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ORIGINAL ARTICLE

Hegel of the gaps? Truth, falsity and conjunction in Hegelian contradictions

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Abstract
I offer here a critical assessment of Beall and Ficara's most recent take on Hegelian contradictions. By interpreting differently some key passages of Hegel's work, I favor, unlike them, a no-gaps approach which leads to a different logic.

Keywords Insufficiency of contradictories · Completion of contradictories · Conjunction Elimination · Speculative content

1 Introduction

I first met Jc Beall in 2015 in Istanbul, during the Fifth World Congress and School on Universal Logic. Within the event, I and the late Carlos César Jiménez organized a workshop for the 25 years of Mortensen's *Inconsistent Mathematics*, and we invited many prominent paraconsistentists, including, of course, Jc. He was kind enough to accept our invitation. Since then, he has been a frequent visitor and supporter of the logic group at UNAM's Institute for Philosophical Research.

Although by then Jc's work on philosophy of logic and philosophy of mathematics was already influential on me, in Istanbul I learned about his interest, shared with Elena Ficara, on formalizing at least some bits of Hegel's thought. That was important for me because at the time I was entertaining the idea, based on independent considerations, that contradictions are not like any other conjunctions; and, on the other hand, I was interested in clearly delineating several contradiction-friendly stances, from "Some contradictions could be true" to "All and only contradictions are true" and many others.

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(See Estrada-Gonzalez (2017) and Estrada-Gonzalez and Martinez-Ordaz (2018), for example.)[1]

Since 2015, the attendees to Ficara's tutorial on Hegel's logic knew that she and Beall had drafted a joint paper on the formalization of Hegelian contradictions, but at least I never saw it, not even a rough draft. Others did, and so, in the meantime, Beall and Ficara's typescript became an important basis for d'Agostini and Ficara's d'Agostini and Ficara (2022) reconstruction of Hegel's analysis of the Liar paradox and d'Agostini's d'Agostini (2023b) reconstruction of Hegel's analysis of the Sorites, for example. Thus, working independently of Beall and Ficara's proposal but building upon the interpretative work of the latter, Elisángela Ramírez-Cámara and I developed a logic we dubbed '**HEGFL**', and we introduced it in a joint paper titled "The many faces of Antiphasis". We presented a draft of it at the 17th Latin American Symposium on Mathematical Logic in 2017, and the full paper was accepted for publication in the *Logic Journal of the IGPL* in 2019. Nonetheless, we were not entirely satisfied with its contents, especially on the conceptual foundations of the semantics proposed, and we withdrew it. (We are still unsatisfied with it, so this piece is not intended as a reboot of **HEGFL** in any way.) A summary of its contents, together with some new results, was published by Ricardo Arturo Nicolás-Francisco in Nicolás-Francisco (2020).[2]

The long-promised paper by Beall and Ficara has finally been published; see Beall and Ficara (2023). As I foresaw, their approach is not entirely satisfactory to me, although it contains many interesting hints. Since I think that one of the ways of showing respect to other philosophers is by stating clearly and honestly our disagreements with them, in this short note I want to present an understanding of Hegelian contradictions different from the one advanced by Beall and Ficara. In particular, I want to advance a 'no-gaps' semantics for certain key Hegelian ideas. This means that some target passages used by Beall and Ficara to motivate their semantics can, and perhaps should, be interpreted in a different way.

The plan of the paper is as follows. In Section 2 I critically discuss Beall and Ficara's interpretation of two passages where Hegel speaks of the union of contradictory propositions. I claim that these should not be isolated from a third passage, considered in earlier works by Ficara, and that in doing so one ends up with logical requirements different from those envisaged by Beall and Ficara. (In general, I will use Ficara's own translations of the target passages, although in some cases I will make comments on possible variations.) In Section 3 I present a logic that meets the desiderata put forward at the end of Section 2 . In Section 4 I give an outline of other options to meet the desiderata and provide some preliminary conclusions.

[1] Some positions reconstructed in those two papers are instances of what nowadays is called —mistakenly, in my view— 'conjunctive paraconsistency'. Franca d'Agostini has recently been making an impressive case for this kind of paraconsistency as a serious philosophical alternative to other forms of paraconsistency; see d'Agostini (2021), d'Agostini (2022), d'Agostini (2023a, b).

[2] More recently, Ficara's interpretative work inspired, at least partially, another logic; see Francez (2023).

2 Interpreting the target passages

Beall and Ficara's aim "is to give an account wherein Hegel embraces 'true contradictions' without thereby embracing either of the 'simple conjuncts', and in fact while thinking that each 'simple conjunct' is itself untrue - insufficient to accept on its own." (Beall & Ficara, 2023, pp. 120f). Two target passages by Hegel underlie Beall and Ficara's account of Hegelian conjunction[3], namely

Target passage 1, Insufficiency of contradictory conjuncts

> In order to be conjoined [vereinigt] the elements of an antinomy have to be recognized as contradictorily opposed to each other, their relation between each other has to be felt and known as an antinomy; but the opposite can be recognized as an opposite only if it has already been unified [vereinigt]; the conjunction [Vereinigung] is the norm through which the comparison takes place and through which the opposites, as such, emerge in their insufficiency. (Hegel, 1797/1798, p. 251)

Besides the fact that Beall and Ficara use this passage to claim that it is only in conjunction —not in mere pairs— that true contradictions can arise, they pay special attention to the "insufficiency" of each conjunct of the contradiction and say: "the elements of an antinomy (a 'true contradiction'), taken in isolation, are 'insufficient', that is, not able to express on their own the whole content at stake." (Beall & Ficara, 2023, p. 121). The fact that true contradictions can arise only in conjunction strongly suggests that it is conjunction which completes contradictories, or at least that it is through conjunction that contradictories can be complete. Beall and Ficara use Hegel's own "Principle of completion", quoted in what follows, as evidence of that.

Target passage 2, Completion of limitations

> […] completing the limitations [i.e. the single thesis or the single antithesis of an antinomy] by affirming their contradictories, as their conditions; the latter limitations need, in turn, the same completion. Hegel, 1801, p. 26)

Parenthetical remark The translation of 'Vereinigung' as 'conjunction' is outstanding, concerning both linguistic correctness and logical suggestiveness. There are other possible translations of 'Vereinigung' which could score almost as well as 'conjunction' concerning both linguistic correctness and logical suggestiveness, namely 'fusion' and 'merging'. The only shortcoming of those translations is that they could lead us too much in the relevance logic direction, and that could opaque the particularities of Hegel's logic. On the other hand, there is a strong tradition of German thought on *Verbinden*, that is, connection, juxtaposition, which is but another name for *synthesis*, i.e. the showing of something *together with* something else. Although not a literal translation, I would not have minded if 'Vereinigung' had been translated

[3] In all fairness, they use one more passage, but it is rather to support the idea that Hegel is at least a dialetheist, that is, that he believes that some contradictions are true, and it has no special bearing on how Hegel understood conjunction.

directly as ‘synthesis’. (But in addition to not being an exact literal translation respecting the German, and the general philosophical suggestiveness gained notwithstanding, the logical suggestiveness would have been probably lost for many readers.) **End of parenthetical remark.**

Thus, it would be that in affirming both a limitation —i.e. a proposition— and its contradictory, completion (of content or truth) would be achieved. Beall and Ficara say: “In order to express the whole content the elements (what we call Hegelian conjuncts) need to be ‘completed’ by their contradictory opposites [in a conjunction].” (Beall & Ficara, 2023, p. 121) One can think that the general idea is clear. A true contradiction is complete or sufficient, and any of its (propositional) components, taken in isolation, is incomplete or insufficient. The problem is that, thus far, we have no clue what it means for a proposition to be complete or incomplete.

I will not reproduce Beall and Ficara’s semantic machinery in detail here. It is enough to say that Beall and Ficara cash the incompleteness or insufficiency of a proposition as its untruth. Moreover, they seem to assume the usual characterization of logical validity as truth preservation (from premises to conclusions in all interpretations). Thus, in their model, while both p and $\sim p$ are untrue, $p \curlywedge \sim p$ is true. This entails that the following hold in their logic (which I will call ‘**BF**’):

$$A, B \models_{\mathbf{BF}} A \curlywedge B \qquad \text{(Adjunction)}$$
$$A \curlywedge B \not\models_{\mathbf{BF}} A, \quad A \curlywedge B \not\models_{\mathbf{BF}} B \qquad \text{(Conjunction Elimination)}$$
$$A \models_{\mathbf{BF}} A \curlywedge A \qquad \text{(Idempotence (i))}$$
$$A \curlywedge A \not\models_{\mathbf{BF}} A \qquad \text{(Idempotence (ii))}$$
$$A, \sim A \models_{\mathbf{BF}} B \qquad \text{(Explosion)}$$
$$A \curlywedge \sim A \not\models_{\mathbf{BF}} B \qquad \text{($\curlywedge$-constrained Explosion)}$$

They also include **FDE**’s conjunction $\wedge$ in their logic, and $A \wedge \sim A \models_{\mathbf{BF}} B$ holds good because the interpretation that could produce the countermodel, namely that A is both true and false, is not available in their semantics. This makes their proposal “not strictly dialetheic” (see d’Agostini, 2021, §4) since a (true) contradiction is not made of two truths, but of two untruths.

I do not feel compelled by Beall and Ficara’s interpretations of the target passages, though, and I do not think that the passages lead us so smoothly to the logic they present. To better explain why I think so, let me endorse an objection anticipated by them ((Beall & Ficara, 2023), p. 128 and then analyze their response to see that it is wanting and that a non-gappy semantics is better motivated by the textual evidence. The objection runs as follows. It is clear, from the target passages, that Hegel says that the conjuncts of these contradictions are false and yet Beall and Ficara have it that they are not in fact, in reality, false, but rather simply gappy in some sense (neither true in reality nor false in reality). So, their semantics is not appealing enough as a capturing of some of Hegel’s ideas.

Beall and Ficara reply that it is plausible that Hegel recognized two notions of falsity: one, the usual truth-of-negation account; the other, the familiar not fully or completely true. Then, Beall and Ficara say that, for them, the latter notion is not implausibly modeled by their semantics, and that it is that notion of falsity that fits the Hegelian account of the dialectical process.

I think their reply is not satisfactory. I do not know whether it is plausible that Hegel recognized two notions of falsity, one of them, falsity$_u$ with the property that being A is false$_u$ iff A is not fully or completely true, with 'A is not fully or completely true' entailing 'A is not true'. (And hence, with the falsity$_u$ of A entailing that A is untrue.) Certainly, some textual evidence is needed to assess such alleged plausibility. But treating the conjuncts as untrue is not well-motivated conceptually, either. To see why, consider the following target passage, not quoted by Beall and Ficara although it was considered in Ficara's book (Ficara, 2021, p. 178):

Target passage 3, Separating contradictories as injustice

> The commonest injustice done to a speculative content is to make it one-sided, that is, to give prominence only to one of the propositions into which it can be resolved. It cannot then be denied that this proposition is asserted; but the statement *is just as false as it is true*, for once *one* of the propositions is taken out of the speculative content, the other must at least be equally considered and stated (Hegel 1832, p. 94, italics in the original.).

Hegel clearly says here that each one of the contradictory propositions "is just as false as it is true", and I think that falsity at least could be taken at face value, not as meaning untruth. The insufficiency of each conjunct would be their being false and not just true, rather than their lacking a truth value. The consequences of the above for Beall and Ficara's approach are clear. According to Target passage 3, 'not fully (or completely) true' does not entail 'not true', on the contrary. Rather, 'A is not fully (or completely) true' entails that A is false, and in fact, that there is falsity, as well as truth, in A. There is no need to appeal to untruth, and there is no need to have a notion of falsity as untruth. Furthermore, there is no reason to suppose that falsity is different from not-A's being true, or there being some truth in not-A. This would entail that true contradictions are true but not also false, because there must be a qualitative difference between true contradictions (the speculative content) and each of its components (to make the speculative content one-sided).

But then logical entailment cannot be understood as truth preservation (in all interpretations). As I have just said, the true contradiction is true (and not also false) and each conjunct "is just as false as it is true"; then, in Conjunction Elimination (for contradictions) one would go from the truth to the truth, and hence, if logical entailment were truth preservation, Conjunction Elimination would be logically valid. But remember that "[t]he commonest injustice done to a speculative content is to make it one-sided". If the logical validity of Conjunction Elimination entails that a true contradiction can be made one-sided, and if making a true contradiction one-sided is "the commonest injustice done to a speculative content [i.e. a true contradiction]" (and injustice in general should be avoided), then Conjunction Elimination, at least for contradictions, cannot be logically valid after all.[4]

[4] As I have said, Beall and Ficara's gappy approach underlies (d'Agostini & Ficara, 2022) reconstruction of Hegel's analysis of the Liar paradox and d'Agostini's (d'Agostini, 2023b) reconstruction of Hegel's analysis of the Sorites. What differences my dialetheic approach would entail for those projects should be left for further work.

There is one more issue to settle before attempting a formal account of Hegel's contradictions, and that is whether contradictions are merely satisfiable (i.e. there is an interpretation in which the conjunction of A and not-A is true), as in typical dialetheism, or whether there are contradictions that are true in all interpretations in the theory. I stand with Ficara's earlier interpretations, in particular, with what she says in Ficara (2013). There, she thinks of Hegel's logic as *Vernunftslogik*, a logic such that "its application is circumscribed to the analysis of the contents and semantic behaviours of rational concepts" (Ficara, 2013, p. 43). Examples of such rational concepts include *being*, *nothingness*, *difference*, *infinite*, *infinite*, etc. And, according to Ficara's reconstruction, which she makes very plausible, every sentence of reason is contradictory for Hegel, and necessarily so. Here is some textual evidence presented by Ficara (I modify slightly her translation):

> The so-called principle of non-contradiction has no formal value for reason, so that every sentence of reason about concepts must entail a violation of it; that a sentence is merely formal means for reason: affirming it alone, without affirming at the same time its contradictory opposite, is false. (…) every true philosophy contains (…) this eternal violation of the principle of non-contradiction (...) (Hegel, 1801, p. 230)

where the "violation" of the principle of non-contradiction is not its mere invalidity, but the validity of a contradiction for every sentence of reason. Summarizing with a bit of formalese: Let A be any sentence of reason, for example, "Being is being". Then both A and not-A are false in all interpretations (but also true, according to Target passage 3), yet A-and-not-A is true (and true only) in all interpretations because an "eternal violation" of the principle of non-contradiction is demanded.

Given what I have said, these are the minimal desiderata of the formal account to be given:

D1. At least some contradictions are logically true.
D2. Conjunction Elimination is not logically valid.
D3. Logical entailment is not truth preservation (from premises to conclusion).

Of course, these are minimal desiderata in the sense that it would be better if the formal account to be given matched other Hegelian ideas. Then, let me proceed now to give formal shape to what we have learned.

3 An alternative to BF

The desiderata mentioned above can be achieved in several ways. In what follows I present a rather straightforward one. Let $\mathcal{L}$ be a formal language built, in the usual way, from a countable set of propositional variables with the connectives $\backsim$ (unary) and $\circledcirc$ (binary). I will use the first capital letters of the Latin alphabet, 'A', 'B', 'C'…, as variables ranging over arbitrary formulas. Recall that this formal language is intended to represent the language of Hegel's dialectical logic, and that it is "limited to a very specific domain, the one of self-referential thought, variously called by Hegel the field of 'pure concepts', 'concepts of reason (*Vernunfterkenntnisse*)', 'the conceptual', 'the logical (*das Logische*)'." (Ficara, 2013, pp. 45f)

A *Dunn model* for $\mathcal{L}$ is a relation σ between propositional variables and values 1 (*truth*) and 0 (*falsity*), that can be extended to cover all formulas as follows[5]:

- $1 \in \sigma(p)$ and $0 \in \sigma(p)$
- $1 \in \sigma(\backsim A)$ iff $0 \in \sigma(A)$
- $0 \in \sigma(\backsim A)$ iff $1 \in \sigma(A)$ or $1 \notin \sigma(A)$
- $1 \in \sigma(A \circledcirc B)$ iff $1 \in \sigma(A)$ and $1 \in \sigma(B)$
- $0 \in \sigma(A \circledcirc B)$ iff $1 \notin \sigma(A)$ or $1 \notin \sigma(B)$, or $1 \in \sigma(A)$ and $1 \in \sigma(B)$ and either $0 \notin \sigma(A)$ or $0 \notin \sigma(B)$

Finally, let Γ and A be a set of formulas and a formula, respectively, of $\mathcal{L}$. The argument $\Gamma \models A$ is *logically valid* if and only if, for every evaluation σ, if $1 \in \sigma(B)$ for every $B \in \Gamma$ then $0 \notin \sigma(A)$. Otherwise, the argument is not logically valid. Accordingly, A is a *logically valid* if and only if, for all σ, $0 \notin \sigma(A)$. I will call '**HC**' the logic induced by the above evaluations and definitions of logical validity.[6]

Although evaluation conditions are the way to understand these connectives, I display here a tabular presentation to make calculations even easier:

A	$\backsim A$
$\{1\}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$
$\{0\}$	$\{1,0\}$

$A \circledcirc B$	$\{1\}$	$\{1,0\}$	$\{0\}$
$\{1\}$	$\{1,0\}$	$\{1,0\}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$	$\{1\}$	$\{0\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

Note that at least the truth conditions for negation and (Hegelian) conjunction are standard. The falsity condition for conjunction, although a bit more complex than the usual one, is not so odd either: it expresses that the conjunction is false iff either one of the conjuncts is untrue, or both are true but not also both false. The falsity condition for negation is such that it entails that $\backsim A$ is always false. The pay-off of such departure from the standard will be apparent soon, but let me say a few words on why I think it is a principled departure. I think that a contradiction is any binding of a formula and a negation of it. And, unlike many other persons, I do not think that a connective must form contradictory pairs, in the sense of the square of opposition, to be a negation; for me, a negation must negate. What is to negate then, if not forming contradictory pairs in the sense of the square of opposition? Following (Avron 2005, p. 160), I think that a connective c negates if it "represents the idea of falsehood within the language", that is, if cA expresses that A is false, provided that A alone expresses the truth of A. Or, and this is not said by Avron, but I consider it by parity of reasoning, a connective c negates if it "represents the idea of truth within the language" in the sense that, if a formula A expresses initially its falsity, cA expresses that A is true.[7] That is, any

[5] For simplicity, I will make a not so damaging abuse of language and use the same symbol for the relation σ and its extension.

[6] This notion of logical validity is an old acquaintance, namely Malinowski's p-consequence; see Malinowski (1990); see also Wansing and Shramko (2008).

[7] Studies on paraconsistency favoring falsity conditions over truth conditions can be found in Mortensen, 1995, Ch. 11, Mortensen (2003) and then continued in Estrada-González (2010) and Estrada-González (2015).

connective c whose evaluation conditions entail at least one of the following clauses can be considered a negation:

- If cA is true then A is false.
- If cA is false then A is true.

The truth condition for $\backsim$ meets at least Avron's desideratum. (A negation meeting the dual condition will be introduced in the next section.) If all this produces or fails to produce contradictories in the sense of the traditional square of opposition is, to me, besides the point.[8]

A final word on the evaluation conditions that were just given. The reason for making propositional variables true and false in any interpretation is that they are the most basic elements from which contradictions are made up, and any split of a contradiction in simpler parts should deliver, as per Target passage 3, a formula that is both true and false. Also, remember that these are intended to be sentences of reason, and they are at least implicitly contradictory.

One is now in a position to assess the consequences of the semantics offered here, and it seems that double negations are a good starting point because of their distinguished status in Hegel's philosophy. Thus, let me make at least some brief comments on how to relate certain features of **HC** with Hegel's thoughts on double negations. If one calls A, $\backsim A$ and $\backsim\backsim A$ 'thesis', 'antithesis' and 'synthesis', respectively, one can see that each propositional variable is a synthesis. Likewise, every negated propositional variable is a synthesis. This is a form of the identity of opposites: p and $\backsim p$ are equivalent, i.e. they have exactly the same interpretations.

Target passage 1 is helpful here too. Although the (propositional) opposites are identical in the sense just mentioned, it is only by conjoining that their truth becomes salient and their falsity disappears. In addition, although each propositional variable (and its negation) is a synthesis, they are not speculative contents yet, because they are still not presented in the right way (i.e. as a contradiction) to be considered so.

Finally, note that, as one would expect from a synthesis, $\backsim\backsim A$ is truer than A and $\backsim A$, in the sense that it is true in more interpretations than the other two propositions. (And these, thesis and antithesis, are true in the same number of interpretations, but not exactly in the same ones.) This is a good feature of the negation I use here, and a good reason to oppose the idea of keeping a standard negation, be it de Morgan, Boolean or a slight modification of any of these, untouched, as in Beall and Ficara's approach.

It is easy to check now that **HC** meets the desiderata mentioned at the end of the previous section:

- D1 is met. For instance, for every propositional variable p, $\models_{\mathbf{HC}} p \circledcirc \backsim p$.
- As for D2, consider Hegel's own example of "injustice to speculative content", i.e. $p \circledcirc \backsim p \not\models_{\mathbf{HC}} p$ or $p \circledcirc \backsim p \not\models_{\mathbf{HC}} \backsim p$. This is not the only countermodel, though. Any $A \circledcirc B$ such that both A and B are true but only one of them is also false provides a countermodel to Conjunction Elimination.

[8] Hegel was not a big fan of the square of opposition, either. Critical remarks on it, both as a theory of logical concepts and as a figure representing that theory can be found in many places through the *Wissenschaft der Logik*. Pluder (2022) is a useful summary of Hegel's stance towards the square of opposition.

- And, well, logical entailment was precisely defined in a way that an argument with true premises and false conclusion is not logically valid, so D3 is met as well.

Let me highlight some additional features of **HC** worth noting.

- **HC is non-reflexive**. For a countermodel to reflexivity, suppose that $\sigma(A) = \{1, 0\}$. Then $A \not\models_{\mathbf{HC}} A$.
- **HC is non-monotonic**. For a countermodel to monotonicity, consider that $\sim (p \circledcirc \sim p) \models_{\mathbf{HC}} \sim (p \circledcirc \sim p)$ is valid but $\backsim (p \circledcirc \sim p), p \models_{\mathbf{HC}} \sim (p \circledcirc \sim p)$ is not.
- **HC is paraconsistent *tout court***. Both $A \circledcirc \sim A \models_{\mathbf{HC}} B$ and $A, \sim A \models_{\mathbf{HC}} B$ are invalid. Consider simply any case when $\sigma(A) = \{1, 0\}$ and $0 \in \sigma(B)$. Note that even if **HC** was expanded with **FDE**'s conjunction, i.e. a connective $\wedge$ with the following evaluation conditions

$A \wedge B$	$\{1\}$	$\{1, 0\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1, 0\}$	$\{0\}$
$\{1, 0\}$	$\{1, 0\}$	$\{1, 0\}$	$\{0\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

one would have that $A \wedge \backsim A \not\models_{\mathbf{HC}_{\{\wedge\}}} B$. Simply consider the case in which $\sigma(A) = \sigma(B) = \{1, 0\}$.
- **Results on double negation**. Although $\backsim\backsim A$ is true in all interpretations, it is not logically valid in **HC** because in some of those interpretations (in fact, in all of them) it is also false. Other invalid principles concerning double negations include:

$$\sim\sim A \models_{\mathbf{HC}} A \tag{DNE}$$

$$A \models_{\mathbf{HC}} \sim\sim A \tag{DNI}$$

$$\sim\sim A \models_{\mathbf{HC}} A \circledcirc \sim A \tag{DDN}$$

$$A \circledcirc \sim A \models_{\mathbf{HC}} \sim\sim A \tag{CDDN}$$

$$A, \sim A \models_{\mathbf{HC}} \sim\sim A \tag{,-CDDN}$$

DDN, "Dialectical double negation", is valid if A is a propositional variable, although the same does not hold for the other principles.[9]

[9] 'CDDN' stands for 'Converse of Dialectical double negation" and ',-CDDN' stands for the comma-constrained version of Dialectical double negation. 'DNE' and 'DNI' stand, as usual, for 'Double negation elimination' and 'Double negation introduction', respectively. DDN was asked by Ficara (2015), p. 34 on the grounds that, for Hegel, "we capture the true nature of concepts only when, by negating their negation, we gain them in their completeness, which is contradictory." That this holds for propositional variables is good enough for me, because one could make a case for propositional variables as the main bearers of (Hegelian) concepts. Nonetheless, I will not press further on that point.

Moreover, if logical validity is understood as follows:

- The argument $\Gamma \models A$ is *logically valid* if and only if, for every evaluation σ, if $0 \notin \sigma(B)$ for every $B \in \Gamma$ then $0 \notin \sigma(A)$.[10]

and one puts that notion on top of the semantics for $\mathcal{L}$, the resulting logic, that I will call '$\mathbf{HC_{ET}}$', is reflexive, monotonic and the five double negation items hold.

The price to pay for this way of validating reflexivity, monotonicity and the principles on double negation would be simply too high, at least for some people, because with the latter notion of logical validity, one obtains that, for any propositional variables p and q, $p \models_{\mathbf{HC_{ET}}} q$ is valid. One can think that this is so just by vacuity; $p \models_{\mathbf{HC_{ET}}} q$ holds simply because there is no σ such that $\sigma(p) = \{1\}$, and that asking for at least one interpretations in which the premises are just true fixes the problem.

Alas, that is not the case. Asking for non-vacuous validity leads to losing again all the double negation principles that motivated the changes in the notion of logical validity in the first place: simply put a propositional variable p instead of A. But even if A is not a propositional variable, the principles fail: there is no interpretation in which $\backsim\backsim A$ is just true, which would rule DNE, DDN and CDDN out; for that very reason, DNI fails even if A is just true; and there is no interpretation that makes both A and $\backsim A$ just true, which rules ,-CCDN out.

Finally, another feature of $\mathbf{HC}$ that might not be so appealing is that, since every propositional variable is both true and false, any two propositional variables form a contradictory pair and, thus, for any p and q, $\models_{\mathbf{HC}} p \circledcirc q$. Let me suppose that the validity of $\models p \circledcirc q$ must be avoided. One way of doing it is by saying that $A \circledcirc B$ is a formula if and only if both A and B are formulas and B is syntactically of the form $\backsim A$. Said otherwise, $\circledcirc$ binds only contradictory propositions. These would open the option for discussing the possibility of expanding the logic with other linguistic premise-binders, like $\wedge$, or whether the (meta-linguistic) comma is going to do the job. Let $\mathbf{HC}_{nf}$ be $\mathbf{HC}$ based on the language with the restriction on formation rules just entertained, and suppose that one adds $\wedge$ to $\mathbf{HC}_{nf}$ to obtain $\mathbf{HC}_{nf\{\wedge\}}$. Then, if $\circledcirc$ binds only contradictory propositions, one could demand that $\wedge$ binds only non-contradictory propositions, and the question about the validity of $A \wedge \backsim A \models_{\mathbf{HC}_{nf\{\wedge\}}} B$ would not arise in that case because the premise would not be a formula.

4 Final remarks

After challenging Beall and Ficara's formal treatment of a couple of Hegel's passages where he discusses conjunction (of contradictory propositions), I proposed what I hope is a sensible alternative and sketched some other ways to achieve the expected ends. I have not tried to be exhaustive in the ways in which some of Hegel's ideas can be accommodated in a relatively simple formal semantics. Even the small list of desiderata given at the end of Section 2 can be met in several ways as a response to different theoretical pressures. For example, one could say that

[10] Again, in a semantics like the one employed here, this notion of logical validity is an old acquaintance, namely the preservation of just truth, studied extensively first in Pietz and Rivieccio (2013) and then, for example, in Shramko et al. (2017, 2019); Shramko (2019) and Belikov and Petrukhin (2020).

the evaluation conditions of $\circledcirc$ should be such that one obtains the following table instead:

$A \circledcirc B$	$\{1\}$	$\{1,0\}$	$\{0\}$
$\{1\}$	$\{0\}$	$\{0\}$	$\{1,*\}$
$\{1,0\}$	$\{0\}$	$\{1,*\}$	$\{1,*\}$
$\{0\}$	$\{1,*\}$	$\{1,*\}$	$\{0\}$

The reason is that, per the evaluation conditions of negation, the pairs $\{1\} - \{0\}$, $\{1,0\} - \{1,0\}$ and $\{0\} - \{1,0\}$ are the only contradictories, and the cells where they coincide are the only ones that should be true. (The stars indicate that it is left open whether falsity should be assigned there as well.)

One can also say that the preceding objection is along the right lines but that the problem starts with negation, because it does not produce the right contradictory pairs. Rather, negation should be evaluated as follows:

A	$\sim A$
$\{1\}$	$\{1,0\}$
$\{1,0\}$	$\{1,0\}$
$\{0\}$	$\{1\}$

and then conjunction should be something as follows:

$A \circledcirc B$	$\{1\}$	$\{1,0\}$	$\{0\}$
$\{1\}$	$\{0\}$	$\{1,*\}$	$\{1,*\}$
$\{1,0\}$	$\{1,*\}$	$\{1,*\}$	$\{0\}$
$\{0\}$	$\{1,*\}$	$\{0\}$	$\{0\}$

Many more options are available, depending on what are the "three-valued" negations one thinks can play the role needed in the formalization of certain Hegelian ideas. (The interested reader can consult (Omori & Wansing, 2022) and Estrada-González and Nicolás-Francisco (2023) for thorough surveys of options for negation.)

Another important discussion that could arise from these concerns about the proper evaluation of contradictions is what is the proper formation rule for contradictions, and more generally what are the right binders for contradictory propositions and whether they should have different logical properties. Beall and Ficara work under the supposition that their binders —人, $\wedge$ and the comma— can bind any kind of proposition. I did as much for the $\circledcirc$ of **HC**. Nonetheless, the notion of logical entailment in **HC** played a crucial role in preventing that the different versions of Explosion, suitably constrained to one or another binder, had different logical statuses: they all are invalid. But getting this good result does not mean that assuming, like Beall and Ficara did, that any binder can bind any two propositions is correct. A logic like $\mathbf{HC}_{nf\{\wedge\}}$ could then be on a better track than **BF** and **HC**, leading us thus to the proverbial synthesis, at least with respect to the correct way of binding propositions.

On the other hand, an important shortcoming of many formalizations of Hegel's ideas is their static character, and **HC** or even $\mathbf{HC}_{nf\{\wedge\}}$ could certainly suffer from a lack of dynamics. Suppose for a moment that Beall and Ficara were right, that contradictory pairs are untrue before they get conjoined. In fact, Target passage 3 does not rule that out; all Target passage 3 demands is that, when detached from the contradiction, each of the conjuncts is both true and false. Then, each of p and $\sim p$ in a true contradiction would be untrue in the premises of $p, \sim p \models p \circledcirc \sim p$, but they would be both true and false in the conclusion of $p \circledcirc \sim p \models p$ and $p \circledcirc \sim p \models \sim p$. Now, thanks to works like those of Russell (2017, 2018) or Fjellstad (2020) we have the means to evaluate the same symbol differently in the premises and in the conclusions to combine Beall and Ficara's approach (for which, I insist, textual evidence is needed) and my treatment of Target passage 3.

Finally, Beall and Ficara say that their conjunction is "extra-logical". The reasons for such a claim elude me. Also, it eludes me whether the connectives I have used in building **HC** and in sketching its variants are also extra-logical according to them. This, and all the other issues mentioned in this last section, deserves further, separate work.

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