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RESEARCH ARTICLE

Applying a Genetic Algorithm to Implement the Fuzzy-MACBETH Method in Decision-Making Processes

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Abstract
This paper describes the development of an evolutionary algorithm for building cardinal scales based on the Fuzzy-MACBETH method. This method uses a triangular fuzzy numbers scale in the MACBETH method to incorporate the subjectivity of a semantic scale into mathematical modeling, which enables circumventing the cardinal inconsistency problem of the classical method, facilitating its application in complex contexts. A genetic algorithm is used in the fuzzy system developed here to build the basic fuzzy scale in a cardinally inconsistent decision matrix. The proposed technique is inspired by crossover and mutation genetic operations to explore potential solutions and obtain a cardinal scale aligned with the decision maker's preferences. Finally, an illustrative example of the application of the proposed decision support system is presented. The results confirm that the FGA-MACBETH method aligns with the classical method. This study's primary contribution is that circumventing the problem of cardinal inconsistency in a semantically consistent decision matrix enabled obtaining a cardinal scale without requiring the decision maker to redo his/her initial assessments.

Keywords Cardinal scale · FGA-MACBETH · Fuzzy-MACBETH · Fuzzy number · Fuzzy system · Genetic algorithm

Abbreviations

FGA-MACBETH	Fuzzy genetic algorithm of measuring attractiveness by a categorical-based evaluation technique
F-LP-MACBETH	Fuzzy linear programming of measuring attractiveness by a categorical-based evaluation technique
FPV	Fundamental points of view
GA	Genetic algorithms
LPP	Linear programming problem
MACBETH	Measuring attractiveness by a categorical-based evaluation technique
MCDM/A	Multi-criteria decision-making/aid
M-MACBETH	Decision support system of measuring attractiveness by a categorical-based evaluation technique

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1 Introduction

Complex decisions require a detailed analysis of the decision-making problem, considering the multiple aspects of its context. In this sense, MCDM/A is a sub-discipline of Operations Research intended to develop multi-criteria evaluation systems based on the knowledge of specialists or decision makers, consisting of important tools for structuring and assessing complex decisions [1].

Among the multi-criteria methods, MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique), developed by Bana and Costa and Vansnick [2], enables transforming ordinal scales based on value judgments expressed by a decision maker into cardinal scales using linear programming problems

(LPP). It is a socio-technical process combining decision-conferencing elements to structure a decision problem [3].

The central idea in the method's mathematical stage is to provide decision makers with a tool that supports evaluations through a semantic scale, converting it to a cardinal scale using a decision support system [4]. Thus, the method's primary differential is allowing decision makers to express their assessments through linguistic terms such as indifferent, very weak, weak, moderate, strong, very strong, and extreme.

The method is used to support multi-criteria decision-making in complex contexts. However, sometimes, the MACBETH method cannot determine a cardinal scale, even if the decision matrix is semantically consistent. In other words, cardinal inconsistency may occur even when the decision maker coherently (semantically) assesses the evaluation elements. This situation occurs because traditional mathematics does not incorporate the uncertainty inherent to semantic scales.

In this case, the decision maker is asked to modify his/her initial assessment so the method can provide a cardinal scale. Considering that this new cardinal scale was not included in the initial decision matrix, the decision maker may feel unsure whether the resulting model is reliable. Furthermore, the context's complexity hinders the decision maker's task when determining a semantic category to account for the difference in attractiveness between two actions or elements. As shown in [5], the decision maker may perceive a difference in attractiveness *somewhere* between two semantic categories.

In this context, the fuzzy sets theory resembles human thinking, allowing the incorporation of uncertainty inherent to linguistic terms into the scale [6]. Additionally, the measurement of assessment elements may be imprecise, uncertain, or inaccurate due to the imprecision and uncertainty present in real-world decision-making [7]. Thus, using fuzzy numbers to include uncertainty in the scale can improve the method.

In this sense, some studies apply the Fuzzy theory to the MACBETH method. Dhouib [8] expands MACBETH by integrating a 2-tuple fuzzy model, considering decision makers' imprecise and linguistic evaluations. The alternatives in the model developed here present input data that can be numerical, interval, or linguistic and are expressed in fuzzy sets called Basic Linguistics Terms Sets (BLTS). A decision matrix is built with the results of the 2-tuple fuzzy model, and the cardinal scale is obtained with the M-MACBETH software. The traditional MACBETH method, in which a decision maker performs pairwise comparisons according to his/her preference, was used to determine the criteria weights. Note that the 2-tuple model objectively assesses the uncertain context in this study; hence, its main contribution is evaluating alternatives without the participation of a decision maker.

Yurtyapan and Aydemir [9] introduce a new ERP (Enterprise Resource Planning) software selection approach in which semantic judgments are converted into interval scales through grey numbers, generating a new assessment that is subsequently applied in the classical MACBETH method. A second approach was also proposed, aiming at group evaluation, in which an intuitionistic fuzzy set was established to assess the effects of decision makers on the research problem, that is, to determine the weight of each decision maker.

Pacumar et al. [10] propose a ranking method based on the MACBETH method expanded to a fuzzy version. Criteria weights were obtained from a triangular fuzzy numbers scale, generating a fuzzy weight vector for each decision maker. The vectors were then aggregated, and the alternatives were assessed through the mathematical formulation proposed. Afterward, the overall fuzzy value of each alternative was obtained, defuzzified, and sorted; i.e., cardinal scales were not obtained for each evaluation element but rather a ranking of alternatives.

However, these studies do not present a new approach to overcoming the cardinal inconsistency problem in a semantically consistent decision matrix presented by the MACBETH method. Finally, the Fuzzy-MACBETH modeling proposed by Bastos et al. [11] and adopted in this study consists of using a fuzzy numbers scale applied to the MACBETH's semantic scale to incorporate the subjectivity of linguistic terms into cardinal scales. Thus, the method's main contribution concerns the possibility of obtaining a cardinally consistent scale in semantically consistent decision matrices, which, according to the classical method, present cardinal inconsistency, thus facilitating the elicitation process for decision makers.

Additionally, computational implementation is important due to the mathematical complexity of the Fuzzy-MACBETH approach. Therefore, this study presents the computational modeling of the Fuzzy-MACBETH method for solving the linear programming problem and developing cardinal scales.

According to [12], innovations and software allow organizations to integrate their activities, contributing to decision-making. Decision support systems contribute to the multi-criteria decision-making process, considering the multidimensional nature of problems and enabling decision makers to incorporate preference systems [13]. Hence, the computational coding of complex algorithms allows the operationalization of multi-criteria methods in a fast and easy-to-handle manner.

Recent studies show the advancements of incorporating soft computing into conventional decision-making methods to deal with uncertainties [14]. Bio-inspired algorithms in evolutionary computation optimize decision-making models in so-called evolutionary systems, which use fuzzy theory to incorporate uncertainties and an evolutionary optimization algorithm, such as genetic algorithms [15].

Given the previous discussion, this paper presents the computational implementation of the Fuzzy-MACBETH method based on a genetic algorithm. The modeling uses genetic crossover and mutation operators to explore potential solutions in a semantically consistent but cardinally inconsistent decision matrix, arriving at a cardinal scale. Therefore, this approach can be quickly and easily operationalized through a decision support system, enabling it to be used in real-world decision-making problems. GA facilitates the implementation of the Fuzzy-MACBETH method, considering that the objective function in the linear programming problem is to minimize the most significant differences in fuzzy attractiveness. Because GA works with discrete and continuous scales, it can be used to operationalize the F-LP-MACBETH linear programming problem.

Thus, the approach developed here enables the development of performance evaluation and multi-criteria decision-making models to assist managers in different contexts. It is applied in the evaluation stage of the MACBETH method with the advantage of overcoming the problem of cardinal inconsistency, preventing the decision maker from having to change his/her initial assessment. Thus, this method uses concepts from evolutionary computing and soft computing to improve a highly regarded method widely used in Operational Research, making it easier to use.

This paper is organized into six sections. Following the introduction, the second section presents the Fuzzy-MACBETH method, the third deals with genetic algorithms (GA), and the computational modeling of the FGA-MACBETH technique proposed here is shown in the fourth section. An illustrative example is described in the fifth section, followed by the sixth section, which presents the conclusions and suggestions for future research, and finally, the references.

2 Fuzzy-MACBETH Method

The value functions in the Fuzzy-MACBETH proposed by Bastos et al. [11] are obtained the same way as in the classical MACBETH, i.e., based on pairwise comparisons of differences of attractiveness between evaluation elements, through a semantic scale composed of six linguistic terms: very weak, weak, moderate, strong, very strong and extreme. Hence, the decision maker assigns an absolute verbal judgment about the difference in attractiveness between x and y for each ordered pair (x,y) in $A \times A$ with xPy (x is preferred to y). Thus, the value functions are based on the decision maker's perceptions and values.

Hence, the method aims to find a basic fuzzy scale for each alternative according to the decision maker's qualitative judgment based on a triangular fuzzy numbers scale. As shown in Fig. 1a, where $x_{i,j}$ is assigned value $\widetilde{A}$ if and only if the decision maker has assigned a category c_k to (x_i, x_j), such that

$$\widetilde{A} = \begin{cases} m_k = k \\ l_k = k-1 \\ u_k = k+1 \end{cases} \iff k = \{2,3,4,5\}, \tag{1}$$

$$\widetilde{A} = \begin{cases} m_k = l_k = k \\ u_k = k+1 \end{cases} \iff k = 1, \tag{2}$$

$$\widetilde{A} = \begin{cases} m_k = u_k = k \\ l_k = k-1 \end{cases} \iff k = 6. \tag{3}$$

The resulting fuzzy scale is presented in Fig. 1b. Thus, the decision maker's qualitative judgments are fuzzified, as shown in Table 1, generating a fuzzified decision matrix.

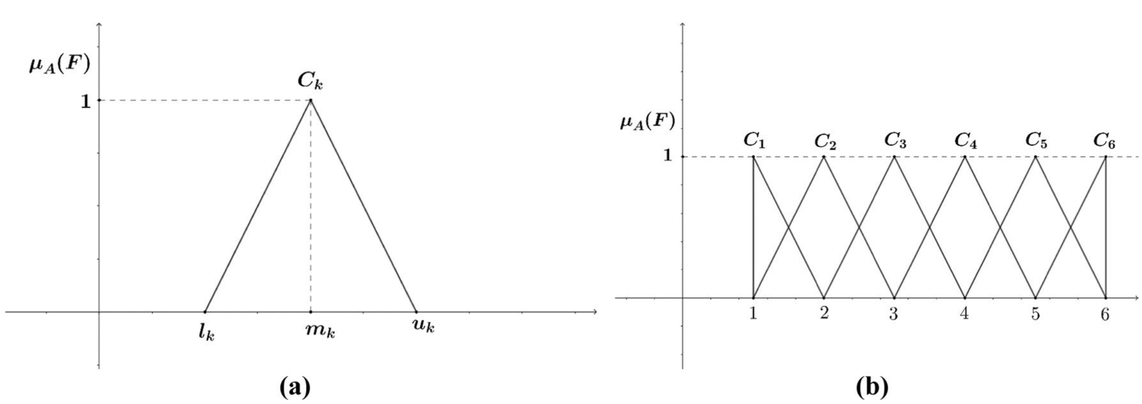


Fig. 1 **a** Triangular Fuzzy-MACBETH number scale. **b** Graphical representation of the scale in the Fuzzy-MACBETH method. Source: [10]

Table 1 Fuzzy-MACBETH method scale

C_k	Semantic Scale	$\widetilde{A}$
C_0	No differences between the alternatives	(0, 0, 0)
C_1	Very weak difference in attractiveness	(1, 1, 2)
C_2	Weak difference in attractiveness	(1, 2, 3)
C_3	Moderate difference in attractiveness	(2, 3, 4)
C_4	Strong difference in attractiveness	(3, 4, 5)
C_5	Very strong difference in attractiveness	(4, 5, 6)
C_6	Extremely strong difference in attractiveness	(5, 6, 6)

Source: [10]

When using the scale proposed here, the decision maker must first order the evaluation elements (actions or alternatives) in descending order according to his/her preference. Evaluation elements, which the decision maker perceives as *indifferent,* are allocated to the same position in the matrix, as they will have the same cardinal scale. In other words, let the elements be a_1, a_2, a_3, a_4 e a_5, such that $a_1 = a_2Pa_3Pa_4Pa_5$, with P being "*preferable to.*" The resulting decision matrix will be assembled as shown in Fig. 2.

Thus, after the first stage, with the evaluation elements already ordered, the decision maker must assign a linguistic term corresponding to the semantic scale from C_1 a C_6 to each pair $x_{i,j}$, filling in the upper part of the matrix $n \times n$ with verbal responses, such that $\forall i \neq j \in 1, 2, \cdots, N : x_iPx_j \Leftrightarrow i > j$.

After fuzzifying the decision matrix, the basic fuzzy scale $\widetilde{v}$ for each alternative, called the pre-cardinal scale, is obtained through the F-LP-MACBETH linear programming problem. The F-LP-MACBETH is based on the linear programming problem proposed in [3], differing in the fourth restriction, which the triangular fuzzy scale can flexibilize.

2.1 F-LP-MACBETH

$$Min\left[\widetilde{v}\left(x^+\right) - \widetilde{v}(x^-)\right] \tag{4}$$

Subject to restrictions:

$$\widetilde{v}(x^-) = (0, 0, 0) \tag{5}$$

$$\widetilde{v}(x) - \widetilde{v}(y) = (0, 0, 0), \forall (x, y) \in C_0, \tag{6}$$

$$\widetilde{v}(x) - \widetilde{v}(y) \geq (1, 1, 2), \forall (x, y) \in C_k, k \in \{1, 2, 3, 4, 5, 6\}, \tag{7}$$

$$\begin{aligned} &\tilde{v}(x) - \tilde{v}(y) \geq \tilde{v}(w) - \tilde{v}(z), \forall (x, y) \in C_k \text{ and} \\ &\forall (w, z) \in C_{k'} \text{ and } k, k' \in \{1, 2, 3, 4, 5, 6\} \text{ and } k > k'. \end{aligned} \tag{8}$$

Such that, $\widetilde{v}\left(x^+\right)$ is the fuzzy value for the element considered most attractive by the decision maker, $\widetilde{v}(x^-)$ is the fuzzy value for the element considered least attractive, and $\widetilde{v}(x) - \widetilde{v}(y)$ is the fuzzy value of the difference in attractiveness between elements x and y for the semantic category C_k.

$$\begin{array}{c} \\ a_1 = a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{cccc} a_1 = a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} \cdot & x_{1,2} & x_{1,3} & x_{1,4} \\ \vdots & \vdots & x_{2,3} & x_{2,4} \\ & & \cdot & x_{3,4} \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array}$$

Fig. 2 Decision matrix

The objective function (4) in the F-LP-MACBETH linear programming problem aims to minimize the most significant differences in the fuzzy values between the most and least attractive alternatives, subject to a set of restrictions that fix the origin of the scale (5), ensuring the ranking order of elements (6) and (7) and dealing with cardinal consistency (8). At the end of this step, the basic fuzzy scale remains a fuzzy number; therefore, the scale is defuzzified using the centroid method to get the basic crisp scale (v_x).

Note that the fuzzy numbers scale enabled to loose up the restriction (8) that requires a cardinal scale. For example, Fig. 1b shows that an element may simultaneously belong to the semantic categories C_3 and C_4 with a certain degree of relevance.

Finally, to obtain the cardinal scale (E_x), the basic scale is anchored by assigning zero to the performance level the decision maker considered neutral and one hundred to the performance level considered good, according to (9), thus enabling the aggregation of local assessments.

$$E_x = \alpha v_x + \beta \tag{9}$$

Such that,

E_x is the cardinal scale,

v_x is the basic scale or pre-cardinal scale obtained from F-LP-MACBETH, and

α and β are the angular and linear coefficients, respectively.

In other words, the decision matrix in Fig. 3 using the Fuzzy-MACBETH approach results in the fuzzy basic scale $\widetilde{v}_x$ presented in Table 2. The basic scale $\widetilde{v}_x$ is obtained when the centroid method is applied.

System (10) is assembled by anchoring the good and neutral levels, which determines the values for α and β applied to Eq. (9) to obtain the cardinal scale. In the hypothetical example presented here, we have $\beta = 0$ and $\alpha = 20$.

$$
\begin{array}{c}
 \\ a_1 = a_2 \\ a_3 \\ a_4 \\ a_5
\end{array}
\begin{array}{c}
\begin{array}{cccc} a_1 = a_2 & a_3 & a_4 & a_5 \end{array} \\
\left[\begin{array}{cccc}
\cdot & C_1 & C_3 & C_4 \\
\vdots & \vdots & C_2 & C_3 \\
 & & \cdot & C_1 \\
\cdot & \cdot & \cdot & \cdot
\end{array}\right]
\end{array}
$$

Fig. 3 Hypothetical Decision Matrix

Table 2 Results for the decision matrix presented in Fig. 3

	$\widetilde{v}_x$	v_x	Anchored	E_x
$a_1 = a_2$	4.0, 4.0, 7.0	5.00	*Good*	100.0
a_3	3.0, 3.0, 5.0	3.67		73.33
a_4	1.0, 1.0, 2.0	1.33		26.67
a_5	0.0, 0.0, 0.0	0.00	*Neutral*	0.00

$$
\begin{cases} 100 = 5\alpha + \beta \\ 0 = 0\alpha + \beta \end{cases} \tag{10}
$$

Figure 4 presents the algorithm of the Fuzzy-MACBETH system used in this study, visually showing all the steps that begin with the decision maker's elicitation process. The decision maker performs pairwise comparisons of the differences in attractiveness between the evaluation elements, creating the initial decision matrix. Based on the triangular fuzzy numbers scale, the decision matrix is fuzzified in the fuzzification module. Afterward, mathematical procedures are performed to obtain the basic fuzzy scale based on the calculation of the F-LP-MACBETH linear programming problem. Finally, the basic scale is defuzzified in the defuzzification module, a crisp basic scale is obtained, cardinalization is performed, and the cardinal scale is achieved.

3 Genetic Algorithms

Genetic Algorithms (GA), developed by John H. Holland [16], are a group of metaheuristics inspired by Darwin's theory of evolution. They are part of nature-inspired computation, which seeks to create intelligent systems that reproduce aspects of human behavior. The central idea in GA is based on the natural selection process that controls the evolution of living beings, in which the most adapted organisms tend to live long enough to reproduce and perpetuate their genetic code.

Many studies show that GA is superior to finding the optimal approximate solution in some situations compared to other heuristic algorithms, such as the Tabu Search Algorithm, the Simulated Annealing, and the Ant Colony Algorithm [17]. In recent decades, researchers have preferred using GA, which has been widely used in several knowledge fields due to its adaptability and versatility [18]. Additionally, a probability optimization method that automatically adjusts the search direction without a pre-established rule is adopted, allowing superior global optimization ability [19].

An initial population in GA, formed by chromosomes, represented by a chain of symbols, is randomly defined. Each population evolves during algorithm iterations and is evaluated by a fitness function. The chromosomes can be encoded using various representations, such as binary, integer, real numbers, or letters [20]. Genetic operators: selection, crossover, and mutation must be applied to obtain each generation [21].

The selection process involves determining the individuals participating in the reproduction phase so that the fittest are more likely to be selected [22]. The selection method proposed by [16] is called the roulette wheel selection method, though other methods are found in the literature, such as the tournament [23] and the elitist selection methods [24].

Crossing consists of recombining part of the code of pairs of chromosomes selected to generate offspring [25]. Several crossing methods differ according to the choice of the locus of the chromosomes that will be exchanged and the way they will be recombined, such as a point [25], multipoint [26], or uniform [19].

Finally, a mutation operator is needed to ensure the population's genetic diversity, obtained by changing one or more randomly chosen locus genes [25]. The correct definition of GA parameters is critical for good performance, as it will define the balance between the search mechanisms and the diversity of solutions, avoiding premature convergence.

4 Proposed FGA-MACBETH Method

The FGA-MACBETH method proposed in this study consists of the computational modeling of the Fuzzy-MACBETH method based on Genetic Algorithms. The genetic concepts of selection, crossover, and mutation were used to solve the F-LP-MACBETH Linear Programming Problem (PPL) in a decision matrix with cardinal inconsistency, as shown in the system in Fig. 5.

Therefore, each chromosome consists of a triangular fuzzy number of three genes, represented by ordered real numbers. The population comprises n chromosomes, where n is the number of evaluation elements that form the decision matrix. An initial population is randomly generated according to the decision maker's semantic categories,

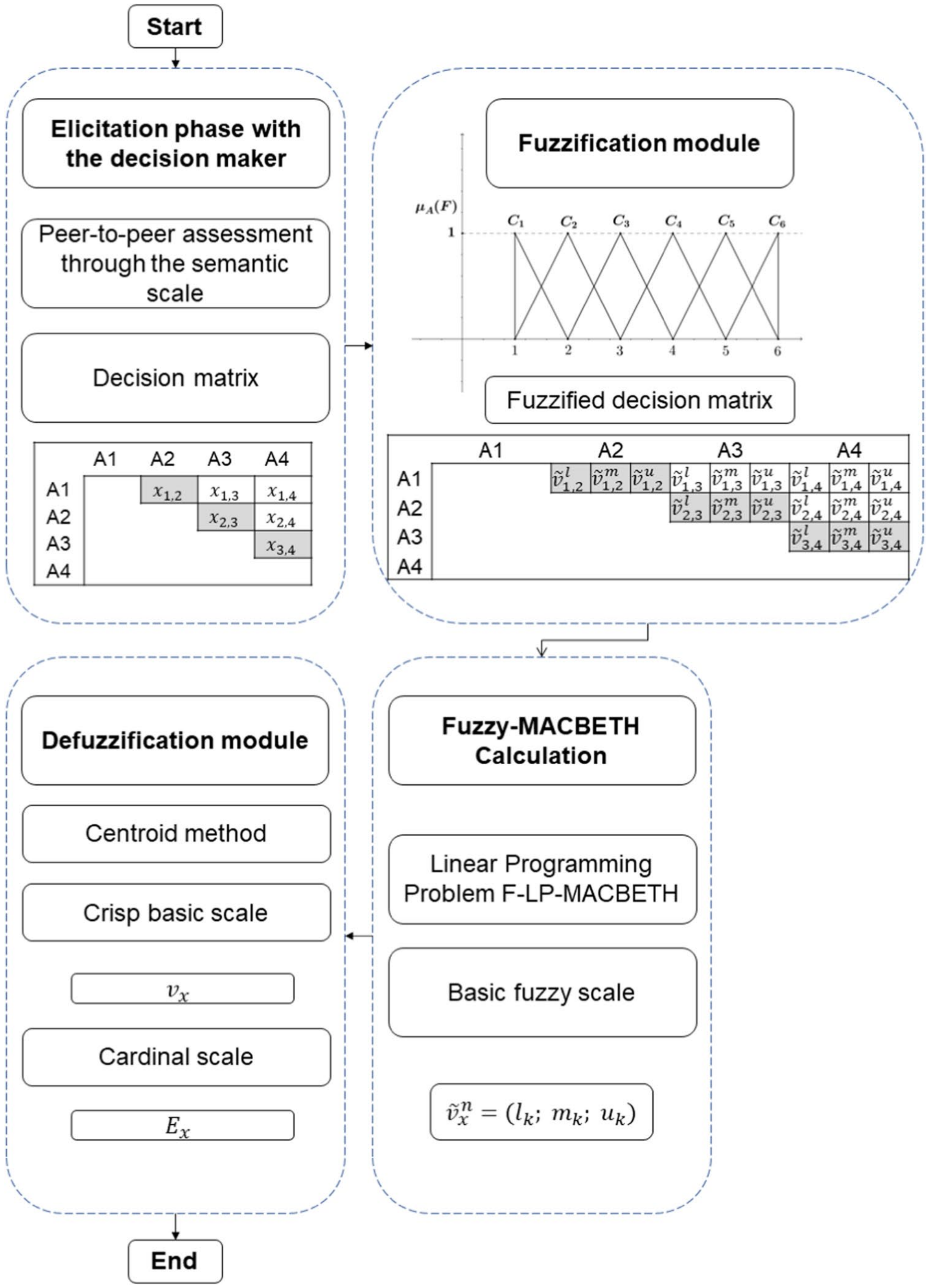


Fig. 4 Algorithm of the Fuzzy-MACBETH system

representing a potential solution to cardinal inconsistency, inserted in the upper diagonal of the decision matrix, highlighted in Fig. 6.

The F-LP-MACBETH linear programming problem is calculated with the matrix containing the values of the initial population of the genetic algorithm, and cardinal consistency is verified. Thus, the fitness function consists of the objective function of the Fuzzy-MACBETH method.

If the F-LP-MACBETH calculation shows cardinal inconsistency, the program initiates genetic algorithm rounds, selecting the chromosomes that will undergo crossover and mutation. The chromosomes in the matrix's upper diagonal, highlighted in Fig. 6, are selected through the tournament method, i.e., each chromosome has the same probability of being selected.

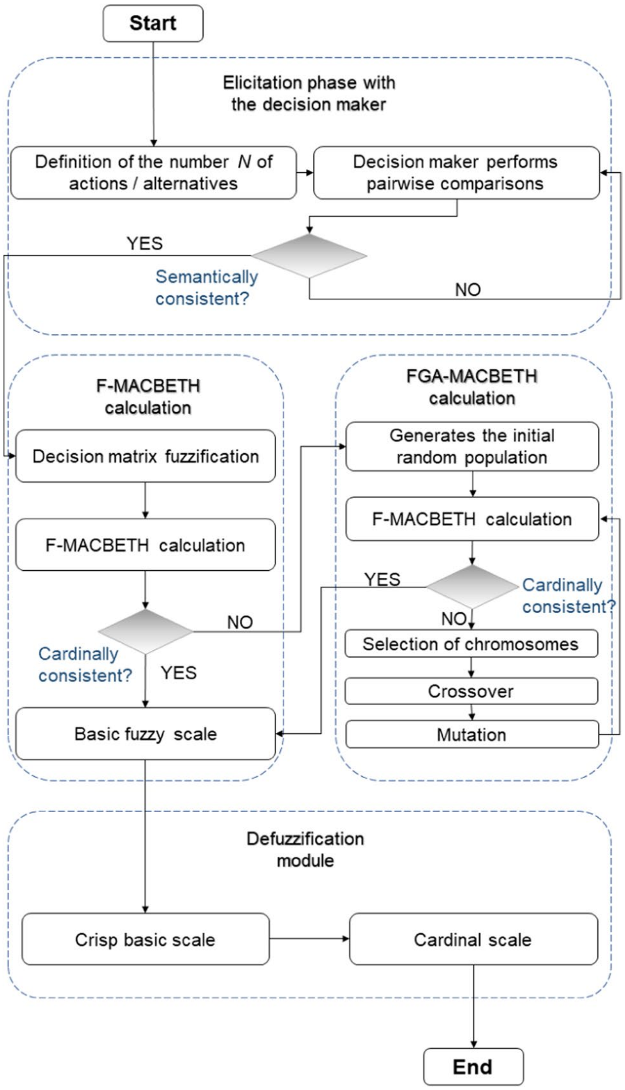


Fig. 5 Algorithm of the FGA-MACBETH system

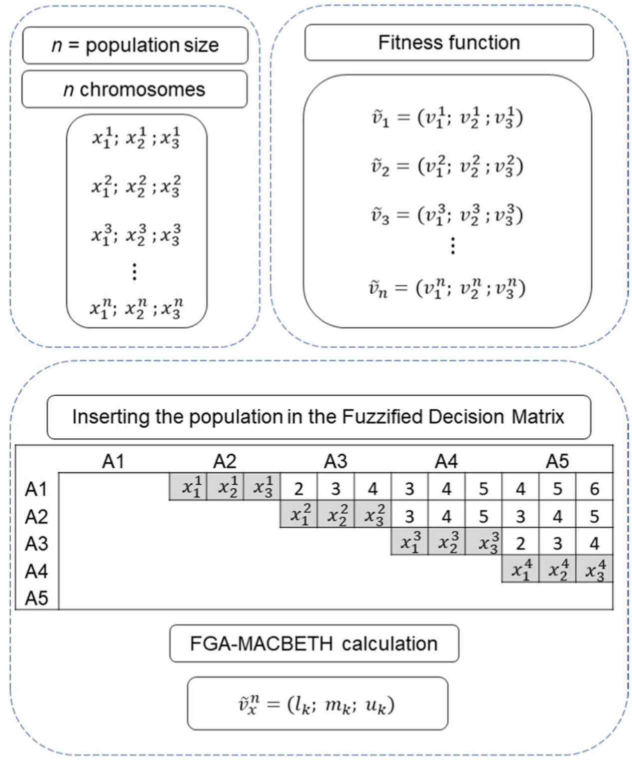


Fig. 6 Details of the genetic operators of the FGA-MACBETH method

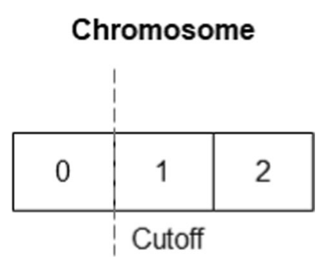


Fig. 7 Detailing the cut point for the crossing operator

As shown in Fig. 7, crossing at a fixed single point between locus 0 and 1 was chosen. The genetic parameters were calibrated through tests. The best configuration obtained was a crossover probability of 0.90 and a mutation probability of 0.35. The cycle repeats until cardinal consistency or the termination parameter is reached.

Python language, version 3.9, through the integrated development environment Pycharm, developed by JetBrains, was used for programming the FGA-MACBETH method. The pseudo-code of Algorithm 1 presents the Fuzzy-MACBETH calculation until the scales are determined in case of cardinal consistency.

Algorithm 1. Fuzzy-MACBETH Calculation.

```
m, n ← number of evaluation elements
judgments ← function defined to receive the pairwise comparisons and return the mxn array
consistentjudgments ← function defined to check semantic consistency
displaymatrix ← function defined to display the array
fuzzifiedmatrix ← function defined to apply the triangular fuzzy scale to the decision
matrixfuzzyquality ← function defined to provide the qualitative fuzzy matrix
calc_fuzzymacbeth ← function defined to perform the Fuzzy-MACBETH calculation
cardinalconsistency ← function defined to check if array is cardinally consistent

judgments (initialmatrix, m, n)
consistentjudgments (initialmatrix, m, n)
displaymatrix (initialmatrix)
fuzzifiedmatrix (fuzzymatrix, m, n)
displaymatrix (fuzzymatrix)
fuzzyquality(fuzzyqualitymatrix, m, n)
calc_fuzzymacbeth(fuzzyqualitymatrix, fuzzymatrix, n)
displaymatrix (fuzzymatrix)
if cardinalconsistency = ‘Not’ then
        write (“Inconsistent matrix”)
else
        write (“Consistent matrix”)
endif
if consistenciacardinal ≠ ‘Not’
        write (“Basic Fuzzy Scale”)
        for each row on the matrix
                Calculate the basic fuzzy scale
                write (fuzzymatrix)
        endfor
        write (“Defuzzified scale”)
        for each row on the matrix
                Calculate the crisp scale
                write (crisp)
        endfor
        write (“Cardinal scale”)
        calculate alpha
        calculate beta
        for each row on the matrix
                Calculate the cardinal scale
                write (cardinal)
        endfor
endif
```

In case of cardinal inconsistency, the calculation of the scales is performed using the method based on a genetic algorithm, as shown in the pseudo-code of Algorithm 2.

Algorithm 2. Calculation FGA-MACBETH.

Table 3 Comparative analysis of the MACBETH method and the FGA-MACBETH modeling for the first example

	FGA-MACBETH			Classic MACBETH	
	$\widetilde{v}_x$	v_x	E_x	v_x	E_x
x_1	11,11,19	13.67	112.12	13	120
x_2	10,10,17	12.33	100.00	11	100
x_3	7,7,12	8.67	66.67	8	70
x_4	2,2,4	2.67	12.12	2	10
x_5	1,1,2	1.33	0.00	1	0
x_6	0,0,0	0.00	− 12.12	0	− 10

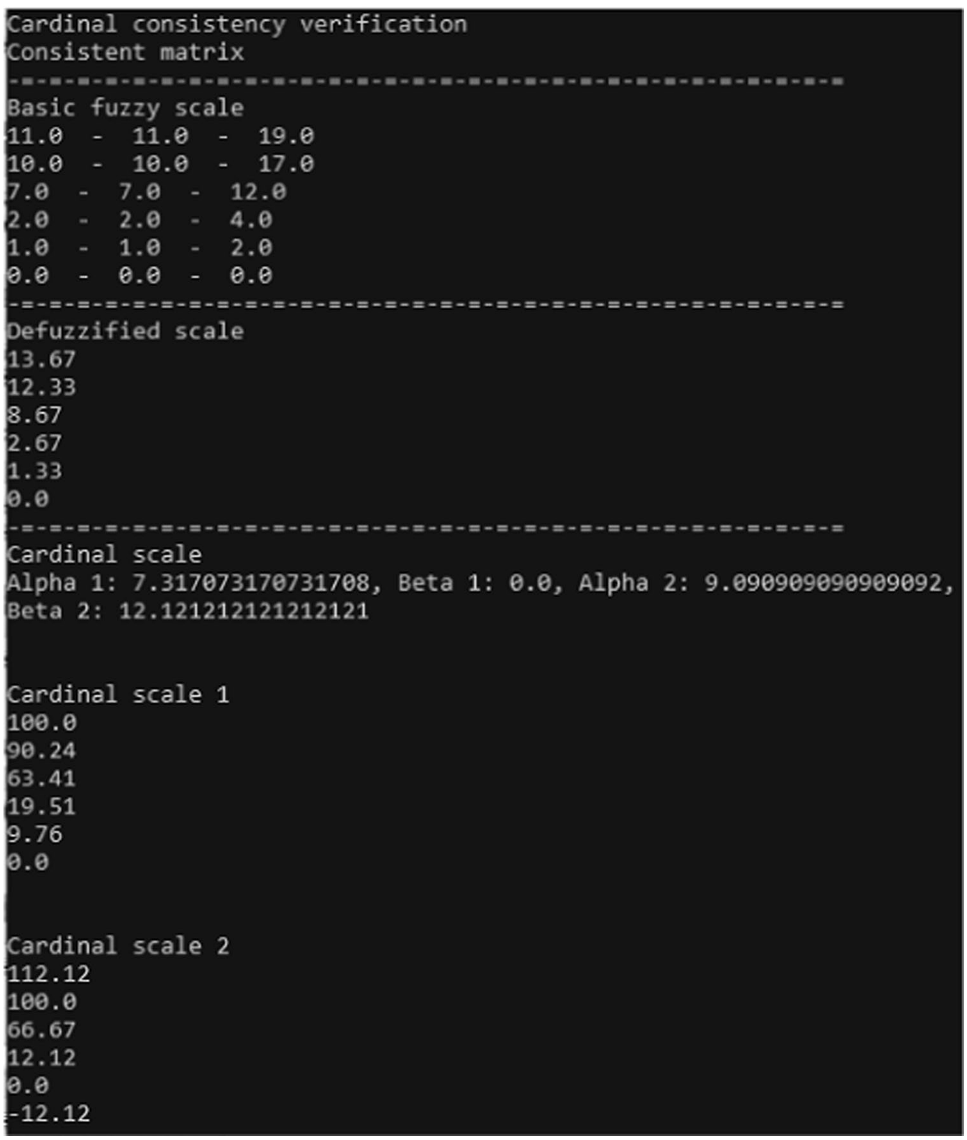
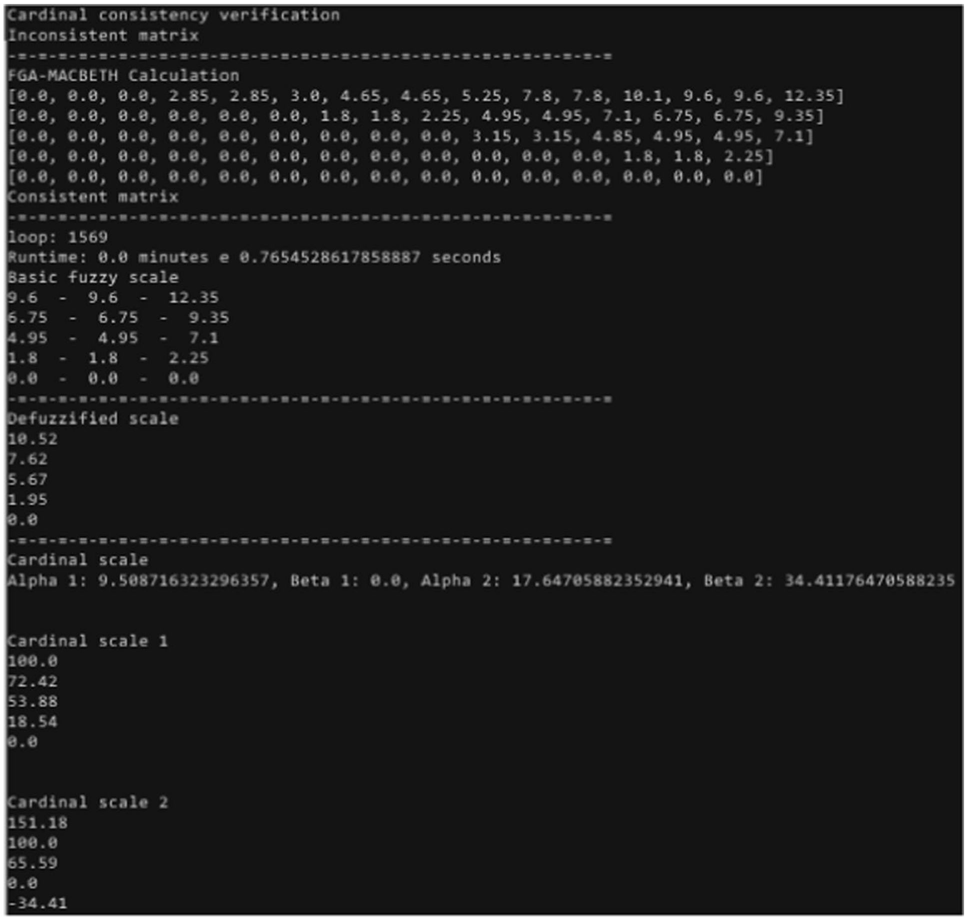


Fig. 8 Application of the FGA-MACBETH method in the first example



```
Cardinal consistency verification
Inconsistent matrix
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
FGA-MACBETH Calculation
[0.0, 0.0, 0.0, 2.85, 2.85, 3.0, 4.65, 4.65, 5.25, 7.8, 7.8, 10.1, 9.6, 9.6, 12.35]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.8, 1.8, 2.25, 4.95, 4.95, 7.1, 6.75, 6.75, 9.35]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 3.15, 3.15, 4.85, 4.95, 4.95, 7.1]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.8, 1.8, 2.25]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
Consistent matrix
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
loop: 1569
Runtime: 0.0 minutes e 0.7654528617858887 seconds
Basic fuzzy scale
9.6  -  9.6  -  12.35
6.75  -  6.75  -  9.35
4.95  -  4.95  -  7.1
1.8  -  1.8  -  2.25
0.0  -  0.0  -  0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
Defuzzified scale
10.52
7.62
5.67
1.95
0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
Cardinal scale
Alpha 1: 9.508716323296357, Beta 1: 0.0, Alpha 2: 17.64705882352941, Beta 2: 34.41176470588235


Cardinal scale 1
100.0
72.42
53.88
18.54
0.0


Cardinal scale 2
151.18
100.0
65.59
0.0
-34.41
```

Fig. 9 Application of the FGA-MACBETH method in the second example

Table 4 Comparative analysis of the MACBETH method and the FGA-MACBETH modeling for the second example

	FGA-MACBETH			Classic MACBETH	
	$\widetilde{v}_x$	v_x	E_x	v_x	E_x
x_1	9.6,9.6,12.35	10.52	100.00	10	100
x_2	6.75,6.75,9.35	7.62	72.42	7	70
x_3	4.95,4.95,7.1	5.67	53.88	5	50
x_4	1.8,1.8,2.25	1.95	18.54	1	10
x_5	0,0,0	0.0	0.00	0	0

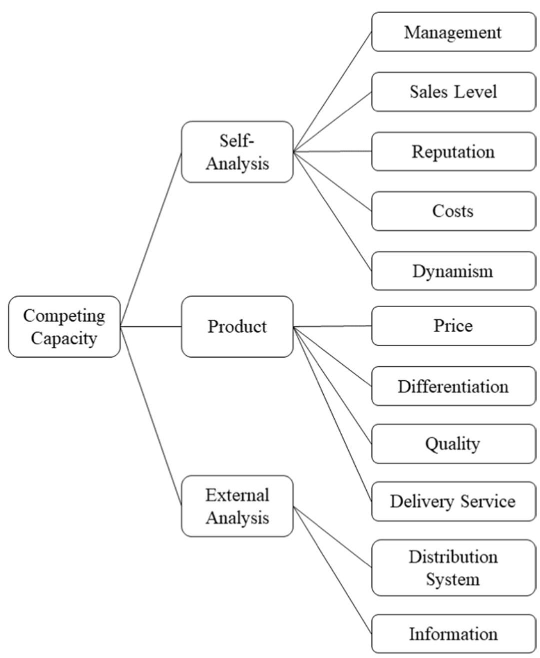


Fig. 10 Tree of Fundamental Points of View. Source: [28]

```
for each row on the matrix
    for each column on the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'VWEAK' then
            Randomly generate a fuzzy number for the very weak scale
            Insert the new fuzzy number into the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'WEAK' then
            Randomly generate a fuzzy number for the weak scale
            Insert the new fuzzy number into the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'MOD' then
            Randomly generate a fuzzy number for the moderate scale
            Insert the new fuzzy number into the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'STRO' then
            Randomly generate a fuzzy number for the strong scale
            Insert the new fuzzy number into the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'VSTRO' then


            Randomly generate a fuzzy number for the very strong scale
            Insert the new fuzzy number into the matrix
        if fuzzyqualitymatrix [l][((l + 1) * 3)] = 'EXT' then
            Randomly generate a fuzzy number for the extreme scale
            Insert the new fuzzy number into the matrix
        endif
    endfor
endfor
calc_fuzzymacbeth(fuzzyqualitymatrix, fuzzymatrix, n)
if consistenciacardinal ≠ 'Not'
    Calculate the scales as demonstrated in Algorithm 1
endif
if consistenciacardinal = 'Not'
    while True
        if consistenciacardinal ≠ 'Not'
            write (fuzzymatrix)
            break
        for each row on the matrix
            for each column on the matrix
                Calculate the crossover value for each category
                Check the crossover probability
                Perform gene selection
                carry out the crossing
            endfor
        endfor
        for each row on the matrix
            for each column on the matrix
                Calculate the mutation value for each category
                Check the probability of mutation
                Perform gene selection
                mutate
            endfor
        endfor
    endwhile
    calc_fuzzymacbeth(fuzzyqualitymatrix, fuzzymatrix, n)
    Calculate the scales as demonstrated in Algorithm 1
endif
```

The results of the method applied to two decision matrices are presented to show how the computational modeling was performed. The decision matrix (M_1) extracted from [3] was chosen in the first example.

$$M_1 = \begin{bmatrix} * & (veryweak) & (weak) & (strong) & (strong) & (verystrong) \\ * & * & (weak) & (moderate) & (moderate) & (strong) \\ * & * & * & (moderate) & (moderate) & (moderate) \\ * & * & * & * & (veryweak) & (veryweak) \\ * & * & * & * & * & (veryweak) \\ * & * & * & * & * & * \end{bmatrix}$$

The results are presented in Fig. 8. Table 3 shows the results of the classical MACBETH method and those obtained through FGA-MACBETH modeling for comparison purposes.

The second example concerns the decision matrix (M_2), extracted from [27], which, although semantically consistent, presents cardinal inconsistency. Thus, the decision maker had to change his/her initial assessment in the classical MACBETH method to correct the inconsistency. As shown in Fig. 9, the FGA-MACBETH modeling can generate a cardinal scale for the decision maker's initial decision matrix. Table 4 presents the results of the two methods, MACBETH and FGA-MACBETH, for comparison purposes.

$$M_2 = \begin{bmatrix} * & (weak) & (moderate) & (strong) & (verystrong) \\ * & * & (veryweak) & (strong) & (strong) \\ * & * & * & (moderate) & (moderate) \\ * & * & * & * & (weak) \\ * & * & * & * & * \end{bmatrix}$$

The two examples show an alignment between the FGA-MACBETH method and the classical MACBETH method. The first example applied the method to a semantically and cardinally consistent decision matrix, reaching a cardinal scale consistent with that obtained by the MACBETH method. In the second example, the FGA-MACBETH method found a cardinal scale consistent with the one in the classical MACBETH method; however, the decision maker did not have to change his/her initial assessments. Therefore, the main advantage of fuzzy modeling based on a genetic algorithm is that the need for context reassessments is decreased, thus facilitating the decision maker's task.

The methods are combined to facilitate the technique's implementation to solve real-world problems. The Fuzzy theory was applied to incorporate the uncertainty inherent to the qualitative terms composing the semantic scale in mathematical modeling. Hence, it contributes to correcting the cardinal inconsistency problem in a semantically consistent decision matrix, facilitating decision makers to use the method. GA was used in the computational implementation, which is vital for using the method in real-world decision-making. Therefore, the decision to combine the methods was not random but intended to incorporate the potential of both methods in this approach's development.

5 Application of the Model

The paper by Bana and Costa et al. [28], which describes an application of multi-criteria decision analysis in a real case study, was used to illustrate the method's application. Such a study was replicated using the FGA-MACBETH method to verify whether the results aligned with the evaluation model built with the M-MACBETH software. This study was chosen because it was validated by the international scientific community, reaching 445 citations from its publication in 1999 to 2023 [29]. Furthermore, the study's data, mainly all the decision-maker pairwise comparison matrices (fundamental for the case study to be replicated here), were accessible, as the paper originated from the master's thesis by [30].

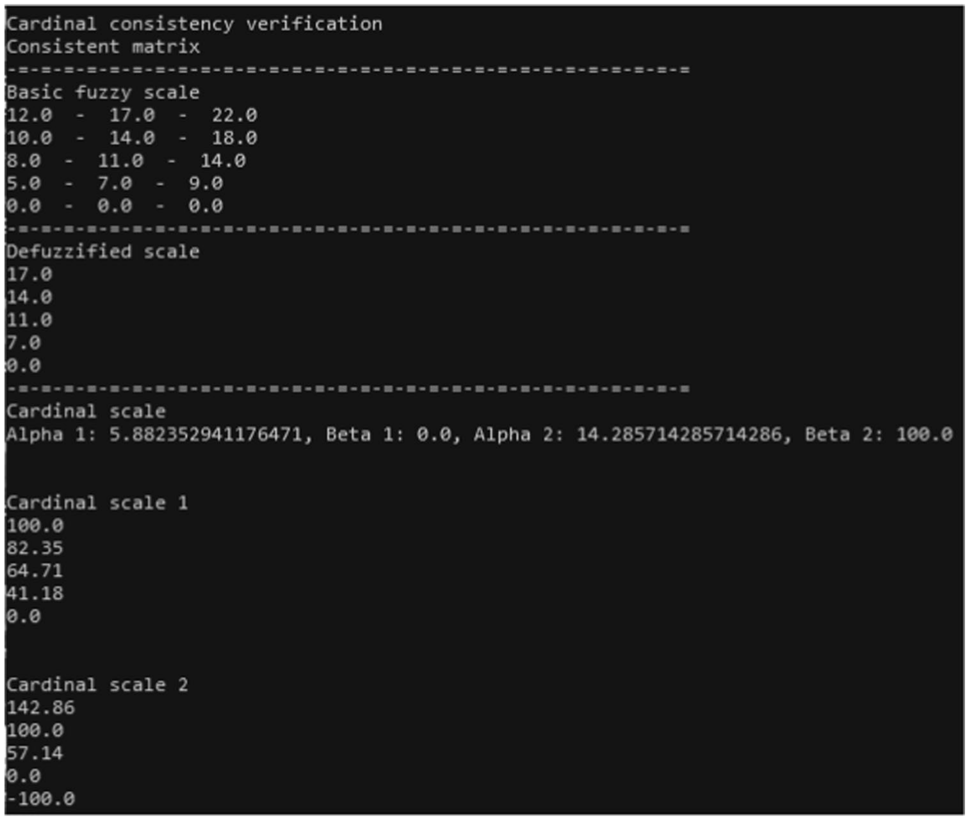
A multi-criteria model was developed in the earlier case study to assess market capacity and suggest strategies for small and medium-sized companies in the textile sector in Santa Catarina, Brazil. The objective was to support these companies in facing the crisis caused in the industry due to globalization and the opening of the market for foreign products. The model was developed using a decision-support constructivist methodology based on the perceptions of managers and specialists in the textile sector through three stages: structuring, evaluation, and recommendations or directions.

The problem was defined in the structuring phase as determining strategies for the small and medium-sized textile industries in Santa Catarina. The critical points of view were listed and organized into 11 Fundamental Points of View (FPV) using cognitive maps and then classified under three areas of interest (Self-Analysis, Product, and External Analysis), as shown in Fig. 10.

The descriptors consist of ordinal scales to operationalize the criteria and measure the context's

Table 5 Comparative analysis of value functions generated by the M-MACBETH and FGA-MACBETH methods for the reputation descriptor

Descriptor	Impact levels	M-MACBETH Cardinal Scale	FGA-MACBETH Cardinal Scale
3 Reputation	N5	100	100.00
	N4	83	82.35
	N3	67	64.71
	N2	42	41.18
	N1	0	0.00



```
Cardinal consistency verification
Consistent matrix
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Basic fuzzy scale
12.0  -  17.0  -  22.0
10.0  -  14.0  -  18.0
8.0  -  11.0  -  14.0
5.0  -  7.0  -  9.0
0.0  -  0.0  -  0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Defuzzified scale
17.0
14.0
11.0
7.0
0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Cardinal scale
Alpha 1: 5.882352941176471, Beta 1: 0.0, Alpha 2: 14.285714285714286, Beta 2: 100.0

Cardinal scale 1
100.0
82.35
64.71
41.18
0.0

Cardinal scale 2
142.86
100.0
57.14
0.0
-100.0
```

Fig. 11 Result obtained by FGA-MACBETH for the Reputation descriptor

properties [31]. Hence, the descriptors are the indicators for the ordinal measurement of performance based on what the decision maker considers relevant to measure, using cognitive maps and an interactive process between the decision maker and facilitator. Thus, 11 descriptors were defined, one for each FPV.

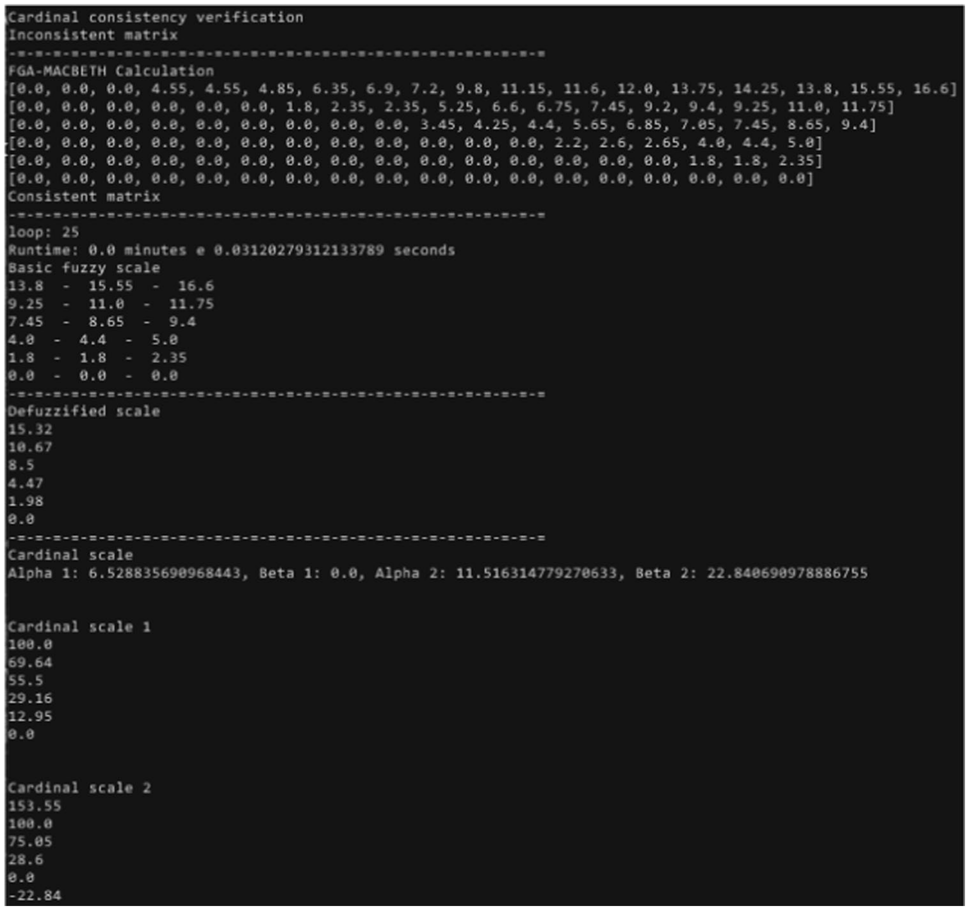
After briefly describing the decision-making context's structuring phase, this paper will focus on the evaluation phase, in which the value functions were obtained from the descriptors' ordinal scales by applying the M-MACBETH software. The cardinal scales obtained with the FGA-MACBETH model developed here show how the results align.

Three companies from the textile industry in Santa Catarina, a state in the southern region of Brazil, were assessed. The first company is a T-shirt and knitwear producer that manufactures 100,000 pieces monthly. The second makes T-shirts and children's clothes, manufacturing 600,000 pieces/month. The third manufactures 40,000 pairs of jeans per month. Because this study's objective was not to compare the companies but individually assess their performance, two fictitious companies served as parameters for the analysis. One fictional company's performance was considered good, and the other's was considered neutral, i.e., the decision maker did not find its performance attractive or repulsive.

Table 6 Comparative analysis of the value functions generated by the M-MACBETH and FGA-MACBETH methods for the differentiation descriptor

Descriptor	Impact levels	M-MACBETH Cardinal Scale	FGA-MACBETH Cardinal Scale
7 Differentiation	N6	100	100.00
	N5	70	69.64
	N4	60	55.50
	N3	35	29.16
	N2	20	12.95
	N1	0	0.00

Pairwise comparisons of the differences in attractiveness between the impact levels of each descriptor for each evaluated company were performed using the MACBETH method's semantic scale. In this study, the ordinal scale based on these comparisons was transformed into cardinal scales, or value functions, through the FGA-MACBETH method. Table 5 and Fig. 11 show the results obtained by both methods for the Reputation descriptor (M_3). This analysis highlights



```
Cardinal consistency verification
Inconsistent matrix
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
FGA-MACBETH Calculation
[0.0, 0.0, 0.0, 4.55, 4.55, 4.85, 6.35, 6.9, 7.2, 9.8, 11.15, 11.6, 12.0, 13.75, 14.25, 13.8, 15.55, 16.6]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.8, 2.35, 2.35, 5.25, 6.6, 6.75, 7.45, 9.2, 9.4, 9.25, 11.0, 11.75]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 3.45, 4.25, 4.4, 5.65, 6.85, 7.05, 7.45, 8.65, 9.4]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 2.2, 2.6, 2.65, 4.0, 4.4, 5.0]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.8, 1.8, 2.35]
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
Consistent matrix
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
loop: 25
Runtime: 0.0 minutes e 0.03120279312133789 seconds
Basic fuzzy scale
13.8  -  15.55  -  16.6
9.25  -  11.0  -  11.75
7.45  -  8.65  -  9.4
4.0  -  4.4  -  5.0
1.8  -  1.8  -  2.35
0.0  -  0.0  -  0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Defuzzified scale
15.32
10.67
8.5
4.47
1.98
0.0
-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Cardinal scale
Alpha 1: 6.528835690968443, Beta 1: 0.0, Alpha 2: 11.516314779270633, Beta 2: 22.840690978886755


Cardinal scale 1
100.0
69.64
55.5
29.16
12.95
0.0


Cardinal scale 2
153.55
100.0
75.05
28.6
0.0
-22.84
```

Fig. 12 Results obtained by FGA-MACBETH for the Differentiation descriptor

the alignment between the cardinal scale generated by the FGA-MACBETH method and the scale generated by the M-MACBETH software.

$$M_3 = \begin{bmatrix} * & (moderate) & (strong) & (very strong) & (\text{extreme}) \\ * & * & (moderate) & (very strong) & (\text{extreme}) \\ * & * & * & (strong) & (verystrong) \\ * & * & * & * & (verystrong) \\ * & * & * & * & * \end{bmatrix}$$

This procedure was performed for all descriptors, highlighting that cardinal scales were obtained for the differentiation descriptor (M_4), which, according to the MACBETH method, presented cardinal inconsistency; the initial evaluation was changed to correct the inconsistency. The cardinal scales were obtained through the FGA-MACBETH method without changing the initial judgments. Table 6 and Fig. 12 present the results, showing the ability of the proposed method to build value functions based on the decision maker's judgments.

$$M_4 = \begin{bmatrix} * & (moderate) & (strong) & (verystrong) & (verystrong) & (extreme) \\ * & * & (weak) & (moderate) & (strong) & (\text{very}strong) \\ * & * & * & (moderate) & (strong) & (verystrong) \\ * & * & * & * & (weak) & (\text{moderate}) \\ * & * & * & * & * & (veryweak) \\ * & * & * & * & * & * \end{bmatrix}$$

Weights, or compensation rates, obtained by the M-MACBETH software and presented in [28] were used to assess each company's overall performance. Hence, the companies' performance was assessed in each descriptor based on the decision maker's evaluations. Figure 13 presents the assessment of local performance based on the MACBETH and FGA-MACBETH methods, showing that the method developed in this study coherently assessed performance while aligned with the MACBETH method.

This method also enabled assessing the companies' overall performance in each area of interest. Figure 14 shows no changes in the results obtained by the FGA-MACBETH method. Hence, only company 3 performed worse in the Self-Analysis area of interest than the one the

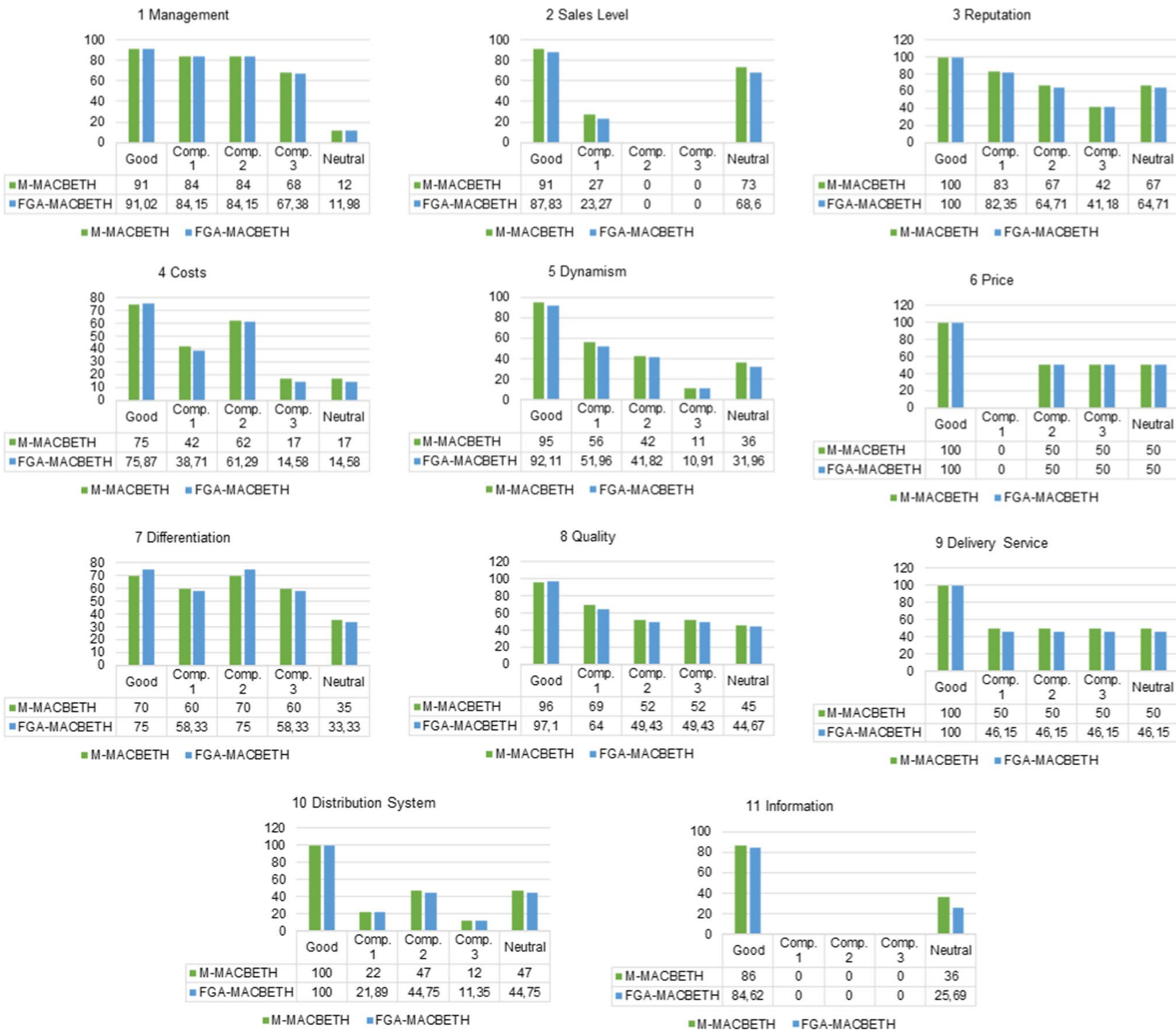


Fig. 13 Comparative analysis of the companies' local evaluation for each descriptor

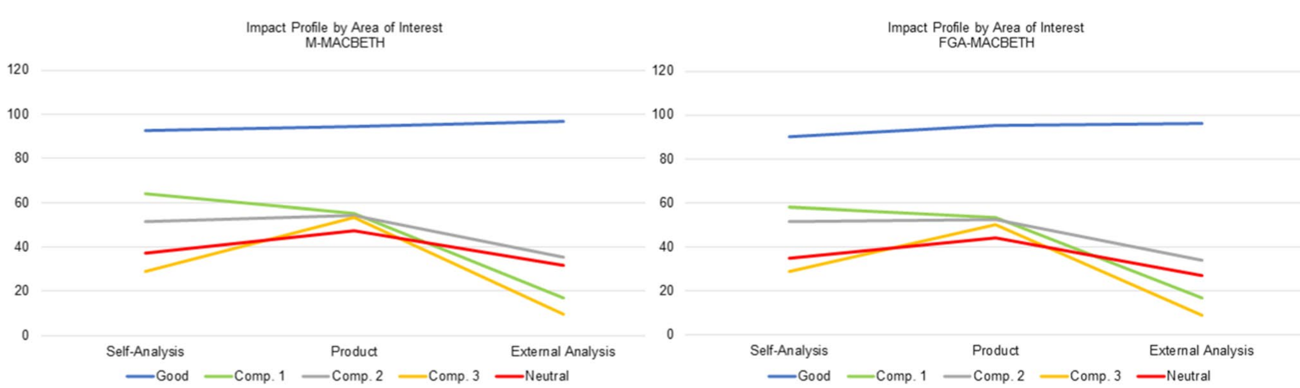


Fig. 14 Comparative analysis of the impact profile by area of interest

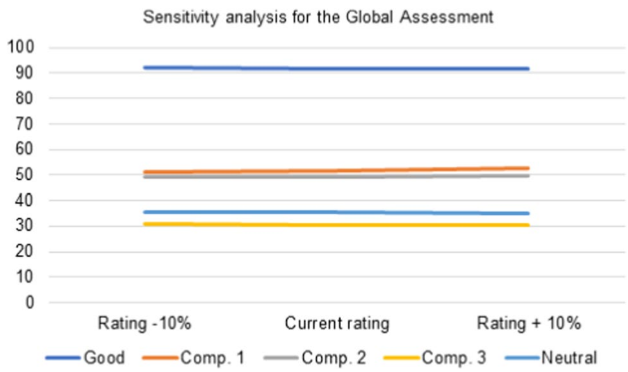


Fig. 15 Sensitivity analysis of the overall assessment

decision maker considered neutral. All companies performed better than the one considered neutral in the Product area of interest. In turn, companies 1 and 3 performed worse than the level considered neutral in the External Analysis area of interest—notably, the overall assessment generated by the FGA-MACBETH method aligned with the original method.

Finally, a sensitivity analysis of the overall evaluation was developed to verify the robustness of the evaluation model obtained by the FGA-MACBETH method. Therefore, the compensation rate of the Self-Analysis area of interest, which the decision maker considered the most important, was reduced by 10% and increased by 10% to verify whether changes in the weights of the areas of interest changed the companies' overall assessment.

Thus, with w_1 being the compensation rate of the Self-Analysis interest area, w_2 the compensation rate of the Product interest area, and w_3 the compensantion rate of the External Analysis interest area, the sensitivity analysis is performed by $w_1' = w_1 - 10\%$ and $w_1''= w_1 + 10\%$, calculating the corresponding values of w_2', w_3', w_2'' and w_3'' without changing the proportions between the compensation rates w_1, w_2 and w_3.

Figure 15 presents the sensitivity analysis results, which showed that minor variations in the compensation rates do not alter the overall assessment of each area of interest, confirming that the model proposed here is indeed robust.

The case study showed that the method presented here is helpful in Operational Research, in multi-criteria decision-making problems in complex contexts, facilitating the task of assessing evaluation elements when using a semantic scale to compare between pairs; hence, the method's applicability is improved when the Fuzzy version is used. At the same time, its implementation using GA allows for reaching the cardinal scale fast and enables the method to be applied to real-world problems. Thus, like the MACBETH method, the FGA-MACBETH method can reach a performance evaluation model to assist managers in their decision-making tasks.

6 Conclusions

The computational implementation of the FGA-MACBETH method was performed in this study through a bio-inspired metaheuristic. The FGA-MACBETH method uses a triangular fuzzy numbers scale to incorporate the subjectivity inherent to the linguistic terms of the semantic scale of the MACBETH method into the mathematical modeling. Hence, the F-LP-MACBETH linear programming problem is used to make the LPP of the original MACBETH method flexible.

Such flexibility, enabled by applying the triangular fuzzy numbers scale, allows incorporating the subjectivity of the linguistic terms of the semantic categories, mathematically translating them into a basic fuzzy scale. This basic fuzzy scale is defuzzified by the centroid method, giving rise to a basic crisp scale consisting of the pre-cardinal scale. Afterward, a cardinal scale is obtained for each evaluation element.

A hybrid method was developed for the computational implementation using a genetic algorithm to solve the cardinally inconsistent decision matrix. Therefore, the genetic concepts of selection, crossover, and mutation were applied to go through the solution space and reach the cardinal solution. The process of solving the cardinally consistent matrix with fuzzy variables through LPP was facilitated using the genetic algorithm. Because it is a stochastic resolution method that analyzes potential solutions randomly originated, the solutions are generated until the inconsistency is resolved.

Two illustrative examples were presented to show how the model works in a semantically and cardinally consistent decision matrix and a decision matrix with cardinal inconsistency. The case study [28] was replicated using the FGA-MACBETH to demonstrate the method's applicability.

The results confirm the alignment between the FGA-MACBETH and the classical MACBETH method. This study's primary contribution is that the method developed here enabled circumventing the problem of cardinal inconsistency in a semantically consistent decision matrix, obtaining a cardinal scale without requiring the decision maker to redo his/her initial evaluation.

Therefore, the modeling developed here is expected to improve the MACBETH method by incorporating the fuzzy theory's ability to mathematically treat linguistic variables, making the LPP more flexible, and facilitating the decision maker's task when making assessments using qualitative terms.

Suggestions for future studies include using other computational metaheuristics to compare the performance of computational implementations. Additionally, combining different metaheuristics is suggested to compare hybrid methods and isolated metaheuristics. Within the group decision scope, we recommend the application of Large-Scale Group Decision Making (LSGDM), applying fuzzy preference relations for LSGDM, and considering the level of agreement among experts before selecting the best alternative [32]. For group decision-making, we also suggest using a linguistic metric for Consensus Reaching Processes (LiCRPs) [33]. In this sense, some studies use the process of obtaining consensus for group decision-making [34–36]. Intuitionistic fuzzy numbers can be adopted to assess non-membership and membership degrees together [33–37]. The use of fuzzy extensions of the ordered weighted average method for the aggregation of evaluations, as proposed in [34–38], is also suggested. Additionally, we intend to improve the computational modeling presented in this study to make its application feasible through an on-site implemented decision support system.

Finally, the scale proposed here comprises linguistic terms corresponding to seven semantic categories, following the linguistic proposition of the MACBETH method. Different decision contexts can be assessed using the semantic terms composing the scale, but different linguistic granularities can be used, which is a suggestion for future studies.

Author contributions TRB developed the mathematical modeling analysis of the algorithm and wrote the manuscript, AAL guided in all stages of development and analysis of the algorithm, and CMSM, LE, SRE, and AD helped to carry out the analysis with constructive discussions.

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Availability of data and material Not applicable.

Declarations

Conflict of interest The authors declare no conflicts of interest.

Ethical approval This study did not involve human or animal subjects.

Consent for publication All the authors agree with this manuscript's publication.

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