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ORIGINAL ARTICLE

No cause for collapse

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Abstract

We investigate a hitherto under-considered avenue of response for the logical pluralist to collapse worries. In particular, we note that standard forms of the collapse arguments seem to require significant *order-theoretic* assumptions, namely that the collection of admissible logics for the pluralist should be closed under *meets* and *joins*. We consider some reasons for rejecting this assumption, noting some *prima facie* plausible constraints on the class of admissible logics which would lead a pluralist admitting those logics to resist such closure conditions.

Keywords Logical pluralism · Collapse argument · Admissible logics

1 Introduction

Various forms of logical pluralism have been objected to on grounds of what have been called *collapse arguments*, which aim to show that the pluralist is, tacitly or unintentionally, actually committed to some kind of logical monism.[1] We'll consider two such, namely *upward* collapse arguments which seek to show that the pluralist is committed to a single *deductively strong* logic, and *downward*[2] collapse arguments, which seek to show that the pluralist is committed to a single *deductively weak* logic.[3] There has been a fairly substantial literature concerning the collapse arguments, to what extent they have force against various forms of pluralism, and how the pluralist

[1] The phrase "collapse argument" has been used by Caret (2017) to unify a broad collection of objections. We'll focus on a couple of variants, but won't try to be exhaustive in covering the variations available.
[2] We adopt the terminology of collapsing "upward" and "downward" from Steinberger (2019a).
[3] Words like "strong" and "weak" as applied to logic are used in various ways, and there are often unhelpful ambiguities. The way we use the terms has it that a logic $\mathbf{L}_1$ is *stronger* than $\mathbf{L}_2$ just in case the former validates all the arguments of the latter (and, perhaps, then some).

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may avoid them. In this paper, we shall argue that such arguments build in certain substantial, and questionable, assumptions about the collections of admissible logics, and their closure conditions. We then argue that the pluralist has ways out of these arguments if they admit collections of logics which satisfy (or, more aptly, fail to satisfy) certain *order-theoretic* properties.

That is, if we consider the collection of admissible logics as *ordered* in terms of deductive strength, then the properties of this ordering are salient to assessing the effectiveness of collapse arguments. In particular, it seems to be assumed in various collapse arguments that the collection of admissible logics will include logics which are *lattice joins* and *lattice meets*, to which the pluralist is then argued to be *really* committed. We note that if one does not require that the collection of admissible logics is closed under meets and joins, then the strength of the collapse arguments seems to dissipate.[4] This leaves the question of why one might not want the collection of admissible logics to be so closed, to which we'll propose a handful of potential answers. Having made these proposals, we shall consider some lingering questions about the normativity of logic, and about the requirement that all admissible logics be globally applicable.

2 Technical preliminaries

We shall assume that logics are *Tarskian consequence relations* over a fixed language.[5] That is, let us fix a set Fm of formulas (constructed, as usual, out of some atomic formulas, connectives, and punctuation), and take a logic $\mathbf{L}$ to be a relation $\vdash_{\mathbf{L}} \subseteq \wp(Fm) \times Fm$ satisfying the following constraints:

- If $A \in \Gamma$ then $\Gamma \vdash_{\mathbf{L}} A$. REFLEXIVITY
- If $\Gamma' \supseteq \Gamma \vdash_{\mathbf{L}} A$ then $\Gamma' \vdash_{\mathbf{L}} A$. MONOTONICITY
- If $\Gamma \vdash_{\mathbf{L}} A$ and for each $B \in \Gamma$, $\Delta \vdash_{\mathbf{L}} B$, then $\Delta \vdash_{\mathbf{L}} A$. CUT
- If $\Gamma \vdash_{\mathbf{L}} A$ then for any $\sigma : Fm \xrightarrow{hom.} Fm$, $\{\sigma B \mid B \in \Gamma\} \vdash_{\mathbf{L}} \sigma A$.[6] STRUCTURALITY

We'll sometimes use the notation $\Gamma \vdash_{\mathbf{L}} \Delta$ to mean that $\Gamma \vdash_{\mathbf{L}} A$ holds for every $A \in \Delta$.

This definition of "logic" does wind up committing us to some presuppositions which are contentious in the literature. For instance, by fixing a language Fm, and assuming that the connectives of Fm have a univocal meaning in all the logics considered, we are skating past difficult issues concerning logical pluralism and *meaning*

[4] A similar critical point has been made by Timo Meier though his reaction to it is very different from ours Meier (n.a.).

[5] For the sake of simplicity, we concern ourselves just with propositional logics.

[6] The notation "$\sigma : Fm \xrightarrow{hom.} Fm$ indicates that σ is a homomorphism: that is, for each n-ary logical connective $\otimes$, $\sigma \otimes (A_1, \ldots, A_n) = \otimes(\sigma A_1, \ldots, \sigma A_n)$. That is to say, σ is any *substitution* of formulas for formulas which preserves logical form. So what this constraint enforces is that logical consequence is a matter of logical form, and doesn't depend on the particular formulas mentioned. Further details on algebraic logic can be found in Font (2016) and Cintula & Noguera (2021), and we'll assume a familiarity with some such basic notions. We'll employ the standard convention of writing $A_1, \ldots, A_n \vdash B$ in place of $\{A_1, \ldots, A_n\} \vdash B$.

change, as have been discussed in Kouri Kissel (2018a), for instance.[7] Furthermore, by restricting our attention to Tarskian consequence relations, we avoid questions concerning whether *substructural* logics, presented in a certain way, are legitimate.[8] We focus on this setup in order to facilitate our argument, though recognise that doing so involves making substantial assumptions that may be brought into question.

The major advantage of this definition of logic is that there are a number of results about Tarskian consequence relations, of a kind not yet proved for alternatives. In particular, it is known that the family of all such logics form a *complete lattice*.[9] Logics are ordered by subset:

$$\mathbf{L}_1 \leq \mathbf{L}_2 \iff \vdash_{\mathbf{L}_1} \subseteq \vdash_{\mathbf{L}_2}$$

In which case we'll say that $\mathbf{L}_2$ is *stronger* than $\mathbf{L}_1$ (and $\mathbf{L}_1$ is *weaker* than $\mathbf{L}_2$).[10] This order, as has been discussed in Wójcicki (1988), is a complete lattice in that each set of logics $\{\mathbf{L}_i\}_{i\in I}$ has a *join* $\bigvee\limits_{i\in I}\mathbf{L}_i$ and a *meet* $\bigwedge\limits_{i\in I}\mathbf{L}_i$. The meet is defined straightforwardly as set intersection:

$$\bigwedge_{i\in I}\mathbf{L}_i = \bigcap_{i\in I}\mathbf{L}_i$$

but the join is not set union, as this may fail to be a logic (for reasons of transitivity), so we require a more involved definition:

$$\bigvee_{i\in I}\mathbf{L}_i = \bigwedge\{\mathbf{L} \mid \bigcup_{i\in I}\mathbf{L}_i \subseteq \mathbf{L}\}$$

So we take not the join, but rather the least logic including the join — that way, we can be sure that what we have is a logic, while still preserving the inferential resources of all the logics to be joined.

To spell out this formalism a bit, what we require for A to follow from Γ in $\bigvee\limits_{i\in I}\mathbf{L}_i$ is that A has to follow from Γ given the inferential resources we have in *any* of the logics $\{\mathbf{L}_i\}_{i\in I}$. One way for this to be the case is if $\Gamma \vdash_{\mathbf{L}_i} A$ holds for some such logic, but another way for this to happen is if we have multiple logics where we can reason from Γ, via a sequence of collections of lemmas (using different logics among $\{\mathbf{L}_i\}_{i\in I}$), to A. The latter option amounts to *chaining together* inferences in the "joined" logics.

7 Issues surrounding whether the same connective means different things in different logics may arise in our setting (though cross-language comparisons will not), but we set those issues aside as tangential for our purposes.

8 In order to accommodate our style of argument beyond the Tarskian setting, we'd need a bit more investigation into other frameworks for consequence relations, which are starting to come up for investigation, for instance in Cintula & Paoli (2021) and Badia et al. (n.a.). We encourage the development of such projects, and expect that a straightforward variation on our work here will transfer there.

9 We'll proceed assuming a basic familiarity with lattice and order theory. All of our results can be found in Davey & Priestley (2002).

10 From here on in, we'll use expressions like $\mathbf{L}$ and $\vdash_{\mathbf{L}}$ more or less interchangeably.

As an example, neither intuitionistic logic $\mathbf{J}$ nor the relevant logic $\mathbf{R}$ can prove *Peirce's law* as a theorem, i.e. for $\mathbf{L} \in \{\mathbf{J}, \mathbf{R}\}$:[11]

$$\varnothing \nvdash_{\mathbf{L}} ((A \to B) \to A) \to A$$

However, in intuitionistic logic we have the provability of:

$$\neg\neg A \to A \vdash_{\mathbf{J}} ((A \to B) \to A) \to A$$

and in $\mathbf{R}$ we have:

$$\varnothing \vdash_{\mathbf{R}} \neg\neg A \to A$$

Thus, we have $\varnothing \vdash_{\mathbf{J} \vee \mathbf{R}} ((A \to B) \to A) \to A$ from chaining these together.[12]

3 Collapse arguments

What Caret (2017) calls *collapse arguments* are among the most venerable points of opposition logical pluralists have faced. The aim of such arguments is to show that the pluralist's view collapses into *logical monism* of some form. A version of this objection, the *upward* collapse argument, has been pressed by Priest (2001) and, following on him, Read (2006), as well as by Keefe (2014).[13] A further version, the *downward* collapse argument, has been proposed by Steinberger (2019a) and Bueno and Shalkowski (2009). The upward collapse arguments are aimed at the conclusion that the pluralist is committed to a relatively *strong* logic, whereas the downward collapse arguments aim to show that the pluralist is committed to a relatively *weak* logic. Let's consider these in some more detail.

3.1 The upward collapse argument

Read (2006), developing on Priest (2001), presses the upward collapse argument in the following terms (we've altered the notation to match that we're using here):

> Graham Priest challenges Beall and Restall as follows: suppose there really are two equally good accounts of deductive validity, $\mathbf{L}_1$ and $\mathbf{L}_2$, that B follows from A according to $\mathbf{L}_1$ but not $\mathbf{L}_2$, and we know that A is true. Is B true? Does the truth of B follow (deductively) from the information presented?…The answer seems clear: $\mathbf{L}_1$ trumps $\mathbf{L}_2$. After all, $\mathbf{L}_2$ does not tell us that B is false; it simply fails to tell us whether it is true.…It follows that in a very real sense, $\mathbf{L}_1$ and $\mathbf{L}_2$

[11] Information about intuitionistic logic can be found, among other places, in Dummett (1977) and Chagrov & Zakharyaschev (1997). Information about relevant logics, including $\mathbf{R}$, and some systems we'll consider later, can be found in Anderson & Belnap (1975) and Routley et al. (1982).

[12] Hence, note, that $\mathbf{J} \vee \mathbf{R} = \mathbf{CL}$, where $\mathbf{CL}$ is classical logic, as both $\mathbf{J}, \mathbf{R} \lneq \mathbf{CL}$ and the axiomatic extension of $\mathbf{J}$ by $((A \to B) \to A) \to A$ (or $\neg\neg A \to A$) is $\mathbf{CL}$. Note that this example serves to illustrate the reason why the join of a set of logics is not just the union.

[13] A formulation of the general problem can already be found in Williamson (1988).

are not equally good. $\mathbf{L}_1$ answers a crucial question that $\mathbf{L}_2$ does not. For Priest's question is the central question of logic.

(Read 2006, pp. 194–195)

By "the central question of logic", Read refers to the question "what follows from what", which Priest, Read, and, it seems, Beall and Restall (2006) all take to be what logic is about. The challenge rests squarely on this notion of *trumping*,[14] and it would seem that strength is the appropriate notion here — in Read's example, if we were comparing **CL** and **J**, the argument would have us conclude that the pluralist is really a monist about **CL**.

We can, it seems, generalise this argument in the following way to consider more than just two logics (which does not seem out of line with how upward collapse arguments have been discussed in the literature).

(P1) The central role of logic is to characterise the correct patterns of inference.
(P2) The strongest logic admitted by the pluralist trumps all the other admitted logics.
(P3) The central role of logic is thus fulfilled by the strongest admitted logic.

The pluralist is committed to monism about their strongest admitted logic.

This just seeks to precisely set out Read's argument, while generalising it to consider comparisons between classes of logics. As we'll mention shortly, we've omitted a key assumption, namely, that there *is* a strongest logic under consideration — there certainly is a logic which is the join of all those under consideration, but this argument requires the assumption that it is *admissible*. We'll come back to this shortly, but first let's consider an argument for collapsing in the opposite direction.

3.2 The downward collapse argument

The upward collapse argument sought to undermine versions of pluralism that assume a form of *global application* of logics — namely that logics seek to do one thing.[15] The downward collapse argument goes in the opposite direction, seeking to undermine versions of pluralism which do not (seem to) make this assumption, but which, rather opt for some kind of *logical relativism*.[16] One such way to go would be to take different admissible logics to apply to different domains of discourse,[17] such as having intuitionistic logic as admissible and applying to mathematics or ethics (or some other thing supposed to be constructive) and having classical logic as admissible for applications to *middle sized dry goods in the middle distance*.[18]

[14] The idea that a stronger logic *trumps* a weaker logic in cases of disagreement depends on the assumptions that both logics exhibit the same kind of normativity for thought and that invalidity-claims don't provide normative guidance at all. Both assumptions have recently been called into question in Blake-Turner (2021) and Evershed (2021) respectively. We will set those issues for now, but we will discuss a related idea in Section 6.

[15] The global scope of logical consequence is thus also mentioned as a premise in the generalised version of the argument found in Stei Rivalry (2020).

[16] For a general formulation of logical relativism see Shapiro (2014, pp. 7–8).

[17] Haack calls this "*local pluralism*" (Haack 1978, p. 223).

[18] Such a view might be desirable for a truth pluralist like Michael (2009), as discussed by Shapiro and Lynch in Shapiro & Lynch (2019) and in a similar vein conceptualised by Pedersen in Pedersen (2014).

Steinberger (2019a) sets out a form of the downward collapse argument, in analogy with upward collapse arguments:[19]

> [T]he natural move here is instead to take the intersection of the logics in question. This is in the spirit of logical modesty: When engaging in cross-domain reasoning, we should draw only on principles sanctioned by all the relevant logics.
>
> (Steinberger 2019a, p. 15)

The key idea seems to be that against the relativist-seeming assumption of multiple domains, we should instead recognise the core importance of universal applicability, and require "capital L" Logic to be applicable to every domain.[20] Assuming that reasoning in different domains will validate different principles of inference, the one true Logic is just that which validates only those principles valid for use in every domain.[21]

Using this as a starting point, we can construct a collapse argument along something like the following lines:

(P1) The domain-relative pluralist admits multiple correct logics, each of which applies to a particular domain.
(P2) The intersection of all the domain-relative pluralist's admitted logics applies to every domain.
(P3) Logic is universal, and hence can be appropriately applied to every domain.[22]

The domain-relative pluralist is committed (via (P1)) to logical monism concerning the intersection of their admitted logics.

[19] A related point has been made by Priest (2006, §10.14) in response to the question of whether logic is global or local. He, however, does not push this as an argument against the pluralist, but rather as reason to think that the dispute between the logical globalist and the logical "localist" is not significant. Questions about whether disagreements over logic are meaningful are rather common, but getting into that topic would take us too far afield. A version of a downward collapse argument that is developed in analogy to the problem of mixed inferences for alethic pluralists can be found in Keefe (2018, p. 441).

[20] Bueno and Shalkowski develop a version of the downward collapse argument. They argue that Beall and Restall's conceptualisation of necessity as enjoining satisfaction preservation in *all* cases naturally leads to the universality of logical consequence. They describe the downward collapse of Beall and Restall's pluralism in the following way: "Only a very weak consequence relation survives this scrutiny, according to their accounting of the necessity constraint as quantification over all cases. This relation is just the intersection of the inferences treated as valid by classical, constructive, and paraconsistent logics. Some fragments of positive logic and some rules of identity will survive." (Bueno and Shalkowski 2009, p. 299).

[21] That logic is general/universal/topic-neutral/etc. is oftentimes seen as part of the traditional and predominant view of logic. So, e.g., Martin and Hjortland (citing Frege and Kant) claim this to be one of "Logic's special properties" that the Anti-exceptionalists reject (Martin & Hjortland 2022, pp. 8–9). Apart from such more traditional views on logic, the universality of logic is also of importance in Russell's argument for nihilism in Russell (2018).

[22] For logical pluralists who consider not differing *domains* but rather something else, like differing *subjects*, please feel free to make the necessary substitutions and assess the argument having done so. Strictly speaking, it seems that the logical relativist must (in our understanding and following Shapiro (2014)) *always* take logic to be relative to some domain. This should then also count for the universal logic. This means that the universal logic is also relative, but to a special domain: the absolute domain or the domain of all domains. Related discussion can be found in Rayo & Uzquiano (2006). It's not clear what difference, if any, this consideration makes for our point, so we won't pursue it further.

This, similarly to the upward collapse argument, seems to make the tacit assumption that the intersection of the pluralist's admissible logics (the meet of all such logics) will itself be admissible.

We can see traces of this kind of consideration motivating Beall's (2017, 2018, 2020) recent turn to logical monism with **FDE** as the one true Logic, and other inferential principles one might adopt as being *extra-logical*, and adopted for particular theoretical purposes. For Beall, any admissible logics will make use of *distributive lattice* properties for conjunction and disjunction, and at least double negation equivalence and the De Morgan laws for negation — hence, **FDE** is the "basement level" consequence relation applying everywhere.

3.3 How to escape collapse arguments

There have been a number of attempts by pluralists (and fellow travelers[23]) to respond to collapse arguments. For instance, we've cited Caret (2017) who proposes a *contextualist* solution. There are surely many such solutions available.

Our focus, however, is on the tacit *order-theoretic* assumptions. In the upward collapse argument, this is the assumption that the join of all the admissible logics will itself be admissible; in the downward collapse argument, it is that the meet of all the admissible logics will itself be admissible. There may be some reasons for the pluralist to want their set of admissible logics to be so closed, but we'll consider some reasons why they might not want that, and propose that if they don't, indeed, want that, then they also have a clear-cut path for escaping the force of the collapse arguments.

The solution proposed is a *schematic* one — we don't provide any universal reason for rejecting the assumption that the class of admissible logics must form a (join/meet semi-)lattice, but rather note that doing so provides an escape hatch. It is up to the individual pluralist to assess whether they have reasons for adopting such a class of admissible logics, and, having done so, thumb their nose at the would-be collapser.

4 Why not have a join-semilattice of admissible logics?

In this section we'll consider two sorts of reasons one might have for resisting admitting the join of one's admissible logics. Both rely on the fact that the join of a class of logics will, if not itself admissible, be inferentially stronger than all of one's admissible logics. Thus, if one is pressed to require their admissible logics not to be *too strong*, then one has reason to push back against this pressure.

[23] We are included at least among these, though whether we are actually pluralists or not, and if so what kind we are, is a question whose answer may differ according to the context of the asking.

4.1 Avoiding triviality

The first reason hearkens back to the by now well-worn caution[24] that *too much strength is a bad thing*, at least when it results in one being committed to the trivial logic:

$$\mathbf{Triv} = \wp(Fm) \times Fm$$

Clearly, commitment to this logic has a number of undesirable results. For one thing, as a tool for studying good inference, and distinguishing this from bad inference, **Triv** is more or less useless — according to it, every inference is good.[25] Furthermore, if one requires something like *closure of commitments* under this logic (in a manner we'll discuss further later, according to which one ought (defeasibly) not to accept all the premises and reject the conclusion of a valid argument), then one is stuck in the tricky position of either having to accept everything, or reject everything. This is not an enviable epistemic position in which to find oneself. Even formally speaking, **Triv** is little more than a convoluted way of studying the language Fm.

It is not too difficult to find examples of seriously proposed logics in the literature which join up to **Triv**. Let's consider two such recipes.

4.1.1 Classical logic and a contraclassical logic

Classical logic **CL** is famously *Post-complete*: any proper extension of **CL** (in the same language) will be trivial. That is, if a logic **L** is such that $\mathbf{CL} \lneq \mathbf{L}$ then $\mathbf{L} = \mathbf{Triv}$. A logic **L** is *contraclassical* in the "superficial" sense of Humberstone (2000) just in case **L** is not a sublogic of **CL** (i.e. $\mathbf{L} \not\leq \mathbf{CL}$). It follows immediately that for any contraclassical logic **L**, $\mathbf{L} \vee \mathbf{CL} = \mathbf{Triv}$, and so if one's class of admissible logics includes any such pair, then their join will be **Triv**.

Classical logic is usually taken for granted as admissible. Indeed, it seems to be the default admissible logic (the one we use when "nothing goes wrong"). So we guess the pluralist who admits classical logic won't have too much work to do in justifying this (perhaps unfortunate) choice. The partisan of a contraclassical logic will have a much harder time — most forms of logical pluralism seem to consider only subclassical logics. Indeed, Beall and Restall (2006) seem to do so in response to Read's (2006) objection that, for upward collapse reasons, adopting a contraclassical logic will force them to admit **Triv**.

There are, however, a few fairly well-developed contraclassical logics (at least by the standard of "well-developedness" appropriate for non-classical (or "deviant", if you follow Haack's (1996) nomenclature, to apply to only those logics which invalidate some classically valid inference forms) logics beyond intuitionistic logic). Some salient examples include *connexive logic* (Wansing, 2022; Francez, 2021) and *Abelian logic* (Meyer & Slaney, 1989, 2002). The former, at least, has been put forward philosophical purposes (some of which are discussed in Wansing (2022)), and so it is

[24] See Robert (1971) for a particularly eloquent and amusing version: for a more recent discussion of strength as a theoretical virtue see Russell (2019).

[25] One upshot of this is that it doesn't allow us to distinguish logically valid from other good forms of argument, such as strong inductive or abductive arguments. Thanks to Anna-Sara Malmgren for noting this point.

conceivable that there may be some logician who wants to adopt both classical logic and some connexive logic as admissible. Any such logician (perhaps working in the vicinity of Bochum, Germany) will have the grist of a response to collapse worries.

4.1.2 An explosive logic and an inconsistent logic

For our purposes, an *explosive* logic $\mathbf{L}$ is one in which (assuming the language includes a negation connective $\neg$) the argument form $A, \neg A \vdash_{\mathbf{L}} B$ is valid.[26] Futhermore, we'll call a logic $\mathbf{L}$ *inconsistent*[27] just in case there is some $A \in Fm$ such that $\varnothing \vdash_{\mathbf{L}} A$ and $\varnothing \vdash_{\mathbf{L}} \neg A$.[28]

It is straightforward that if $\mathbf{L}_1$ is an explosive logic and $\mathbf{L}_2$ is an inconsistent logic then $\mathbf{L}_1 \vee \mathbf{L}_2 = \mathbf{Triv}$, for note the following reasoning:

$$\dfrac{\dfrac{\varnothing \vdash_{\mathbf{L}_2} A \qquad \dfrac{\varnothing \vdash_{\mathbf{L}_2} \neg A \qquad A, \neg A \vdash_{\mathbf{L}_1} B}{A \vdash_{\mathbf{L}_1 \vee \mathbf{L}_2} B}\,\textsc{Cut}}{\varnothing \vdash_{\mathbf{L}_1 \vee \mathbf{L}_2} B}\,\textsc{Cut}}{\Gamma \vdash_{\mathbf{L}_1 \vee \mathbf{L}_2} B}\,\textsc{Monotonicity}$$

and Structurality ensures that for any $\Gamma \cup \{A\} \subseteq Fm$, $\Gamma \vdash_{\mathbf{L}_1 \vee \mathbf{L}_2} A$, and thus $\mathbf{L}_1 \vee \mathbf{L}_2 = \mathbf{Triv}$.

Now, while not all connexive logics are inconsistent, some, like Wansing's $\mathbf{C}$ (Niki & Wansing, n.a.) are. Abelian logic (Meyer & Slaney, 1989) is also an inconsistent logic, as are connexive *extensions* of relevant logics (Mortensen, 1984). Furthermore, there are interesting subtleties concerning how the inconsistencies of these logics spread (or don't spread) to premise sets, as has been investigated in Mangraviti & Tedder (n.a.). So it is, again, not inconceivable that one may desire to admit some inconsistent logic. Furthermore, many of the most popular logics on the scene are explosive (including **CL**, of course, but also intuitionist logic, fuzzy logics, and so on). So, as with **CL**, the pluralist seeking to admit an explosive logic will probably not run into too much in the way of challenges to their admission.

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26 One might prefer to widen this definition to call a logic explosive if it validates arguments like the following, making use of some different vocabulary:

$$\neg A \vdash A \to B$$
$$\varnothing \vdash (\neg A \wedge A) \to B$$
$$\varnothing \vdash \neg A \to (A \to B)$$

but we'll stick to the simple presentation for now.

27 Note that we use the phrase "inconsistent logic" differently than some in the literature — such as Font (2016) — who use this phrase to pick out what we have called the trivial logic **Triv**. Our terminology follows Wansing (n.a.), Niki & Wansing (n.a.), and Mangraviti & Tedder (n.a.), who study logics with inconsistent sets of theorems.

28 As with "explosive logic" there are a handful of different options for cashing out this notion — such as requiring that there be some A such that $\varnothing \vdash_{\mathbf{L}} A \wedge \neg A$ — but we stick with this simple presentation.

4.2 Preserving systematic properties

Concern with triviality is, however, just one reason to oppose admitting logics which are too strong. Another reason to be skeptical of such admission would be a desire that all admissible logics satisfy some systematic properties which limits the joint validities of these systems. Let's consider some examples.

4.2.1 Multiple routes to paraconsistency

In trying to avoid explosion, while preserving as much else of classical logic as possible, there are a few natural avenues one will land on. We can delineate some such options by considering a version of Lewis' (1932) "proof" of explosion:

$$\dfrac{\dfrac{\dfrac{A, \neg A \vdash A \qquad A \vdash A \vee B}{A, \neg A \vdash A \vee B} \qquad A, \neg A \vdash \neg A}{A, \neg A \vdash (A \vee B) \wedge \neg A}\,(\star) \qquad (A \vee B) \wedge \neg A \vdash B}{A, \neg A \vdash B}$$

The two most popular options are to reject the inference rule $(\star)$, leading to such non-adjunctive paraconsistent logics as Schotch et al. (2009). Another popular option is to reject disjunctive syllogism $(A \vee B) \wedge \neg A \vdash B$, obtaining paraconsistent logics such as **LP** (Priest, 1979), and the usual relevant logics (Anderson & Belnap, 1975).

As the proof shows, any logic including all these inferences will be explosive. So if one is a particularly hardcore paraconsistentist, who holds that *every* admissible logic should be non-explosive, then if one adopts the non-adjunctive strategy for some purposes (such as reasoning about *combining* sets of beliefs or commitments) and another for some other purposes (such as reasoning about *true contradictions*, assuming there are any), then one has good reason to reject join closure.

4.2.2 Preserving relevance

The most common way of formalise the notion of *relevance* in relevant logics is in terms of the *variable sharing property* (VSP), namely: a logic **L** has VSP just in case $\varnothing \vdash_{\mathbf{L}} A \to B$ holds only if there is an atomic formula in common between A and B. While there are other ways of formalising the notion of relevance, this is the standard: it assumes that the implication connective $\to$ captures the *real* notion of entailment, and that Tarskian consequence relations are, at best, an instrument for investigating logics.

Probably the most famous relevant logic (**FDE** aside) is the logic **R**, discussed earlier. A proof that **R** has VSP is one of the earliest results in the field, but there are a number of other systems, including some logics which are *not* subsystems of **R**, which also have VSP. One salient example is the logic **TM**, which comprises the extension of the logic **T** of *ticket entailment*, discussed in Anderson and Belnap (1975, Ch. 5) by the so-called *mingle* axiom:

$$A \to (A \to A)$$

In the extension of **R** by this axiom, obtaining the system **RM**, we can quickly obtain results which injure VSP, such as the following theorem:[29]

$$\vdash_{\mathbf{RM}} (A \wedge \neg A) \rightarrow (B \vee \neg B)$$

So **TM** is not a subsystem of **R**, but it has been shown, in Méndez et al. (2012), that **TM** has VSP. So it is compatible with being a *relevantist pluralist* to admit both **TM** and **R**, but admitting their join is to admit the irrelevant system **RM**.[30]

4.2.3 Preserving the disjunction property

Another property that many have taken seriously, this time for reasons of constructivity, is the *disjunction property*, enjoyed by a logic **L** just in case whenever $\varnothing \vdash_{\mathbf{L}} A \vee B$ then either $\varnothing \vdash_{\mathbf{L}} A$ or $\varnothing \vdash_{\mathbf{L}} B$. This is a feature of intuitionistic logic **J** (a proof can be found in Chagrov & Zakharyaschev 1997, §2.8), but it also holds of the *contraction-free* relevant logic **RW**, as was proved in Slaney (1984). So a pluralist of a certain constructivist stripe may well have grounds to admit both of these logics, while rejecting their join $\mathbf{CL} = \mathbf{J} \vee \mathbf{RW}$ which, famously, lacks the disjunction property.

These are some examples of systematic properties close to (at least one of the) authors' heart(s), but there are likely to be others which similarly militate against admitting the join of a collection of admitted logics. This provides us a couple of avenues for the logic-join resister, so let us briefly turn our attention to meets.

5 Why not have a meet-semilattice of admissible logics?

In a sense, the considerations in favour of resisting closing the set of admissible logics under meet are dual to those for joins. In this case, the *limiting* logic is not **Triv** but rather the logic of *conclusion*:

$$\mathbf{Incl} = \{\langle \Gamma, A \rangle \in \wp(Fm) \times Fm \mid A \in \Gamma\}$$

In the Tarskian setting, this is the minimum logic, admitting as valid only those inferences which literally just *recapitulate* a premise in the conclusion. While not as obviously undesirable as **Triv**, there are reasons to be skeptical of **Incl**. For one thing, under almost any way of reading the phrase "normative guidance" (a range of such options are surveyed in MacFarlane (n.a.)), **Incl** provides no normative guidance. What guidance it provides boils down, more or less, to the rule *don't accept and reject the same proposition*, which is likely to be, at best, otiose (depending on your views about commitment and acceptance/rejection).

There are, however, some salient differences. In particular, it seems to require *more radical differences* among one's set of admissible logics in order for them to meet

[29] Salient information of **RM** can be found in Anderson and Belnap (Anderson & Belnap 1975, §29.5).

[30] It has been argued, e.g., in Dunn (2021) and Shramko (2022) that while not satisfying VSP, **RM** is still *good enough* for many of the purposes of relevantists/paraconsistentists. For a response which argues that **RM** is a bad choice for the relevantist, see Tedder (2022).

to **Incl** than for them to join to **Triv**. In order for this to happen, one must already admit some logics which have no *substantive* validities in common, so to speak. Given that there is fairly widespread agreement on the inferential behaviour of at least some connectives (most notably conjunction), this seems like an unlikely scenario.

Having said this, however, it seems that there are some reasons one might not want to admit the meet of one's admissible logics. One such reason is that the meet might be *too weak* to fulfill one's goals; another is that one might take the various applications of one's logics to be *incongruent* to such an extent that one should not expect there to be any overlap. Let's consider some variations on these themes arising in the literature, and how they put pressure on meet closure.

5.1 Not *meeting* the goals of logic

If one admits logics on the grounds of their ability to facilitate some *goal*, then it may well be that rather different logics can do so successfully, while their meet may not. There is a considerable number of logical pluralists putting forward such a goal-oriented view of logic, each of which possibly lends itself to endorse a certain collection of logics but not admit their meet, as shown by the following examples.[31]

5.1.1 Logical eclecticism

Consider Shapiro's (2014) *logical eclecticism*, according to which logics are to be admitted on the basis of their capacity to provide for formal frameworks for fruitful mathematical theories. On the grounds of this, Shapiro admits **CL** and **J**, but leaves the door open that many other logics may also work for similar aims.

On one hand, it seems that one could build logics in a way quite different from **CL** or **J**, so long as one adapted one's mathematical axioms to suit, and still obtain interesting mathematical theories, where the logic allows one to draw significant conclusions beyond those assumed in the axiomatic basis. On the other hand, however, it seems that if one's logic is too weak, then one will be *unable* to draw much of anything that isn't already assumed (**Incl** is the limit case of this tendency for weak logics). For example, a pluralism that admitted sufficiently different logics would find a rather weak meet: for instance, the meet of *quantum logic* (for details, see the classic Birkhoff & von Neumann (1936) or Goldblatt (1974)), intuitionistic logic, and some weak paraconsistent or relevant logic such as **FDE**, would lead one to a rather minimal extension of lattice logic, by a negation and conditional which obeys very few prop-

[31] If one decides that the global scope of logic is in fact a necessary criterion for something to be a capital "L" Logic, then our argument will lead to nihilism rather than pluralism. Starting from the point of logical relativism to begin with, a pluralism that doesn't admit the meet of its accepted logics to itself be admissible seems (at least to us) like the more natural solution. Thanks to Teresa Kouri Kissel for pushing us on that point.

erties. While not quite the minimal Tarskian consequence relation, this would still be extremely weak.[32]

This provides at least an indication that taking meets may not provide the basis for interesting mathematical theorising. For instance, such considerations have caused some consternation in the area of *relevant* and *inconsistent* formal mathematics, where it is a common refrain that very weak logics provide little to no inferential power to draw significant consequences not already apparent in the axioms.[33]

Another broader form of elective pluralism was developed by Teresa Kouri Kissel (2018b, 2019). Her approach generalises Shapiro's pluralism to account for deductive goals other than formalising mathematics. This generalising move makes it, if anything, even less plausible that the meet of a collection of logics will itself be suitable for some goal. If we broaden the kinds of goals we can use logics for, it makes it, *prima facie*, less likely that there will be substantial overlap between the logics used to satisfy those different goals.

5.1.2 Telic pluralism

Leon Commandeur (2022) has suggested that logic might not only meaningfully serve one (exceptional) goal, but is adequately described as fulfilling a plurality of goals resulting in his "Telic Pluralism". This view on logic straightforwardly results in a logical pluralism when different admissible goals are attained by using different logics. The examples Commandeur uses to motivate his position suggest that this is the case. Some such goals are: providing a formalisation of logical consequence in natural languages, capturing the structure of mind- and language-independent reality, or model information-flow.[34] It is reasonable to assume that these goals are attainable only by using different logics. However, taking the meet of all so admitted logics will most likely not be of instrumental use to attain any of said goals. This provides the telic pluralist with a reason to admit a plurality of different logics while rejecting to endorse their meet.

5.2 Pespectival pluralism

The second possibility alluded to above is that the things to which we apply different logics might be so different that one would expect, *prima facie*, that there is no significant overlap.

[32] A related point has been made by Bueno and Shalkowski (2009, pp. 299–300), related to the passage we cited earlier.

[33] For salient discussion, see Weber (2021), where a relevant implication connective is combined with an irrelevant, but still non-contractive, implication in order to draw significant consequences from a naïve theory of sets.

[34] For examples of logic as modelling consequence in natural language see Priest (2006) and Cook (2010), for the mentioned kind of realism about logic see McSweeney (2019), and for logic as modelling information-flow see van Benthem (2011) and Wansing (2022).

Recently, Roy Cook (2023) aimed to fill a lacuna in the landscape of logical pluralism by promoting the idea of different *perspectives* being the source of plurality in logics. The general idea is that, accepting insights from *standpoint epistemology*, epistemic subjects might reason in different, but in the context of their respective perspectives *correct*, ways. Standpoints are usually taken to be influenced by certain social and political circumstances, but the general idea of a plurality of epistemic perspectives is sufficient for our purpose and does not depend on any one way of accounting for standpoints. If the difference in perspectives can be taken to also affect the logical parts of reasoning, then pluralism about logic is correct, assuming a close tie between logic and reasoning. It is, however, easy to see that the meet of all so admitted logics will only by chance correspond to the logic used by a group of epistemic subjects from their specific standpoint. Thus, also perspectival pluralism provides reasons for rejecting the meet of all admitted logics.[35]

5.3 The difference between rejecting joins and meets

So far the provided reasons for resisting meet closure are, in a sense, less concrete than the kinds of considerations we bring to bear in the case of rejecting join closure. Nonetheless it seems that there is the grist here for a case to be made that meet closure need not be required of the admissible logics of every kind of logical pluralist.

Having said this, we can also find some systematic properties not preserved under meets, and thus obtain some concrete examples as in the join case. In general, this will turn not on what becomes *provable*, as in the case of taking a join, but rather with what becomes *unprovable* when we take the meet. This is because, dual to the join case, if an inference form is invalid in any of the meet-ed logics, then it will be invalid in the meet, which is where we can run into the sorts of problems with weakness mentioned above. This can, however, break some desired properties. We'll just go into one example in detail, concerning our old friend the disjunction property.[36]

5.3.1 Example: The disjunction property again

We saw above how taking the join of logics satisfying the disjunction property might result in a logic which doesn't enjoy this property, and taking the meet of some such

[35] Note that only the case of eclectic pluralism really depends on the meet of all admitted logics being deductively *weaker*, while the other two examples provide reasons independent of logical strength. Thanks to Franci Mangraviti for pointing this out and suggesting perspectival pluralism as a reason for rejecting meets.

[36] An example which involves a bit more technical complexity concerns whether a logic has *finite characteristic matrix* or is *tabular* — information on matrix semantics can be found, e.g., in Font (2016, Ch. 4). The meet of a collection of tabular logics need not be tabular. As an example, consider the class of logics induced by some finite Heyting algebra. Because **J** has the finite model property (see Ono 2019, §7.4), it is the meet of all such logics, and yet, famously, **J** does not have a finite characteristic matrix.

logics can also have this result. For example, take $\mathbf{RW} \wedge \mathbf{J}$, as above. Note then if $\mathbf{L} \in \{\mathbf{RW}, \mathbf{J}\}$:

$$\varnothing \vdash_{\mathbf{L}} (\neg\neg A \to A) \vee (A \to (\neg A \to B))$$

because we have:

$$\varnothing \vdash_{\mathbf{RW}} \neg\neg A \to A \qquad \text{and} \qquad \varnothing \vdash_{\mathbf{J}} A \to (\neg A \to B)$$

and in both of these systems:

$$A \vdash_{\mathbf{L}} A \vee B$$

Yet, clearly:

$$\varnothing \nvdash_{\mathbf{RW} \wedge \mathbf{J}} \neg\neg A \to A \qquad \text{and} \qquad \varnothing \nvdash_{\mathbf{RW} \wedge \mathbf{J}} A \to (\neg A \to B)$$

This is an eminently generalisable failure, as it will hold whenever we take the meet of a collection of independent logics all of which validate the inference rule of *disjunction-introduction*. So if one is a pluralist who insists on the disjunction property, then one also has as much reason to be suspicious of meet closure as of join-closure.[37]

6 A lingering thought about normativity

We've set out our main point — that there is a, to our knowledge, unexplored avenue of response for pluralists against collapse objections, and furthermore that this avenue may well be desirable for pluralists of a certain stripe. Admittedly, this stripe may well be uncommon, and perhaps uninhabited, nonetheless it seems to provide a live option. Furthermore, it points to a lacuna in discussions surrounding this issue. Before concluding let's consider one remaining point, and a potential problem, to which we were led by the preceding discussion.

In our treatment of the upward collapse argument, we've noted a straightforward formal way around the problem, but this may seem quite unsatisfactory to the would-be collapse-pusher. Indeed, there is something to the intuitive appeal of the argument which we have not addressed. In particular, it seems that there is a case to be made that if one admits a collection of logics, then in endorsing all of them one is granting a salient claim about normativity: namely, that one will not go wrong in reasoning in accordance with each of these logics. If one further admits that all the logics ought to apply *globally*, then it seems one is committed to the claim that one can't ever go wrong *in any context* by reasoning in accordance with any of the admitted logics.

It seems to us that this intuition is trading on a plausible account of the *normativity* of logic: namely, if one admits the logic $\mathbf{L}$ and $\Gamma \vdash_{\mathbf{L}} A$, then one ought (defeasibly) not to accept all of Γ while (simultaneously) rejecting A. If we take this as the salient kind of normativity of logic, then there is a sense in which one can *never* go wrong by reasoning in the join of all one's admitted logics. To do so just requires that one *chain together* inferences in one's admitted logics. Furthermore, it seems that if one

[37] Thanks to Shay Logan for suggesting this point.

cannot go wrong with any individual step, then one cannot go wrong in chaining the steps together.[38] So on this kind of picture of the normativity of logic, one cannot go wrong by using the join of some admitted logics (as doing so either involves using one of the admitted logics or using just such a chaining together of inferences from admitted logics) and so one has little ground to reject its admission. This seems to put pressure on the pluralist who wants to avail themselves of our suggestion of resisting the join-closure of the collection of admissible logics.

When discussing the normativity of logic, there are some salient distinctions to draw, which we'll draw following (loosely) on the heels of Steinberger (2019b). The above charactersation of normativity seems to best fit the third-person *appraisal* kind discussed in Steinberger, but there is another which seems to be salient, especially when considering the kinds of proposals given in Section 4.2. The constraints considered there, especially as concerns relevance, can be seen as involving directives about *how (not) to reason*. This sort of *directive* normativity has a more first-person flavour, embodying rules salient for how an agent goes about drawing inferences, not about how their inferences, or indeed their total set of commitments, are assessed from the outside.

According to this distinction, there seems to be a way to explain the salient intuition. One reason why we might want to rule out such chaining, even while admitting something like global applicability of all the admissable logics, is that while one can't get into trouble of the commitment-position-incoherence sort, nonetheless one *ought not* reason that way. According to the relevantist pluralist, while irrelevant reasoning may not wind up with you doing some bad accepting and rejecting, nonetheless the reasoning itself is bad *qua* being irrelevant.

This solution (or at least explanation) may share some affinity with some recent proposals to solve the collapse problem such as proposed by Blake-Turner (2021) or Tajer (n.a.). The general idea behind both of theirs and our solution is to attribute different normative roles to different logics. This avoids the "trumping"-mechanism described in Section 3.1, since the different kinds of normativity can still be thought of as trumping each other in a practical sense, but no longer render any of the logics normatively idle. Following Tajer, we could assign different bridge-principles to the logics endorsed by the relevantist pluralist and their non-relevant join. In the proposed scenario, the explicitly endorsed logics encompass a normativity close to MacFarlane's (n.a.) principle WO+, while their join only exhibits something close to WP+.[39]

So while the intuition that reasoning in the join of a collection of logics each of which one admits won't get one into trouble, nonetheless there can be a principled reason to be skeptical of such reasoning.

[38] There is a related discussion surrounding the rule of CUT in non-transitive logics (for instance in Ripley (2013)), but we set this issue aside for now.

[39] Tajer in his Tajer (n.a.) avoids the upward collapse by pairing up stronger logics with weaker bridge-principles and vice versa. It is interesting to note the affinity to our proposal here for the kind of normativity one might contravene by inferring "in the join", so to speak, and suggests a potential avenue for further comparison which we'll not pursue here.

7 Concluding remarks

This paper has been aimed at problematising key order-theoretic assumptions which are (often tacitly) employed in collapse arguments. Our goal is not to argue directly against the monist, or even for a specific form of pluralism, but rather to intervene in the debate, arguing that these assumptions need an explicit defense if the collapse arguments are to carry weight. Even if there were no actual pluralists who adopted a class of logics appropriate to avoid both directions of the collapse argument, according to our lights, our points will still, hopefully, be helpful as providing some important points to note in adjudicating collapse debates in the future.

There is a great deal more to be said here, including, perhaps, some reasons for adopting the kinds of pluralism we have suggested here which go beyond avoiding collapse problems. Our lingering last comments on normativity also beg for more spelling out. We'll not attempt to do either of these things here, but note these as potentially fruitful avenues of research.

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