An Archetypal Vibration Isolator with Quasi-zero Stiffness in Multiple Directions

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Abstract

To avoid the failure under the longitudinal, shear, and mixed wave from earthquake, an archetypal vibration isolator based on a smooth and discontinuous (SD) oscillator is proposed. This model comprises a lumped mass mounted by a vertical spring and a horizontal elastic continuum, which separately provides positive and the negative stiffness forming the stable quasi-zero stiffness (SQZS) in all vertical and horizontal directions. The equation of motion is formulated by employing the Lagrange equation, and the SQZS condition is obtained by optimizing the geometrical parameters of the system. The analysis shows that the system admits remarkable performance in vibration isolation with low resonant frequency and a large stroke of SQZS interval. The results also demonstrate the system has improved aseismic behaviour under the complex excitation of seismic wave significantly.

Keywords SD oscillator · Multi-DOF quasi-zero stiffness · Vibration isolation · Aseismic

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1 Introduction

This work is motivated by the increasing concern for the seismic hazard resulted from the low-frequency seismic waves [1–4], which can be equivalent to longitudinal and tangential motion waves. Traditional scheme is mainly based on the linear theory to separate the structure from the basement [5–10], which is just effective for high-frequency waves. Recently, attentions have been made on nonlinear low-frequency vibration isolation structure, which significantly improves the isolation performance [11–16]. However, these works mainly focus on vertical damage using stable quasi-zero stiffness (SQZS) system with single degree-of-freedom (DOF), which is invalid for horizontal direction.

The most acceptable model of nonlinear structure with SQZS is based upon the SD oscillator [17, 18], which was firstly introduced in 2006 by Cao [19] and is deeply developed recently [20–26]. The SD oscillator is described in Fig. 1, which comprises a lumped mass, m , linked by a pair of inclined elastic springs, each spring with stiffness k pinned to a rigid support. The equation of motion can be written as,

$$\frac{d^2X}{dt^2} + 2KX \left(1 - \frac{L}{\sqrt{X^2 + l^2}} \right) = 0, \quad (1)$$

where X , L and l represent the mass displacement, the equilibrium length of the spring, and the half distance between the rigid supports, separately.

It is clear that the SQZS can be achieved [27] by letting $L=l$, which is called the SQZS condition of the SD oscillator, and lead (1) into the following form.

$$\frac{d^2X}{dt^2} + \frac{K}{l^2}X^3 - \frac{3K}{4l^4}X^5 + \dots = 0, \quad (2)$$

which indicates a meritorious low-frequency vibration isolation performance, because its zero natural frequency term [28].

In this paper, we propose an archetypal vibration isolator with SQZS in multiple directions based on the SD oscillator to protect structure failure from the complex waves vertically and horizontally. The investigations to the system are carried on to get the response under the harmonic excitation from all vertical and horizontal directions.

Fig. 1 The model of the SD oscillator [19]

The evaluations are given for the system under the real earthquake ground acceleration record to get the mechanism for structure failure of the system, which enable the scheme to ensure the safety.

2 SQZS Analysis

Considering the mechanism of an aseismic structure, shown in Fig. 2, which comprises a lumped mass, m , linked by a horizontal elastic continuum and a vertical supporting spring. Establish coordinate system with the initial position of the mass as the origin. The direction of Z axis is called the vertical direction which is defined to opposite to the direction of gravity, and the directions of X and Y axes are the horizontal directions which are fixed in the plane perpendicular to gravity and are determined by right-hand rule. The continuum in Fig. 2 can be regarded as elastic disc with equilibrium radius L_h , the radius stiffness along θ ($0 < \theta \leq 2\pi$) direction is assumed $K_h d\theta$, where $d\theta$ represented the differentiation of θ , and l_h is regarded as the assembled radius of the continuum. The equilibrium length of the vertical supporting spring is L_v with stiffness K_v and $l_v = L_v - \frac{mg}{K_v}$ is the compressed length to balance the weight of mass at the equilibrium position, respectively.

Considering the central symmetry of the isolator, the motion of the mass can be simplified as the motion in XOZ plane, and the restoring force provided by the vertical supporting spring and any equivalent spring of the elastic continuum can be obtained by using the similar way as in [19]. Then the restoring force of the whole continuum in X and Z directions can be obtained as the following

$$\begin{cases} F_{hh} = 2 \int_0^\pi K_h (X - l_h \cos \theta) \cdot \left(1 - \frac{L_h}{\sqrt{(X - l_h \cos \theta)^2 + (l_h \sin \theta)^2}} \right) d\theta, \\ F_{hv} = 2 \int_0^\pi K_h Z \cdot \left(1 - \frac{L_h}{\sqrt{Z^2 + l_h^2}} \right) d\theta, \end{cases} \quad (3)$$

Fig. 2 Mechanism model of the anti-seismic isolator with SQZS in multiple directions

Similarly, the restoring forces resulted by the vertical supporting spring in X and Z directions are

$$\begin{cases} F_{vh} = K_v X \left(1 - \frac{L_v}{\sqrt{X^2 + l_v^2}} \right), \\ F_{vv} = K_v Z. \end{cases} \quad (4)$$

Therefore, the restoring force vector of the system can be obtained after simplification, written as

$$\vec{F} = \begin{bmatrix} F_h \\ F_v \end{bmatrix} = \begin{bmatrix} F_{hh} + F_{vh} \\ F_{hv} + F_{vv} \end{bmatrix} = \begin{bmatrix} \left(K_v + 2\pi K_h - \frac{K_v L_v}{\sqrt{l_v^2 + X^2}} \right) X - \frac{2K_h L_h}{X} I(X) \\ \left(K_v + 2\pi K_h - \frac{2\pi K_h L_h}{\sqrt{l_h^2 + Z^2}} \right) Z \end{bmatrix}, \quad (5)$$

where $I(X) = (X + l_h) \text{EllipticE}\left(\frac{4l_h X}{(X + l_h)^2}\right) + (X - l_h) \text{EllipticK}\left(\frac{4l_h X}{(X + l_h)^2}\right)$, $\text{EllipticK}(x)$ and $\text{EllipticE}(x)$ represent the complete elliptic integral of the first and the second kind, separately.

The SQZS condition in X and Z direction can be obtained by letting $\hat{K}_h(0) = \frac{dF_h}{dX}\bigg|_{X=0} = 0$ and $\hat{K}_v(0) = \frac{dF_v}{dZ}\bigg|_{Z=0} = 0$, written as

$$\begin{cases} \alpha_v + 2\alpha_h = 2, \\ \begin{bmatrix} K_h \\ K_v \end{bmatrix} = C \begin{bmatrix} \alpha_h \\ \pi\alpha_v \end{bmatrix}, \end{cases} \quad (6)$$

where $\alpha_v = \frac{l_v}{L_v}$, $\alpha_h = \frac{l_h}{L_h}$, and C is arbitrary constant.

Substituting (6) into (5) and writing $\alpha = \alpha_v$, $K = K_v$, $l = l_v = l_h$, the equivalent stiffnesses in horizontal and vertical directions are shown as following

$$\vec{\hat{K}} = \begin{bmatrix} \hat{K}_h \\ \hat{K}_v \end{bmatrix} = \begin{bmatrix} \frac{dF_h}{dX} \\ \frac{dF_v}{dZ} \end{bmatrix} = \begin{bmatrix} \frac{2}{\alpha} \left(1 - \frac{l^3}{2(l^2 + X^2)^{3/2}} + \frac{l}{\pi X^2} J(X) \right) K \\ \frac{2}{\alpha} \left(1 - \frac{l^3}{(l^2 + Z^2)^{3/2}} \right) K \end{bmatrix}, \quad (7)$$

where $J(X) = (X + l) \text{EllipticE}\left(\frac{4lX}{(X + l)^2}\right) - \frac{X^2 + l^2}{X + l} \text{EllipticK}\left(\frac{4lX}{(X + l)^2}\right)$. The curves of the equivalent stiffnesses in horizontal and vertical directions with different geometric parameter $\alpha = 0.7, 0.8$, and 0.9 are shown in Fig. 3a and b respectively, in which the

Fig. 3 Equivalent stiffness curve in horizontal and vertical directions with the parameters $K=1 \times 10^7$ N/m, $l=0.1$ m, and $\alpha=0.7, 0.8$, and 0.9 , plotted by dotted, dashed and solid lines, respectively. **a** For the equivalent in horizontal, and **b** for the equivalent in vertical

physical parameters $K=1 \times 10^7$ N/m and $l=0.1$ m to provide sufficient support stiffness for the system and meet the assembly space constraints, respectively [29, 30]. On the premise of sufficient carrying capacity of the system, the equivalent stiffness of the system equals to zero at the equilibrium position, and it is close to zero near the equilibrium position, which indicates the stiffness characteristic of a quasi-zero stiffness system.

Let $R = \sqrt{X^2 + Y^2}$ express the displacement of the oscillator in horizontal direction and establish dynamic equation with SQZS condition, which can be obtained as following

$$\begin{cases} m \frac{d^2 R}{dt^2} + c \frac{dR}{dt} + \frac{2}{\alpha} \left(1 - \frac{l}{2\sqrt{l^2 + R^2}} - \frac{l}{\pi R^2} I(R) \right) KR = 0, \\ m \frac{d^2 Z}{dt^2} + c \frac{dZ}{dt} + \frac{2}{\alpha} \left(1 - \frac{l}{2\sqrt{l^2 + Z^2}} \right) KZ = 0, \end{cases} \quad (8)$$

m and c represent the mass of the oscillator and the damping coefficient, respectively. And the dimensionless equation is obtained by letting $x = \frac{X}{l}$, $y = \frac{Y}{l}$, $z = \frac{Z}{l}$, $\omega_0^2 = \frac{K}{m}$, $\zeta = \frac{c}{2m\omega_0}$, $\tau = \omega_0 t$, written as

$$\begin{cases} \ddot{r} + 2\zeta\dot{r} + \frac{2}{\alpha} \left(1 - \frac{1}{2\sqrt{1+r^2}} - \frac{1}{\pi r^2} i(r) \right) r = 0, \\ \ddot{z} + 2\zeta\dot{z} + \frac{2}{\alpha} \left(1 - \frac{1}{\sqrt{1+z^2}} \right) z = 0, \end{cases} \quad (9)$$

where $\dot{r}$ and $\dot{z}$ represents the derivative of r and z to τ , and $i(r) = (r+1)\text{EllipticE}\left(\frac{4r}{(r+1)^2}\right) + (r-1)\text{EllipticK}\left(\frac{4r}{(r+1)^2}\right)$.

Write the dimensionless dynamic Eq. (9) into the form of two simultaneous first-order differential equations in horizontal and vertical directions, obtained by (10) and (11) separately,

$$\begin{cases} \dot{r} = v_r, \\ \dot{v}_r = -2\zeta v_r - \frac{2}{\alpha} \left(1 - \frac{1}{2\sqrt{1+r^2}} - \frac{1}{\pi r^2} i(r) \right) r, \end{cases} \quad (10)$$

$$\begin{cases} \dot{z} = v_z, \\ \dot{v}_z = -2\zeta v_z - \frac{2}{\alpha} \left(1 - \frac{1}{\sqrt{1+z^2}} \right) z. \end{cases} \quad (11)$$

The phase diagrams of the non-conservative autonomous systems are shown in Fig. 4. It can be aware that with the increase in time, the phase point tends to the origin, and the system will do a decay motion, which means that the equilibrium point of the system is the asymptotically stable focus, and the damped system is stable in all horizontal and vertical directions. Thus, this kind of system is called stable quasi zero stiffness (SQZS) system with multi-DOF.

3 Isolation Analysis

In this section, the isolation analysis is given for both the case of longitudinal and the case of tangential excitation.

Firstly, the dimensionless equation of motion with longitudinal excitation can be written as

$$\ddot{z} + 2\zeta\dot{z} + \frac{2}{\alpha} \left(1 - \frac{1}{2\sqrt{1+z^2}} \right) z = f_z \cos(\omega_z \tau), \quad (12)$$

Fig. 4 Phase diagrams in horizontal and vertical directions with the damping ratio $\zeta=0.1$ and the geometric parameters $\alpha=0.8$. **a** For the phase locus in horizontal directions, and **b** for the phase locus in vertical direction

which implies the amplitude-frequency response by employing the averaging technique as the following

$$f_z^2 = (2\zeta a\omega_z)^2 + \left(\frac{2a}{\alpha} - a\omega_z^2 - \frac{8}{\pi\alpha}Z(a) \right)^2, \quad (13)$$

where a is the dimensionless amplitude of the response in vertical direction, and $Z(a) = \text{EllipticE}(-a^2) - \text{EllipticK}(-a^2)$.

Figure 5 shows the amplitude-frequency curves in vertical direction with different parameters α and ζ , in which (a) for $\zeta=0.05$ and $\alpha=0.7, 0.8$, and 0.9 , plotted by dotted, dashed and solid lines, respectively, and (b) for $\alpha=0.8$ and $\zeta=0.05, 0.10$, and 0.15 , denoted by dotted, dashed and solid lines, respectively. It can be seen from Fig. 5a that the resonance amplitude increases and the resonance frequency decreases as the parameter α increases, while both the resonance amplitude and frequency decrease as the damping ratio increases, seen from Fig. 5b.

Secondly, we can get the dimensionless equations of motion by using Taylor series expansion regarding X and Y , respectively.

$$\begin{cases} \ddot{x} + 2\zeta\dot{x} + A(x^3 + xy^2) = f_x \cos(\omega_x\tau), \\ \ddot{y} + 2\zeta\dot{y} + A(y^3 + x^2y) = f_y \cos(\omega_y\tau), \end{cases} \quad (14)$$

where $A = \frac{3}{8\alpha}$.

The external excitation can be classified in two cases, that is the case of $\omega_x = \omega_y = \omega$, which is equivalent to the case along a fixed direction; and the other case of $\omega_x \neq \omega_y$.

The amplitude-frequency relation can be obtained by harmonic balance method for the first case when $\omega_x = \omega_y = \omega$, written as

$$f_x^2 + f_y^2 = \left(\frac{3}{4}Aa^3 - a\omega^2 \right)^2 + (2\zeta a\omega)^2, \quad (15)$$

where a is the dimensionless amplitude of the response in horizontal directions.

Fig. 5 Amplitude-frequency curve in vertical direction with the parameters $f_z = 0.05$, different α and ζ . **a** For $\zeta=0.05$ and $\alpha=0.7, 0.8$, and 0.9 , plotted by dotted, dashed and solid lines, respectively, and **b** for $\alpha=0.8$ and $\zeta=0.05, 0.10$ and 0.15 , denoted by dotted, dashed and solid lines, respectively

The amplitude-frequency curves with different parameters α and ζ are plotted in Fig. 6a for $\zeta=0.05$ and $\alpha=0.7, 0.8$, and 0.9 , plotted by dotted, dashed and solid lines, respectively, and Fig. 6b for $\alpha=0.7$ and $\zeta=0.05, 0.10$ and 0.15 , denoted by dotted, dashed and solid lines, respectively. It can be seen from Fig. 6a that the resonance amplitude increases and the resonance frequency decreases as the parameter α increases, while both the resonance amplitude and frequency are decreased as the damping ratio increases, seen from Fig. 6b.

With the analysis results of Figs. 5 and 6, it can be found that under the condition of excitation with the same magnitude energy and the same parameter configuration, both the amplitude and the frequency of the resonance in horizontal directions is higher than vertical direction. Therefore, to improve the vibration isolation performance in all directions vertical and horizontal, it is necessary to further analyze and optimize the horizontal vibration isolation performance of the system.

Define the force transmissibility $T = \frac{|F_{\max}|}{\sqrt{f_x^2 + f_y^2}}$, where $F_{\max}$ is the maximum value of the force response to the mass. Accordingly, the transmissibility for the forced SQZS system can be written as

$$T^2 = \frac{\left(\frac{3A(f_x^2 + f_y^2)}{4\omega^4} T^2 \right)^2 + (2\zeta\omega)^2}{\left(\frac{3A(f_x^2 + f_y^2)}{4\omega^4} T^2 - \omega^2 \right)^2 + (2\zeta\omega)^2}, \quad (16)$$

Figure 7 plots the transmissibility curves of (16), the relative linear system (vertical spring only, in Fig. 2) and the Runge–Kutta simulation for (14), denoted by blue curves, black curves and the discrete points, respectively, with the parameters of $\alpha=0.8$ and $f_x = f_y = 0.1$. All the curves are plotted by solid, dot-dashed and dashed with damping ratio $\zeta=0.05, 0.10$ and 0.15 , respectively, while the simulated plots for the same damping ratio marked in cyan, red and green,

Fig. 6 Amplitude-frequency curve in horizontal directions with the parameters $\sqrt{f_x^2 + f_y^2} = 0.05$, different parameter α and ζ . **a** For $\zeta=0.05$ and $\alpha=0.7, 0.8$, and 0.9 , plotted by dotted, dashed and solid lines, respectively, and **b** for $\alpha=0.8$ and $\zeta=0.05, 0.10$ and 0.15 , denoted by dotted, dashed and solid lines, respectively

Fig. 7 Force transmissibility curve of linear and SQZS system with different $\zeta=0.05, 0.10$, and 0.15 in horizontal directions

respectively. And the discrete points denoted by “ $\triangleleft$ ” and “ $\triangle$ ” separately represent the forward and the reverse sweep results.

It is found from Fig. 7 that both the resonance amplitude and frequency of SQZS system are lower than the relative linear system, and also decrease as the damping ratio increase. Particularly, the bifurcation occurs in the low-frequency band, the transmissibility response will increase along the large amplitude branch and then falls down to the small amplitude branch at the downward jumping point, denoted by green arrow direction as the excitation frequency increases from zero gradually. In the other case, when the excitation frequency decreases from 1.5 gradually, the transmissibility response will increase along the small amplitude branch and then jumps up to the large amplitude branch at the upward jumping point, denoted by red arrow direction. It is found that the downward jumping point shifts left, and the upward jumping point shifts right as the damping ratio increases, approaching to a fixed point $\left(\sqrt[6]{\frac{3}{2}A(f_x^2 + f_y^2)}, 1\right)$. Here in this

paper, we define the $\frac{3}{2}A(f_x^2 + f_y^2)$ is the initial isolation frequency of the SQZS system similar with the linear system ($\omega_{Linear}^* = 2$). It can be seen from Fig. 7 that $\omega_{SQZS}^* \approx 0.491$ is substantially lower than ω_{Linear}^* , which implies that a better isolation performance at the lower frequency band for the SQZS system. This is because the stiffness of the system near the static equilibrium position is close to 0, so that the natural frequency of the system under a small amplitude vibration is also close to 0. It is widely known that the initial vibration isolation frequency of a support system is directly affected by its natural frequency, thus, the SQZS system expresses a better low-frequency vibration isolation performance and a wider effective vibration isolation frequency band than linear system.

Assuming $\beta = \frac{\omega_x}{\omega_y}$, ($\beta \neq 1$) for the second case, the excitation is distributed over the $X-Y$ plane. In the same way as previous, the analytic relationship between frequency and transmissibility in $X-Y$ plane can be obtained as following

$$\left\{ \begin{array}{l} T_x^2 = \frac{\left(\frac{3Af_x^2}{4\omega_x^4} T_x^2 + \left(\frac{1}{2} + \frac{\omega_x(\omega_x^2 - 2\omega_y^2) \sin \frac{2\pi\omega_y}{\omega_x}}{4\pi\omega_y(\omega_x^2 - \omega_y^2)} \right) \frac{Af_y^2}{\omega_y^4} T_y^2 \right)^2 + (2\zeta\omega_x)^2}{\left(\frac{3Af_x^2}{4\omega_x^4} T_x^2 + \left(\frac{1}{2} + \frac{\omega_x(\omega_x^2 - 2\omega_y^2) \sin \frac{2\pi\omega_y}{\omega_x}}{4\pi\omega_y(\omega_x^2 - \omega_y^2)} \right) \frac{Af_y^2}{\omega_y^4} T_y^2 - \omega_x^2 \right)^2 + (2\zeta\omega_x)^2} \\ T_y^2 = \frac{\left(\frac{3Af_y^2}{4\omega_y^4} T_y^2 + \left(\frac{1}{2} + \frac{\omega_y(\omega_y^2 - 2\omega_x^2) \sin \frac{2\pi\omega_x}{\omega_y}}{4\pi\omega_x(\omega_y^2 - \omega_x^2)} \right) \frac{Af_x^2}{\omega_x^4} T_x^2 \right)^2 + (2\zeta\omega_y)^2}{\left(\frac{3Af_y^2}{4\omega_y^4} T_y^2 + \left(\frac{1}{2} + \frac{\omega_y(\omega_y^2 - 2\omega_x^2) \sin \frac{2\pi\omega_x}{\omega_y}}{4\pi\omega_x(\omega_y^2 - \omega_x^2)} \right) \frac{Af_x^2}{\omega_x^4} T_x^2 - \omega_y^2 \right)^2 + (2\zeta\omega_y)^2} \end{array} \right. \quad (17)$$

The intersectional surfaces marked orange and blue in Fig. 8a–e express the first and the second Eqs. (17) respectively, which also can be used to assist the representation of the spatial trend of the curve. The force transmissibility curves with the different frequency ratio $\beta = 2, \sqrt{2}, 1, \frac{1}{\sqrt{2}},$ and $\frac{1}{2}$ (Because the excitation is in the $X-Y$ plane, the trajectory of excitation is always a closed curve, if β is a rational number; or the trajectory of excitation will never be closed and will cover the whole $X-Y$ plane, if β is an irrational number and the action time is long enough. Therefore, we choose β separately depends on 2 and $\sqrt{2}$ as the examples of the rational and irrational frequency ratios, to investigate the action form of complex excitation.), marked red, green, blue, cyan, and black, respectively, which are indicated by the intersection of two surfaces. Fig. 8f–h show all spatial curves in the same coordinate using their original color, which indicate that the initial vibration isolation frequency of this system ($\omega_{SQZS}^* < 0.5$) is much lower than it of linear system

Fig. 8 Force transmissibility curve under mixed wave with the parameters $\alpha=0.8$ and $f_x=f_y=0.1$

($\omega_{Linear}^* = \sqrt{2}$). We noticed that there are two resonant peaks separately appear in the X and Y directions which are depended on β , in this case. When $\beta > 1$, the component of the first peak along x axis is greater than y axis, by contraries in the case of $\beta < 1$ the larger component of the first peak appears along y axis. As β approaches to 1, the two peaks are gradually approaching together, meanwhile the module of the sum vector of the transmissibility peaks on both axes are gradually increasing. Particularly, when $\beta = 1$ shown by the black one, the two peaks are merged into one, that is agree with the analysis for the first case. It can be found from 8(h) that the maximum initial vibration isolation frequency of the multi-DOF system is obtained while $\beta = 1$. In the other words, under the same excitation condition, the low-frequency vibration isolation performance in the horizontal directions is the worst when $\beta = 1$. According to the previous study of the first case ($\beta = 1$), it is proved that the SQZS system represent a remarkable low-frequency vibration isolation performance than linear system.

4 Aseismic Analysis

In this section, the effective aseismic performance of the system is verified numerically using a seismic acceleration. The equation of motion under seismic wave can be written as

$$m\ddot{Z} + 2c\dot{Z} + \frac{2}{\alpha}Z - \left(\frac{2}{\alpha} - 1\right) \frac{KLZ}{\sqrt{Z^2 + \left(1 - \frac{\alpha}{2}\right)^2 L^2}} = mS(t), \quad (18)$$

where $S(t)$ is the function of a seismic acceleration record.

Runge–Kutta Method is used to analyse the response of this system, and the numerical solution is shown in Fig. 9a for the time history and (b) for the frequency domain. In this figure, the real earthquake ground acceleration record is plotted with black and the response accelerations for conventional linear system and the SQZS system proposed in this paper are marked green and red, respectively. It is found

Fig. 9 Curve of acceleration responses under the seismic wave

from Fig. 9b that the linear system has a good aseismic performance in higher frequency band over 1.5 Hz, but the response is enlarged greatly in the frequency band from 0.4 to 1.5 Hz. And it is apparently from Fig. 9a that the linear system even engenders the vibration amplification over the time $20\text{ s} < t < 25\text{ s}$ and $30\text{ s} < t < 43\text{ s}$ for this earthquake record which is of the remarkable destructive power to the structures. However, the SQZS system reflect the pre-eminent aseismic performance during the whole action time of this earthquake.

It is noticed that the seismic wave used in these analyses with the peak acceleration is 1 m/s^2 , and the average acceleration of seismic wave is about 0.1632 m/s^2 . The maximum and average accelerations' response of the conventional linear aseismic system are 2.1447 m/s^2 and 0.3503 m/s^2 , which shows that there is no obvious aseismic availability from this seismic wave. While the excellent aseismic ability of SQZS system can be shown by examining the maximum response and average acceleration, which is 0.4538 m/s^2 and 0.0389 m/s^2 , respectively. It can be seen that the peak and the average acceleration are separately reduced by 54.62% and 76.19%, which shows that the significant aseismic performance under the earthquake ground acceleration. On the other hand, because the actual seismic wave contains a large amount of broadband vibration energy, the effective aseismic frequency band of a isolation system is also an important index to investigate its performance. It can be seen from Fig. 9b that the SQZS system reduces the initial vibration isolation frequency to 0.4 Hz, which is much lower than the linear system (about 1.5 Hz, reduced by about 73.33%), which shows that the SQZS system significantly improves the effective aseismic frequency band and greatly improves the aseismic performance.

5 Conclusion

In conclusion, we have proposed an archetypal vibration isolator, based on SD oscillator, with SQZS in vertical and all horizontal directions to suppress the destruction caused by seismic waves. Utilizing Lagrange equation to establish the mathematical model and to obtain the SQZS condition by optimizing the geometrical parameters of the system. It is interesting that a fixed point is found for the proposed SQZS which is much lower than the linear one, showing the remarkable performance in vibration isolation with low resonant frequency and a large stroke of SQZS interval. The results obtained herein this paper also demonstrate the significantly improved aseismic behaviour under the complex excitation of seismic wave. The presented archetypal vibration isolator is being actively studied by the authors in two main directions. The first direction is to set up an experiment rig for the vibration isolation system for validating the theoretical results presented in this paper further. The second proposed direction is to ameliorate and optimize the structure such that SQZS can be achieved in all directions to suppress the damage from any form of seismic wave.

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