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RESEARCH ARTICLE

Neutrosophic Log-Logistic Distribution Model in Complex Alloy Metal Melting Point Applications

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Abstract
The log-logistic distribution is more comprehensively applied in the area of survival and reliability engineering analysis for modeling the lifetime data practices of both human and electronic designs. The goal of this paper is to develop a generalization of the classical pattern log-logistic distribution, known as the neutrosophic log-logistic distribution (NLLD), to model various survival and reliability engineering data with indeterminacies. The developed distribution is especially useful for modeling indeterminate data that is roughly positively skewed. This paper discusses the developed NLLD's main statistical properties such as neutrosophic survival function, neutrosophic hazard rate, neutrosophic moments, and neutrosophic mean time failure. Furthermore, the neutrosophic parameters are estimated using the well-known maximum likelihood (ML) estimation method in a neutrosophic environment. A simulation study is carried out to establish the achievement of the estimated neutrosophic parameters. As a final point, the proposed NLLD applications in the real world have been discussed with the help of real data. The real data illustrated that the efficiency of the proposed model as compared with the existing models.

Keywords Simulation · Neutrosophic statistics · Maximum likelihood · Log-logistic distribution · Indeterminacy

Abbreviations

NLLD	Neutrosophic log-logistic distribution
LLD	Log-logistic distribution
PDF	Probability density function
CDF	Cumulative density function
MGF	Moment generating function
CF	Characteristic function
MLE	Maximum likelihood estimation
NAB	Neutrosophic average bias
NMSE	Neutrosophic Mean Square Error
NWD	Weibull distribution
NED	Neutrosophic exponential distribution
LL	Log-likelihood value
AIC	Akaike Information Criteria
BIC	Bayesian information criteria
KS	Kolmogorov–Smirnov

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1 Introduction

The log-logistic distribution (LLD) is introduced and developed under the name of Fisk distribution by [11] for an applications of economics. Due to the increasing and decreasing hazard function nature of log-logistic distribution, it is more applicable in survival analysis [13]. Studied the log-logistic distribution in survival analysis specifically mortality rate of cancer patients undergo analysis or treatment [4]. Developed the log-logistic distribution for survival data analysis using MCMC [38]. Presented the systems of frequency curves generated by transformations of logistic variables, discussed the nature of the distribution and found that LLD is a special case of Burr XII. Sampling properties of estimators of the log-logistic distribution with application to Canadian precipitation data discussed by [29]. They showed that LLD has more applicable in hydrology to model stream flow and precipitation. Some more applications of LLD in reliability estimation and quality control aspects please refer [2, 16, 17, 23–25].

The objective of this study is to create a novel neutrosophic log-logistic distribution for use in material engineering, specifically in applications involving alloy metal melting points. The melting point of a substance is very important in determining the quality of that material. Calculating the melting point is not just quantitatively challenging, but also conceptually challenging. Finding the hotness at which the solid and liquid stages are equal is the method for determining the melting point at constant pressure. While, the correctness of the current possibilities is strongly believed to determine the grade of the achieved melting hotness. Hence, the material engineer who wish to choose a pragmatic prospective for their definite resources and are fascinated in the melting temperature constituted by this possible for more details refer [22].

Nevertheless, it is more sensible to consider that the LLD is the best depicted distribution for the alloy metal melting point data based on interval set of quantities for vague parameters. In this situation, neutrosophic statistics is the better environment to address alloy metal melting point based on interval data. The concept of the neutrosophic theory is invented and studied extensively by [30] for indeterminacies in the data. The new school of thought on neutrosophic theory is an expansion of fuzzy logics or fuzzy sets for more details, see [7, 26, 31–33, 35, 37]. Furthermore, the neutrosophic statistics is pioneered by [34] and it is an expansion to the classical statistics which addresses through uncertain or vague data and corresponding statistical probability distributions. The generalization of interval statistics is neutrosophic statistics and also studies fuzzy interval sets. The neutrosophic statistics becomes classical statistics when data is known or deterministic. Whereas, in real-world application, most of the data sets are vague or undeterministic or unclear, partially unknown or incomplete than determinate data in this situations neutrosophic statistical procedures are desirable for more details refer [5, 19, 21].

This study aims to extend the traditional log-logistic distribution for modeling industrial data in a neutrosophic context. Few scholars have been drawn to work on neutrosophic probability distributions in recent years; for additional information, see [6] developed neutrosophic Weibull distribution. Neutrosophic exponential distribution applications for complex data analysis was studied by [10, 28] presented neutrosophic beta distribution with properties and applications, [1] studied neutrosophic Kumaraswamy distribution with engineering application, [27] discussed on neutrosophic extension of the Maxwell model, [18] attempted on statistical development of neutrosophic gamma distribution with applications to complex data analysis [3]. Developed a neutrosophic conditional probabilities with theories and applications [14]. Studied the Basics of neutrosophic simulation for converting random numbers associated with a uniform probability distribution into random variables follow an exponential distribution. To begin the log-logistic distribution's neutrosophic adaptation, a new model is being created. Particularly when the data presented interval statistics, this sort of approach is competent of solving real-world practical challenges when dealing with indeterminate data in either a univariate or multivariate circumstance. The neutrosophic statistics is an extension of interval statistics, for more details refer [36]. He discussed the neutrosophic statistics deals with all types of indeterminacy, while interval statistics deals only with indeterminacy that may be represented by intervals. Furthermore, he pointed out that neutrosophic statistics is based on set analysis, while interval statistics on interval analysis, therefore, the interval statistics is a particular case of the neutrosophic statistics (that uses all types of sets, not only intervals).

The remaining paper is reported as under: A description about NLLD is given in Sect. 2. The various statistical properties of NLLD are presented in Sect. 3. In Sect. 4, the estimation of neutrosophic parameters is explained. The extensive simulation study is carried out in Sect. 5. An industrial application of the developed NLLD using the alloy melting points data is given in Sect. 6 and in Sect. 7, the concluding remarks and future study are presented.

2 Neutrosophic Log-Logistic Distribution

The neutrosophic statistics concept is introduced by [34] and extensively studied by [8]. According to these authors, the neutrosophic variable could be expressed as: $X_N = X_L + I_N X_U; I_N \in [I_L, I_U]$, where X_L and $I_N X_U$ denote the determined and indeterminate parts, correspondingly. Note that $I_N \in [I_L, I_U]$ is an undetermined interval. Assume that the neutrosophic random variable $X_N \in [X_L, X_U]$ follows to the NLLD having neutrosophic scale parameter $\sigma_N \in [\sigma_L, \sigma_U]$ and neutrosophic shape parameter $\beta_N \in [\beta_L, \beta_U]$. Reminder that X_L, σ_L, β_L and X_U, σ_U, β_U are the lower values and the upper values, respectively. The neutrosophic probability density function (PDF) of NLLD is given by

$$f(x_N; \beta_N, \sigma_N) = \frac{\beta_N}{\sigma_N} \frac{(x_N/\sigma_N)^{\beta_N - 1}}{\left[1 + (x_N/\sigma_N)^{\beta_N}\right]^2}; x_N \geq 0, \beta_N > 0, \sigma_N > 0 \tag{1}$$

And neutrosophic cumulative density function (CDF) is given below:

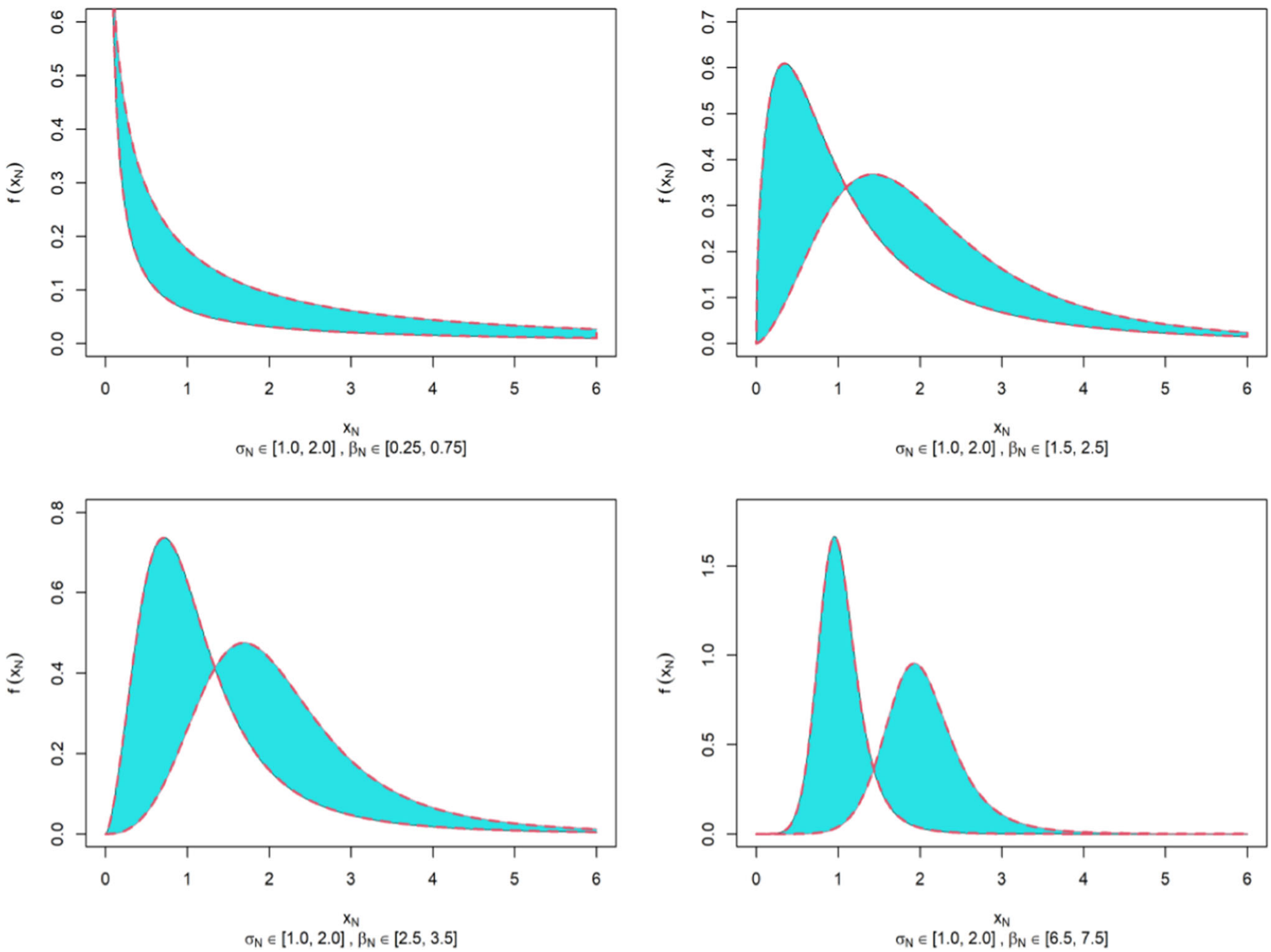

Fig. 1 The pdf curves of NLLD for various neutrosophic parameters

$$F(x_N;\beta_N,\sigma_N)=\frac{(x_N/\sigma_N)^{\beta_N}}{1+(x_N/\sigma_N)^{\beta_N}};x_N\geq 0,\beta_N>0,\sigma_N>0. \tag{2}$$

The nature of PDF curves for various neutrosophic parametric combinations are given in Fig. 1. The NLLD is very bendy owing to different neutrosophic patterns of the neutrosophic density function. The neutrosophic distribution is unimodel and its dispersion decreases as β_N increases. The structures of CDF curves for various neutrosophic parametric combinations are given in Fig. 2.

Survival function $s(x_N)$ is given by

$$s(x_N)=1-F(x_N)=\left[1+(x_N/\sigma_N)^{\beta_N}\right]^{-1}. \tag{3}$$

Hazard rate function $h(x_N)$ is obtained as

$$h(x_N)=\frac{f(x_N)}{s(x_N)}=\frac{\beta_N}{\sigma_N}\frac{(x_N/\sigma_N)^{\beta_N-1}}{1+(x_N/\sigma_N)^{\beta_N}}. \tag{4}$$

The nature of survival function curves for different neutrosophic parametric combinations are given in Fig. 3 and hazard rate function curves depicted in Fig. 4. From Fig. 4, it is interesting to note that hazard function is decreasing monotonically when $\beta_N\leq[1,1]$ and hazard function is increasing monotonically when $\beta_N>[1,1]$.

3 Statistical Properties

This section deals with some statistical properties of the NLLD and the outcome are publicized as follow:

Theorem 1 *The* k^{th} *moment about origin of NLLD is*

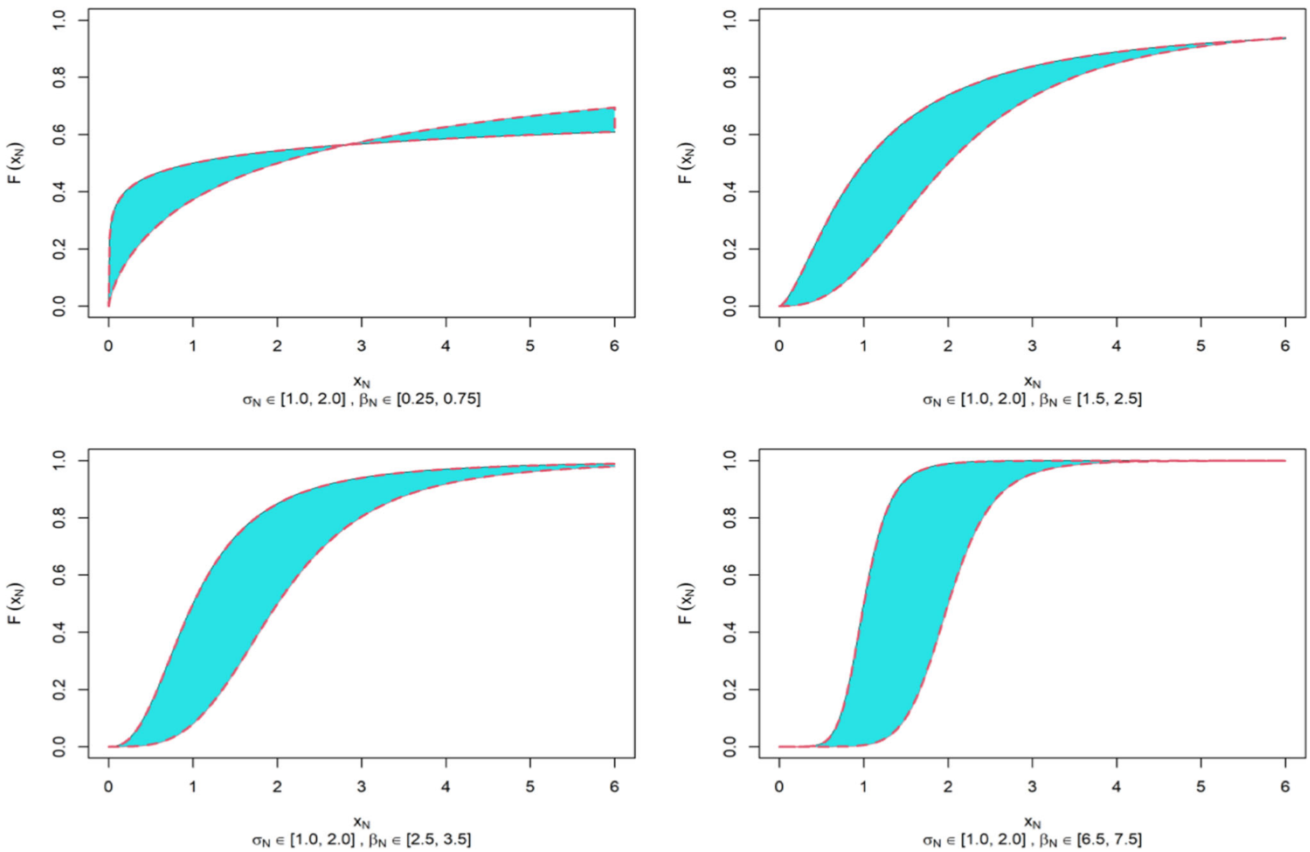

Fig. 2 The cdf curves of NLLD for various neutrosophic parameters

$$\left[\sigma_L^k\Gamma(1-k/\beta_L)\Gamma(1+k/\beta_L), \sigma_U^k\Gamma(1-k/\beta_U)\Gamma(1+k/\beta_U)\right].$$

Proof By definition, the kth raw moment is given as

$$\mu'_{kN} = E\left(X_N^k\right) = \int_0^\infty x_N^k f(x_N;\beta_N,\sigma_N)dx_N$$
$$= \int_0^\infty x_N^k \frac{\beta_N}{\sigma_N}\frac{(x_N/\sigma_N)^{\beta_N-1}}{\left[1+(x_N/\sigma_N)^{\beta_N}\right]^2}dx_N$$
$$= \left[\int_0^\infty x_L^k \frac{\beta_L}{\sigma_L}\frac{(x_L/\sigma_L)^{\beta_L-1}}{\left[1+(x_L/\sigma_N)^{\beta_L}\right]^2}dx_L, \int_0^\infty x_U^k \frac{\beta_U}{\sigma_U}\frac{(x_U/\sigma_U)^{\beta_U-1}}{\left[1+(x_U/\sigma_U)^{\beta_U}\right]^2}dx_U\right]. \quad (5)$$

After simplification, the Eq. (5) becomes

$$\mu'_{kN} = \left[\sigma_L^k\int_0^1 z_L^{(1-k/\beta_L)-1}(1-z_L)^{(1+k/\beta_L)-1}dz_L, \sigma_U^k\int_0^1 z_U^{(1-k/\beta_U)-1}(1-z_U)^{(1+k/\beta_U)-1}dz_U\right].$$

where $z_L = \left[1+(x_L/\sigma_L)^{\beta_L}\right]^{-1}$ and

$$z_U = \left[1+(x_U/\sigma_U)^{\beta_U}\right]^{-1}$$
$$= \left[\sigma_L^k \mathrm{B}(1-k/\beta_L, 1+k/\beta_L), \sigma_U^k \mathrm{B}(1-k/\beta_U, 1+k/\beta_U)\right].$$

$$Hence,\ \mu'_{kN} = \left[\sigma_L^k\Gamma(1-k/\beta_L)\Gamma(1+k/\beta_L), \sigma_U^k\Gamma(1-k/\beta_U)\Gamma(1+k/\beta_U)\right]. \quad (6)$$

The above results holds good when $k < \beta_N \in [\beta_L, \beta_U]$.

In particular, the first four raw moment are obtained on substituting k = 1, 2, 3 and 4 in Eq. (6).

The neutrosophic mean (NM) of NLLD is obtained as

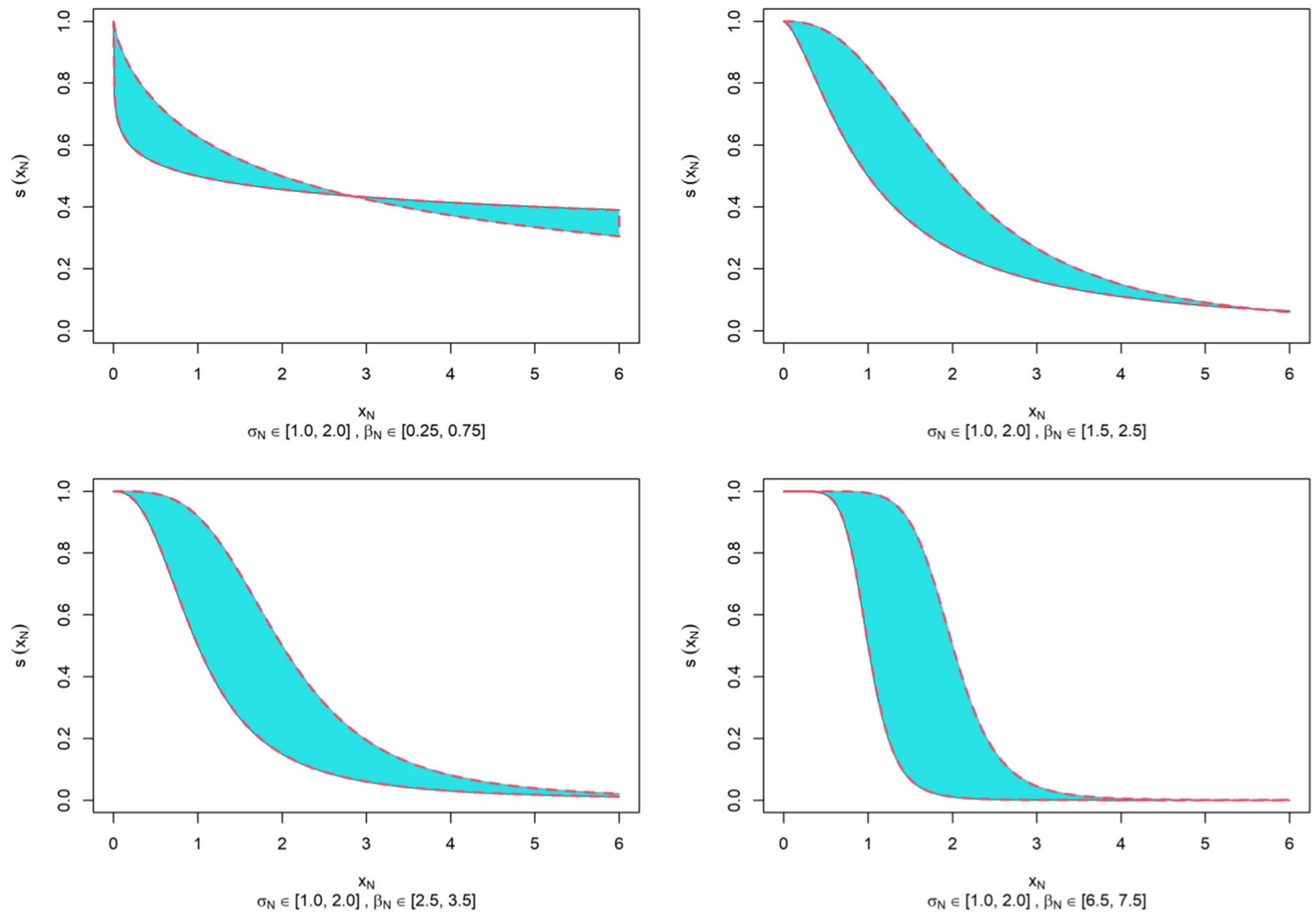

Fig. 3 The survival function curves of NLLD for various neutrosophic parameters

$$\begin{aligned}NM = \mu'_{1N} = [&\sigma_L \Gamma(1 - 1/\beta_L)\Gamma(1 + 1/\beta_L),\\ &\sigma_U \Gamma(1 - 1/\beta_U)\Gamma(1 + 1/\beta_U)].\end{aligned} \tag{7}$$

And neutrosophic variance (NV) of NLLD is obtained as

$$\begin{aligned}NV &= \mu_{2N} = \mu'_{2N} - \left(\mu'_{1N}\right)^2\\ &= \begin{bmatrix} \sigma_L^2\left\{\Gamma(1-2/\beta_L)\Gamma(1+2/\beta_L) - (\Gamma(1-1/\beta_L)\Gamma(1+1/\beta_L))^2\right\},\\ \sigma_U^2\left\{\Gamma(1-2/\beta_U)\Gamma(1+2/\beta_U) - (\Gamma(1-1/\beta_U)\Gamma(1+1/\beta_U))^2\right\}\end{bmatrix}.\end{aligned} \tag{8}$$

Theorem 2 *The moment generating function (MGF) of NLLD is*

$$\begin{aligned}M_{X_N}(t) = \Bigg[&\sum_{k=0}^{\infty} \frac{(t\sigma_L)^k}{k!} \mathrm{B}(1 - k/\beta_L, 1 + k/\beta_L),\\ &\sum_{k=0}^{\infty} \frac{(t\sigma_U)^k}{k} \mathrm{B}(1 - k/\beta_U, 1 + k/\beta_U)\Bigg].\end{aligned}$$

Proof By definition MGF is given as

$$\begin{aligned}M_{X_N}(t) &= E(e^{tx_N}) = \int_0^\infty e^{tx_N} f(x_N; \beta_N, \sigma_N) dx_N\\ &= \int_0^\infty e^{tx_N} \frac{\beta_N}{\sigma_N} \frac{(x_N/\sigma_N)^{\beta_N - 1}}{\left[1 + (x_N/\sigma_N)^{\beta_N}\right]^2} dx_N\\ &= \Bigg[\int_0^\infty e^{tx_L} \frac{\beta_L}{\sigma_L} \frac{(x_L/\sigma_L)^{\beta_L - 1}}{\left[1 + (x_L/\sigma_N)^{\beta_L}\right]^2} dx_L,\\ &\qquad \int_0^\infty e^{tx_U} \frac{\beta_U}{\sigma_U} \frac{(x_U/\sigma_U)^{\beta_U - 1}}{\left[1 + (x_U/\sigma_U)^{\beta_U}\right]^2} dx_U\Bigg]\\ &= \Bigg[\sum_{k=0}^{\infty} \frac{t^k}{k!} \int_0^\infty x_L^k \frac{\beta_L}{\sigma_L} \frac{(x_L/\sigma_L)^{\beta_L - 1}}{\left[1 + (x_L/\sigma_N)^{\beta_L}\right]^2} dx_L,\\ &\qquad \sum_{k=0}^{\infty} \frac{t^k}{k!} \int_0^\infty x_U^k \frac{\beta_U}{\sigma_U} \frac{(x_U/\sigma_U)^{\beta_U - 1}}{\left[1 + (x_U/\sigma_U)^{\beta_U}\right]^2} dx_U\Bigg].\end{aligned} \tag{9}$$

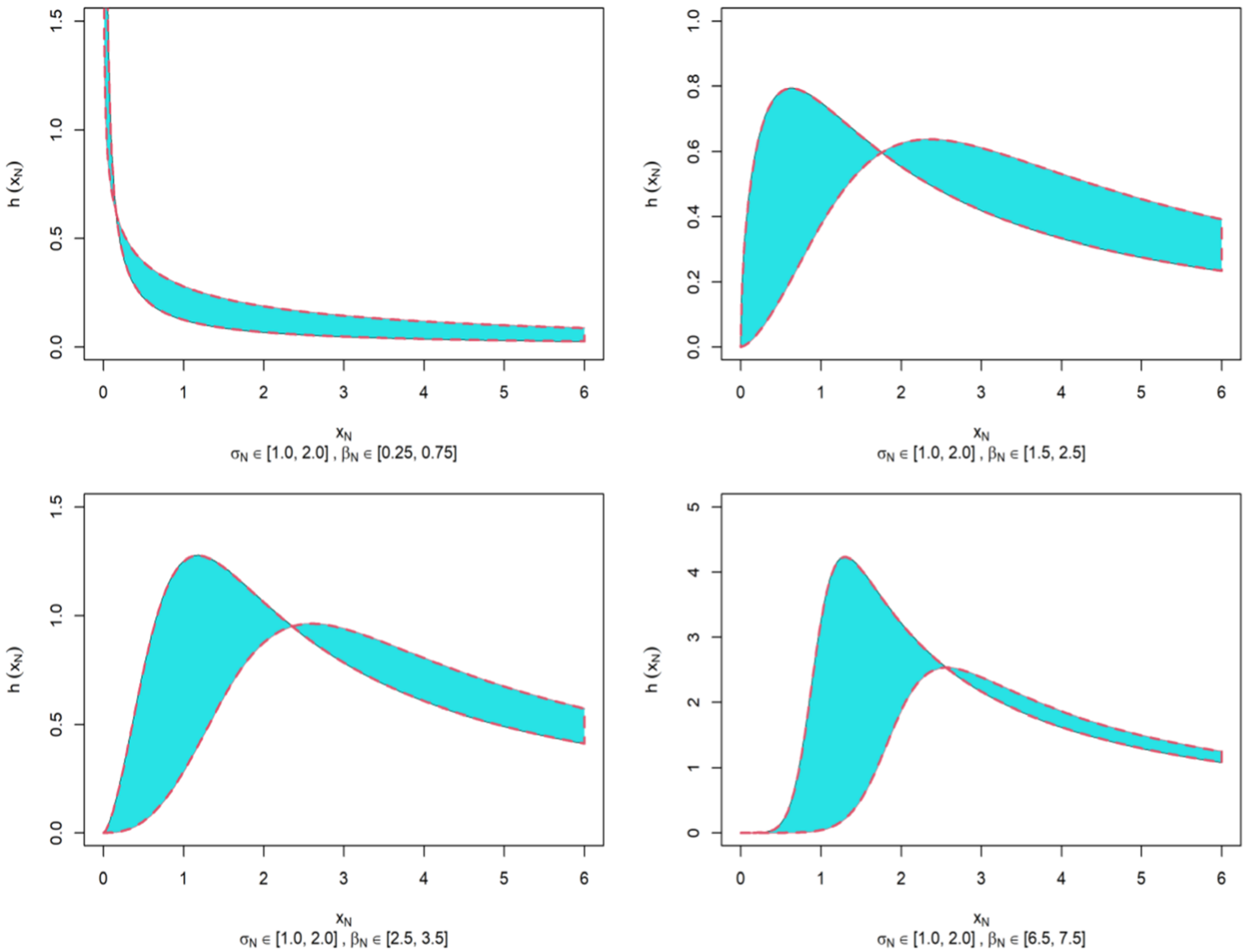

Fig. 4 The hazard rate function curves of NLLD for various neutrosophic parameters

After simplification, the Eq. (9) becomes

$$M_{X_N}(t) = \left[\sum_{k=0}^{\infty}\frac{t^k}{k!}\sigma_L^k\int_0^1 z_L^{(1-k/\beta_L)-1}(1-z_L)^{(1+k/\beta_L)-1}dz_L,\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{t^k}{k!}\sigma_U^k\int_0^1 z_U^{(1-k/\beta_U)-1}(1-z_U)^{(1+k/\beta_U)-1}dz_U\right]$$

where $z_L = \left[1+(x_L/\sigma_L)^{\beta_L}\right]^{-1}$ and $z_U = \left[1+(x_U/\sigma_U)^{\beta_U}\right]^{-1}$

$$= \left[\sum_{k=0}^{\infty}\frac{(t\sigma_L)^k}{k!}\mathrm{B}(1-k/\beta_L, 1+k/\beta_L),\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(t\sigma_U)^k}{k!}\mathrm{B}(1-k/\beta_U, 1+k/\beta_U)\right]$$

Hence, $$M_{X_N}(t) = \left[\sum_{k=0}^{\infty}\frac{(t\sigma_L)^k}{k!}\Gamma(1-k/\beta_L)\Gamma(1+k/\beta_L),\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(t\sigma_U)^k}{k!}\Gamma(1-k/\beta_U)\Gamma(1+k/\beta_U)\right]. \tag{10}$$

Theorem 3 *The characteristic function (CF) of NLLD is*

$$\Phi_{X_N}(t) = \left[\sum_{k=0}^{\infty}\frac{(it\sigma_L)^k}{k!}\mathrm{B}(1-k/\beta_L, 1+k/\beta_L),\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(it\sigma_U)^k}{k}\mathrm{B}(1-k/\beta_U, 1+k/\beta_U)\right].$$

Proof By definition CF is given as

Table 1 The Neutrosophic statistical constants of NLLD

Statistic	$\sigma_N \in [1,2]$ $\beta_N \in [2.25, 2.75]$	$\sigma_N \in [1,2]$ $\beta_N \in [2.5, 3.25]$	$\sigma_N \in [1,2]\beta_N \in [3.75, 4.25]$	$\sigma_N \in [1,2]$ $\beta_N \in [6.5, 7.5]$	$\sigma_N \in [2.0, 2.75]\beta_N \in [2.5, 3.25]$
Mean	[1.418, 2.512]	[1.321, 2.349]	[1.127, 2.195]	[1.040, 2.059]	[2.643, 3.230]
Variance	[5.784, 6.155]	[2.530, 2.752]	[0.414, 1.123]	[0.093, 0.267]	[5.204, 10.119]
Mode	[0.654, 1.516]	[0.713, 1.645]	[0.864, 1.787]	[0.953, 1.930]	[1.425, 2.261]
Median	[1, 2]	[1, 2]	[1, 2]	[1, 2]	[2.00, 2.750]
Skewness	[0.197, 0.239]	[0.167, 0.216]	[0.129, 0.145]	[0.073, 0.084]	[0.167, 0.216]
Kurtosis	[1.417, 1.472]	[1.386, 1.441]	[1.353, 1.366]	[1.321, 1.326]	[1.386, 1.441]

Table 2 Simulated average neutrosophic biases and MSEs of parameters when $\sigma_N \in [1,2]$ and $\beta_N \in [2.25, 2.75]$

n	NAB		NMSE	
	σ_N	β_N	σ_N	β_N
20	[0.0150, 0.0200]	[0.1641, 0.2005]	[0.0313, 0.0824]	[0.2593, 0.3873]
30	[0.0105, 0.0143]	[0.1057, 0.1292]	[0.0207, 0.0548]	[0.1515, 0.2263]
40	[0.0080, 0.0109]	[0.0777, 0.0950]	[0.0152, 0.0405]	[0.1033, 0.1544]
50	[0.0065, 0.0089]	[0.0642, 0.0784]	[0.0121, 0.0323]	[0.0821, 0.1226]
100	[0.0032, 0.0043]	[0.0327, 0.0399]	[0.0061, 0.0162]	[0.0384, 0.0574]
150	[0.0023, 0.0031]	[0.0213, 0.0260]	[0.0039, 0.0105]	[0.0253, 0.0379]
200	[0.0018, 0.0025]	[0.0163, 0.0200]	[0.0030, 0.0079]	[0.0186, 0.0278]
250	[0.0015, 0.0021]	[0.0138, 0.0168]	[0.0023, 0.0063]	[0.0150, 0.0224]
300	[0.0013, 0.0019]	[0.0115, 0.0140]	[0.0020, 0.0053]	[0.0123, 0.0184]

Table 3 Simulated average neutrosophic biases and MSEs of parameters when $\sigma_N \in [1,2]$ and $\beta_N \in [3.75, 4.25]$

n	NAB		NMSE	
	σ_N	β_N	σ_N	β_N
20	[0.0054, 0.0084]	[0.2735, 0.3099]	[0.0109, 0.0339]	[0.7202, 0.9250]
30	[0.0039, 0.0061]	[0.1762, 0.1997]	[0.0073, 0.0226]	[0.4207, 0.5404]
40	[0.0030, 0.0048]	[0.1295, 0.1467]	[0.0054, 0.0168]	[0.2871, 0.3687]
50	[0.0025, 0.0039]	[0.1069, 0.1212]	[0.0043, 0.0134]	[0.2280, 0.2929]
100	[0.0012, 0.0019]	[0.0545, 0.0617]	[0.0022, 0.0067]	[0.1067, 0.1371]
150	[0.0009, 0.0014]	[0.0354, 0.0402]	[0.0014, 0.0044]	[0.0704, 0.0904]
200	[0.0007, 0.0011]	[0.0272, 0.0309]	[0.0011, 0.0033]	[0.0517, 0.0664]
250	[0.0006, 0.0010]	[0.0229, 0.0260]	[0.0008, 0.0026]	[0.0416, 0.0534]
300	[0.0006, 0.0009]	[0.0191, 0.0217]	[0.0007, 0.0022]	[0.0342, 0.0439]

Table 4 Simulated average neutrosophic biases and MSEs of parameters when $\sigma_N \in [2.0, 2.75]$ and $\beta_N \in [2.5, 3.25]$

n	NAB		NMSE	
	σ_N	β_N	σ_N	β_N
20	[0.0243, 0.0197]	[0.1823, 0.2370]	[0.1003, 0.1105]	[0.3201, 0.5409]
30	[0.0172, 0.0142]	[0.1175, 0.1527]	[0.0666, 0.0736]	[0.1870, 0.3160]
40	[0.0131, 0.0109]	[0.0863, 0.1122]	[0.0492, 0.0546]	[0.1276, 0.2156]
50	[0.0106, 0.0089]	[0.0713, 0.0927]	[0.0391, 0.0435]	[0.1014, 0.1713]
100	[0.0052, 0.0043]	[0.0363, 0.0472]	[0.0196, 0.0218]	[0.0474, 0.0802]
150	[0.0037, 0.0032]	[0.0236, 0.0307]	[0.0127, 0.0142]	[0.0313, 0.0529]
200	[0.0029, 0.0025]	[0.0182, 0.0236]	[0.0096, 0.0107]	[0.0230, 0.0388]
250	[0.0025, 0.0022]	[0.0153, 0.0199]	[0.0076, 0.0085]	[0.0185, 0.0312]
300	[0.0022, 0.0019]	[0.0128, 0.0166]	[0.0064, 0.0071]	[0.0152, 0.0257]

Table 5 The neutrosophic parametric estimation and goodness of fit for alloy melting points data

Model	Parameter	MLE	2LL	AIC	BIC	KS	
						D-value	p-value
NLLD	σ_N	[492.696, 512.952]	**[164.287, 165.162]**	**[168.287, 169.162]**	**[170.067, 170.943]**	**[0.101, 0.117]**	**[0.942, 0.984]**
	β_N	[36.418, 38.934]					
NWD	Scale α_N	[505.091, 525.665]	[167.005, 167.536]	[171.001, 171.533	[172.782, 173.317]	[0.146, 0.115]	[0.789, 0.950]
	Shape β_N	[21.267, 22.494]					
NED	σ_N	[461.958, 491.642]	[259.349, 260.784]	[261.349, 262.784]	[262.239, 263.674]	[0.6156, 0.62181]	[3.032e−07, 4.289e−07]

$$\Phi_{X_N}(t) = E\left(e^{itx_N}\right) = \int_0^\infty e^{itx_N} f(x_N; \beta_N, \sigma_N)\ dx_N$$
$$= \int_0^\infty e^{itx_N} \frac{\beta_N}{\sigma_N} \frac{(x_N/\sigma_N)^{\beta_N - 1}}{\left[1 + (x_N/\sigma_N)^{\beta_N}\right]^2} dx_N$$
$$= \left[\int_0^\infty e^{itx_L} \frac{\beta_L}{\sigma_L} \frac{(x_L/\sigma_L)^{\beta_L - 1}}{\left[1 + (x_L/\sigma_N)^{\beta_L}\right]^2} dx_L, \int_0^\infty e^{itx_U} \frac{\beta_U}{\sigma_U} \frac{(x_U/\sigma_U)^{\beta_U - 1}}{\left[1 + (x_U/\sigma_U)^{\beta_U}\right]^2} dx_U\right]$$
$$= \left[\sum_{k=0}^{\infty} \frac{(it)^k}{k!} \int_0^\infty x_L^k \frac{\beta_L}{\sigma_L} \frac{(x_L/\sigma_L)^{\beta_L - 1}}{\left[1 + (x_L/\sigma_N)^{\beta_L}\right]^2} dx_L, \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \int_0^\infty x_U^k \frac{\beta_U}{\sigma_U} \frac{(x_U/\sigma_U)^{\beta_U - 1}}{\left[1 + (x_U/\sigma_U)^{\beta_U}\right]^2} dx_U\right]. \quad (11)$$

After simplification, the Eq. (11) becomes

$$\phi_{X_N}(t) = \left[\sum_{k=0}^{\infty} \frac{(it)^k}{k!} \sigma_L^k \int_0^1 z_L^{(1-k/\beta_L)-1} (1 - z_L)^{(1+k/\beta_L)-1} dz_L, \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \sigma_U^k \int_0^1 z_U^{(1-k/\beta_U)-1} (1 - z_U)^{(1+k/\beta_U)-1} dz_U\right]$$

where $z_L = \left[1 + (x_L/\sigma_L)^{\beta_L}\right]^{-1}$ and $z_U = \left[1 + (x_U/\sigma_U)^{\beta_U}\right]^{-1}$.

$$= \left[\sum_{k=0}^{\infty} \frac{(it\sigma_L)^k}{k!} \mathrm{B}(1 - k/\beta_L, 1 + k/\beta_L), \sum_{k=0}^{\infty} \frac{(it\sigma_L)^k}{k!} \mathrm{B}(1 - k/\beta_U, 1 + k/\beta_U)\right].$$

$$\text{Hence, } \Phi_{X_N}(t) = \left[\sum_{k=0}^{\infty}\frac{(it\sigma_L)^k}{k!}\Gamma(1-k/\beta_L)\Gamma(1+k/\beta_L), \sum_{k=0}^{\infty}\frac{(it\sigma_U)^k}{k!}\Gamma(1-k/\beta_U)\Gamma(1+k/\beta_U)\right]. \quad (12)$$

Theorem 4 *The mode of NLLD is* $\sigma_N\left(\frac{\beta_N-1}{\beta_N+1}\right)^{1/\beta_N}$.

Proof By definition mode is the value of the variate which obtained on solving $\frac{\partial f(x_N)}{\partial x_N} = 0$ *and* $\frac{\partial^2 f(x_N)}{\partial x_N^2} < 0$.

By taking logarithm on both sides of Eq. (1), we get

$$\log(f(x_N)) = \log\left(\frac{\beta_N}{\sigma_N}\right) + (\beta_N - 1)\log(x_N/\sigma_N) - 2\log\left(1 + (x_N/\sigma_N)^{\beta_N}\right)$$

$$\Rightarrow \frac{\partial f(x_N)}{\partial x_N} = \left\{\frac{(\beta_N - 1)}{x_N} - \frac{2\beta_N}{\sigma_N}\frac{(x_N/\sigma_N)^{\beta_N-1}}{1+(x_N/\sigma_N)^{\beta_N}}\right\}f(x_N). \quad (13)$$

$$\frac{\partial f(x_N)}{\partial x_N} = 0 \Rightarrow \left\{\frac{(\beta_N - 1)}{x_N} - \frac{2\beta_N}{\sigma_N}\frac{(x_N/\sigma_N)^{\beta_N-1}}{1+(x_N/\sigma_N)^{\beta_N}}\right\} = 0$$

$$\Rightarrow \frac{(\beta_N - 1)}{x_N} = \frac{2\beta_N}{\sigma_N}\frac{(x_N/\sigma_N)^{\beta_N-1}}{1+(x_N/\sigma_N)^{\beta_N}}$$

$$\Rightarrow \frac{(\beta_N - 1)}{2\beta_N} = \frac{(x_N/\sigma_N)^{\beta_N}}{1+(x_N/\sigma_N)^{\beta_N}}$$

$$\Rightarrow x_N = \sigma_N\left(\frac{\beta_N - 1}{\beta_N + 1}\right)^{1/\beta_N}. \quad (14)$$

On differentiating Eq. (13), we can easily shown that $\frac{\partial^2 f(x_N)}{\partial x_N^2}|_{x_N} < 0$.

Hence, the mode is $\sigma_N\left(\frac{\beta_N-1}{\beta_N+1}\right)^{1/\beta_N}$ where $\sigma_N \in [\sigma_L, \sigma_U]$ *and* $\beta_N \in [\beta_L, \beta_U]$.

3.1 Quantile Function

A quantile function, another useful statistical characteristic of NLLD, is crucial to the Monte Carlo simulation method. To produce the random numbers for the probability distribution model, this function is also helpful. The formula for the NLLD quantile function is

$$Q_N(p) = F_N^{-1}(p) = \sigma_N\left(\frac{p}{1-p}\right)^{1/\beta_N} \text{ where } \sigma_N \in [\sigma_L, \sigma_U] \textit{ and } \beta_N \in [\beta_L, \beta_U]. \quad (15)$$

When $p = 0.5$, the median of the NLLD is σ_N.

3.2 Measures of Skewness and Kurtosis Based on Quantile Function

The quantitative measure of skewness and Kurtosis based on quantile function is defined by [12] and [20], respectively. The following formula is used to determine the neutrosophic skewness and kurtosis of NLLD under neutrosophic environment.

$$\text{Skewness } Sk_N = \frac{Q_N(6/8) - 2Q_N(4/8) + Q_N(2/8)}{Q_N(6/8) - Q_N(2/8)}. \quad (16)$$

$$\text{Kurtosis } Ku_N = \frac{Q_N(7/8) - Q_N(5/8) + Q_N(3/8) - Q_N(1/8)}{Q_N(6/8) - Q_N(2/8)}. \quad (17)$$

For different neutrosophic scale and shape parameters, Table 1 shows the neutrosophic mean, neutrosophic variance, neutrosophic mode, neutrosophic median, neutrosophic skewness, and neutrosophic kurtosis. According to Table 1's findings, for given neutrosophic scale parameters, various statistic values fall when neutrosophic share parametric values grow.

4 Estimation of Parameters

The neutrosophic parameters of the developed NLLD are obtained by using the maximum likelihood estimation (MLE) technique. Suppose $X_{N1}, X_{N2}, \ldots, X_{Nn}$ be a neutrosophic random sample of NLLD then the log-likelihood function could be expressed as:

$$l(\sigma_N, \beta_N) = n\ln(\beta_N) - n\ln(\sigma_N) + (\beta_N - 1)\sum_{i=1}^{n}\ln(x_{Ni}/\sigma_N) - 2\sum_{i=1}^{n}\ln\left[1 + (x_{Ni}/\sigma_N)^{\beta_N}\right]. \quad (18)$$

$$\frac{\partial l(\sigma_N, \beta_N)}{\partial \sigma_N} = 0 \Rightarrow -\frac{n}{\sigma_N} - \frac{n(\beta_N - 1)}{\sigma_N} + \frac{2\beta_N}{\sigma_N}\sum_{i=1}^{n}\frac{(x_{Ni}/\sigma_N)^{\beta_N}}{\left(1 + (x_{Ni}/\sigma_N)^{\beta_N}\right)} = 0 \quad (19)$$

$$\frac{\partial l(\sigma_N,\beta_N)}{\partial \beta_N}=0$$
$$\Rightarrow \frac{n}{\beta_N}+\sum_{i=1}^{n}\ln(x_{Ni}/\sigma_N) - 2\sum_{i=1}^{n}\frac{(x_{Ni}/\sigma_N)^{\beta_N}\ln(x_{Ni}/\sigma_N)}{\left(1+(x_{Ni}/\sigma_N)^{\beta_N}\right)}$$
$$=0. \qquad (20)$$

The MLEs of (σ_N,β_N) say $\left(\hat{\sigma}_N,\hat{\beta}_N\right)$ where $\sigma_N \in [\sigma_L,\sigma_U]$ *and* $\beta_N \in [\beta_L,\beta_U]$ are obtained on solving the Eqs. (19) and (20).

5 Simulation

In this section, a simulation study is carried out to examine the effectiveness of neutrosophic MLEs of the neutrosophic parameters σ_N and β_N of the proposed NLLD. For simulation, a random sample of sizes, n = 20, 30, 40, 50, 100, 150, 200, 250 and 300 are generated from NLLD with various amalgams of neutrosophic parameters. Using simulated data, estimated MLEs of the neutrosophic parameters of various sample sizes for 5000 replications. Hence, the neutrosophic average of the estimators, the neutrosophic average bias (NAB) and neutrosophic Mean Square Error (NMSE) are obtained for all sample sizes. The estimates of NAB and NMSE are employed to assess the characteristics of the superior neutrosophic estimator. In Table 2, the simulated results for $\sigma_N \in [1,2]$ and $\beta_N \in [2.25,2.75]$ presented. In Table 3, the simulated results for $\sigma_N \in [1,2]$ and $\beta_N \in [3.75,4.25]$ presented. In Table 4, the simulated results for $\sigma_N \in [2.0,2.75]$ and $\beta_N \in [2.5,3.25]$ presented. In Tables 2, 3 and 4, the simulated NAB and NMSE are reported. From Tables 2, 3 and 4, it is noticed that as sample sizes increase the NAB and NMSE are decreases for both neutrosophic parameters as expected. Furthermore, when neutrosophic shape parameter increases from $\beta_N \in [2.25,2.75]$ to $\beta_N \in [3.75,4.25]$ at fixed neutrosophic scale parameter $\sigma_N \in [1,2]$ the NAB and NMSE are decreases across all sample sizes. Also, same phenomenon is observed when neutrosophic scale parameter increases.

6 Industrial Utilization

Comparing the proposed novel neutrosophic probability distribution to existing neutrosophic probability distributions based on data set, the current part shows how useful and thin it is. The well thought-out data set is regarding the alloy melting points data taken from [9] and these data were first used by [15]. A combination of material constituents, including at least one metal, makes up an alloy. These alloys might have characteristics that make them beneficial for being distinct from pure metals and useful for increasing strength or hardness while also lowering the cost of the material. Examples of alloys include red gold, which is composed of a copper and gold alloy, white gold, which is composed of a silver and gold alloy, etc. The data on alloy melting points are frequently taken from a distribution with a set of aggregate melting points by manufacturing engineers involved in the production of bimetals. In general, evaluating melting points is quite challenging, therefore observations are indeterministic and can be communicated in intervals. For ready reference, the 18 uncertain data observations of melting points of alloys are as follows:

[563.3, 545.5], [529.4, 511.6], [523.1, 503.5], [470.1, 449.2], [506.7, 489.0], [495.6, 479.1], [495.3, 467.9], [520.9, 495.6], [496.9, 472.8], [542.9, 519.1], [505.4, 484.0], [550.7, 525.9], [517.7, 500.9], [499.2, 483.0], [500.6, 480.0], [516.8, 499.6], [535.0, 515.1], [489.3, 464.4].

The model adequacy of the proposed NLLD is compared with neutrosophic Weibull distribution (NWD) developed by [6] and neutrosophic exponential distribution (NED) applications for complex data analysis studied by [10]. The decision about the best model fit would be considered by means of the following criterion selection measures: log-likelihood value (*LL*), Akaike Information Criteria (AIC), Bayesian information criteria (BIC) and Kolmogorov–Smirnov (KS) test. The criterion for best fitted model is that highest values of $2LL$ and smallest values of AIC, BIC, KS statistic. Also, for larger p-value indicates that the best fitted model for the neutrosophic data. The neutrosophic maximum likelihood estimators and model adequacy measures are given in Table 5. The results indicate that the developed NLLD is more efficient for melting points of alloys data than the contemporary NWD and NED. The bold values in the table shows that the efficiency of the proposed model.

7 Conclusion

A new generalized version of distribution namely NLLD has been recommended in this paper. This developed model is helpful in various survival and reliability engineering data for indeterminacies. The chief statistical properties of the developed NLLD such as neutrosophic survival function, neutrosophic hazard rate, neutrosophic moments, and neutrosophic mean time failure have been discussed. The neutrosophic MLEs have been formulated and depicted neutrosophic average bias and MSEs for various sample sizes. Additionally, neutrosophic random sample concept is established and hence capable of use authenticate the systematic outcome of the developed NLLD. The simulation study has been conducted to investigate achievement of the computed neutrosophic parameters. The outcome from simulations depict that the sample size and neutrosophic parametric value play a vital role in perfectly estimating an unsuspected parameter. The melting point of alloy material usage also confirms the validity of the NLLD in applied material science under neutrosophic cases. It is considered that by augmentation of the capacity of NLLD in stress-strength reliability studies and quality control studies, also broadened for further neutrosophic distributions.

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