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RESEARCH ARTICLE

Relative Basic Uncertain Information in Preference and Uncertain Involved Information Fusion

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Abstract
Basic uncertain information is a newly proposed normative formulation to express and model uncertain information. This study further generalizes this concept by introducing the concept of refined interval of discourse in which the true value is known to be included. Hence, we define some new definitions of relative basic uncertain information, relative certainty/uncertainty degree and comprehensive certainty/uncertainty with some related measurements and analysis. With the introduced uncertain data type, we define two corresponding aggregation operators, namely, the relative basic uncertain information valued weighted arithmetic mean operator and the interval-induced relative basic uncertain information valued ordered weight averaging operator. An application of the proposed concepts and methods in multi-agents evaluation is provided.

Keywords Aggregation operator · Basic uncertain information · Information fusion · Multi-criteria evaluation · Preference involved evaluation

1 Introduction

Information fusion techniques are particularly important in numerous areas such as multi-criteria evaluation and decision making [1–6]. The theory of information fusion largely includes two aspects. The first one is the rigorous theory of aggregation operators [7] which has been significantly developed during the past decades [8–13]. The second concern is the development and study of the diverse data types used in information fusion which includes a large number of uncertainty involved data types.

Uncertainties with different types have been ever increasingly more involved in multi-criteria evaluation and decision-making problems. For example, interval information, fuzzy information, probability information and possibility information are some representative types. A large number of further extensions and applications such as basic uncertain information have been developed with fast speed [14–17].

With the space provided by uncertainties, the subjective preferences of decision makers can be involved in multi-criteria evaluation and decision-making problems. Notwithstanding with some extent of subjectivity, the preferences usually are indeed the appropriate reflection and embodiment of the decision makers' long-time working experiences or extraordinary management intuitions. A typical preference concerned in a myriad of practical problems is the bi-polar preferences such as the bi-polar optimism–pessimism preference, and the bi-polar strong–weak, strong–neutral, or weak–neutral preference of any type of aversions.

Recently, Jin et al. proposed the concept of basic uncertain information (BUI) [16, 17], which can generalize many types of uncertainties using some paradigmatic expression, and soon this concept has been further studied and applied in different areas [18–27]. We use a real pair $(a, c) \in [0, 1]^2$ to express a BUI granule in which a is the concerned individual assessment

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value, while c is the certainty degree of a; and $1 - c$ is called the uncertain degree of a. Certainty degree can have different meanings in different decisional scenarios. For example, a large certainty degree may indicate that decision maker has large confidence over the obtained value a, or show the extent to which the value a is precisely measured or exactly collected.

Without loss of generality, the defined BUI is based on the domain of discourse [0, 1], which implies that for a BUI granule (a, c) , once certainty degree $c = 0$ is offered from an evaluator, then the value a is absolutely uninformative. Although it indeed provides an ideal model and frame for modeling many decision-making and evaluation situations, in practice the preset domain of discourse sometimes can be narrowed and thereby properly included in [0, 1], still without sacrificing the generality. For example, for a given object under evaluation, an approximate range that is smaller than [0, 1] for the evaluation value has been known or been preset by a decision maker before he possibly invites some other evaluators to make more precise assessment. As another example, when an expert provides an evaluation value with certainty degree (which can be transformed into a BUI granule), he/she may also give the range in which the true evaluation value will definitely not fall, e.g., $[0, u) \cup (v, 1]\,(u < v)$. Clearly, such requirements can be easily realized in practice. In other words, sometimes the domain of discourse is no longer [0, 1], but a narrowed sub-interval of it $[u, v] \subset [0, 1]$. Meanwhile, the concept of BUI can still be applied to such narrowed sub-interval.

Against this background, we will discuss some new normative expression based on a narrowed sub-interval and analyze its related basic operations, measurements and applications. The results of this study will provide both theoretical contributions and good application potentials for practitioners.

The remainder of this work is organized as follows. Section 2 reviews or recapitulates some basic concepts of BUI and related aggregation operators. In Sect. 3, we propose the new concept of relative basic uncertain information and relative certainty degree with some related analysis and numerical examples. Section 4 discusses some of the new defined concepts in bi-polar preference aggregations and rules-based decision making. Section 5 concludes and remarks this study.

2 Basic Uncertain Information and Some Related Aggregation Operators

This section only reviews a minimum knowledge of basic uncertain information, weighted arithmetic mean for BUI vector and interval valued vectors.

Definition 1 [16, 17] A BUI granule is in expression of a pair $(a, c) \in [0, 1]^2$ in which a is the concerned value, while c is the certainty degree of a; and $1 - c$ is called the uncertain degree of a. The set of all BUI granules (a, c) is denoted by $\mathcal{B}$.

In this work, all of the closed intervals $[a, b] \subseteq [0, 1]$ are denoted by $\mathcal{I}$. Conventionally, the extended interval $[a, a]$ is sometimes identified with real number a, while all the interval operators can be retained. We also consider the lattice $(\mathcal{I}, \leq_{Int})$ in which the partial order $\leq_{Int}$ may be used for comparison of two intervals and is defined such that $[a_1, b_1] \leq_{Int} [a_2, b_2]$ if and only if $a_1 \leq a_2$ and $b_1 \leq b_2$; we take the notation $[a_1, b_1] <_{Int} [a_2, b_2]$ if and only if $[a_1, b_1] \leq_{Int} [a_2, b_2]$ and $[a_1, b_1] \neq [a_2, b_2]$.

We agree on the following notations even with some possible abusements when no risk of misunderstanding can arise. Let $\mathbf{a} = (a_i)_{i=1}^n \in [0, 1]^n$ and $\mathbf{b} = (b_i)_{i=1}^n \in [0, 1]^n$ be two real vectors, and then a vector of BUI granules $((a_i, c_i))_{i=1}^n \in \mathcal{B}^n$ is denoted by $(\mathbf{a}, \mathbf{c})$; and a vector of closed intervals $([a_i, b_i])_{i=1}^n \in \mathcal{I}^n$ is denoted by $[\mathbf{a}, \mathbf{b}]$.

Definition 2 (IWAM operators for vectors of intervals and BUI granules)

(i) The interval valued weighted arithmetic mean (IvWA) operator with weight vector $\mathbf{w} = (w_i)_{i=1}^n$ is a mapping $IvWA_{\mathbf{w}} : \mathcal{I}^n \to \mathcal{I}$ such that

$$IvWA_{\mathbf{w}}([\mathbf{a}, \mathbf{b}]) = \sum_{i=1}^{n} w_i [a_i, b_i] = \sum_{i=1}^{n} [w_i a_i, w_i b_i]. \tag{1}$$

(ii) [16] The BUI weighted arithmetic mean (BUIWAM) operator with weight vector $\mathbf{w} = (w_i)_{i=1}^n$ for BUI vector $BWA_{\mathbf{w}} : \mathcal{B}^n \to \mathcal{B}$ is defined by

$$BWA_{\mathbf{w}}(\mathbf{a}, \mathbf{c}) = \sum_{i=1}^{n} w_i (a_i, c_i) = \sum_{i=1}^{n} (w_i a_i, w_i c_i). \tag{2}$$

3 Relative Basic Uncertain Information and Relative Certainty Degree

As we discussed in Introduction, sometimes the domain of discourse for BUI granules should be further narrowed down into a sub-interval. We next directly present a strict paradigmatic form for the desired generalization since we assume one is familiar with the concept of BUI.

Definition 3 (*Relative basic uncertain information*) Let $g(\omega) \in [0, 1]$ be the true value of a certain object under evaluation (which might not be known). A relative basic uncertain information (RBUI) granule is expressed by a triple $([u, v], a, c) \in \mathcal{I} \times [0, 1]^2$, where $[u, v]$ is a refined interval (of discourse) in which the true value $g(\omega)$ is known to be included, $a \in [u, v]$ is the plausible value (of the true value

$g(\omega)$), and c is called the relative certainty degree of a; and $1-c$ is called the relative uncertainty degree of a. The set of all RBUI granules ([u, v], a, c) is denoted by $\mathcal{RB}$.

Remark In the above definition, the true value of a certain object $g(\omega)$ has fallen into the narrowed interval [u, v] before making further assessment. This implies that it is impossible for $g(\omega)$ to be within $[0, 1]\backslash[u, v]$. Besides, the plausible value should also fall into [u, v] as a tacit restriction.

In Definition 3, even if we may have a very small relative certainty degree c, the "actual comprehensive uncertainty" contained in ([u, v], a, c) may not be high. As an extreme case, even if $c = 0$ but the refined interval degenerates into an extended interval of a real number [u, u], then both the plausible value and the true value "have to" equal to u; in this situation, the value of relative certainty degree c actually no longer has effect. Particularly note that, although in this case c seems irrelevant, we still let it remain in its position for the consistencies in both mathematics and expressions. We recall that for the standard BUI granule in Definition 1, once certainty degree $c = 0$, then the true value may be at any point of the unit interval [0,1] irrespective of the plausible value a, meaning that the BUI granule (a, 0) provides the same information as the unit interval [0,1]. Similarly, for the RBUI granule ([u, v], a, 0), the true value can be at any point in [u, v] and thus ([u, v], a, 0) becomes equivalent to interval [u, v]. As another extreme case, if the refined interval is [0,1] (i.e., without any refinement), then ([0, 1], a, c) in fact degenerates into the standard BUI in Definition 1. In addition, if $c = 1$, then the plausible value and the true value of a certain object coincide, which implies ([u, v], a, 1) in actual degenerates into real value a. To make the above analysis clearer, we take the following illustrative formulations where "⇒" represents "degenerates into":

(a) $([u, u], a, c) \Rightarrow [u, u] \Rightarrow u = a$;
(b) $([u, v], a, 0) \Rightarrow [u, v]$;
(c) $([0, 1], a, c) \Rightarrow (a, c)$;
(d) $([u, v], a, 1) \Rightarrow a$.

In an RBUI granule ([u, v], a, c), it actually contains two different types of uncertainties which may interact to generate a resulting effect of uncertainty. The first type is the uncertainty within interval [u, v] which usually can be measured by the length of it, and the other type is just the relative uncertainty degree $1-c$. It seems natural in form to express these two types of uncertainty by a pair $(v-u, 1-c) \in [0, 1]^2$. However, when either of the two values $v-u$ and $1-c$ is zero, the RBUI granule ([u, v], a, c) degenerates into the real value a as we just discussed, which implies the whole/comprehensive uncertainty involved in ([u, v], a, c) is zero, i.e., the whole/comprehensive certainty involved is 1. Hence, in practice we may define the following definition to provide different comprehensive certainty/uncertainty measurements using any semi-copula [7, 28]. Recall a semi-copula $(x, y) \mapsto x \circ y$ is a binary aggregation operator which is monotonic non-decreasing w. r. t. each parameter and satisfies $1 \circ x = x \circ 1 = x$. One may differentiate it from the relative certainty/uncertainty of an RBUI granule.

Definition 4 The comprehensive uncertainty of an RBUI granule ([u, v], a, c) is defined by

$$CU([u, v], a, c) = (v-u) \circ (1-c). \tag{3}$$

The comprehensive certainty of an RBUI granule ([u, v], a, c) is defined by

$$CC([u, v], a, c) = 1 - (v-u) \circ (1-c), \tag{4}$$

where $\circ$ can be any semi-copula.

Example 1 We firstly adopt the product as the desired semi-copula.

(i) $CU([0.1, 0.7], 0.5, 0.6) = (0.7-0.1) \cdot (1-0.6) = 0.24$,
$CC([0.1, 0.7], 0.5, 0.6) = 1-(0.7-0.1) \cdot (1-0.6) = 0.76$.
(ii) $CU([0.5, 0.8], 0.6, 0.5) = (0.8-0.5) \cdot (1-0.5) = 0.15$,
$CC([0.5, 0.8], 0.6, 0.5) = 1-(0.8-0.5) \cdot (1-0.5) = 0.85$.
(iii) $CU([0, 1], 0.6, 0.2) = (1-0) \cdot (1-0.2) = 0.8$,
$CC([0, 1], 0.6, 0.2) = 1-(1-0) \cdot (1-0.2) = 0.2$.
(iv) $CU([0.5, 0.5], 0.5, 0) = (0.5-0.5) \cdot (1-0) = 0$,
$CC([0.5, 0.5], 0.5, 0) = 1-(0.5-0.5) \cdot (1-0) = 1$.

Next, we consider using min operator $\wedge$, i.e., $x \wedge y = \min(x, y)$. Accordingly,

(i) $CU([0.1, 0.7], 0.5, 0.6) = (0.7-0.1) \wedge (1-0.6) = 0.4$,
$CC([0.1, 0.7], 0.5, 0.6) = 1-(0.7-0.1) \wedge (1-0.6) = 0.6$.
(ii) $CU([0.5, 0.8], 0.6, 0.5) = (0.8-0.5) \wedge (1-0.5) = 0.3$,
$CC([0.5, 0.8], 0.6, 0.5) = 1-(0.8-0.5) \wedge (1-0.5) = 0.7$.
(iii) $CU([0, 1], 0.6, 0.2) = (1-0) \wedge (1-0.2) = 0.8$,
$CC([0, 1], 0.6, 0.2) = 1-(1-0) \wedge (1-0.2) = 0.2$.
(iv) $CU([0.5, 0.5], 0.5, 0) = (0.5-0.5) \wedge (1-0) = 0$,
$CC([0.5, 0.5], 0.5, 0) = 1-(0.5-0.5) \wedge (1-0) = 1$.

Recall that given a BUI granule (a, c), we can transform it into an interval by the following transformation $T : \mathcal{B} \rightarrow \mathcal{I}$ [29–31]:

$$T(a, c) = [a - a(1-c), a + (1-a)(1-c)]. \tag{5}$$

Though BUI granule can be directly used in some rules-based decision making [32–35], the derived interval value sometimes might be more suitable for the interval-based decision making. Correspondingly, we have the following transformation $T : \mathcal{RB} \to \mathcal{I}$ to convert an RBUI granule $([u, v], a, c)$ into an interval:

$$T([u, v], a, c) = [a - (a - u)(1 - c), a + (v - a)(1 - c)]. \quad (6)$$

Note that when $[u, v] = [0, 1]$, (6) degenerates into (5).

Example 2

(i) $T([0.1, 0.7], 0.5, 0.6) = [0.5 - (0.5 - 0.1)(1 - 0.6), 0.5 + (0.7 - 0.5)(1 - 0.6)] = [0.34, 0.58]$.
(ii) $T([0.5, 0.8], 0.6, 0.5) = [0.6 - (0.6 - 0.5)(1 - 0.5), 0.6 + (0.8 - 0.6)(1 - 0.5)] = [0.55, 0.7]$.
(iii) $T([0, 1], 0.6, 0.2) = [0.6 - (0.6 - 0)(1 - 0.2), 0.6 + (1 - 0.6)(1 - 0.2)] = [0.12, 0.92]$.
(iv) $T([0.5, 0.5], 0.5, 0) = [0.5 - (0.5 - 0.5)(1 - 0), 0.5 + (0.5 - 0.5)(1 - 0)] = [0.5, 0.5]$.

For any BUI granule (a, c), let $T(a, c) = [g, h]$ be the derived interval by (5), then it is easy to observe the relation $1 - c = h - g$. For any RBUI granule $([u, v], a, c)$ we can have the corresponding relation when the semi-copula chosen in Definition 4 is product $\circ = \cdot$.

Proposition 1 *For any RBUI granule $([u, v], a, c)$, the comprehensive uncertainty degree $CU([u, v], a, c) = (v - u) \cdot (1 - c)$ defined with semi-copula $\cdot$ being the product, and the interval $[g, h] = T([u, v], a, c)$ derived from (6) has the relation $h - g = (v - u) \cdot (1 - c)$.*

Proof

$$\begin{aligned} h - g &= (a + (v - a)(1 - c)) - (a - (a - u)(1 - c)) \\ &= (v - a)(1 - c) + (a - u)(1 - c) \\ &= v - vc - a + ac + a - ac - u + uc = v - vc - u + uc \\ &= (v - u) \cdot (1 - c). \end{aligned}$$

Remark If the semi-copula adopted is not the product, then the corresponding result as in Proposition 1 may not hold.

Remark With the same a and c, an RBUI granule $([u, v], a, c)$ can provide better or more refined information than the standard BUI granule (a, c) in the sense that if we take the transformations in (5) and (6), respectively, we easily check $T([u, v], a, c) \subseteq T(a, c)$.

4 Bi-polar Preference Aggregations and Rules-Based Decision Making in RBUI Environment

Ordered weighted averaging (OWA) operators [6], induced ordered weighted averaging (IOWA) operators [36] and some of their extensions [37, 38] can well model bi-polar references with different types. The whole information fusion process and the type modeled can be decided by a vector of inducing variable related to input vector which can be magnitudes of inputs, chronological orders of inputs or the amounts of uncertainty contained in inputs, etc.

Recently, scholars proposed some formulations based on three-set method [37] which can very effectively perform the IOWA operators with interval inducing variables, without using any permutations as should be used in the original IOWA operators where it is difficult to handle tied or incomparable inducing values. In what follows we review as minimum related knowledge as we can before we discuss some bi-polar preference aggregations and rules-based decision making [32–35] in RBUI environment.

Similar to BUI vector and interval vector, a vector of RBUI granules is denoted by $([\mathbf{u}, \mathbf{v}], \mathbf{a}, \mathbf{c}) = (([u_i, v_i], a_i, c_i))_{i=1}^n$. The RBUI weighted arithmetic mean operator is formally defined as follows.

Definition 5 The relative basic uncertain information valued weighted arithmetic mean (RvWA) operator with weight vector $\mathbf{w} = (w_i)_{i=1}^n$ for RBUI vector $RvWA_{\mathbf{w}} : (\mathcal{RB})^n \to \mathcal{RB}$ is defined by

$$\begin{aligned} &RvWA_{\mathbf{w}}([\mathbf{u}, \mathbf{v}], \mathbf{a}, \mathbf{c}) = \sum_{i=1}^n w_i\left(\left[u_i, v_i\right], a_i, c_i\right) \\ &= \left(\left[\sum_{i=1}^n w_i u_i, \sum_{i=1}^n w_i v_i\right], \sum_{i=1}^n w_i a_i, \sum_{i=1}^n w_i c_i\right). \end{aligned} \quad (7)$$

Remark For the reasonability of multiplying the corresponding entries by w_i four times, respectively, one may refer to literature [16] where the reasonability of defining BUIWAM in (2) has been discussed. In addition, it is not difficult to check that $\sum_{i=1}^n w_i a_i \in [\sum_{i=1}^n w_i u_i, \sum_{i=1}^n w_i v_i]$ and hence the resulting form in (7) is still an RBUI granule.

Definition 6 [39]A BUM function $Q : [0, 1] \to [0, 1]$ is a non-decreasing function with $Q(0) = 0$ and $Q(1) = 1$.

The orness of a BUM function is defined by $orness(Q) = \int_0^1 Q(t)dt$ and the andness of a BUM function is defined by $andness(Q) = 1 - orness(Q)$ [40]. When the adopted BUM functions are convex (e.g., $Q(y) = y^n$ with $n \geq 1$) or concave (e.g., $Q(y) = 1 - (1 - y)^2$ with $n \geq 1$), a larger BUM function can well model a stronger preference extent exhibited by decision maker, and vice versa [37, 41]. When the inducing vector is about magnitudes of input, the orness of a BUM function embodies its original literal meaning of logic OR; that is, a large orness corresponds to an optimism preference in evaluation, and vice versa.

As a combined method, we adopt the three-set method [37] as a powerful tool to perform OWA/IOWA operators with interval valued inducing vectors using the language of WA expressions of IOWA operators [38]. One significant advantage of using this weighting method lies in that it can perfectly handle the situation where tied inducing values appear which cannot be ideally or conveniently tackled by existing induced weighting methods.

Definition 7 An interval-induced relative basic uncertain information valued ordered weight averaging (IvIRvOWA) operator $IvIRvOWA_{Q;[\mathbf{x},\mathbf{y}]} : (\mathcal{RI})^n \to \mathcal{RI}$ with an inducing interval vector $[\mathbf{x},\mathbf{y}] = ([x_i, y_i])_{i=1}^n$ and a BUM function $Q : [0,1] \to [0,1]$ is defined by an RBUI valued weighted arithmetic mean operator (with $\mathbf{w}$) $RvWA_{\mathbf{w}} : (\mathcal{RB})^n \to \mathcal{RB}$,

$$RvWA_{\mathbf{w}}([\mathbf{u},\mathbf{v}],\mathbf{a},\mathbf{c}) = \sum_{i=1}^n w_i([u_i, v_i], a_i, c_i), \tag{8}$$

in which $\mathbf{w}$ is defined in the following steps:

Step 1: For each $[x_i, y_i]$, define three disjoint subsets of $\{1,\dots,n\}$: $A_i, B_i, E_i \subseteq \{1,\dots,n\}$ such that

$A_i = \{j \in \{1,\dots,n\} : [x_i, y_i] <_{Int} [x_j, y_j]\}$,

$B_i = \{j \in \{1,\dots,n\} : [x_j, y_j] <_{Int} [x_i, y_i]\}$,

$E_i = \{1,\dots,n\} \backslash (A \cup B)$.

Step 2: Form an intermediate vector $\mathbf{s} = (s_i)_{i=1}^n \in [0,1]^n$ (which is not necessarily normalized) such that

$$s_i = \frac{Q(1-\frac{|B_i|}{n}) - Q(\frac{|A_i|}{n})}{|E_i|}, \tag{9}$$

where $|S|$ is the cardinality of any finite set S.

Step 3: It can be shown that $\mathbf{s} \neq \mathbf{0} = (0,\dots,0)$ [37], and then after normalizing $\mathbf{s}$, we obtain a normalized weight vector $\mathbf{w} = (w_i)_{i=1}^n$ by

$$w_i = \frac{s_i}{\sum_{k=1}^n s_k}. \tag{10}$$

Remark Similar to induced OWA aggregation for BUI, the reason why we do not define the OWA version of IvIRvOWA is because the inducing vector about substantial magnitudes in the vector of RBUI is not clear due to the existence of uncertainties. When the inducing interval vector $[\mathbf{x},\mathbf{y}]$ is clearly determined by a magnitude vector in relation to the RBUI inputs, the corresponding bi-polar preference aggregation with the type of input magnitudes can be defined by Definition 7 as the following numerical example shows.

Example 3 Suppose a company needs to decide whether or not to put into production a new product according to a prediction of the next year's market share of this product. The manager invites $n = 4$ experts $\{E_i\}_{i=1}^4$ to give their individual predictions which are expressed by an RBUI vector $([\mathbf{u},\mathbf{v}],\mathbf{a},\mathbf{c}) = (([u_i, v_i], a_i, c_i))_{i=1}^4$. That is, $([u_i, v_i], a_i, c_i)$ is provided by expert E_i, indicating that he feels a plausible market share is $100a_i\%$ with confidence (relative certainty degree) c_i, and it is impossible that the market share will be larger than $100v_i\%$ or lower than $100u_i\%$.

If the manager has a moderate pessimistic preference which, for example, is embodied by a convex BUM function $Q(t) = t^2$, then he prefers the RBUI granules that indicate predictions with lower market share. We will firstly apply the transformation in (6) to derive a vector of intervals $[\mathbf{x},\mathbf{y}] = ([x_i, y_i])_{i=1}^4$ from the given RBUI vector, and then perform IvIRvOWA operator with inducing vector $[\mathbf{x},\mathbf{y}]$ and BUI function Q.

Assume the manager wishes to take the rules-based decision making [32–35] to automatically judge whether the new product can be put into production. If the final aggregation result $([u, v], a, c)$ satisfies at least one of the following set of rules, the new product should be produced, else it cannot.

Set of rules A: $(u \geq 0.3)$ or $(u < 0.3, a \geq 0.6$ and $c \geq 0.5)$.

Set of rules B: $(u \geq 0.3)$ or $(u < 0.3, a \geq 0.4$ and $c \geq 0.75)$.

Set of rules C: $(u \geq 0)$ and $(a \geq 0.6$ and $c \geq 0.7)$.

Set of rules D: $(u \geq 0)$ and $(a \geq 0.4$ and $c = 1)$.

Note that the above sets of rules are majorly for illustration, and in practice decision makers can design suitable rules according to their own situations.

Suppose $([u_1, v_1], a_1, c_1) = ([0.1, 0.7], 0.5, 0.6)$, $([u_2, v_2], a_2, c_2) = ([0.5, 0.8], 0.6, 0.5)$, $([u_3, v_3], a_3, c_3) = ([0, 1], 0.6, 0.2)$, $([u_4, v_4], a_4, c_4) = ([0.4, 0.6], 0.5, 0.5)$.

With the transformation defined in (6), we have

$[x_1, y_1] = T([0.1, 0.7], 0.5, 0.6) = [0.34, 0.58]$,

$[x_2, y_2] = T([0.5, 0.8], 0.6, 0.5) = [0.55, 0.7]$,

$[x_3, y_3] = T([0, 1], 0.6, 0.2) = [0.12, 0.92]$,

$[x_4, y_4] = T([0.4, 0.6], 0.5, 0.5) = [0.45, 0.55]$.

Next, by formula (9), we obtain $s_1 = \frac{Q(1-\frac{|B_1|}{4}) - Q(\frac{|A_1|}{4})}{|E_1|}$

$$s_1 = \frac{Q(1-\frac{|B_1|}{4}) - Q(\frac{|A_1|}{4})}{|E_1|} s_2 = \frac{1}{8}, s_3 = \frac{1}{4}, s_4 = \frac{5}{16}.$$

Since the intermediate vector $\mathbf{s} = (s_i)_{i=1}^4 = (\frac{5}{16}, \frac{1}{8}, \frac{1}{4}, \frac{5}{16})$ is coincidentally already normalized, we obtain the normalized weight vector $\mathbf{w} = (w_i)_{i=1}^4 = \mathbf{s} = (s_i)_{i=1}^4 = (0.3125, 0.125, 0.25, 0.3125)$.

Finally, with (8) we carry out the RvWA and obtain

$$\begin{aligned} &RvWA_{\mathbf{w}}([\mathbf{u},\mathbf{v}],\mathbf{a},\mathbf{c}) \\ &= \sum_{i=1}^4 w_i([u_i, v_i], a_i, c_i) \\ &= ([0.21875, 0.75625], 0.5375, 0.45625). \end{aligned}$$

Since the result $([u, v], a, c) = ([0.21875, 0.75625], 0.5375, 0.45625)$ satisfies none of the predetermined four sets of rules, the decision suggests the product cannot be produced.

5 Conclusions

The RBUI granule is with the form $([u, v], a, c)$ in which the refined interval of discourse can convey better information than the standard BUI granule (a, c). The fact that RBUI is one perfect generalization of BUI can be observed by four degeneration relations (a) $([u, u], a, c) \Rightarrow [u, u] \Rightarrow u = a$; (b) $([u, v], a, 0) \Rightarrow [u, v]$; (c) $([0, 1], a, c) \Rightarrow (a, c)$; (d) $([u, v], a, 1) \Rightarrow a$. Some new concepts of relative certainty/uncertainty degree and comprehensive certainty/uncertainty have been defined, which further show the difference of RBUI from BUI.

The relative basic uncertain information valued weighted arithmetic mean operator has been defined, which further serves as an intermediate tool to well define the interval-induced relative basic uncertain information valued ordered weight averaging operator. To define IvIRvOWA, we combine the three-set method with the WA expressions of IOWA operators. The RBUI granule can be also applied in rules-based decision making, adding more decision making flexibility. An application with numerical example in market share opinions related bi-polar preference aggregation has been presented.

This work may provide both theoretical value to aggregation theory and good application potential for practitioners. In future work we may further extend or generalize RBUI so as to be suitable in linguistic and large scale decision making environments.

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Declarations

Conflict of interest The authors declare that they have no competing interests.

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable

Availability of data and material The data that support the findings of this study are available on request from the corresponding authors, [Y. Q. Xu & Z. S. Chen].

List of Abbreviations No abbreviations have been used in this paper.

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