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Generalized topology optimization for architectural design

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Abstract

In recent years, topology optimization has become a popular strategy for creating elegant and innovative forms for architectural design. However, the use of existing topology optimization techniques in practical applications, especially for large-scale projects, is rare because the generated forms often cannot satisfy all the design requirements of architects and engineers. This paper identifies the limitations of commonly used assumptions in topology optimization and highlights the importance of having multiple solutions. We show how these limitations could be removed and present various techniques for generating diverse and competitive structural designs that are more useful for architects. Unlike conventional topology optimization, we may include load and support conditions as additional design variables to enhance the structural performance substantially. Furthermore, we show that varying the design domain provides a plethora of opportunities to achieve more-desirable design outcomes.

Keywords: Topology optimization, Architectural design, Multiple solutions, Support location, Load location, Design domain

1 Introduction

In recent decades, computational generative design strategy has been increasingly adopted in the field of architecture. Awareness of this strategy is growing rapidly, as it allows users to conveniently create, control, and modify 3D geometries through computational algorithms. Among the various generative design tools, *topology optimization* has attracted the attention of many architects because of its capability to create elegant organic forms by determining voids in continuum structures. More significantly, topology optimization is a performance-based design tool that aims to find the most-efficient structural form, meaning the generated configuration corresponds to an optimized material layout under a set of defined conditions and a given volume.

Two of the most commonly used topology optimization techniques in architectural design are the evolutionary structural optimization (ESO) method and the bidirectional evolutionary structural optimization (BESO) method. The optimization strategies of the ESO and BESO methods were inspired by naturally occurring structures—such as bones, shells, and trees—which acquire excellent properties through an evolutionary process of "survival of the fittest". The original ESO method, first proposed by Xie and Steven (1993, 1997), was based on the simple concept of gradually removing inefficient material from a structure so that the resulting shape would evolve toward the optimum. The more robust method, BESO, allows the material to be removed and added simultaneously by redistributing the underutilized material to the most-needed locations (Huang & Xie, 2007; Querin et al., 1998). The ESO and BESO methods have been widely used to design highly efficient and strikingly elegant buildings and bridges (Burry & Burry, 2010; Huang & Xie, 2010).

Notwithstanding the growing popularity of topology optimization among architects, the number of practical applications in large-scale projects is still relatively small. A key reason is that the generated forms often cannot meet all the design requirements of architects and

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engineers. This paper endeavors to identify the limitations of conventional topology optimization and show how these limitations could, and should, be removed so that we can obtain solutions that are more useful for architectural applications.

Traditionally, topology optimization is defined as a technique to find *the best design* of the material layout of a structure under *prescribed load and support conditions* within a *given design domain* while satisfying various other constraints. As a paradigm shift, we propose that all the italicized items in this definition could be amended or even abandoned, thereby achieving design outcomes that better meet practical requirements.

The rest of the paper is organized as follows. Section 2 discusses the importance of and techniques for finding diverse and competitive designs, instead of searching for *the best design*. Section 3 shows the techniques for optimizing support locations, rather than having *prescribed support conditions*. Section 4 discusses our latest research on treating load locations as design variables, instead of accepting *prescribed load conditions*. Section 5 illustrates the benefits of changing the design domain, rather than having a *given design domain*. Section 6 concludes the discussion and highlights relevant future research.

2 Techniques for achieving diverse and competitive designs

While many engineers and mathematicians are obsessed with the "unique" and "globally optimal" solution from topology optimization algorithms, architects are less inclined to readily accept the "optimal solution" because it often deviates significantly from their artistic intuitions and does not satisfy many functional requirements. The "best design" based purely on structural performance is usually of low value in architectural practice. Engineers who extol the virtues of such a unique solution without considering other options may inadvertently contribute to the breakdown of the collaboration with the architect (or client).

To overcome this bottleneck, we have developed a series of techniques capable of producing multiple designs that have distinctly different configurations but possess similar structural performance to that of the optimal solution (He et al., 2020; Yang et al., 2019). We show that vastly different designs could be obtained by sacrificing a small amount (e.g., 3%) of the structural performance. Such *diverse* and *competitive* designs will provide the architect with a wide range of design options while maintaining a high level of structural efficiency.

2.1 Varying parameters in the optimization algorithm

The most straightforward way to obtain diverse and competitive designs is to vary one or more input parameters in the optimization algorithm. For example, in the BESO method, one of the commonly used parameters is the filter radius R (Huang & Xie, 2007), which controls the minimum member size in the optimized structure. Figure 1 shows the results of generating multiple designs of a cantilever beam by varying the filter radius R from 4.5 to 1.0 in the BESO algorithm. The beam is fixed on the left-hand side and is subjected to a vertical load at the midpoint of the right-hand side. We have obtained five distinctly different designs. Using a larger R value has resulted in fewer members but larger sizes (see Fig. 1b). In contrast, a smaller R value has led to more members but smaller sizes (see Fig. 1f). For the same amount of material, the five geometrically different designs have less than 1.4% difference in their structural performance in terms of the overall stiffness. This simple technique of varying the input parameter is very useful for architectural applications.

2.2 Penalizing precedent designs

We present two slightly different methods to create diverse and competitive designs based on penalizing some elements of previously optimized designs. Here, penalization means artificially reducing the "fitness" or importance of certain parts (or elements) of the structure. In the first method, we penalize a prescribed percentage of the least-efficient solid elements in the initial optimized design obtained by the conventional BESO process. We may achieve a series of new designs by setting different penalty percentages.

Figure 2 shows the application of the first method using a long-span structure example. Its load and support conditions are given in Fig. 2a. The initial optimized design is shown in Fig. 2b, and a subsequent design generated by penalizing the precedent design is given in Fig. 2c. The penalty percentage in this example is set at 100% of the remaining solid elements shown in Fig. 2b. Under the same volume, the overall stiffness of the subsequent design (Fig. 2c) is only 9% lower than that of the initial optimized design (Fig. 2b).

It is noted that a magnificent structure very similar to the initial optimized design has been constructed at the Qatar National Convention Centre, designed by the renowned architect Arata Isozaki and his team using an extended ESO method (Burry & Burry, 2010; Cui et al., 2003; Sasaki, 2005). The subsequent design in Fig. 2c would provide an innovative and elegant alternative structural form for a new project that has similar or identical load and support conditions.

In the second method, *all* precedent designs are included in the penalization process when searching for subsequent structural forms. A certain percentage of the least-efficient solid elements in each precedent design is

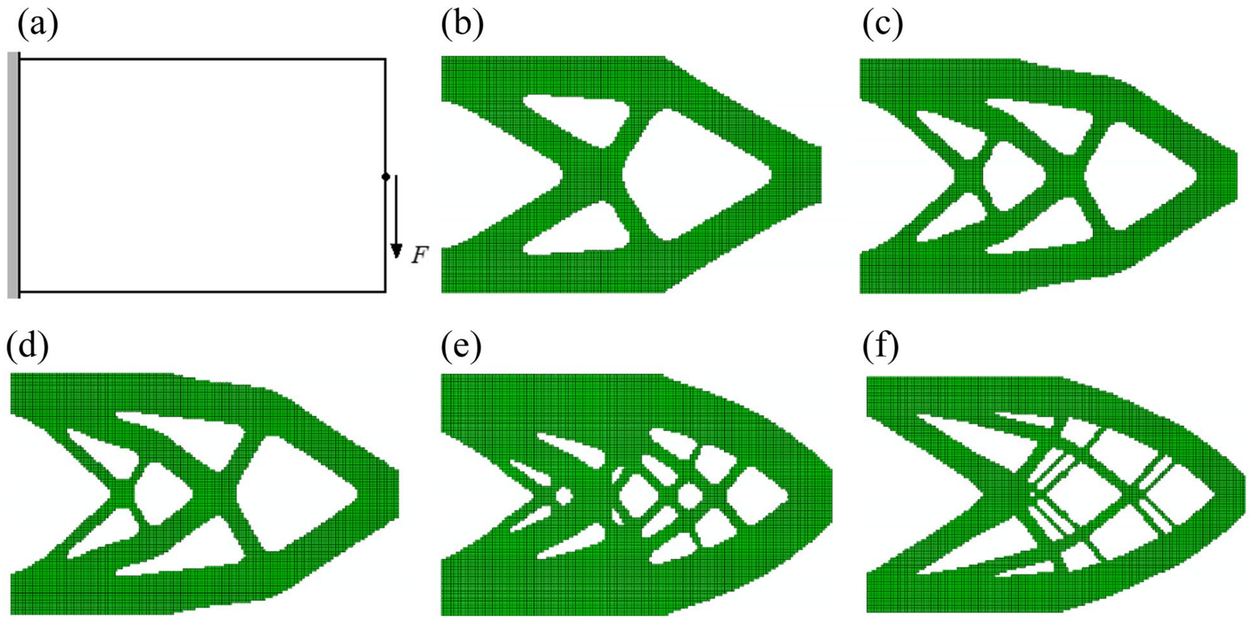

Fig. 1 Multiple designs of a cantilever beam obtained by varying the filter radius *R*: **a** load and support conditions; **b** $R=4.5$; **c** $R=3.5$; **d** $R=3.0$; **e** $R=2.0$; **f** $R=1.0$

penalized during the subsequent design cycle, so all the least-efficient elements in the previous designs would be less likely to reappear in the new design.

Figure 3 shows the application of the second method using a bridge structure example. Its load and support conditions are given in Fig. 3a. The initial optimized design is shown in Fig. 3b. The first subsequent design (Fig. 3c) is obtained by penalizing the only precedent form so far (Fig. 3b), while the second subsequent design (Fig. 3d) is obtained by penalizing both precedent forms (Fig. 3b and c). The penalty percentage is set at 10% of the remaining solid elements in each precedent design. The three generated designs in Fig. 3b–d clearly show that the arches of the bridge have substantially different arrangements. All three designs are structurally efficient and aesthetically pleasing. They could be used under different traffic conditions as their lanes change from the middle (see Fig. 3b) to the two sides (see Fig. 3d). More significantly, the overall stiffness of the two subsequent designs is only 3% lower than that of the initial optimized design (Yang et al., 2019).

2.3 Introducing randomness into the structural optimization model

A different approach to influencing the design outcome is by penalizing random elements in a structural model. In doing so, many unexpected designs could be obtained. Using this technique, the designer might get a distinctly different structural form every time the optimization process is rerun. There are various ways to introduce randomness into structural optimization models (He et al., 2020; Xie et al., 2019), and a simple technique is presented here.

Figure 4a shows a cantilever beam and its conventional optimized design. By introducing some random voids in the initial finite element model (see the two images on the left-hand side of Fig. 4b and c), we deliberately divert the optimization process toward "local optima". Traditionally, such suboptimal results are frowned upon by mathematicians and engineers, as the local optima are usually regarded as "bad solutions". However, when it comes to finding diverse and competitive designs, the local optima could be considered an excellent opportunity for exploring alternative structural forms. Figure 4b and c show two new designs generated by setting random voids in the initial finite element model. It should be noted that some of these voids are changed into solid elements in later iterations at locations where the material is needed for structural efficiency. Compared to the conventional optimized design (see Fig. 4a), the overall stiffness of the two distinctly different designs shown in Fig. 4b and c is only 4% and 3% lower, respectively.

2.4 Other techniques

We have developed many other techniques for achieving diverse and competitive designs through topology optimization in recent years. Interested readers may refer to the following publications for more detail: Xie et al.

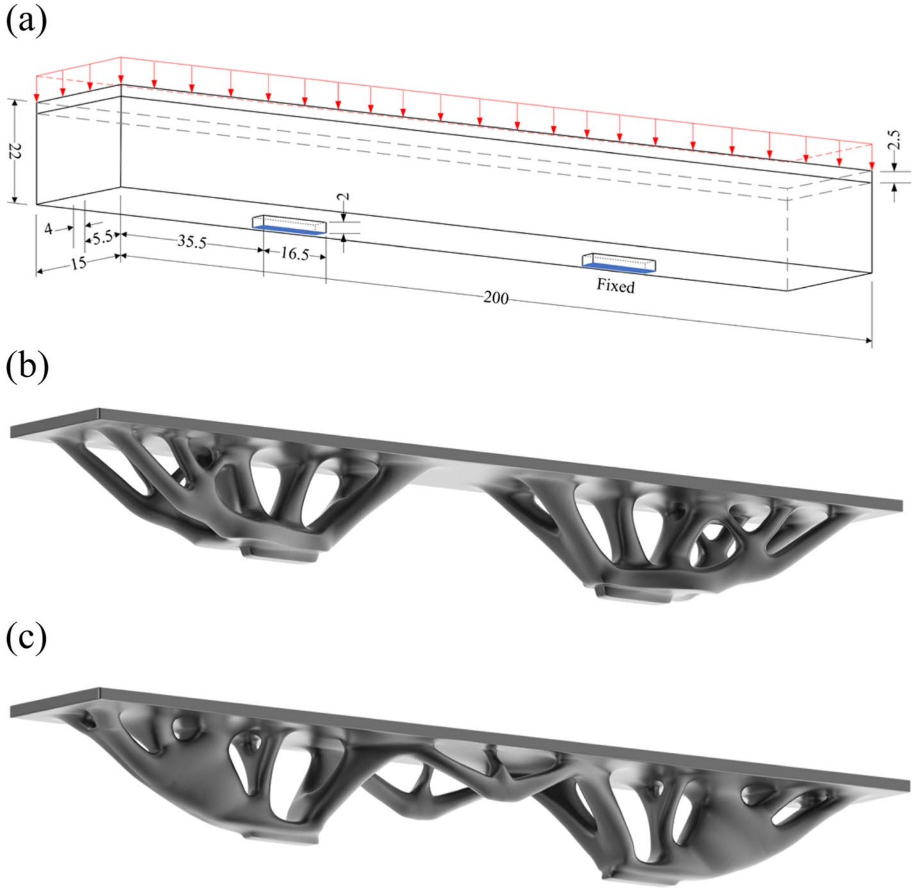

Fig. 2 Multiple designs of a long-span structure (Yang et al., 2019): **a** load and support conditions; **b** initial optimized design; **c** subsequent design generated by penalizing the precedent design

(2019), Yang et al. (2019), He et al. (2020), Zhao et al. (2020), Cai et al. (2021), Yan et al. (2022), and He et al. (2022). Furthermore, from a broader perspective, all the techniques presented in Sections 3, 4 and 5 could be considered additional methods for generating diverse and competitive designs.

3 Optimizing support locations

In traditional structural design processes, support locations of a structure are typically prescribed. A structure with improved performance may be obtained by manually adjusting its support locations through a trial-and-error process. However, such an approach is tedious and time-consuming, and the results can be far from optimal. We have developed methods to automatically find the optimal support locations of a structure under given load conditions. The computational algorithms are based on a combination of an optimality criteria method and the BESO method. We have considered two classes of problems: optimizing the distribution of a given number of vertical columns under a roof structure (Meng et al., 2021) and simultaneously optimizing the topology of a structure and its support locations (Lee & Xie, 2021). We demonstrate that treating support locations as additional design variables provides new opportunities to improve the structural performance significantly, thereby reducing the environmental impact of bridges, buildings, and other artifacts. Two examples are given below.

Figure 5a shows the optimization process of a bean-shaped flat roof supported by vertical columns. Using our optimization technique, we can find the optimal locations of the columns. In this example, we set the number of final columns to be seven. The optimization process starts from 1159 candidate columns within an allowable area and gradually finds the optimal locations for the seven columns. The optimized design is illustrated in Fig. 5b. The final column locations fulfill both engineering and architectural requirements: the structural performance is optimized, and the number of columns and their allowable locations are predetermined by the designer. A set of manual arrangements of columns are tested, as shown in Fig. 5c. The optimized design has the

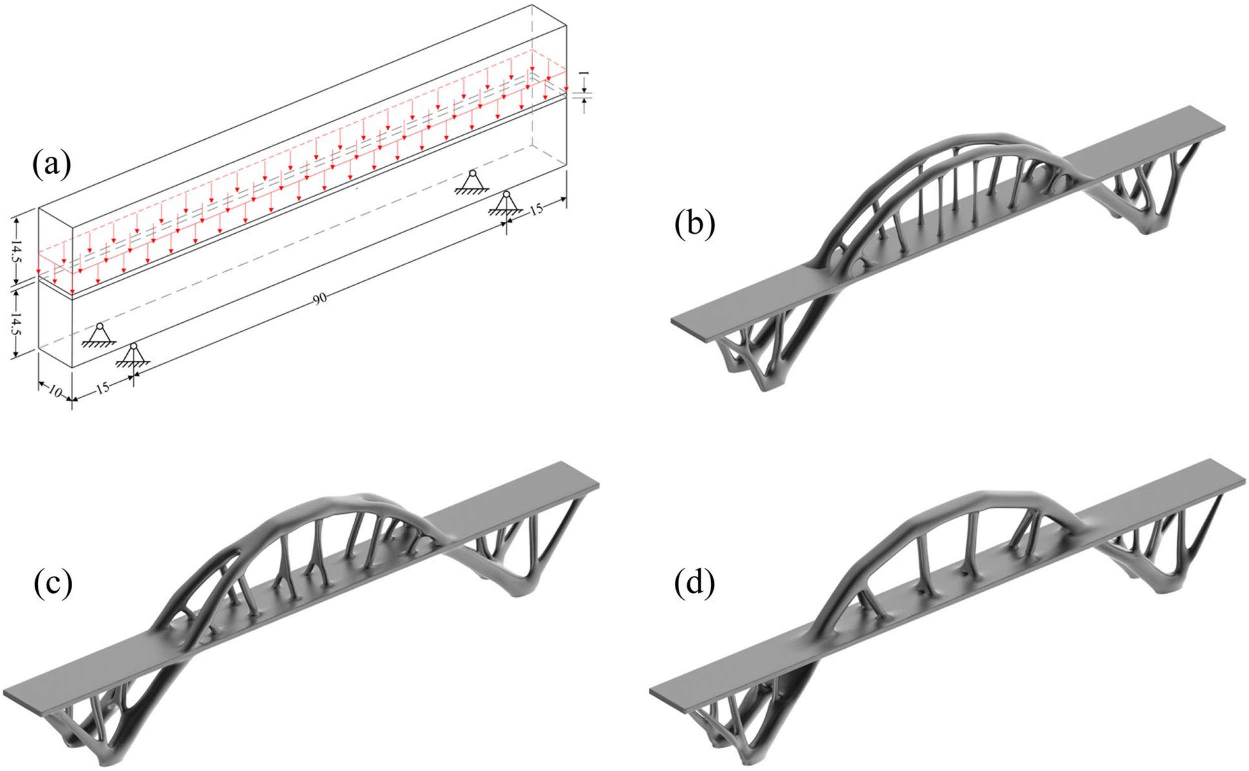

Fig. 3 Multiple designs of a bridge structure (Yang et al., 2019): **a** load and support conditions; **b** initial optimized design; **c** and **d** subsequent designs generated by penalizing all precedent designs

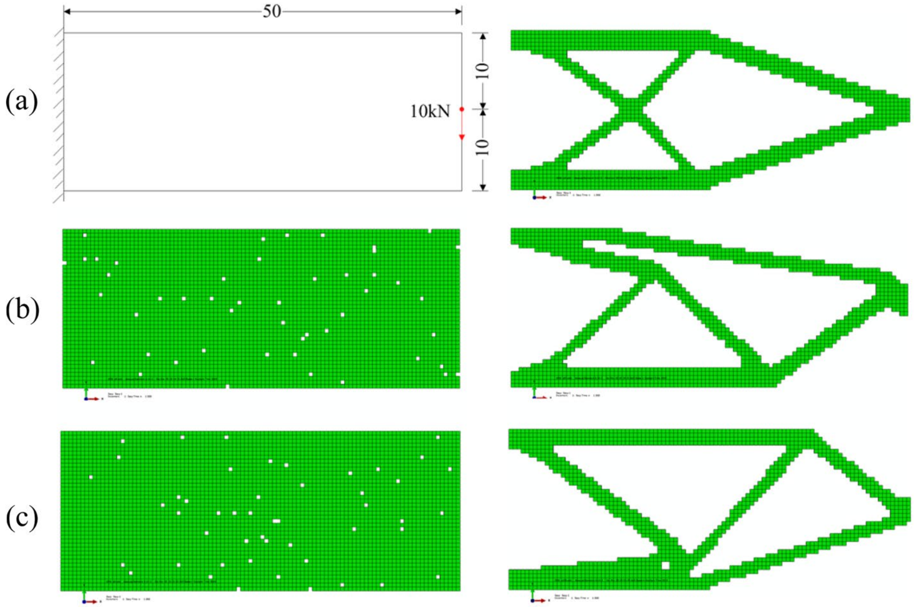

Fig. 4 Multiple designs of a cantilever beam (Xie et al., 2019): **a** conventional optimized design; **b** and **c** alternative designs generated by setting random voids in the initial model

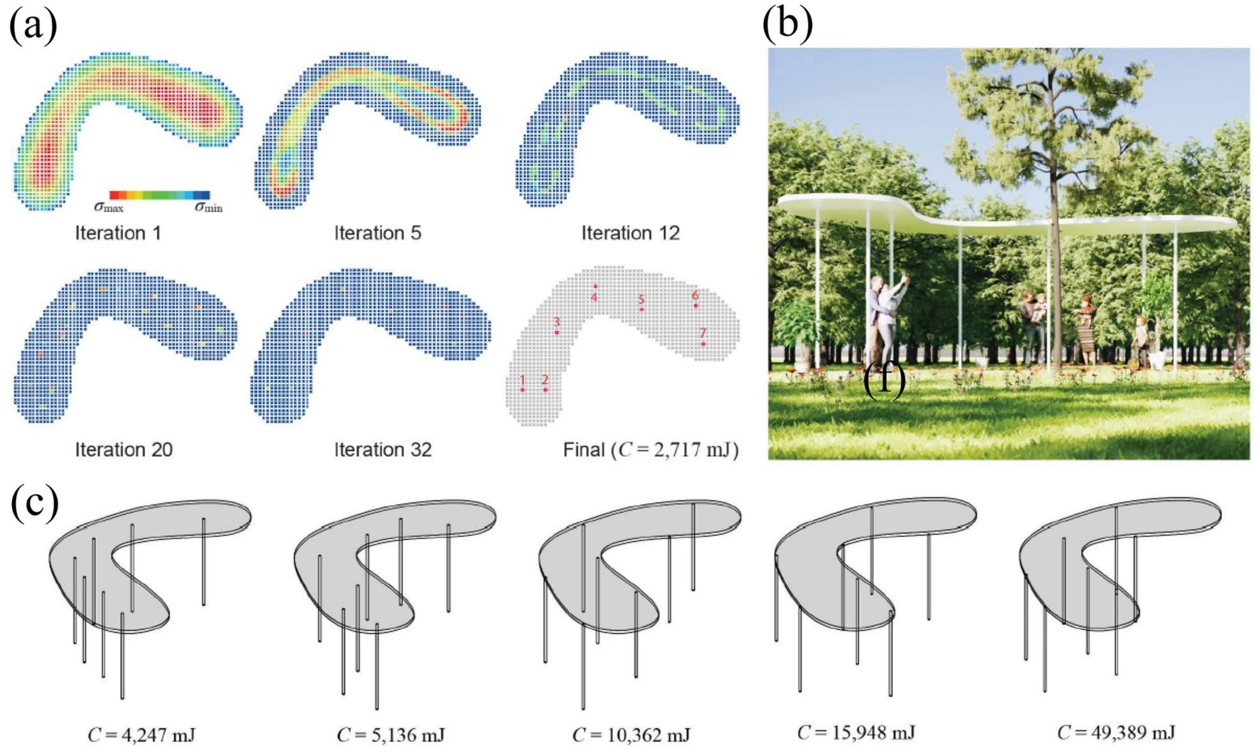

Fig. 5 Designing column locations under a bean-shaped roof (Meng et al., 2021): **a** optimization process; **b** rendering of the optimized design; **c** manual designs

lowest compliance C (the inverse of the overall stiffness). This observation confirms that our optimization technique can effectively optimize the column locations and improve the structural performance.

Figure 6 shows three designs of a bridge structure obtained by simultaneously optimizing the support locations and the structural topology. It is seen that the number of supports significantly affects the geometries of the designs. This example also demonstrates that treating the number of supports and the support locations as additional design variables is an effective strategy to generate geometrically diverse and structurally competitive designs.

4 Optimizing load locations

Existing optimization techniques are typically performed with prescribed load conditions. However, changing the load locations or directions may significantly affect the static and dynamic responses of a structure. Recently, we have developed a new optimization method that treats

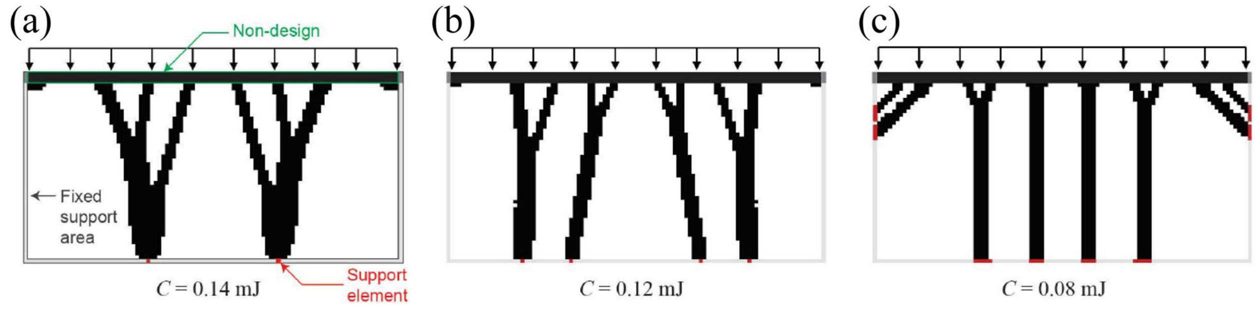

Fig. 6 Simultaneous optimization of the support locations and the structural topology of a bridge structure for (**a**) 2, (**b**) 4, and (**c**) 34 support elements, in addition to fixed supports at two ends of the deck (Lee et al., 2021)

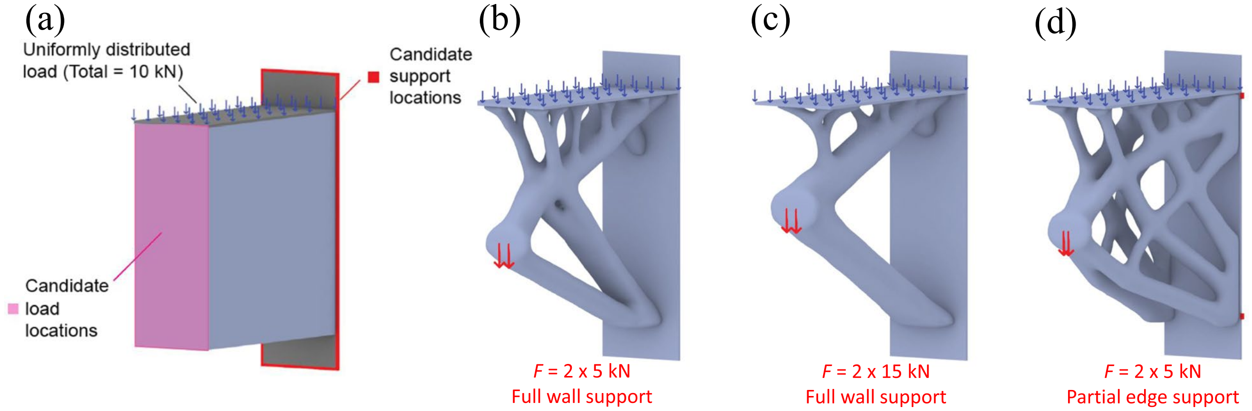

Fig. 7 Simultaneous optimization of load locations, support locations and the structural topology of a multifunctional structure (Lee & Xie, 2022): **a** variable load and support conditions; **b** and **c** optimized load locations and the most-efficient topologies under different load magnitudes with full wall support; **d** optimized load and support locations and the most-efficient topology with partial edge support

load conditions as design variables. This approach can optimize load locations and directions automatically to improve structural efficiency. Furthermore, this method can be extended to the simultaneous optimization of load locations and directions, support locations, and the structural topology to achieve new types of high-performance structures (Lee & Xie, 2022).

Figure 7 shows an example of simultaneous optimization of load locations, support locations, and the structural topology of a multifunctional structure. In addition to the fixed pressure load on the top surface, two concentrated loads are applied to the front surface in the vertical direction. The locations of the two concentrated loads are optimized to minimize the overall deformation of

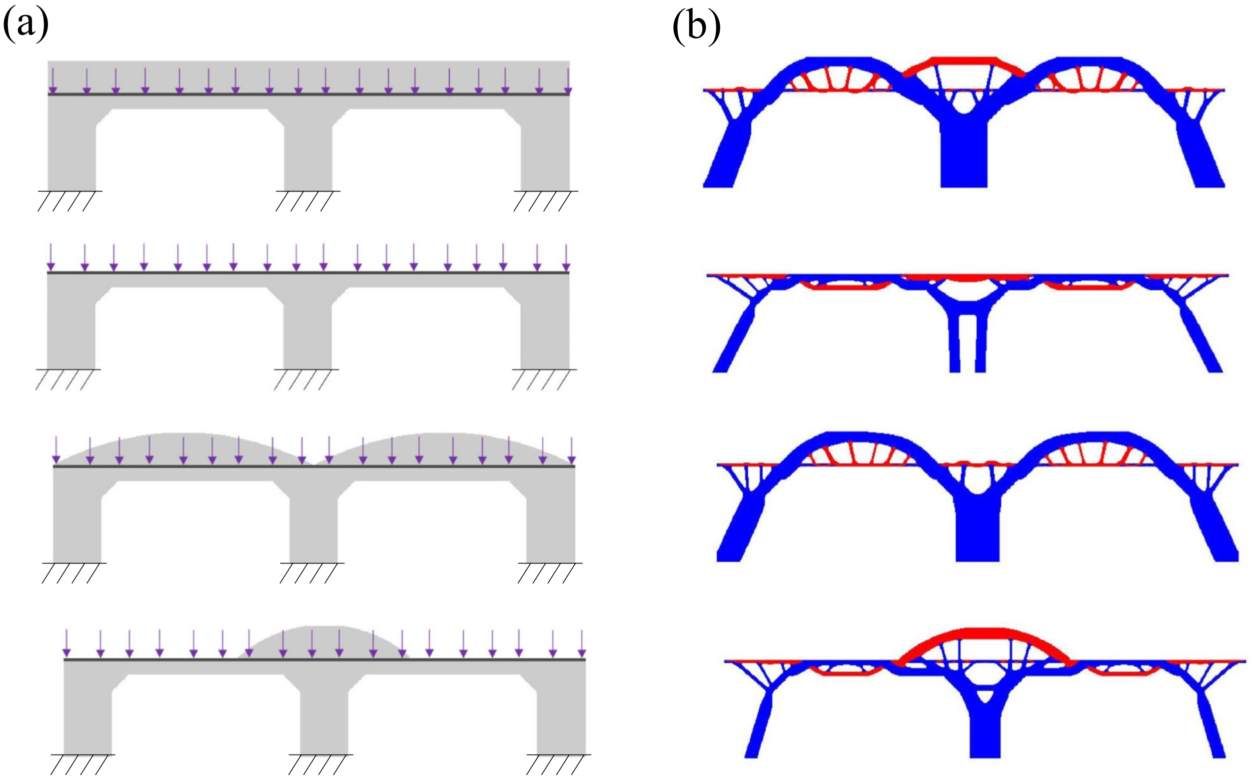

Fig. 8 Exploration of different design domains of a footbridge: **a** load and support conditions, and different design domains (gray); **b** distributions of steel (red) and concrete (blue) obtained by applying the multimaterial BESO method developed by Li and Xie (2021a, 2021b)

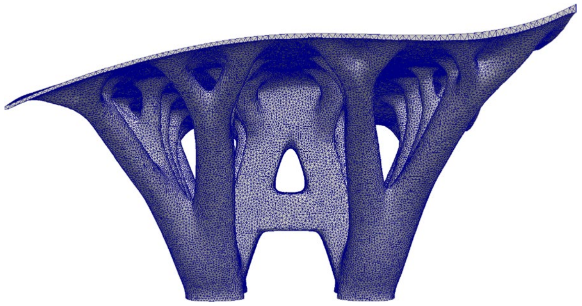

Fig. 9 Fixing the A-shaped part of the design space during the topology optimization process (Li et al., 2022)

the structure. The back surface of the structure is either fully fixed at every point or partially fixed at a prescribed number of points on the edges. Figure 7b and c show the optimized load locations and the most efficient topologies when the back wall is fully fixed. It is seen that when the magnitude of the concentrated loads is increased, the geometry of the structure changes significantly. Furthermore, we may treat support locations as additional design variables and obtain a distinctly different design, as shown in Fig. 7d. By varying the number of supports and the magnitude of loads, we can generate a series of diverse and competitive designs.

5 Redefining the design domain

Traditionally, structural optimization is carried out within a prescribed, fixed design domain. However, such a commonly adopted approach could severely hinder the "creativity" of the emerging digital design tools that use topology optimization. This section shows how the design domain could be easily redefined and used as an effective *driver* for design innovation. Four simple strategies are presented: (1) exploring different design domains, (2) fixing part of the design space, (3) setting part of the design space as a prohibited region, and (4) adaptive design domain.

5.1 Exploring different design domains

Changing the design domain has an enormous effect on the resulting structural form. This simple approach provides a great opportunity to the engineer when working with the architect or client, especially at the early stage of a project when the final design is unclear. By exploring various design domains, the engineer may propose multiple structurally efficient and geometrically different solutions.

Figure 8 shows an example of the first set of designs we proposed to our collaborator (a leading bridge design firm) for a real footbridge to be constructed of steel and concrete. By simply varying the design domain (see Fig. 8a), we quickly produced a series of distinctly different preliminary designs (see Fig. 8b) for further discussion with our collaborator. Here, we have used the multimaterial BESO method developed by Li and Xie (2021a, 2021b), which judiciously puts steel in tensile regions and concrete in compressive areas to achieve structural efficiency.

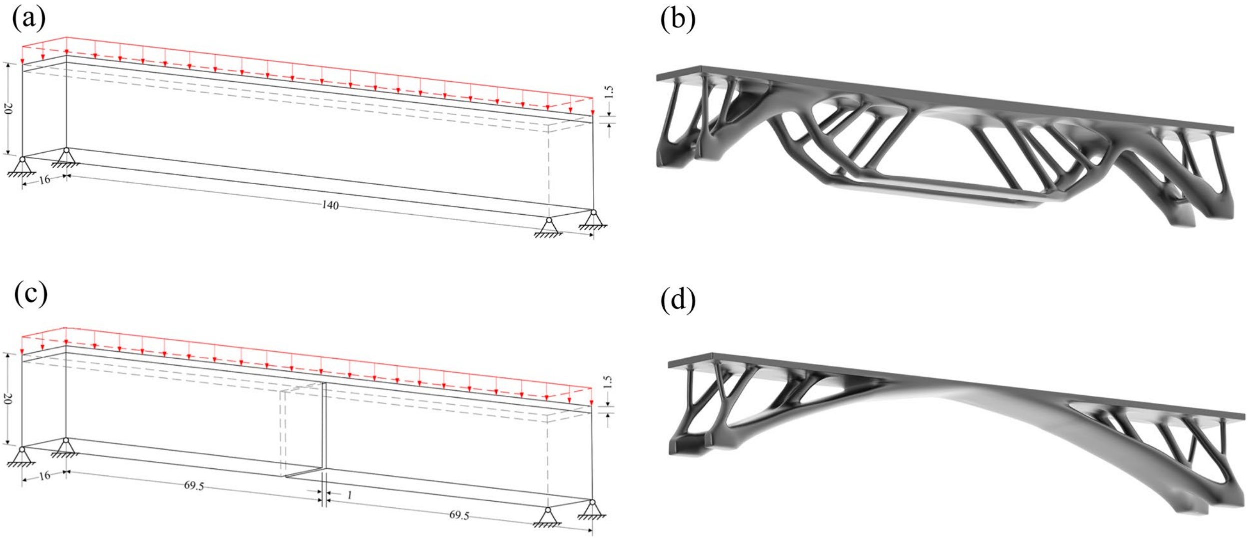

Fig. 10 Setting part of the design space as a prohibited region (Yang et al., 2019): **a** and **c** load and support conditions, and different design domains; **b** and **d** optimized designs without and with setting a prohibited region

5.2 Fixing part of the design space

In architectural (and engineering) design, a certain part of a structure (e.g., the deck of a bridge) often must be kept intact even if it is not fully required for maintaining the structural performance. There may also be some specific geometric features that the designer wishes to have in the structure. In such cases, we may set part of the structure as a "non-design domain" so that it will not be altered during the topology optimization process. Figure 9 shows an example by Li et al. (2022), which was inspired by an architectural design test case of Ameba—a BESO-based topology optimization tool (Zhou et al., 2018; https://ameba.xieym.com). Here, an A-shaped part is set as a non-design domain within a large block of design space. While inefficient material around the A could be freely removed, the A-shaped part is kept unchanged throughout the optimization process.

Other commonly used strategies similar to fixing part of the design space include imposing symmetric and periodic constraints on the geometry (Huang & Xie, 2008; Thomas et al., 2021). These strategies can be easily implemented in topology optimization tools and are very useful for architectural applications.

5.3 Setting part of the design space as a prohibited region

Here, we introduce the concept of a "prohibited region" in topology optimization, a region of gap or voids to be kept empty in the design space. Setting part of the design space as a prohibited region sounds like a trivial operation, but it may drastically affect the design outcome. For example, when we want to design a bridge using topology optimization, we may set a cuboid as the design domain, with the four bottom corners fixed and a uniformly distributed load applied to the top surface, as shown in Fig. 10a. The optimized result is given in Fig. 10b. Unfortunately, this design does not satisfy the basic functional requirement of a bridge, as the two bars at the bottom would prevent traffic from passing underneath the deck. To address this problem, we set a narrow gap below the deck as a prohibited region, as shown in Fig. 10c. In doing so, we obtain a conceptually different design (see Fig. 10d), which looks like an interesting and elegant arch bridge. The overall stiffness of the design in Fig. 10d is 14% lower than that of the solution in Fig. 10b. However, the structurally worse solution in Fig. 10d would be far more likely to be accepted by the architect or client as a viable design for a bridge.

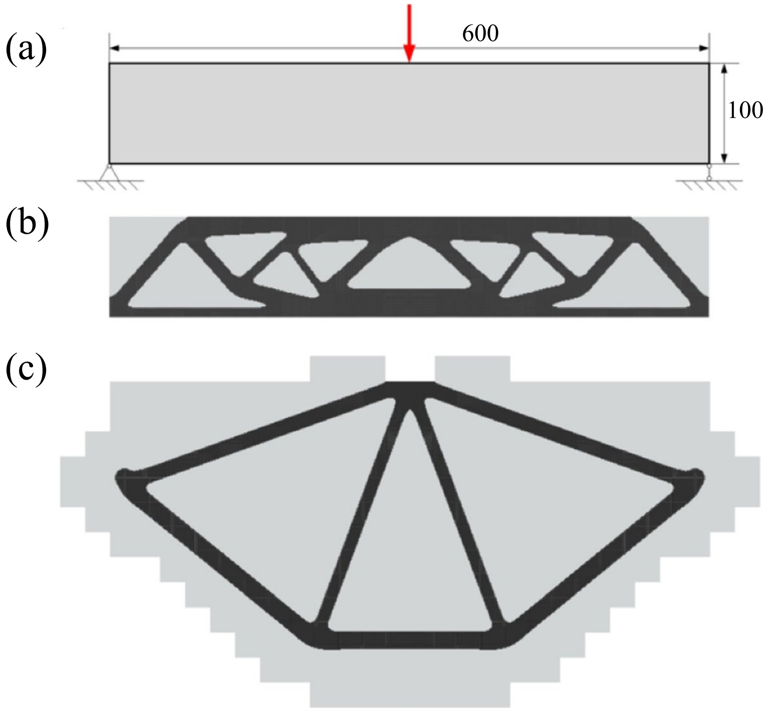

Fig. 11 Design of a beam using an adaptive design domain (Rong et al., 2022): **a** load and support conditions; **b** traditional solution; **c** new solution from an adaptive design domain

5.4 Adaptive design domain

Instead of predefining or fixing the design domain, a more intelligent and beneficial approach is to allow the design domain to adapt automatically according to structural or functional requirements. This approach provides a powerful tool to generate diverse and competitive designs.

Figure 11 shows an example of the drastic effect of having an adaptive design domain on the design outcome. Figure 11b gives the traditional topology optimization solution to a classical beam problem shown in Fig. 11a. Using an adaptive design domain, we obtain a new design (see Fig. 11c) that is much simpler in geometry and far more efficient in structural performance; for the same amount of material, the overall stiffness of the latter is 3.1 times that of the former.

6 Concluding remarks

In this paper, we have proposed and demonstrated that to make topology optimization more useful for architectural design, we should reconsider most of the assumptions commonly used in conventional topology optimization. Such assumptions include: (1) seeking the unique and globally optimal "best design", (2) following prescribed load and support conditions, and (3) using predetermined design domain. We show that these assumptions are unnecessary, imposing severe limitations on design outcomes, and hindering creativity in architectural design. Throughout this paper, we have demonstrated the importance of multiple solutions and presented a variety of techniques for creating diverse and competitive designs.

The generalized topology optimization approach we have systematically advocated and demonstrated in this paper departs fundamentally from much of the traditional thinking of topology optimization. The new approach is expected to enable topology optimization to find more practical applications in architectural design.

Although most of the techniques and examples discussed in this paper have been developed based on the BESO method, the same ideas are equally applicable to other topology optimization techniques such as the solid isotropic material with penalization (SIMP) (Bendsøe & Sigmund, 2003). In the future, we aim to make topology optimization more intelligent and valuable for architectural design, for example, by developing an advanced generative design tool that would allow close interaction between the designer and the computer *during* the optimization process. Such an interactive topology optimization tool would lead to design outcomes that exhibit a fine balance between the designer's artistic (subjective) preferences and the structure's technical (objective) requirements.

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Author's contributions
Yi Min Xie: Conceptualization, Methodology, and Writing. The author(s) read and approved the final manuscript.

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Availability of data and materials
The datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests
The author is the member of Academic Committee for Architectural Intelligence but were not involved in the journal's review, or any decisions, related to this submission.

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