International Journal of Computational Intelligence Systems (2023) 16:21
https://doi.org/10.1007/s44196-023-00195-9

RESEARCH ARTICLE

Fuzzy-MACBETH Hybrid Method: Mathematical Treatment of a Qualitative Scale Using the Fuzzy Theory

Tatiane Roldão Bastos[1] · **André Andrade Longaray**[1] · **Catia Maria dos Santos Machado**[1] · **Leonardo Ensslin**[2] · **Sandra Rolim Ensslin**[3] · **Ademar Dutra**[2]

Received: 5 October 2022 / Accepted: 30 January 2023

Abstract

This paper describes the research procedures adopted in developing a triangular fuzzy number scale based on the semantic scale of MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique). The objective was to mathematically treat the uncertainty and subjectivity of linguistic variables used to assess a decision problem. A matrix was initially obtained based on a decision maker's assessment of a given context analysis. This decision matrix was then fuzzified based on a triangular Fuzzy numbers scale. Next, the inference process was performed using F-LP-MACBETH linear programming problem proposed here, resulting in a Fuzzy scale. This scale was then defuzzified using the centroid method, from which a crisp basic scale emerged, which was then cardinalized. The results show that the MACBETH Fuzzy method proposed here can overcome the classical method's cardinal inconsistency problem, which facilitates its application in complex contexts. Hence, the MACBETH Fuzzy Hybrid method generated numerical values based on the decision makers' semantically consistent assessments in a decision matrix, which by the classical method presents cardinal inconsistency. Therefore, the advantage of the proposed method consists in the possibility of obtaining a cardinal scale aligned to the decision makers' preferences without the need to reassess the context.

Keywords MACBETH method · Semantic scale · Cardinal scale · Fuzzy logic · Fuzzy-MACBETH

Abbreviations

ANP	Analytic network process
AHP	Analytic hierarchy process
BWM	Best–worst method
DSS	Decision support systems
ELECTRE	Elimination and choice translating reality
ERP	Enterprise resource planning
FITradeoff	Flexible and interactive tradeoff
F-LP-MACBETH	Fuzzy linear programming of measuring attractiveness by a categorical based evaluation technique
ICW	Interval criterion weights
LP-MACBETH	Linear programming of measuring attractiveness by a categorical based evaluation technique
MACBETH	Measuring attractiveness by a categorical based evaluation technique
MAUT/UTA	Multi-attribute utility theory/additive utility
MCDA	Multi-criteria decision aid
MCDM	Multi-criteria decision-making
MCDM/A	Multi-criteria decision-making/aid
MODM	Multi-objective decision-making
MOLP	Multi-objective linear programming
PRISMA	Preferred reporting items for systematic reviews and meta-analyses
PROMETHEE	Preference ranking organization method for enrichment evaluation
SMART	Simple multi-attribute rate technique
STEM	Step method

✉ André Andrade Longaray
andrelongaray@gmail.com

1 Universidade Federal do Rio Grande—FURG, Campus Carreiros, Rio Grande, Rio Grande do Sul CEP 96.203-900, Brazil

2 Programa de Pós-Graduação em Administração, Universidade do Sul de Santa Catarina—UNISUL, Florianópolis, Santa Catarina CEP 88.010-010, Brazil

3 Programa de Pós-Graduação em Engenharia de Produção, Universidade Federal de Santa Catarina—UFSC, Florianópolis, Santa Catarina CEP 88.010-970, Brazil

Published online: 26 February 2023

Supp	Support
TACTIC	Treatment of the alternatives according to the importance of criteria
TODIM	Multi-criteria interactive decision-making
TOPSIS	Technique for order preference by similarity to ideal solution
TRIMAP	Tricriterion multi-objective linear programming
VIKOR	Multi-criteria optimization and compromise solution

1 Introduction

Decision-making permeates people's daily lives as we are constantly making choices. Most decisions are routine and intuitive and do not require further analysis. However, many of the variables involved in complex contexts are subjective and demand trade-offs, requiring individuals to make subjective judgments. As noted by [1], when a decision task is complex, or the impact of actions is relevant, one should resort to a formal or informal decision support technique.

Multi-criteria techniques stand out among formal techniques supporting decision-making. Even though it is sometimes more convenient to consider a single objective and make decisions based on its optimization subject to a set of restrictions, in most cases, multiple objectives or criteria affecting a given context should be considered, as reality is multidimensional [2].

Hence, the various factors composing a decision problem should be modeled based on facts, values, scientific knowledge, and human judgment, which, when considered together, enable a broader view and improved understanding of the potential impact of actions on the context under analysis. Many formulas can be used to combine these factors [1]. A method is chosen among the various multi-criteria techniques based on various factors, such as the type of variables composing a given problem, the nature of such a decision problem, and the compensation of criteria.

In this context, MCDM (Multi-criteria Decision Making) and MCDA (Multi-criteria Decision Aid) are two criteria decision paradigms that, despite epistemological differences, support decisions and gather important tools to structure and assess complex decisions [3]. Furthermore, MCDM/A techniques enable the development of assessment systems based on experts' or decision makers' knowledge, with mathematical procedures and advanced computational methods that support decision-making modeling and solving decision problems [4].

According to [5, 6], Decision Support Systems (DSS) enable the development and use of computational tools to support managers when making decisions. The use of support systems was driven by the advancement of information technologies such as personal computers and communication networks. Therefore, informatics enables computational coding of complex algorithms and the creation of applications and software, able to operationalize multi-criteria methods quickly and easily. Hence, methods that were previously unfeasible due to a significant level of complexity can currently be used with the support of DSS.

A systematic literature review was performed using the simplified PRISMA Protocol (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) to better understand how meta-heuristic-based DSS is used to support multi-criteria decision-making. It resulted in a Bibliographic Portfolio of 54 papers, and a meta-synthesis of these studies revealed that Fuzzy Logic is applied to treat subjectivity by supporting decision-making in complex environments.

The subjectivity permeating decision-making contexts is a concern. Some methods of the Electre Family, for instance, take into account the subjectivity of judges when making assessments by incorporating the credibility index $\sigma(a,b)$, which conceptually resembles Fuzzy sets, where $\sigma(a,b)$ values between 0 and 1 are assigned with gradual agreement and disagreement thresholds. This overclassification in Electre III is also used in Electre TRI-nB, in which the categories are defined by limiting profiles [7].

The literature describes several methods for dealing with uncertainty, among which the intuitionistic fuzzy theory [8], the neutrosophic set theory [9], and the grey number theory [10] stand out. A linguistic metric for Consensus Reaching Processes (LiCRPs) is proposed by [11] for group decision-making. This metric is built upon a Comprehensive Minimum Cost Consensus (CMCC) model based on ELICIT (Extended Comparative Linguistic Expressions with Symbolic Translation) that models decision makers' uncertainty through a Fuzzy 2-Tuple model.

Comparative Linguistic Expression Preference Relations (CLEPR) are presented by [12] to represent decision makers' uncertainty in group decisions, considering multiple self-confidence levels and presenting numerical scales of linguistic terms. In turn, [13] present an FMEA (Failure Modes and Effects Analysis) approach based on Personalized Individual Semantics, with linguistic distribution assessments and incomplete preference information used to model decision makers' uncertain opinions.

Among these, the Fuzzy sets theory prevailed in the papers included in the bibliographic portfolio to deal with the uncertainty and subjectivity permeating the decision-making process, allowing decision makers to include their doubts in the assessments.

Fuzzy logic and its extensions have also been used to incorporate decision makers' assessments into group decision-making as they enable reaching a common solution in complex environments. For example, the multi-criteria decision model based on Hesitant Fuzzy linguistic term sets presented by [14] shows the ability to incorporate experts' preferences when they hesitate between several linguistic terms, allowing group evaluation in qualitative environments. In the context of Large-Scale Group Decision Making (LSGDM), [6] reveals that multi-criteria methods for classical group decision-making, such as AHP, TOPSIS, MULTIMOORA, and ELECTRE, have been extended to solve LSGDM problems. In this sense, [15] applies Fuzzy preference relations for LSGDM, considering the level of agreement among experts before selecting the best alternative.

Thus, considering that the differential of the Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH) is that it supports multi-criteria decisions in complex environments, facilitating decision-making assessment by using qualitative scales in the analyses, the use of fuzzy numbers to include uncertainty aspects to the scale can improve the method.

According to [16], MACBETH has not yet been expanded to its fuzzy versions, which is corroborated by the literature review and reveals a research opportunity. MACBETH presents a collaborative value-modeling framework in which the context structuring is performed through a socio-technical process, while the value functions for measuring the decision problem are obtained through a linear programming problem based on qualitative assessments [17]. Therefore, the method's main characteristic is to help decision makers to build knowledge about a decision-making context, enabling assessment through uncertain terms that resemble Fuzzy linguistic terms.

Although MACBETH is intended to assess decision problems through qualitative scales, facilitating decision makers' assessments, especially in complex contexts, traditional mathematical techniques do not allow the incorporation of uncertainties arising from linguistic terms. Therefore, the Fuzzy Theory enables the mathematical treatment of such linguistic terms, improving the method.

In this context, this study aims to apply a triangular Fuzzy number in the MACBETH scale of semantic judgments to incorporate human judgment's aspects, such as uncertainty and subjectivity in the scale of linguistic terms. The mathematical modeling proposed here is expected to contribute to the improvement of the classical method by making the scale more flexible in evaluations considered to present cardinal inconsistency.

2 MCDM/A Methods

MCDM/A is the field of Operational Research intended to build evaluation systems based on the knowledge of experts or decision makers, considering multiple criteria that affect the decision-making problem, constituting important tools for structuring and evaluating complex decisions [3]. Choosing among the various MCDM/A methodologies depend on various factors, such as the type of variables that compose the problem, the nature of the decision problem, and compensation criteria.

Table 1 summarizes the classification of methods in the three multi-criteria approaches, according to what [18, 19] proposed. Regarding the matter of discrete variables, the decision problem consists of choosing one from a discrete set of alternatives, in contrast to continuous methods, where the alternatives are implicitly described through a set of restrictions [20].

As for the nature of the decision problem, it can be classified as a choice problem ($P.\propto$), which consists of helping decision makers choose or select a subset that contains the best solutions; classification problem ($P.\beta$), the objective of which is to categorize actions distributed according to pre-defined categories; ordering problem ($P.\gamma$), in which actions are ordered according to the decision makers' order of preference, or a description problem ($P.\delta$), which consists of aiding decision makers to formally and systematically describe actions and their consequences or to develop a cognitive procedure [18].

Finally, regarding the compensation approach, the criteria presenting inferior performance can be compensated by other criteria with good performance. For example, an alternative with higher costs can be considered a good alternative if it presents superior quality and durability, though, in non-compensatory strategies, poor performance cannot be counterbalanced by good ones [20].

As for preference relationships, the methods can be aggregation, outranking, or interactive. The aggregation methods are based on the Single Synthesis Criterion or the Multi-attribute Utility Theory. They are based on the concept of the utility function, that is, how much a given action provides utility to decision makers in the criterion assessed. In the outranking approach, the methods are called outranking or subordination, in which action a is considered to subordinate action b ($a\ S\ b$); a case in which the decision maker considers action a at least as good as action b, proceeding a pairwise comparison through a binary relation, through thresholds.

In addition to the aggregation and outranking methods, interactive methods also adopt the interactive local judgment approach. Also known as Multi-Objective Decision Making (MODM), they originated in mathematical programming, in

Table 1 Synthesis of multi-criteria methods

Approach	Method	Types of problem	Examples of applications
Aggregation Methods Based on Single Synthesis Criterion or Multi-attribute Utility Theory Assumes a multi-attribute utility function or value function to assess the decision maker's preferences It assumes that the relations of preference and indifference are transitive All alternatives are comparable due to the standardization process Performs an analytical aggregation to establish a score for each alternative, allowing comparisons between alternatives Aggregates into a global assessment Aggregation methods Compensatory methods	AHP ANP SMART TODIM TOPSIS MACBETH MAUT/UTA FITradeoff	α, γ, β (AHP-Sort) α, γ α, γ α, γ α, γ α, γ α, γ, β (UTA-DIS) α, β	[21–24] [25–27] [28, 29] [30, 31] [32–34] [17, 35, 36] [37, 38] [39, 40]
Outranking methods Based on the outranking approach It does not require that the relations of preference and indifference be transitive Allows incomparability between alternatives, using an overclassification between alternatives that are not transitive The criteria weights assume the notion of degree of importance, as there is no transformation of scales in the intra-criterion assessment, in which a global score is assigned to each alternative Allows aggregation into a global assessment Constructivist approach Non-compensatory methods	ELECTRE FAMILY PROMETHEE FAMILY TACTIC REGIME	α, γ, β α, β (Flow-Sort), γ (GAIA) α, γ α, γ	[41–44] [44–46] [47] [48, 49]
Interactive methods Origin in Multi-objective Mathematical Programming It combines computational steps and interaction with the decision maker Does not allow aggregation into a global assessment It seeks to optimize more than one objective function simultaneously	STEM ICW PARETO RACE TRIMAP	α, γ α α, γ α, γ	[50–52] [53] [54, 55] [56, 57]

which Multi-Objective Linear Programming (MOLP) stands out.

Therefore, Operational Research, developed from several related fields of knowledge, gave rise to methods based on different decision-making paradigms for solving problems in different contexts. Thus, the choice of a method to solve a problem or support decision-making depends on the characteristics of the decision-making problem, the context under analysis, and the decision makers' profiles. Keep in mind that the focus of the decision-making process can be optimal or near-optimal or to build the decision maker's knowledge.

The literature shows that MCDM/A methods are used in different contexts. A ranking range based on the MCDM/A approach under an incomplete context is proposed by [58]. The minimum and maximum rankings of each alternative of the multi-attribute decision matrix with incomplete weight information are obtained by a series of mixed 0–1 linear programming models. To obtain the average position of each alternative in the ranking, the Monte Carlo Simulation is applied, and each alternative's minimum, maximum, and mean ranking information is integrated. The model is then applied to a case of assessing risk investment with risk attitudes.

In [59], proposes a hybrid method based on the grey theory, genetic algorithm, and dematel (GA-GDEMATEL) for human resources management. In turn, [60] present the identification of the optimal conceptual design through the hybridization of two MCDM/A models, the Fuzzy Analytical Hierarchy Process (F-AHP) and Fuzzy Grey Relational Analysis (F-GRA). The Fuzzy-Delphi method is used by [61] to refine supplier evaluation criteria, determine the weight of criteria and prioritize suppliers based on green and resilient indexes. The Grey and TOPSIS (GC-TOPSIS) correlation methods are used to analyze the results.

In [62] propose an integrated and comprehensive fuzzy multi-criteria model for supplier selection in the digital supply chain. The fuzzy MULTIMOORA, fuzzy COPRAS, and fuzzy TOPSIS methods are used as prioritization methods to sort suppliers. The maximize agreement heuristic (MAH) method aggregates the supplier rankings obtained from the prioritization methods into a consensus ranking.

The task of personnel selection is modeled by [63] as a multi-criteria decision-making problem, considering several competencies and abilities to assess candidates for a specific position. The Electre III method is built into the software to develop a fuzzy outranking relation, while a multi-objective evolutionary algorithm explores this relationship and generates a ranking as a recommendation. In [64] develops a hybrid method combining MCDM decision-making with a fuzzy method called COMET (Characteristic Objects Method).

A new integrated decision-making model is developed by [65] based on MACBETH to calculate criteria weights. Additionally, Weighted Aggregated Sum Product Assessment (WASPAS) methods under a fuzzy environment with Dombi norms were intended to assess alternatives to solve a selection problem concerning the recovery center location, considering technical, environmental, economic, and social aspects.

The literature presents the studies implementing Fuzzy-MACBETH; although Table 2 does not present an exhaustive list, it summarizes the most representative ones.

2.1 MACBETH Method

Among the multi-criteria methods used to support decision-making, MACBETH, created by Bana e Costa and Vansnick [72], enables transforming ordinal scales, obtained from value judgments expressed by a judge, into cardinal scales using linear programming. It consists of a multi-criteria mathematical method composed of four phases, which begin with an analysis of the decision-making context and structuring of the problem, establishment of the evaluation elements, development of the evaluation model, and analysis of sensitivity and recommendations [73]. Thus, the MACBETH method can be understood as a socio-technical process that combines technical elements with the social aspects of decision conferencing [74]. According to [75], decision conferencing consists of an interactive group technique, i.e., meetings are held with key actors in the decision-making process, who, assisted by a facilitator, arrive at relevant judgments that help them to understand and structure the issues affecting a given organization (Table 3).

MACBETH's central idea is to offer judges a method to assist in the construction of a numerical scale; thus, it is a tool to convert semantic judgments into cardinal scales [72]. Therefore, this method's main differential is the use of a semantic scale to express the decision maker's assessment, to compare pairs of alternatives, using the categories *indifferent, very weak, weak, moderate, strong, very strong,* and *extreme.* The questioning procedure requires decision makers to assign an absolute verbal judgment about the difference in attractiveness between two alternatives or actions, x and y, for each ordered pair (x,y) in $A \times A$ with xPy (x preferable to y).

Where $A = x_1, x_2, \cdots, x_n$ is a set of elements such that $\forall i \neq j \in 1, 2, \cdots, N : x_i P x_j \Leftrightarrow i > j$. Therefore, the elements of set A must be ordered in descending order, that is, $x_n P x_{n-1} P x_{n-2} P \cdots P x_2 P x_1$ and then the upper part of the matrix $n \times n$ is filled with the verbal responses, where $x_{i,j}$ is assigned value $k \in 1, 2, 3, 4, 5, 6$ if, and only if, the decision maker has assigned (x_i, x_j) to a category c_k, according to (1) [76].

$$A_{n\times n} = \begin{bmatrix} . & x_{n,n-1} & x_{n,n-2} & \cdots & x_3 & x_2 & x_1 \\ . & . & x_{n-1,n-2} & \cdots & x_{n-1,3} & x_{n-1,2} & x_{n-1,1} \\ . & . & . & \cdots & x_{n-2,3} & x_{n-2,2} & x_{n-2,1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ . & . & . & \cdots & . & x_{3,2} & x_{3,1} \\ . & . & . & \cdots & . & . & x_{2,1} \\ . & . & . & \cdots & . & . & . \end{bmatrix} \quad (1)$$

where:

- $C_1 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **very weak**;
- $C_2 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **weak**;
- $C_3 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **moderate**;
- $C_4 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **strong**;
- $C_5 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **very strong**;
- $C_6 = \{(x, y) \in A \times A / xPy\}$ and the difference of attractiveness between x and y is **extreme**;

Two conditions must be met for the decision maker's answers be semantically consistent:

- $\forall i = 3, 4, \cdots, n$ values x_{ij} on line i cannot decrease when j decreases; and
- $\forall j = 1, 2, \cdots, n-2$ values x_{ij} in column j cannot increase as i decreases.

Thus, the ordinal attractiveness measure consists of assigning a numerical value—a real number $v(x)$—to each alternative x that satisfies the conditions:

- $\forall x, y \in A : xPy \Leftrightarrow v(x) > v(y)$
- $\forall x, y \in A : xIy \Leftrightarrow v(x) = v(y)$

Table 2 Synthesis of studies presenting Fuzzy-MACBETH

Reference	Method	Approach
[66]	MACBETH and Fuzzy 2-tuple, linguistic model	The analysis was performed using an extended version of the MACBETH methodology to take into account the imprecise and linguistic assessments provided by the decision maker when integrating the 2-tuple model, which deals with non-homogeneous information data. This model objectively assesses the alternatives, exempting decision makers' evaluation. Thus, the alternatives present input data that can be numeric, interval, or linguistic values. Afterward, input data are expressed in Fuzzy sets over a specific linguistic domain, called Basic Linguistic Term Set (BLTS). The resulting information is converted into a numerical value β through a function χ. Afterward, β is translated into a 2-tuple linguistic model $(S\_k, \propto)$. A decision matrix and cardinal scale are obtained with the result of the 2-tuple model using the M-MACBETH software. Finally, the traditional MACBETH method is used to determine the criteria weights, in which the decision maker performs pairwise comparisons according to his/her preference. In this study, uncertain contexts are objectively evaluated by the 2-Tuple model. Its main contribution is the assessment of alternatives without the decision maker's participation. Therefore, the modeling presented in [66] differs from the model proposed here, as the purpose of the hybrid Fuzzy-MACBETH method is to allow that decision makers' preferences to be incorporated through a triangular Fuzzy scale, including the uncertainty of linguistic terms to the mathematical modeling
[67]	MACBETH and AHP-Fuzzy	The authors use two multi-criteria methods based on linguistic evaluations, MACBETH and AHP-Fuzzy, to evaluate the alternatives of wind, solar, biomass, geothermal and hydroelectric renewable energies. The objective was to compare the two approaches. The authors conclude that MACBETH and Fuzzy-AHP resulted in the same ranking order of energy alternatives. Even though the methods are based on different approaches, they allow decision makers to make more confident decisions in selecting an alternative. If the ranking order were different, decision makers would need deeper analysis, examining the sources of differences. The above study differs from the one proposed here, as it does not incorporate the Fuzzy Theory into the MACBETH method
[16]	Literature review	The authors performed a literature review to understand how Fuzzy Set Theory has been used in conjunction with multi-criteria decision-making methods. The results showed a strong tendency to combine Fuzzy with multi-criteria methods, such as Fuzzy-AHP, Fuzzy-ANP, Fuzzy-TOPSIS, Fuzzy-ELECTRE, Fuzzy-PROMETHEE, and Fuzzy-VIKOR. Only two papers used MACBETH though, [66] and [67]. Therefore, the authors conclude that the MACBETH method has not yet been expanded to its Fuzzy versions. The result presented in [16] is aligned with the literature review performed for this study. It shows that applying a fuzzy number scale for mathematically treating qualitative scales of the MACBETH method represents a research opportunity
[68]	MACBETH and Fuzzy type-2	Based on the literature, the authors prepared a list of performance indicators for hub airports and performed prioritization based on the Best–Worst method (BWM). The Fuzzy Type-2 method was used to obtain the criteria weights. Afterward, the five criteria obtaining the highest weights were used to evaluate the 19 Iranian international airports using the MACBETH method. Thus, in the study performed by [68], the Fuzzy Theory was not incorporated into MACBETH to encompass the uncertainties of the context in the decision problem assessment, differing from the present study

Table 2 (continued)

Reference	Method	Approach
[69]	MACBETH, Fuzzy-TOPSIS, and Fuzzy Shannon Entropy	The authors used MACBETH to obtain the weights of subjective criteria based on the decision makers' preferences and Fuzzy Shannon Entropy to obtain the weights of objective criteria. Next, Fuzzy-TOPSIS was used to obtain the classification of alternatives, and finally, PROMETHEE II, ELECTRE III, and Fuzzy-VIKOR were used to validate the method. In this study, they used the Fuzzy Theory jointly with other MCDA/M methods to assess alternatives. MACBETH was used separately to obtain criteria weights rather than being expanded to a Fuzzy version
[70]	MACBETH, Intuitionist Fuzzy Set, and Grey Number	The authors introduce a new approach to select ERP (Enterprise Resource Planning) software based on MACBETH, considering grey numbers and intuitionistic fuzzy sets under uncertainty. First, the semantic judgments in the MACBETH method, based on grey numbers, are transformed into an interval scale through grey numbers, after which whitenization is performed, generating a new semantic judgment that is applied in the classical MACBETH method. The second approach proposed is to establish an intuitionistic fuzzy set to evaluate the effects of decision makers on a research problem, that is, to determine the weight of each decision maker. Therefore, [70] applied a grey number scale to MACBETH semantic scale to generate a new numeric scale for the method. Note that the method proposed by the authors above is not intended to provide a mathematical treatment for semantically consistent and cardinally inconsistent decision matrices, as we propose here
[71]	MACBETH and Fuzzy numbers	The authors propose a method to rank alternatives based on the MACBETH method extended to a Fuzzy version. Initially, the decision matrix *mxn* is assembled with *n* criteria and *m* alternatives. In the first step, the weighted values of each criterion are obtained for each decision maker. Next, the criteria are evaluated based on a triangular fuzzy numbers scale, generating a vector of fuzzy weights for each decision maker. These vectors are aggregated, generating, for each criterion, a single fuzzy weight vector. Next, the alternatives are evaluated using a mathematical formula, generating a crisp number for each alternative. Afterward, the MACBETH fuzzy score is calculated for each alternative, multiplying the value of the alternative by the vector of fuzzy weights for each criterion. The global fuzzy value of each alternative is obtained, and this value is then defuzzified, resulting in a rank. This study's objective is not to obtain cardinal scales for each evaluation element but to provide an aggregation method to rank the alternatives based on Fuzzy-MACBETH modeling. Although [71] present an extension of the MACBETH method to a Fuzzy version, the modeling differs from the one presented here, as it does not use a Fuzzy numbers scale to obtain a cardinal scale but to obtain a ranking of the alternatives being assessed

To determine the cardinal value measure for each alternative x, in addition to the two previous conditions, it is necessary that:

- $\forall w, x, y, z \in A$ with an x more attractive than y and a w more attractive than z, ratio $\left[v(x) - v(y)\right]/[v(w) - v(z)]$ corresponds to the measure of attractiveness between x and y, having the difference in attractiveness between w and z as a unit of measure.

Thus, while the ordinal value measure indicates the preference order of alternatives, the cardinal value measure reflects the difference in attractiveness between the alternatives [74]. The pre-cardinal scale, called basic MACBETH scale, can then be obtained from the linear programming problem LP-MACBETH, presented in [74], with x^+ e x^- being the most attractive and least attractive elements of A, respectively.

LP-MACBETH

$$Min\left[v\left(x^+\right) - v(x^-)\right]$$

Subject to restrictions:

1. $v(x^-) = 0$ (arbitrarily chosen value).
2. $v(x) - v(y) = 0, \forall(x,y) \in C_0$.
3. $v(x) - v(y) \geq 1, \forall(x,y) \in C_i \cup \cdots \cup C_s$, with $i,s \in \{1,2,3,4,5,6\}$ and $i \leq s$.
4. $v(x) - v(y) \geq v(w) - v(z) + i - s', \forall(x,y) \in C_i \cup \cdots \cup C_s$ and $\forall(w,z) \in C'_i \cup \cdots \cup C'_s$ with $i,s,i',s' \in \{1,2,3,4,5,6\}$ and $i \leq s, i' \leq s'$ and $i > s'$.

The basic MACBETH scale is anchored by values $v\left(x^+\right) = 100$ and $v(x^-) = 0$. Hence, MACBETH cardinal scale (μ_x) is obtained from Eq. (2).

$$\mu_x = \alpha v_x + \beta. \tag{2}$$

3 Fuzzy Logic

The Fuzzy set theory was conceived by Lofti A. Zadeh [77] to consider the uncertain aspects of human thought. Differently from randomness, the uncertainty considered by the Fuzzy theory comprises the imprecision of the linguistic variables, which are disregarded in classical mathematical treatment [78]. Hence, imprecise terms such as “small”, “large”, “a lot”, and “a little”, among others, are taken into account in the Fuzzy theory [79].

Including Fuzzy logic in decision-making enables considering ambiguous aspects, uncertainty, or indecision permeating real-world problems [69]. By mathematically formalizing such imprecisions, [77] expanded the image set of the characteristic function of a classic (or crisp) set, in which the Fuzzy subset is characterized by a membership function μ_F. Thus, being U a crisp set, a fuzzy subset is characterized by the membership function (3).

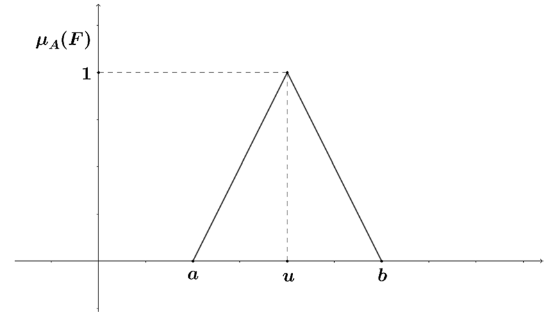

Fig. 1 Triangular fuzzy number

$$\mu_F : U \to [0.1]. \tag{3}$$

This membership function indicates the degree to which element x of set U belongs to subset F, where $\mu_F(x) = 0$ indicates the non-membership of element x to subset F and $\mu_F(x) = 1$ indicates full membership. According to [80], fuzzy numbers are a fuzzy subset of real numbers, representing the expansion of the notion of confidence intervals. Hence, Fuzzy numbers allow the mathematical modeling of the inaccuracy present in complex systems, the main ones being the triangular Fuzzy number, the trapezoidal Fuzzy number, and bell-shaped Fuzzy number; the triangular number is the most frequently found in the literature [81].

Therefore, a Fuzzy number A is said to be triangular if its membership function is of Eq. (4).

$$\mu_A(x) = \begin{cases} \frac{x-a}{u-a}; & if\ a < x \leq u \\ \frac{x-b}{u-b}; & if\ u < x \leq b \\ 0; & otherwise \end{cases} \tag{4}$$

Whose membership function presents a graph in triangular format, with a base $[a,b]$ and vertex $(u,1)$, as shown in Fig. 1. Thus, the triangular fuzzy number is represented by the ordered triplet $(a; u; b)$ or $a/u/b$ and is the closure of the support of A $\overline{suppA}$), corresponding to the α-level [A]^0.

Arithmetic operations with fuzzy numbers are linked to interval arithmetic operations, and the following definitions can be stated, which can be considered particular cases of the Zadeh Extension Principle, where $A_1 = (l_1, m_1, u_1)$ and $A_2 = (l_2, m_2, u_2)$ two fuzzy numbers and λ a real number.

- Sum of Fuzzy numbers: $(l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$
- Multiplication of a real number by a fuzzy number: $(l_1, m_1, u_1) \odot (\lambda, \lambda, \lambda) = (\lambda l_1, \lambda m_1, \lambda u_1)$
- Difference of Fuzzy numbers: $(l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1 - l_2, m_1 - m_2, u_1 - u_2)$
- Multiplication of Fuzzy numbers: $(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)$
- Division of Fuzzy numbers: $(l_1, m_1, u_1) \oslash (l_2, m_2, u_2) = (l_1 \times \frac{1}{l_2}, m_1 \times \frac{1}{m_2}, u_1 \times \frac{1}{u_2})$

Thus, using a Fuzzy number scale, aligned with MACBETH's semantic scale, can include the uncertainty inherent to linguistic terms used to measure differences in attractiveness between elements assessed in the method's mathematical modeling. Uncertainty in the mathematical representation of such terms is related to the meaning they assume for each individual, which does not correspond to an exact value but to a subjective dimension.

The cardinal scales' dimensionality arising from a qualitative assessment based on the decision maker's preferences can be obtained through the Fuzzy Theory by making the idea of number more flexible, adding uncertainty aspects. Thus, this study provides the MACBETH method with a new approach extended to its Fuzzy version.

4 Proposed Method

According to [20], facts, values, science, and human judgment are essential ingredients for decision-making. Hence, the main feature that prompted the choice of the MACBETH method was the use of qualitative scales to assess decision problems, in which the decision maker assigns semantic judgments to ordered pairs of alternatives and criteria. Hence, value functions are obtained based on the decision makers' perceptions and values.

A value function is the mathematical representation of human judgments, which allows an analytical study of preferences and value judgments [20]. Therefore, to obtain the value functions, the alternatives are compared based on the semantic categories of [72], in which the decision maker assigns an absolute verbal judgment on the difference in attractiveness between x and y, for each ordered pair (x,y) in $A \times A$ with xPy.

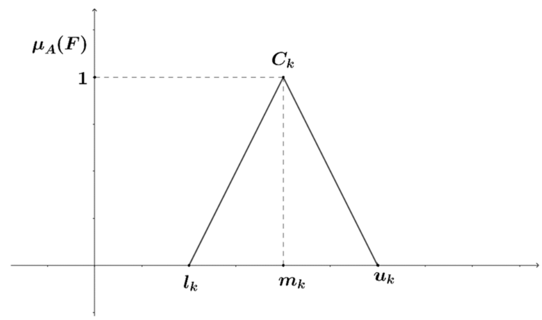

Fig. 2 Triangular fuzzy-MACBETH number

Considering that the objective in the MACBETH method's linear programming problem is to find a numerical value for each alternative that is as close as possible to the center of the interval of the corresponding category, that is, the maximum degree of membership is in the center of the interval, the Fuzzy triangular number was used to build the scale of the Fuzzy-MACBETH method proposed here, as shown in Fig. 2, where $x_{i,j}$ is assigned value $\widetilde{A}$ if and only if the decision maker has assigned (x_i, x_j) to a category c_k, such that:

$$\widetilde{A} = \begin{cases} m_k = k \\ l_k = k - 1 \\ u_k = k + 1 \end{cases} \iff k = \{2, 3, 4, 5\}$$

$$\widetilde{A} = \begin{cases} m_k = l_k = k \\ u_k = k + 1 \end{cases} \iff C_1$$

$$\widetilde{A} = \begin{cases} m_k = u_k = k \\ l_k = k - 1 \end{cases} \iff C_6.$$

Therefore, the resulting scale can be represented as shown in Fig. 3. The semantic scale is fuzzified based on the decision maker's semantic judgments. In other words, the first task of the Fuzzy-MACBETH method proposed here is to determine the differences between the alternatives in terms

Table 3 Scale of the proposed Fuzzy-MACBETH method

C_k	Semantic scale	$\widetilde{A}$
C_0	Indifference between alternatives	(0, 0, 0)
C_1	Difference concerning very weak attractiveness	(1, 1, 2)
C_2	Difference concerning weak attractiveness	(1, 2, 3)
C_3	Difference concerning moderate attractiveness	(2, 3, 4)
C_4	Difference concerning strong attractiveness	(3, 4, 5)
C_5	Difference concerning very strong attractiveness	(4, 5, 6)
C_6	Difference concerning extreme attractiveness	(5, 6, 6)

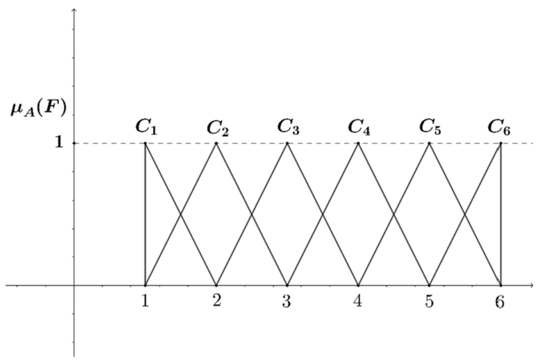

Fig. 3 Graphic scale of the proposed Fuzzy-MACBETH method

of attractiveness, using triangular fuzzy numbers, generating a fuzzy decision matrix.

Having the Fuzzy decision matrix, the basic Fuzzy scale can be obtained through the linear programming problem that we will call F-LP-MACBETH, resulting in a pre-cardinal scale $\tilde{v}$ for each alternative.

F-LP-MACBETH

$$Min\left[\tilde{v}(x^+) - \tilde{v}(x^-)\right]$$

Subject to restrictions:

1. $\tilde{v}(x^-) = (0, 0, 0)$.
2. $\tilde{v}(x) - \tilde{v}(y) = (0, 0, 0), \forall(x, y) \in C_0$.
3. $\tilde{v}(x) - \tilde{v}(y) \geq (1, 1, 2), \forall(x, y) \in C_k$, with $k \in \{1, 2, 3, 4, 5, 6\}$.
4. $\tilde{v}(x) - \tilde{v}(y) \geq \tilde{v}(w) - \tilde{v}(z), \forall(x, y) \in C_k$ and $\forall(w, z) \in C_{k'}$, with $k, k' \in \{1, 2, 3, 4, 5, 6\}$ and $k > k'$.

Thus, in the F-LP-MACBETH linear programming problem, the objective function aims to minimize the largest difference in fuzzy values between the more and less attractive alternatives. In the first restriction, the scale origin is fixed, and the second and third restrictions ensure that the ranking order of the elements is preserved. The fourth restriction, which deals with cardinal inconsistency, was made more flexible using the fuzzy number scale, which has gradual and transposed thresholds. This flexibility becomes viable because, in the Fuzzy Theory, an element can simultaneously belong to a subset and its complement, with a certain degree of membership.

The basic Fuzzy scale obtained is still Fuzzy numbers. Hence, an approach to transforming fuzzy numbers into crisp numbers is needed. These approaches include the mean of the maximum, the centroid method, the α-cut methods, and the maximization and minimization sets (MAX–MIN) [82]. In this study, we chose the centroid method, in which the basic crisp scale is obtained by calculating the centroid of the triangular fuzzy number.

This defuzzified basic scale, however, does not yet correspond to the cardinal scale. For that, the scales are first anchored, assigning *zero* to the level of performance the decision maker considered *neutral* and *one hundred* to the level of performance considered *good*, thus enabling aggregate local assessments. Equation (2) is applied to determine the cardinal scale (μ_x).

5 Results and Discussion

This section presents the results of the Fuzzy-MACBETH method proposed here according to two situations: a consistent ordinal and cardinal decision matrix and an inconsistent cardinal decision matrix.

First Example The first example was developed to show the applicability of the method proposed. The procedure to solve these examples is supported by an electronic spreadsheet. The decision matrix M_1, extracted from [74] was used in the first example. This matrix presented semantic and cardinal consistency.

$$M_1 = \begin{bmatrix} * & (veryweak) & (weak) & (strong) & (strong) & (verystrong) \\ * & * & (weak) & (moderate) & (moderate) & (strong) \\ * & * & * & (moderate) & (moderate) & (moderate) \\ * & * & * & * & (veryweak) & (veryweak) \\ * & * & * & * & * & (veryweak) \\ * & * & * & * & * & * \end{bmatrix}.$$

First, the decision matrix is fuzzified, assigning the values of the α-levels [A]^0 of the triangular Fuzzy-MACBETH scale corresponding to each category the decision maker assigned to the differences in attractiveness between the alternatives, as follows:

$$\tilde{A}_{n\times n} = \begin{bmatrix} * & (1,1,2) & (1,2,3) & (3,4,5) & (3,4,5) & (4,5,6) \\ * & * & (1,2,3) & (2,3,4) & (2,3,4) & (3,4,5) \\ * & * & * & (2,3,4) & (2,3,4) & (2,3,4) \\ * & * & * & * & (1,1,2) & (1,1,2) \\ * & * & * & * & * & (1,1,2) \\ * & * & * & * & * & * \end{bmatrix}.$$

Table 4 presents the results of the Fuzzy-MACBETH method for the first example. Having the basic Fuzzy scale ($\tilde{v}_x$) the defuzzification was performed using the centroid method, obtaining the basic scale (v_x). Finally, the cardinal scales (μ_x) were obtained using Eq. (2).

For comparison purposes, Table 5 shows the results obtained by the method proposed here and the classical MACBETH method.

The cardinal scale obtained with the Fuzzy-MACBETH method is coherent with the scale generated in the classical

Table 4 Results of the Fuzzy-MACBETH method concerning the first example

	x_1	x_2	x_3	x_4	x_5	x_6	$\widetilde{v}_x$	v_x	μ_x
x_1		1,1,2	4,4,7	9,9,15	10,10,17	11,11,19	11,11,19	13.67	112.12
x_2			3,3,5	8,8,13	9,9,15	10,10,17	10,10,17	12.33	100.00
x_3				5,5,8	6,6,10	7,7,12	7,7,12	8.67	66.68
x_4					1,1,2	2,2,4	2,2,4	2.67	12.15
x_5						1,1,2	1,1,2	1.33	0.00
x_6							0,0,0	0.00	– 12.09

MACBETH method. For example, going from level x_5 to x_4 corresponds to a loss of 10 points in the classical method and 12.15 points in the Fuzzy-MACBETH method, while going from x_5 to x_6, corresponds to an improvement of 70 points according to the MACBETH and 66.68 points according to the Fuzzy-MACBETH. It shows that the mathematical modeling is aligned with the source method.

Second Example In the second example, the decision matrix M_2 presented in [76] was chosen. Although this matrix was originally solved by the MACBETH first version, the latest version of the M-MACBETH software was applied and cardinal inconsistency was also found in the most current model. Even though the decision matrix is semantically consistent, cardinal inconsistency was found, so the decision maker was suggested to change the assessment of the difference in attractiveness between alternatives x_1 and x_3 from *Very Strong* to *Strong*.

Table 6 presents the final matrix obtained from the application of the Fuzzy-MACBETH method. Finally, Table 7 presents the results of the Fuzzy-MACBETH method used for the original decision matrix and of the classical MACBETH method, changing the decision maker's value judgments to correct the cardinal inconsistency.

The cardinal inconsistency resulting from the mathematical modeling based on the problem of numerical representation of multiple semiorders with constant thresholds does not emerge in the modeling based on Fuzzy numbers. The reason is that conventional mathematical treatment cannot incorporate the subjectivity of linguistic variables. In other words, it is impossible to incorporate the meaning that concepts such as *weak*, *strong*, *very strong,* or *extreme* assume for each individual into the modeling through classical mathematical methods. Thus, the main contribution of Fuzzy Logic is verified in this type of uncertainty.

$$M_2 = \begin{vmatrix} * & (strong) & (v.strong) & (v.strong) & (v.strong) & (extreme) & (extreme) & (extreme) \\ * & * & (v.weak) & (weak) & (moderate) & (strong) & (v.strong) & (extreme) \\ * & * & * & (weak) & (weak) & (strong) & (strong) & (v.strong) \\ * & * & * & * & (weak) & (weak) & (strong) & (strong) \\ * & * & * & * & * & (weak) & (moderate) & (strong) \\ * & * & * & * & * & * & (weak) & (weak) \\ * & * & * & * & * & * & * & (v.weak) \\ * & * & * & * & * & * & * & * \end{vmatrix}.$$

In this study, the Fuzzy-MACBETH method was applied to the original decision matrix, without correcting the inconsistency. The decision matrix was then fuzzified, as follows:

$$\widetilde{A}_{n\times n} = \begin{bmatrix} * & (3,4,5) & (4,5,6) & (4,5,6) & (4,5,6) & (5,6,6) & (5,6,6) & (5,6,6) \\ * & * & (1,1,2) & (1,2,3) & (2,3,4) & (3,4,5) & (4,5,6) & (5,6,6) \\ * & * & * & (1,2,3) & (1,2,3) & (3,4,5) & (3,4,5) & (4,5,6) \\ * & * & * & * & (1,2,3) & (1,2,3) & (3,4,5) & (3,4,5) \\ * & * & * & * & * & (1,2,3) & (2,3,4) & (3,4,5) \\ * & * & * & * & * & * & (1,2,3) & (1,2,3) \\ * & * & * & * & * & * & * & (1,1,2) \\ * & * & * & * & * & * & * & * \end{bmatrix}.$$

Table 5 Results of the Fuzzy-MACBETH and classical MACBETH methods concerning the first example

	Fuzzy-MACBETH			Classic MACBETH	
	$\widetilde{v}_x$	v_x	μ_x	v_x	μ_x
x_1	11,11,19	13,67	112.12	13	120
x_2	10,10,17	12,33	100.00	11	100
x_3	7,7,12	8,67	66.68	8	70
x_4	2,2,4	2,67	12.15	2	10
x_5	1,1,2	1,33	0.00	1	0
x_6	0,0,0	0,00	– 12.09	0	– 10

Table 6 Final matrix of Fuzzy-MACBETH method for the second example

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1		5,55/5,55/8,25	7,05/7,05/10,75	8,55/8,55/13,25	10,05/10,05/15,75	12,05/12,05/18,75	14,1/14,1/21,5	15,6/15,6/24
x_2			1,5/1,5/2,5	3/3/5	4,5/4,5/7,5	6,5/6,5/10,5	8,55/8,55/13,25	10,05/10,05/15,75
x_3				1,5/1,5/2,5	3/3/5	5/5/8	7,05/7,05/10,75	8,55/8,55/13,25
x_4					1,5/1,5/2,5	3,5/3,5/5,5	5,55/5,55/8,25	7,05/7,05/10,25
x_5						2/2/3	4,05/4,05/5,75	5,55/5,55/8,25
x_6							2,05/2,05/2,75	3,55/3,55/5,25
x_7								1,5/1,5/2,5
x_8								

Table 7 Results of the Fuzzy-MACBETH method for the second example

	Fuzzy-MACBETH			Classic MACBETH	
	$\widetilde{v}_x$	v_x	μ_x	v_x	μ_x
x_1	15,6/15,6/24	18,4	163,76	36	157,14
x_2	10,05/10,05/15,75	11,95	100,00	24	100
x_3	8,55/8,55/13,25	10,11	81,88	21	85,71
x_4	7,05/7,05/10,75	8,28	63,76	17	66,67
x_5	5,55/5,55/8,25	6,45	45,63	13	47,62
x_6	3,55/3,55/5,25	4,11	22,57	8,5	26,19
x_7	1,5/1,5/2,5	1,83	0,00	3	0
x_8	0/0/0	0,00	−18,08	0	− 14,29

In this study, the mathematical modeling allowed us to mathematically incorporate the subjectivity of the linguistic scale of the MACBETH method, generating a cardinal scale based on the decision maker's preferences. The reason is that when the cardinal inconsistency is corrected by changing the decision maker's assessment, one may be at the risk of obtaining a consistent cardinal result, however, at odds with the decision maker's understanding of the decision problem.

The results suggest that the Fuzzy-MACBETH modeling allows the mathematical treatment of linguistic variables of the semantic scale through the application of Fuzzy Logic. Therefore, the uncertainty permeating human thinking can be mathematically incorporated, resulting in a scale aligned with the decision maker's preferences. Considering that the MACBETH differential is the use of a semantic scale to assess alternatives, the method's main benefit is overcoming cardinal inconsistencies arising from traditional mathematical modeling.

Note that there are other multi-criteria methods using Fuzzy scales in the literature, among which the Fuzzy-AHP stands out. However, although both methods, Fuzzy-MACBETH and Fuzzy-AHP, are aggregation methods, they present important differences. The scale generated from the Fuzzy-MACBETH is a cardinal scale obtained a posteriori, based on the differences in attractiveness between pairs of alternatives, using linear programming. That is, *zero* in the cardinal scale obtained from the Fuzzy-MACBETH is not the origin of the scale, allowing us to work with the bipolar concept of attractiveness and repulsiveness. On the other hand, the Fuzzy-AHP presents a ratio scale determined a priori, using eigenvalues and eigenvectors, where *zero* represents the scale's natural threshold, which according to [72] restricts its field of application.

Regarding studies applying the Fuzzy Theory together with the MACBETH method presented in Table 1, note that [70] apply a new approach to the scale of the MACBETH method, in which semantic judgments were transformed into grey numbers. However, the Fuzzy Set Theory was used only to determine the weights the decision makers assigned to the group assessment by applying the Intuitionist Fuzzy Theory. However, in the Fuzzy-MACBETH method proposed here, the problem of cardinal inconsistency was circumvented by transforming it into a semantically consistent decision matrix. [71] propose a new Fuzzy-MACBETH approach for alternative ranking problems, not aiming at the development of scales.

5.1 Sensitivity Analysis

A sensitivity analysis was performed by comparing the results obtained from different methods [65]. Hence, the decision matrix M_3 extracted from [70] was used. The results obtained from the Fuzzy-MACBETH, MACBETH, and Grey-MACBETH methods [70] are shown in Table 8. Additionally, the methods presented in the literature that develop value functions were used, i.e., cardinal scales can be obtained from these methods, which were used to compare the results.

Table 8 Results of the Fuzzy-MACBETH, MACBETH, and Grey-MACBETH methods

	Classic MACBETH		Fuzzy-MACBETH				Grey-MACBETH	
	v_x	μ_x	$\widetilde{v}_x$	v_x	μ_x	Difference (%)	μ_x	Difference (%)
x_1	5	100	5/5/9	6.3333	100.000	0.00	100.0000	0.00
x_2	2	40	2/2/4	2.6667	42.105	5.26	57.1428	42.85
x_3	1	20	1/1/2	1.3333	21.052	5.26	28.5714	42.85
x_4	0	0	0/0/0	0.0000	0.000	0	0.0000	0

$$M_3 = \begin{bmatrix} * & (weak) & (moderate) & (strong) \\ * & * & (veryweak) & (veryweak) \\ * & * & * & (veryweak) \\ * & * & * & * \end{bmatrix}.$$

The results in Table 7 show the alignment of the Fuzzy-MACBETH method with the MACBETH method. Note that the result obtained from the Fuzzy-MACBETH method was closer to that obtained by the original MACBETH. Additionally, the cardinal scale resulting from the Fuzzy-MACBETH showed a percentage difference of 5.26, and the Grey-MACBETH method showed a percentage difference of 42.85 in relation to the MACBETH cardinal scale, revealing the robustness of the proposed method.

6 Conclusions

The mathematical modeling developed in this study led us to the Fuzzy-MACBETH method presented in Sect. 4, which applies a triangular Fuzzy scale to MACBETH semantic scale. It consists of four steps: first, the decision matrix obtained from the decision maker's verbal responses concerning attractiveness differences between pairs of alternatives is fuzzified. Next, the Fuzzy inference procedure is performed based on the F-LP-MACBETH, resulting in a basic Fuzzy scale. Then, the basic fuzzy scale is defuzzified in the third step using the centroid method, which results in a basic crisp scale. Finally, the basic scale is cardinalized.

The results presented and discussed in Sect. 5 show the potential of the Fuzzy-MACBETH method in modeling the imprecision of human speech. Uncertain concepts, such as Very Weak, Weak, Moderate, Strong, Very Strong, and Extreme, which compose the semantic scale of the MACBETH method, can be easily understood in interpersonal interactions; however, translating these into mathematical and computational concepts is not a simple task. The reason is that these are uncertain concepts linked to individual value systems; the meaning of such concepts varies among individuals. Thus, Fuzzy Logic is the tool used to translate the uncertainty of linguistic variables.

Two examples were presented here showing an alignment of the Fuzzy-MACBETH method proposed here with the classical MACBETH method; cardinal scales consistent with those obtained by the classical method were generated when the model was applied to a semantic and cardinal consistent decision matrix. In its application in a semantically consistent decision matrix, which presents cardinal inconsistency with the classical method, the Fuzzy-MACBETH showed the ability to generate a consistent cardinal scale. Overcoming the cardinal inconsistency was possible due to the semantic scale's flexibility obtained with the application of fuzzy numbers, which can incorporate the subjectivity of linguistic variables into the mathematical modeling.

In future studies, we intend to implement the Fuzzy-MACBETH hybrid method in a DSS to facilitate its use by decision-making specialists. Bio-inspired metaheuristics [59, 83, 84] are suggested to facilitate its implementation, considering that evolutionary algorithms enable working both with optimization problems in interactive methods and methods based on the single criterion of synthesis [85]. Another suggestion is to expand the method to allow group evaluation. In this sense, some studies in the literature use the process of obtaining consensus for group decision-making [86–88].

Authors' contributions TRB developed the mathematical modeling, analysis of the algorithm and wrote the manuscript, AAL guided in all stages of development and analysis of the algorithm and CMSM, LE, SRE and AD helped to carry out the analysis with constructive discussions.

Funding Not applicable.

Availability of data and materials Not applicable.

Declarations

Conflict of interest The authors declare no conflicts of interest.

Ethics approval This study did not involve human or animal subjects.

Consent for publication All the authors agree with this manuscript's publication.

References

1. Bouyssou, D., Marchant, T., Pirlot, M., Perny, P., Tsoukiàs, A., Vincke, P.: Evaluation and decision models: A critical perspective. Kluwer Academic Publishers (International series in operations research and management Science) (2000)
2. Bana Costa, C.: Introdução geral às abordagens multicritério de apoio à tomada de decisão. Investigação Operacional. **8**(1), 117–139 (1988)
3. Pérez-Gladish, B., Ferreira, F.A.F., Zopounidis, C.: Mcdm/a studies for economic development, social, cohesion and environmental sustainability: introduction. Int. J. Sustain. Dev. World Ecol. (2020). https://doi.org/10.1080/13504509.2020.1821257
4. Zak, J., Weglinski, S.: The selection of the logistics center location based on mcdm/a methodology. Transp. Res. Proc. **3**, 555–564 (2014)
5. Stabell, C.B.: Decision support systems: Alternative perspectives and schools. Decis. Support Syst. **3**(3), 243–251 (1987)
6. García-Gamorra, D., Labella, A., Ding, W., Rodríguez, R.M., Martínez, L.: Large-scale group decision making: A systematic review and a critical analysis. IEEE/CAA J. Autom. Sin. **9**(6), 949–966 (2022). https://doi.org/10.1109/JAS.2022.105617
7. Bouyssou, D., Marchant, T., Pirlot, M.: A theoretical look at electre tri-nb and related sorting models. Q. J. Oper. Res. (2022). https://doi.org/10.1007/s10288-022-00501-9
8. Atanassov, K., Gargov, G.: Interval valued intuituinistic fuzzy sets. Fuzzy Sets Syst. **31**(3), 343–349 (1989)
9. Smarandache, F.: Neutrosophic logic and sets. (1995). http://fs.gallup.unm.edu/neutrosophy.htm
10. Deng, J.-L.: Control problems of grey systems. Syst. Control Lett. **1**(5), 288–294 (1982)
11. García-Zamora, D., Labella, A., Rodríguez, R.M., Martínez, L.: A linguistic metric for consensus reaching processes based on ELICIT comprehensive minimum cost consensus models. IEEE Trans. Fuzzy Syst. (2022). https://doi.org/10.1109/TFUZZ.2022.3213943
12. Zhang, H., Li, C.-C., Liu, Y., Dong, Y.: Modeling personalized individual semantics and consensus in comparative linguistic expression preference relations with self-confidence: An optimization-based approach. IEEE Trans. Fuzzy Syst. (2021). https://doi.org/10.1109/TFUZZ.2019.2957259
13. Zhang, H., Dong, Y., Xiao, J., Chiclana, F., Herrera-Viedma, E.: Personalized individual semantics-based approach for linguistic failure modes and effects analysis with incomplete preference information. IISE Trans. (2020). https://doi.org/10.1080/24725854.2020.1731774
14. Rodríguez, R.M., Martínez, L., Herrera, F.: Hesitant fuzzy linguistic term sets for decision making. IEEE Trans. Fuzzy Syst. (2012). https://doi.org/10.1109/TFUZZ.2011.2170076
15. Rodríguez, R.M., Labella, A., Dutta, B., Martínez, L.: Comprehensive minimum cost models for large scale group decision making with consistent fuzzy preference relations. Knowl.-Based Syst. (2021). https://doi.org/10.1016/j.knosys.2021.106780
16. Kahraman, C., Onar, S.C., Oztaysi, B.: Fuzzy multicriteria decision-making: A literature review. Int. J. Comput. Intell. Syst. **8**(4), 637–666 (2015)
17. Bana Costa, C.A., Oliveira, M.D., Vieira, A.C., Freitas, L., Rodrigues, T.C., Bana Costa, J., Freitas, A., Santana, P.: Collaborative development of composite indices from qualitative value judgements: The EURO-HEALTHY Population Health Index model. Eur. J. Oper. Res. (2023). https://doi.org/10.1016/j.ejor.2022.05.037
18. Roy, B.: Multicriteria Methodology for Decision Aiding. 1. ed. [S.l.]: Kluwer Academic Publishers (Springer Science plus Business Media Dordrecht) (1996)
19. Almeida, A.T.: Processo de Decisão nas Organizações: Construindo Modelos de Decisão Multicritério. 1. ed. São Paulo: Editora Atlas (2013)
20. Beinat, E.: Value Functions for Environmental Management. 1. ed. [S.1.]: Kluwer Academic Publishers (Springer Science plus Business Media Dordrecht) (1997)
21. Chen, T.-Y., Chang, H.-F.: Critical success factors and architecture of innovation services models in data industry. Exp. Syst. Appl. (2023). https://doi.org/10.1016/j.eswa.2022.119014
22. Singh, R., Majumder, C.B., Vidyarthi, A.K.: Assessing the impacts of industrial wastewater on the inland surface water quality: An application of analytic hierarchy process (AHP) model-based water quality index and GIS techniques. Phys. Chem. Earth. (2023). https://doi.org/10.1016/j.pce.2022.103314
23. Krenicky, T., Hrebenyk, L., Chernobrovchenko, V.: Application of concepts of the analytic hierarchy process in decision-making. Manag. Syst. Prod. Eng. **30**(4), 304–310 (2022). https://doi.org/10.2478/mspe-2022-0039
24. Iaria, J., Susca, T.: Analytic Hierarchy Processes (AHP) evaluation of green roof- and green wall- based UHI mitigation strategies via ENVI-met simulations. Urban Clim. (2022). https://doi.org/10.1016/j.uclim.2022.101293
25. AbdolkhaniNezhad, T., Monavari, S.M., Khorasani, M., Robati, N., Farsad, F.: Comparative analytical study of the results of environmental risk assessment of urban landfills approach: bowtie, network analysis techniques (ANP), TOPSIS (case study: Gilan Province). Environ. Monit. Assess. (2022). https://doi.org/10.1007/s10661-022-10513-x
26. Tseng, S.-H., Chen, H.-C., Nguyen, T.S.: Key success factors of sustainable organization for traditional manufacturing industries: A case study in Taiwan. Mathematics. **10**, 4389 (2022). https://doi.org/10.3390/math10224389
27. Chen, Q., Zhang, W., Jin, N., Wang, X., Dai, P.: Digital transformation evaluation for small and medium-sized manufacturing enterprises using the fuzzy synthetic method DEMATEL-ANP. Sustainability. **14**, 13038 (2022). https://doi.org/10.3390/su142013038
28. Ghodusinejad, M.H., Ghodrati, A., Zahedi, R., Yousefi, H.: Multi-criteria modeling and assessment of PV system performance in different climate areas of Iran. Sustain. Energy Technol. Assess. (2022). https://doi.org/10.1016/j.seta.2022.102520
29. Nakhaei, J., Bitarafan, M., LaleArefi, S., Kaplinski, O.: Model for rapid assessment of vulnerability of office buildings to blast using swara and smart methods (A case study of swiss re tower). J. Civ. Eng. Manag. **22**(6), 831–843 (2016). https://doi.org/10.3846/13923730.2016.1189457
30. Wang, L., Zhang, Z.-X., Ishizaka, A., Wang, Y.-M., Martínez, L.: TODIMSort: A TODIM based method for sorting problems. Omega (2023). https://doi.org/10.1016/j.omega.2022.102771
31. Elidolu, G., Uflaz, E., Aydin, M., Celik, E., Akyuz, E., Arslan, O.: Numerical risk analysis of gas freeing process in oil/chemical tanker ships. Ocean Eng. (2022). https://doi.org/10.1016/j.oceaneng.2022.113082

32. Corrente, S., Tasiou, M.: A robust TOPSIS method for decision making problems with hierarchical and non-monotonic criteria. Exp. Syst. Appl. (2023). https://doi.org/10.1016/j.eswa.2022.119045
33. Du, S.: Hybrid Kano-DEMATEL-TOPSIS model based benefit distribution of multiple logistics service providers considering consumer service evaluation of segmented task. Exp. Syst. Appl. (2023). https://doi.org/10.1016/j.eswa.2022.119292
34. Fan, R., Zhang, H., Gao, Y.: The global cooperation in asteroid mining based on AHP, entropy and TOPSIS. Appl. Math. Comput. (2023). https://doi.org/10.1016/j.amc.2022.127535
35. Tavares, R.S., Angulo-Meza, L., Rangel, L.A.D., Sant Anna, A.P.: Interdisciplinary graduate programs: Application of the MACBETH multicriteria method for assessing their performances. Ann. Oper. Res. **316**, 1383 (2022). https://doi.org/10.1007/s10479-021-04108-x
36. Gkouvitsos, I., Giannikos, I.: Using a MACBETH based multicriteria approach for virtual weight restrictions in each stage of a DEA multi-stage ranking process. Oper. Res. Int. Journal **22**, 1787–1811 (2022). https://doi.org/10.1007/s12351-020-00619-w
37. Campos, V.R., Moreira, D.J.S.: Risk assessment with multi-attribute utility theory for building projects. J. Build. Pathol. Rehabil. **7**, 98 (2022). https://doi.org/10.1007/s41024-022-00241-7
38. Zidi, S., Hamani, N., Kermad, L.: New metrics for measuring supply chain reconfgurability. J. Intell. Manuf. **33**, 2371–2392 (2022). https://doi.org/10.1007/s10845-021-01798-9
39. Marques, A.C., Frej, E.A., de Almeida, A.T.: Multicriteria decision support for project portfolio selection with the FITradeoff method. Omega **111**, 2 (2022). https://doi.org/10.1016/j.omega.2022.102661
40. Correia, L.M.A.M., Silva, J.M.N., Leite, W.K.S., Lucas, R.E.C., Colaço, G.A.: A multicriteria decision model to rank workstations in a footwear industry based on a FITradeof-ranking method for ergonomics interventions. Oper. Res. **22**, 3335–3371 (2022). https://doi.org/10.1007/s12351-021-00671-0
41. Darko, A.P., Liang, D., Xu, Z., Agbodah, K., Obiora, S.: A novel multi-attribute decision-making for ranking mobile payment services using online consumer reviews. Exp. Syst. Appl. **213**, 2 (2023). https://doi.org/10.1016/j.eswa.2022.119262
42. Akram, M., Noreen, U., Deveci, M.: Enhanced ELECTRE II method with 2-tuple linguistic *m*-polar fuzzy sets for multi-criteria group decision making. Exp. Syst. Appl. **213**, 2 (2023). https://doi.org/10.1016/j.eswa.2022.119237
43. Tu, Y., Shi, H., Chen, K., Liang, Y., Zhou, X., Lev, B.: Three-reference-point based group ELECTRE III method for urban flood resilience evaluation. Exp. Syst. Appl. **210**, 2 (2022). https://doi.org/10.1016/j.eswa.2022.118488
44. Emamat, M.S.M.M., de Mota, C.M.M., Mehregan, M.R., Moghadam, M.R.S., Nemery, P.: Using ELECTRE-TRI and flowsort methods in a stock portfolio selection contexto. Financial Innovation. Finan. Innov. **8**, 11 (2022). https://doi.org/10.1186/s40854-021-00318-1
45. Akram, M., Sultan, M., Alcantud, J.C.R., Al-Shamiri, M.M.A.: Extended fuzzy N-Soft PROMETHEE method and its application in robot butler selection. Math. Biosci. Eng. **20**(2), 1774–1800 (2022). https://doi.org/10.3934/mbe.2023081
46. Zolfani, S.H., Taheri, H.M., Gharehgozlou, M., Farahani, A.: An asymmetric PROMETHEE II for cryptocurrency portfolio allocation based on return prediction. Appl. Soft Comput. (2022). https://doi.org/10.1016/j.asoc.2022.109829
47. Vansnick, J.-C.: On the problem of weights in multiple criteria decision making (the noncompensatory approach). Eur. J. Oper. Res. **24**(2), 288–294 (1986). https://doi.org/10.1016/0377-2217(86)90051-2
48. Della Spina, L.: Revitalization of inner and marginal areas: A multi-criteria decision aid approach for shared development strategies. Valori e Valutazioni. **25**, 37–44 (2020)
49. Boggia, A., Rocchi, L.: Water use scenarios assessment using multicriteria analysis. J. Multi-Criteria Decis. Anal. **17**(5–6), 125–135 (2010). https://doi.org/10.1002/mcda.457
50. Mitrovic, S., Lecoutere, L., Weerdt, J.D.: A Comparison of Methods for Link Sign Prediction with Signed Network Embeddings. IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining, Vancouver, Canada. (2019) DOI: https://doi.org/10.1145/3341161.3345335
51. Peric, T., Babic, Z., Matejas, J.: Comparative analysis of application efficiency of two iterative multi objective linear programming methods (MP method and STEM method). CEJOR. **26**, 565–583 (2018). https://doi.org/10.1007/s10100-018-0543-x
52. Bare, B.B., Mendoza, G.: Multiple objective forest land management planning: An illustration. Eur. J. Oper. Res. **34**, 44–55 (1988)
53. Korhonen, P., Wallenius, J.: A Pareto race. Naval Res. Logist. **35**, 615–623 (1988)
54. Steuer, R.: An interactive multiple objective linear programming procedure. TIMS Stud. Manag. Sci. **6**, 225–239 (1977)
55. Steuer, R.: Multiple Criteria Optimization: Theory Computation and Application. Wiley, Boca Raton (1986)
56. Machado, L.G., Carlos, I.C., Angulo-Meza, L., de Mello, J.C.C.B.S.: A Multiobjective data envelopment analysis model applied to the evaluation of Brazilian electrical distributors. Simposio Brasileiro de Sistemas Eletricos (2018). https://doi.org/10.1109/SBSE.2018.8395590
57. de Mello, J.C.C.B.S., Clímaco, J.C.N., Angulo-Meza, L.: Efficiency evaluation of a small number of DMUs: An approach based on Li and Reeves's model. Pesquisa Operacional. **29**(1), 97–110 (2009)
58. Liu, Y., Zhang, H., Wu, Y., Dong, Y.: Ranking range based approach to MADM under incomplete context and its application in venture investment evaluation. Technol. Econ. Dev. Econ. **25**(5), 877–899 (2019)
59. Nguyen, P.-H.: GA-GDEMATEL: A novel approach to optimize recruitment and personnel selection problems. Math. Probl. Eng. (2022). https://doi.org/10.1155/2022/3106672
60. Olabanji, O.M., Mpofu, K.: Appraisal of conceptual designs: Coalescing fuzzy analytic hierarchy process (F-AHP) and fuzzy grey relational analysis (F-GRA). Results Eng. **9**, 100194 (2021). https://doi.org/10.1016/j.rineng.2020.100194
61. Goodarzi, F., Abdollahzadeh, V., Zeinalnezhad, M.: An integrated multi-criteria decision-making and multi-objective optimization framework for green supplier evaluation and optimal order allocation under uncertainty. Decis. Anal. J. **4**, 100087 (2022). https://doi.org/10.1016/j.dajour.2022.100087
62. Tavana, M., Shaabani, A., Di Caprio, D., Amiri, M.: An integrated and comprehensive fuzzy multicriteria model for supplier selection in digital supply chains. Sustain. Oper. Comput. **2**, 149–169 (2021). https://doi.org/10.1016/j.susoc.2021.07.008
63. Lopez-Espin, J.J., Aparicio, J., Gimenez, D., Pastor, J.T.: Benchmarking and data envelopment analysis. an approach based on metaheuristics. In: 14th International Conference on Computational Science - Procedia Computer Science (2014). DOI: https://doi.org/10.1016/j.procs.2014.05.035
64. Kaczyńska, A., Kołodziejczyk, J., Sałabun, W.: A new multi-criteria model for ranking chess players. Proc. Comput. Sci. **192**, 4290–4299 (2021). https://doi.org/10.1016/j.procs.2021.09.205
65. Pamucar, D., EbadiTorkayesh, A., Deveci, M., Simic, V.: Recovery center selection for end-of-life automotive lithium-ion batteries using an integrated fuzzy WASPAS approach. Exp. Syst. Appl. **206**, 117827 (2022). https://doi.org/10.1016/j.eswa.2022.117827

66. Dhouib, D.: An extension of macbeth method for a fuzzy environment to analyze alternatives in reverse logistics for automobile tire wastes. Omega **42**, 25–32 (2014)
67. Ertay, T., Kahraman, C., Kaya, I.: Evaluation of renewable energy alternatives using macbeth and fuzzy ahp multicriteria methods: The case of turkey. Technol. Econ. Dev. Econ. **19**(1), 38–62 (2013)
68. Pishdar, M., Ghasemzadeh, F., Antucheviciene, J.: A mixed interval type-2 fuzzy best-worst macbeth approach to choose hub airport in developing countries: Case of iranian passanger airports. Transport **34**(6), 639–651 (2019)
69. Carnero, M.C.: Developing a fuzzy topsis model combining macbeth and fuzzy shannon entropy to select a gamification app. Mathematics. **9**(1034), 1–34 (2021). https://doi.org/10.3390/math9091034
70. Yurtyapan, M.S., Aydemir, E.: Erp software selection using intuitionistic fuzzy and interval grey number-based macbeth method. Grey Syst. Theory Appl. (2021). https://doi.org/10.1108/GS-01-2021-0002
71. Pacumar, D., Behzad, M., Bozanic, D., Behzad, M.: Designing a fuzzy decision support framework for assessing solid waste management in the South European region. Environ. Sci. Pollut. Res. **29**, 42862–42882 (2022)
72. Bana e Costa, C., Vansnick, J.-C.: Macbeth—an interactive path towards the construction of cardinal value functions. Int. Trans. Oper. Res. **1**, 4 (1994)
73. Bana Costa, C., Angulo-Meza, L., Oliveira, M.D.O.: O método MACBETH e a aplicação no Brasil. Ana Esteira Engevista. **15**, 1 (2013)
74. Bana Costa, C., de Corte, J.M., Vansnick, J.C.: MACBETH. Int. J. Inform. Technol. Decis. Mak. **11**, 2 (2012)
75. Vieira, A.C., Oliveira, M.D., Bana Costa, C.: Enhancing knowledge construction processes within multicriteria decision analysis: The collaborative value modelling framework. Omega **94**, 2 (2020). https://doi.org/10.1016/j.omega.2019.03.005
76. Bana Costa, C., Vansnick, J.-C.: Uma nova abordagem ao problema da construção de uma função de valor cardinal: MACBETH. Investigação Operacional. **1**, 15 (1995)
77. Zadeh, L.A.: Fuzzy sets. Inform Control. **8**, 338–353 (1965)
78. Petrovic-Lazarevic, S.: Personnel selection fuzzy model. Int. Trans. Oper. Res. **8**, 89–105 (2001)
79. Radojevic, D., Petrovic, S.: A fuzzy approach to preference structure in multicriteria ranking. Int. Trans. Oper. Res. **4**(5/6), 419–430 (1997)
80. Deliktas, D., Ustun, O.: Student selection and assignment methodology based on fuzzy multimoora and multichoice goal programming. Int. Trans. Oper. Res. **24**, 1173–1195 (2017)
81. Zhang, Y., Xu, Z.: Efficiency evaluation of sustainable water management using the hf-todim method. Int. Trans. Oper. Res. **26**, 747–764 (2019)
82. Shiu, J.-Y., Lu, S.-T., Chang, D.-S., Wu, K.-W.: Fuzzy multicriteria decision-making tools for selecting a professional property management company. Int. Trans. Oper. Res. **26**, 1527–1557 (2019)
83. Xidonas, P., Doukas, H., Sarmas, E.: A python-based multicriteria portfolio selection DSS. RAIRO Oper Res. **55**, S3009 (2021). https://doi.org/10.1051/ro/2020140
84. Leyva-López, J.C., Solano-Noriega, J.J., Gastélum-Chavira, D.A., Gaxiola-Valenzuela, T.: A personnel selection model for a software development company based on the ELECTRE III method and a variant of NSGA-II. Innovar (2022). https://doi.org/10.15446/innovar.v32n85.100657
85. Zhang, H., Deng, Y., Chan, F.T.S., Zhang, X.: A modified multicriterion optimization genetic algorithm for order distribution in collaborative supply chain. Appl. Math. Model. **37**, 7855–7864 (2013). https://doi.org/10.1016/j.apm.2013.05.021
86. Labella, A., Liu, H., Rodríguez, R.M., Martínez, L.: A cost consensus metric for consensus reaching processes based on a comprehensive minimum cost model. Eur. J. Oper. Res. **281**, 316–331 (2020). https://doi.org/10.1016/j.ejor.2019.08.030
87. Dong, Y., Zha, Q., Zhang, H., Herrera, F.: Consensus reaching and strategic manipulation in group decision making with trust relationships. IEEE Trans. Syst. Man Cybern. Syst. (2021). https://doi.org/10.1109/TSMC.2019.2961752
88. Zha, Q., Dong, Y., Zhang, H., Chiclana, F., Herrera-Viedma, E.: A personalized feedback mechanism based on bounded confidence learning to support consensus reaching in group decision making. IEEE Trans. Syst. Man Cybern. Syst. (2021). https://doi.org/10.1109/TSMC.2019.2945922