of scale qualities like discriminating power, reliability, validity, etc. [26].

Converted data to ratios of female to male are truncated at equality benchmark (EB) = 1, except Number of female births -over-Number of male births (EB = 0.994) and Life expectancy at birth (EB = 1.06) [27].

Dimension scores in [0, 1] are taken as weighted sum where weights to components of a dimension are obtained with SD in denominator. Weights using SD are not invariant under change of scale. Weighted sum depends heavily on method of finding weights. Its assumption that weights reflecting relative importance of dimensions/components are same for different countries may not be justified.

The Pena's method (known as P2-distance or DP2 method) satisfies many the desirable properties. It is an iterative method assigning weights to the partial indicators depending on their correlation with a global index [28]. P2 distance methodology in gender equality studies has been used by [29] to study Gender Equality Index, and observed that countries in the northern Europe are more egalitarian than the countries in eastern Europe [30]. Considered enrollment of women in technology-related programs at the tertiary level of education and for female STEM graduates to investigate reduction of gender-gap, if any during 2011–2020 in 29 European countries. The authors found low level of reduction of gender inequalities in technology-related education which resulted in increasing gender gaps on high-tech employment. A measure for classification of female education and child health was proposed using weights derived from P2-distance with estimated values of relative importance of the variables for ordering of the partial indicators [31]. However, DP2 is very sensitive to the order in which the constituent variables (whose linear aggregation yields the synthetic indicator) are arranged [32] and thus, DP2-based composite (synthetic) indices are indeterminate and arbitrary. Triangle inequality is not satisfied by DP2 i.e. $d(A, B) \neq d(T, A) + d(T, B)$ where *T* denotes the target vector [33]. Moreover, DP2 weights are not a convex linear combination and do not behave efficiently when indicators are poorly correlated with the composite index.

As per [34], no weighting system is above criticism. GGI-score of a country in [0, 1] is the AM of the dimension scores. Selection of weights to component variables and GGI as aggregation of dimensions scores with equal weights to the dimensions are not beyond criticism.

Social Institutions and Gender Index (SIGI): Unlike other measures of gender inequality, SIGI considers the factors underlying gender inequality, measuring social institutions (expressed by societal practices and legal norms) which discriminate against women. SIGI is computed as an un-weighted CI reflecting degree of discrimination against women in social institutions. It is comprised of following dimensions (sub-indices) and components:

*Discrimination in the family (*DFC): Minimum age of marriage for girls and separately for boys, Percentage of girls in the age group 15 to 19 years who are married, divorced, widowed, in informal union; legal rights on inheritance of moveable and non-moveable assets, to initiate divorce, decision-making and responsibilities within the household.

*Restricted physical integrity(*RPI) Legal protection of women from (i) violence, intimate partner violence, sexual harassment including rape, in a comprehensive approach, (ii) reproductive health and rights and family planning; Percentage of women in the age group 15–49 years who are married or in-union relationships and want to stop or delay pregnancy but not using any contraception method, consider a husband is justified in hitting or beating his wife and aware of female genital mutilation and believe in continuation of the practice; Number of male children per 100 female children in the age group zero to four years.

*Restricted access to productive and financial resources (*RCL) Uniform legal rights to secure access to land assets, access to non-land assets, opening bank account and obtain credit from financial institution; to have equal opportunities in the workplace; Percentage of male in the population aged 15 years and more having account at a financial institution, employed in management; Percentage of population who oppose an woman in their family to have an outside paid job.

While SIGI = 0 implies complete equality, SIGI = 1 means complete inequality. Most of the dimension scores of SIGI are in [0, 1] depending on level of discrimination. But for political representation, the range is 0–0.5 since equality is achieved if at least 50% of parliament members are women. Components based on legal issues are scored considering all legal aspects relating to civil, religious, customary and traditional laws. Variables like Parental authority (PA) consider more than one variable as average of PA during marriage and PA after divorce.

Qualitative variables in SIGI (like Laws on: domestic violence, rape, sexual harassment, etc.) are scored using coding manual. Example: 0: Same rights in law and practice are enjoyed by women as well as by men; 0.25:Non-implementation of legislations; 0.5: Discrimination against women by customary laws and practices; 0.75: Contradictory or non-specific or limited scope and definition of legislations; 1: Same rights in the legal framework are not enjoyed by men and women. Scores based on such codes are not equidistant and latent degree of discrimination between 0 and 0.25 may be different between (say) 0.50 and 0.75.

Dimension score involves aggregation of the variables as weighted sum with weights from Principal component analysis (PCA) or Min–Max transformation.

SIGI is a computed as an un-weighted average of square of dimension scores by

$$SIGI = \frac{1}{5}\left(DFC^2 + RPI^2 + Sonbias^2 + RRA^2 + RCL^2\right)$$

The squared terms allow only partial compensation between dimensions with high and low inequality and more egalitarian performance supporting principle of equality in dimensions [35].

5 Limitations of SIGI

Equal weight are assigned to the dimensions despite different values of inter-dimension correlations, dimension-total correlations and different factor loadings (weights) from PCA, each showing dimensions have different importance. Weights to components/dimensions could be found by different methods with different assumptions like PCA, Factor analysis, Budget allocation process, Analytical hierarchy process, Benefit of the doubt approach, etc. Evaluations of the components/dimensions are different for different weighting methods and produce different results [36]. Found that no weight or equal weights are wrong.

To bring all variables as unit-less in the interval [0, 1], the ratios are normalized by $I_X = \frac{X - X_{Min}}{X_{Max} - X_{Min}}$. However, for components like MMR, unmet need for contraception, etc. $I_X = \frac{X - X_{Max}}{X_{Min} - X_{Max}}$ since higher value of MMR indicates poorer maternal health. I_X depends on the extreme values which may be unreliable outliers. In addition, X–I_X curve is not linear [37]. I_X-score of a country is a relative measure, depends on the performance of other countries and may increase in a subsequent period exclusively due to poor performance of other countries. Change in X_{Min} may change marginal rates of substitution and thus change ranks of the countries [38]. Difference in variance is not fully eliminated [39].

Avoiding normalization and selection of weights, the variables may be converted to unit-less numbers in the interval [0, 1] by considering the ratios $\frac{F_i}{M_i}$ where F_i and M_i denote achievement levels in i-th variable for female and male respectively.

SIGI scores classify countries into five groups depending on discrimination levels. However, test of significance of $\frac{Withinclassvariance}{Betweenclassvariance}$ may be required to assess effectiveness of classification.

New Gender Equality Index (GEI): considers gender-differentiated outcomes and also social institutions that contribute to gender gaps, the institutional dimension, and the social and economic dimensions, as factors in the gender gap. Clearly, data for GEI are obtained from many sources. It quantifies gender gaps with respect to access to resources and opportunities and in laws of a country. Countries are ranked based on such gender gaps and not on achieved levels of development. GEI is composed of 3 dimensions, 6 sub-categories and 38 component variables. Each component variable relates to specific gender gap that contributes to gender inequality in development outcomes.

Data for GEI are drawn from international organizations like International Labour Organization, World Health Organization, World Economic Forum, Organization for Economic Co-operation and Development, UNESCO Institute for Statistics, Global Findex, Inter-Parliamentary Union, and Women, Business and the Law. Dimensions, sub-dimensions and 38 component variables of GEI are given in Table 1:

GEI-score of a country in [0, 100] is AM of the dimension scores, where dimension scores are taken as AM of the rescaled component variables obtained by normalizing the truncated ratios by *Min–Max* transformation i.e. $Z = \frac{X - X_{Min}}{X_{Max} - X_{Min}}$ where $0 \leq Z \leq 1$. Such transformation indicates relative performance (not absolute performance) and is dependent heavily on X_{Min} and X_{Max} which are likely to be unreliable outliers. Moreover, gain in Z per unit increase in X is not uniform for such a normalization procedure. Change in value of X_{Min} may change ranking and relative valuations. The GEI score of a country in a given year is calculated as

$$GEI = \frac{(\text{economic opportunity index} + \text{social development index} + \text{laws and institutions index})}{3}$$

Equal weights of the three dimensions may be criticized for compensatory approach, no differentiation of essential and less important dimensions [36]. Enactment of progressive law without distinguishing men and women may improve Laws and Institution index and the improved value may increase GEI even if economic opportunities of women decline.

Barnat et al. [17] Compared GII, GGI and SIGI using PCA to avoid the problem of variable incommensurability and to rank the factors or dimensions in terms of proportion of variation explained by the dimensions. The authors found four principal components of gender equality as Education and Social Conditions (PC1); Economic and Labour Participation (PC2); Political Empowerment (PC3); and Health (PC4) which explained 72% of total variance in the 15 indices that comprise the GGI, GII and SIGI.

However, PCA assumes normal distribution. To achieve normality, first step of PCA is to normalize each variable to have zero mean and SD of one, which again require meaningful addition of raw scores of the variables.

There are a number of other GRIs like *Women, Business and the Law Index* (WBLI) [40], Women's Economic Opportunity Index (WEOI) [41], European Gender Equality Index (https://eige.europa.eu/gender-equality-index/2015), Relative Status of Women (RSW) by [42] as AM of ratios of achievement levels of women and men in education, health, income, etc. with different component variables, data sources, methodologies and coverage of countries. To accommodate various gender related issues, new gender equality indicators at regional, national and international are likely to continue to emerge.

However, need is felt to construct better GRI avoiding the problems of scaling and selection of weights along with satisfaction of the following desired properties to achieve better methodology of combining component variables or dimensions to get composite measure:

P_1 : Continuous function (reflecting GRI-score of a country by a continuous variable).

P_2 : Symmetric over arguments (independent of order of the dimensions/components).

P_3 : Independent of change of scale.

P_4 : GRI as aggregation of dimensions = GRI as aggregation of components ensuring minimum trade-off among the dimensions or components.

P_5 : Reciprocity. If women have 40% disadvantages over men, then men to have 60% advantages over women.

P_6 : Easy assessment of relative importance of dimensions and components.

P_7 : Meaningful aggregation of Components → dimension score, Dimensions → GRI score and Country-wise GRI scores → World GRI so that aggregated scores at each stage are monotonically increasing i.e. an improvement in a domain or component ⇒ Improvement in GRI.

P_8 : For longitudinal data, if $X_{(t+1),i} > X_{t,i}$ for i-th variable, then $GRI_{(t+1)} > GRI_t$

P_9 : Identification of key dimensions/components where performances have not increased and require attention of the policy makers.

P_{10} : Satisfy Time–reversal test i.e. $GRI_{c0} \times GRI_{0c} = 1$ where GRI_{c0} is the GRI of a country at period C with respect to the base period.

P_{11} : Facilitate formation of chain indices i. e. $GRI_{20} = GRI_{21} * GRI_{10}$.

P_{12} : Responsiveness i.e. ability to quantify progress made by a country over time.

P_{13} : Computation of $\overline{GRI}$ and SD (GRI) for a sample of countries at a given time period.

P_{14} : Statistical testing of equality of mean GRI of (i) two countries for a given year (ii) a country at two time periods.

Table 1 Dimensions, sub-dimensions and component variables of GEI

Dimensions	Sub-dimensions	Components
Equality in Economic opportunity	Business and employment opportunity	Labour participation ratio, Ratio of: Wage equality, Wages and salaries, Actual income earned (estimated), Access to loan from financial institutions
Equality in Social Development	Educational opportunities	Ratio of: Rate of Literacy, School enrollment (separately for primary, secondary and tertiary)
	Access to reproductive health services	Maternal mortality ratio (MMR), Receipt of help of skilled health staff during delivery, Fulfillment of need for contraception, Prenatal care to pregnant women
Equality in laws & institutions	Women's participation	Ratio of parliament seats by women and men, Ratio of number of seats at ministerial level by women and men, Proportion of women justices
	Women's legal rights	Non-discrimination clauses in constitution, Proportion of married woman among passport applicants, Married woman chooses where to live, Same weight in court for woman's testimony, Law on rape and sexual harassment, Women's access to financial services, Women's access to land, and non-land assets, Equal remuneration to men and women for work of equal value, Law on non-discrimination of gender in hiring
	Women's household rights	Whether a married woman is required by law to obey her husband, Whether married woman can be the head of household, Parental authority in marriage and divorce, Legal minimum age of marriage for girls and boys, Married woman confers citizenship to her children, Jointly sharing of legal responsibility for family's expenses by married couples, Administration of property during marriage, Inheritance rights of daughters and widows, Laws on domestic violence

6 Proposed methods

6.1 Set up

Let $X_1, X_2, \ldots\ldots, X_n$ are *n*-components relating to gender-differentiated outcomes including social and economic aspects that take into account levels of achievement of women vis-a-vis men. For components measured by ordinal scale, like attitudes obtained from survey using *K*-point scales, transform raw scores of such components to continuous, monotonic, equidistant scores following say $N(30, 10^2)$ to attain comparable results as suggested by [43].

6.2 Method 1: Multiplicative aggregation

Consider the unit-free ratios $\frac{X_{F,i}}{X_{M,i}}$ for $i = 1.2.\ldots\ldots n$ where $X_{M,i} > 0; X_{F,i} > 0$. For components like life expectancy at birth where women has natural advantages over men, $X_{F,i} > X_{M,i}$. For such components, consider reciprocal (i.e. smaller value in the numerator and higher value in denominator).

In line with [44] who proposed Gender Relative Status index (GRS) as geometric mean of ratios of achievements by women and men, the proposed GRI reflecting gender inequality combines the ratios by geometric mean i.e.

$$GRI_{GM} \sqrt[n]{\frac{X_{F1}, X_{F2}, \ldots\ldots .., X_{Fn}}{X_{M1} X_{M2} \ldots\ldots .. X_{Mn}}} \tag{3}$$

$$\text{or by avoiding the} n - \text{th root, } GRI_{GM} = \frac{X_{F1}, X_{F2}, \ldots\ldots .., X_{Fn}}{X_{M1} X_{M2} \ldots\ldots .. X_{Mn}} \tag{4}$$

Clearly, Eq. (3) and (4) are equivalent. 100*GRI_{GM} as per (4) indicates percentage gender inequality.

GRI_{GM} is multiplicative aggregation of all gender-gaps, expressed in ratios. GRI_{GM} <1indicates men are better off than women; and GRI_{GM} >1 indicates that women are better off than men.

6.3 Method 2: Angular similarity approach

N-number of chosen components for women and men can be represented respectively by two vector $\boldsymbol{X_F} = (X_{F1}, X_{F2}, \ldots\ldots .., X_{Fn})^T$ and vector $\boldsymbol{X_M} = (X_{M1} X_{M2} \ldots\ldots .. X_{Mn})^T$

Gender similarity index (*GSI*) can be assessed by evaluating cosine of the angle between the

$$Vectors\, X_F \, and \, X_M \, as \, GSI = Cos\theta = \frac{X_F^T X_M}{\|X_F\| \|X_M\|} \tag{5}$$

where $\|X_F\|$ and $\|X_M\|$ are length of $\boldsymbol{X_F}$ and $\boldsymbol{X_M}$ respectively and $0 \le Cos\theta \le 1$ for acute θ

$Cos\theta = 1 \Leftrightarrow \boldsymbol{X_F} = \boldsymbol{X_M}$ since $\|X_F\| = \sqrt{\sum_{i=1}^{n} {X_{Fi}}^2} = \|X_M\| = \sqrt{\sum_{i=1}^{n} {X_{Mi}}^2}$ i.e. there is no gender related difference in any component. Thus, $Cos\theta = 1$ is the ultimate goal.

$Cos\theta$ is a measure of similarity. Lower value of θ ⇒ higher value of $Cos\theta$ ⇒ higher value of GSI. Measure of gender inequality in terms of angular dissimilarity is $Sin\theta = \sqrt{1 - Cos^2\theta}$ where $0 \le Sin\theta \le 1$

7 Calculations

Consider the following hypothetical data of two countries and four components and calculations of GRI_{GM} and *GSI*.

Observations:

GRI_{GM} of country A, was higher than the country B. Same is true for *GSI*

Male group was better off than the female group for both the countries. But, gender inequality was higher for B than A.

7.1 Properties (Method 1)

The proposed GRI_{GM} satisfies:

GRI_{GM} of dimensions = GRI_{GM} of components
Reciprocity i. e. $\frac{X_{F1},X_{F2},\ldots\ldots..,X_{Fn}}{X_{M1}X_{M2}\ldots\ldots..X_{Mn}} * \frac{X_{M1}X_{M2}\ldots\ldots..X_{Mn}}{X_{F1},X_{F2},\ldots\ldots..,X_{Fn}} = 1$
Trade-off among the dimensions or components are reduced significantly
Relative importance of *i*-th dimension and *j*-th component can be assessed respectively by $\frac{d(GRI_{GM})}{dD_i}$ and $\frac{d(GRI_{GM})}{dX_j}$

$$From\,(4),\ log\,GRI_{GM} = \sum_{i=1}^{n} \log X_{F,i} - \sum_{i=1}^{n} \log X_{M,i} \tag{6}$$

For *n*-countries, Eq. (6) helps to find mean and variance of $logGRI_{GM}$ and also to find average GRI_{GM} for the world as antilog of $\frac{\sum_{i=1}^{n} log_{GM_i}}{n}$
$logGRI_{GM}$ of countries can be transformed by $Z_i = \frac{logGRI_{GMi} - \overline{logGRI_{GM}}}{SD(logGRI_{GM})} \sim N(0,1)$. Z- Scores are transformed to Y by linear transformation as $Y = (99)\left[\frac{Z_i - Min_{Z_i}}{Max_{Z_i} - Min_{Z_i}}\right] + 1$ so that $Y \in [1, 100]$. Countries with Y-scores in common range can be used for better ranking and classification. Normally distributed Y-scores in fixed range help in meaningful addition and parametric analysis including estimation of population mean (μ), variance (σ^2), confidence interval of μ and to test statistical hypothesis of equality of mean GRI_{GM} of countries in different regions. Average GRI_{GM} for the world can also be found as AM of country-wise Y-scores.

For longitudinal data, progress made by Female group and Male group of a country at current period over the base period can be taken respectively as $PF_{c0} = \frac{XF_{1c}.XF_{2c}\ldots\ldots\ldots..XF_{nc}}{XF_{10}.XF_{20}\ldots\ldots\ldots..XF_{n0}}$ and

$$PM_{C0}\frac{XM_{1c}.XM_{2c}\ldots\ldots\ldots..XM_{nc}}{XM_{10}.XM_{20}.\ldots\ldots\ldots..XM_{n0}}$$

GRI_{GMc0} as geometric aggregation of PF_{c0} and PM_{c0} is

$$GRI_{GMC0} = \prod PF_{c0} * PM_{c0} = \prod_{i=1}^{n} \frac{XF_{ic}}{XF_{i0}} * \frac{XM_{ic}}{XM_{i0}} \tag{7}$$

$GRI_{GMc0} < 1 \Rightarrow$ Improvement in the current year over the base year.
Satisfies Time–reversal test since $GRI_{GMc0} \times GRI_{GM0c} = 1$ for a country.
Facilitates formation of chain indices since $GRI_{GM20} = GRI_{GM21} * GRI_{GM10}$
The *i*-th component will be *critical* if inequality widens for the component in *t*-th period over *(t*-1*)*-th period i. e. if $\frac{XF_{i,t}}{XM_{i,t}} < \frac{XF_{i,(t-1)}}{XM_{i,(t-1)}}$ and appropriate corrective action may be initiated regarding the factors influencing the *i*-th component.
Progress of the *i*-th country in *t*-th period *t* over (*t*-1)-th period is assessed by positive value of $(GRI_{GMi_{(t-1)}} - GRI_{GMi(t)})$ or by $\frac{GRI_{GMi_t}}{GRI_{GMi_{(t-1)}}}$. The latter is preferred since $\left(\frac{GRI_{GMi_{(t-1)}} - GRI_{GMi_t}}{GRI_{GMi_{(t-1)}}}\right) * 100$ gives percentage improvement of the GRI_{GM}(PI-GRI). One can draw graph of PI-GRI for $t = 0, 1, 2, 3, \ldots$ and so on to depict progress/decline of GRI_{GM} for a country since the base period. Comparison of two countries with respect to such PI-GRI graphs may be explored.
From Eq. (7), we get

$$logGRI_{GMC0} = \sum_{i=1}^{n} logXF_{ic} + \sum_{i=1}^{n} logXM_{ic} - \sum_{i=1}^{n} logXF_{i0} - \sum_{i=1}^{n} logXM_{i0} \tag{8}$$

Equation (8) helps to find $\sum_{Allcountries} logGRI_{GMc0} = logGRI_{GMc0}$ for the world, from which GRI_{GMc0} of the world can be obtained using antilogarithm. In addition, Eq. (8) helps to find mean and variance of $logGRI_{GMc0}$ for the countries under observations, which in turn can help to express mean, variance of GRI_{GMc0}.

7.2 Properties (Method 2)

The GSI index satisfies:

- Assessment of gender similarity and overall progress of a country in *t*-th period over the previous period or the base period by a continuous variable taking positive values.
- GSI is not always monotonically increasing. Increase in Literacy rate of females from 18 to 19% for the country B in Table 2, resulted in reduced GSI.
- Countries can be ranked and classified in terms of GSI_{c0}.
- Substitutability among components/dimensions, effect of outliers is reduced significantly and thus, GSI is not biased for developed or under-developed countries.
- GSI of a dimension can be computed by focusing on components belonging to that dimension.
- Critical dimensions requiring attention of the policymakers can be found by arranging GSI of the dimensions in increasing order and considering the dimensions with low values of $GSI_{i-thDimension}$
- GSI for sub-groups like economically backward groups, religious groups, certain age-group, etc. can also be computed.

Association between two countries can be evaluated by $Cos\theta_{ij} = \frac{Cos\theta_i{}^T Cos\theta_j}{\|Cos\theta_i\|\|Cos\theta_j\|} > 0$ where

$$\text{Cos}\theta_i = \left(Cos\theta_{i0}, Cos\theta_{i1}, Cos\theta_{i2}, \ldots\ldots., Cos\theta_{im}\right)^T \text{ and}$$

$$Cos\theta_j = \left(Cos\theta_{j0}, Cos\theta_{j1}, Cos\theta_{j2}, \ldots\ldots., Cos\theta_{jm}\right)^T$$

However, *Cos*θ does not satisfy triangle inequality law. For *k*-countries, mean and variance of $Cos\theta_i$'s are obtained following [45] where angles are $\theta_1, \theta_2, \ldots\ldots\ldots, \theta_k$, and each vector is transformed to have unit length i.e. $\boldsymbol{\pi_{Fi}} = \sqrt{\frac{X_{Fi}}{\|X_{Fi}\|}}$ and $\boldsymbol{\pi_{Mi}} = \sqrt{\frac{X_{Mi}}{\|X_{Mi}\|}}$ so that

$$\|\pi_{Fi}\|^2 = \|\pi_{Mi}\|^2 = 1$$

$$\text{Mean}\,\bar{\theta} = Cot^{-1}\frac{\sum_{i=1}^{k} Cos\theta_i}{\sum_{i=1}^{k} Sin\theta_i} \tag{9}$$

$$Variance = \left[\left(\frac{\sum Cos\theta_i}{k}\right)^2 + \left(\frac{\sum Sin\theta_i}{k}\right)^2\right] \tag{10}$$

It is difficult to find probability distribution of $Cos\theta$. For large number of countries, using Eq. (9) and (10), $Cos\theta_i$ can be transformed to *Z*-scores by $Z_i = \frac{Cos\theta_i - \bar{\theta}}{SD(Cos\theta)} \sim N(0,1)$ and $Y = (99)\left[\frac{Z_i - Min_{Z_i}}{Max_{Z_i} - Min_{Z_i}}\right] + 1$ so that $Y \in [1,100]$ and $Y \sim N(\mu, \sigma)$ where μandσ can be estimated from data. Normally distributed *Y*-scores can help in statistical inferences like estimation and hypothesis testing.

Average GSI for the world can be found by Eq. (9) and also by AM of country-wise *GSI*-scores.

For longitudinal data of *i*-th country, one can compute $Cos\theta_{ti}$ for $t = 0$ (base period), 1, 2… and so on facilitating drawing of graph of GSI across time for the country. This may help to study growth pattern of GSI for the country with possibility of forecasting GSI in future years. Association between two countries can be evaluated by correlation between $Cos\theta_{ti}$ and $Cos\theta_{tj}$ $(r_{Cos\theta_{ti}, Cos\theta_{tj}})$

Table 2 Data: Two countries each having four components

Country	Female Group				Male Group			
	% Share of Parliament seats (Com.1)	% of population with min. secondary education (Com.2)	Participation in Labour market (%) (Com.3)	Literacy rate (%) (Com.4)	% Share of Parlia ment seats (Com.1)	% of population with min. secondary education (Com.2)	Participation in Labour market (%) (Com.3)	Literacy rate (%) (Com.4)
A	27.9	61.0	56.6	91.7	72.1	68.3	78.7	98.0
B	17.5	24.3	23.6	18.0	82.5	34.8	69.5	29.4

Table 3 Calculations of GRI_{GM} and *GSI*

Country	Ratios (Female: Male)				GRI_{GM}	*GSI*
	Com. 1	Com. 2	Com. 3	Com. 4		
A	0.3869626	0.893118594	0.719286788	0.93571429	0.232575	0.964862
B	0.2121212	0.698275862	0.339568345	0.6122449	0.0307983	0.903106

The ratio $\frac{Cos\theta_{ti}}{Cos\theta_{0i}}$ >1 indicates overall progress of the *i*-th country in *t*-th period over the base period and $\frac{Cos\theta_{ti}}{Cos\theta_{0i}}$ <1 indicates decline. Year-to-Year changes of a country can be reflected by $\frac{Cos\theta_{ti}}{Cos\theta_{(t-1)i}}$ i.e. replacing base period data by the previous year. Similarly, replacing the base period data by the target vector (say, SDG goals), $GSI_{c,SDG}$ will indicate current distance of a country from the SDG goals (Table 3).

8 Summary of properties

Each GRI_{GM} and *GSI* satisfies:

- Continuous score
- Considers all components in different units
- Avoids scaling and calculation of weights
- Reduces trade-off among dimensions/components
- Facilitates scoring of a dimension and finds relative importance of dimensions.
- Aggregation of country-wise scores to rank, classify countries and find world average
- Mean and variance of sample countries
- Identification of critical components
- Progress of a country in successive periods
- Y-scores in [0,1] $\sim N(\mu, \sigma^2)$ help in statistical inferences and to find meaningful world average

However, assessment of progress made by female group and separately by male group is possible for GRI_{GM} bit is not relevant for *GSI*. Same is true for Reciprocity. Aggregation of dimensions = Aggregation of components is satisfied by GRI_{GM}. Moreover, GRI_{GM} scores are monotonically increasing since it increases with increase in (F: M) ratio. Time—reversal test and formation of Chain indices are satisfied by GRI_{GM} method. However, association between two countries in quantitative term is possible for *GSI* but is not straight forward for GRI_{GM}.

Based on the above, GRI_{GM} is preferred than *GSI*.

9 Relationship between GRI_{GM} and GSI

$$GRI_{GM} = \frac{X_{F1}, X_{F2}, \ldots\ldots.., X_{Fn}}{X_{M1}X_{M2}\ldots\ldots..X_{Mn}} by (4)$$

$$GSI = Cos\theta = \frac{X_F^T X_M}{X_F X_M} by (5)$$

$$logGRI_{GM} = \sum_{i=1}^{n} logX_{Fi} - \sum_{i=1}^{n} logX_{Mi} = \log\left[\frac{X_F}{X_M}\right]$$

$$2logX_{F2} + \ldots\ldots\ldots\ldots + 2logX_{Fn}] = \sum_{i=1}^{n} logX_{ic}$$

Since $log\|X_F\| = \frac{1}{2}[2logX_{F1} + 2logX_{F2} + \ldots\ldots\ldots\ldots + 2logX_{Fn}] = \sum_{i=1}^{n} logX_{ic}$

Taking antilog, $GRI_{GM} = \frac{\|X_F\|}{\|X_M\|}$

$$\text{Now } GSI = \frac{X_F^T X_M}{GRI_{GM} X_M X_M} = \frac{\sum_{i=1}^{n} X_{Fi}X_{Mi}}{GRI_{GM} \cdot \sum_{i=1}^{n} X_{Mi}^2} \Rightarrow \rightarrow GSI.GRI_{GM} = \rightarrow \frac{\sum_{i=1}^{n} X_{Fi}X_{Mi}}{\sum_{i=1}^{n} X_{Mi}^2} \quad (11)$$

Thus, gender similarity by *GSI* and gender inequality by GRI_{GM} is negatively related and slope of the relationship vary for different countries.

10 Discussion

Calculations of GRI_{GM} and GSI are simple. Each gives equal weights to the components, but enables quantification of relative importance of the components/dimensions from the data. Relative importance of the components may vary across countries. Each helps in identification of critical components/dimensions and allows policy makers to focus on those areas for corrective action and policy changes. Both measures quantify responsiveness (changes with time). GRI_{GM} assesses progress/decline in successive periods for women and separately for men. Both facilitate finding mean and SD for a sample of countries, world average at a given time period and statistical testing of equality of mean of (i)two countries for a given year (ii) of a country at two time periods in terms of *Y*-scores obtained from the proposed measures by linear transformations.

Equation (11) gives inverse relationship between gender inequity by GRI_{GM} and gender similarity by GSI. Thus, correlation between GRI_{GM} and GSI is likely to be high with negative sign.

The proposed methods avoiding scaling and selecting weights cannot be compared with CI using weighted sum like DP2.

However, each of the proposed methods has following limitations:

1. Assumes no missing data.
2. Fails if any component score is ≤ 0. In those cases, each zero score may be replaced by a small number $\varepsilon > 0$ (say 0.001).
3. Accommodating new component involves estimation of values of the same for the base period and each subsequent period.

11 Conclusions

After reviewing problems of construction of GRI, the paper proposes non-parametric measure of gender inequality through multiplicative aggregation (GRI_{GM}) and a gender similarity index (GSI) by cosine similarity, avoiding scaling and selecting weights or dimensionality reduction. The proposed, measures consider all chosen components and give overall progress by a country across time.

Inverse relationship between the two proposed methods was derived. Each measure reduces significantly the trade-off among the dimensions/components, effect of outliers, and is not biased to developed or under-developed countries. In addition, each measure enables computation of mean and SD for the countries considered in empirical

studies. *Y*-scores in [0, 1] ~ $N(\mu, \sigma^2)$ help to estimate population mean and SD; test $H_0 : \overline{GRI_{i-thcountry}} = \overline{GRI_{j-thcountry}}$ for a given year and equality of mean GRI of a country at two time periods. Normally distributed *Y*-scores in a fixed range also help in meaningful arithmetic aggregation of country-wise scores to find world average. All such results using *Y*-scores can be translated back to GRI_{GM} score or GSI scores.

Better ranking and classification of countries can be made by each index. Index for dimensions can also be obtained by considering the components related to a dimension. GSI in terms of cosine similarity can indicate current distance of a country from the ultimate goal of zero gender related inequality. GRI_{GM} is preferred for easy interpretation and additional features like time-reversal test, easy identification of critical areas and contribution of the components or dimensions to GRI_{GM}.

Empirical verification of the proposed indices and their properties including robustness could be taken as future studies.

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Declarations

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