Enhanced Tunicate Swarm Algorithm for Solving Large-Scale Nonlinear Optimization Problems

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Abstract

Nowadays optimization problems become difficult and complex, traditional methods become inefficient to reach global optimal solutions. Meanwhile, a huge number of meta-heuristic algorithms have been suggested to overcome the shortcomings of traditional methods. Tunicate Swarm Algorithm (TSA) is a new biologically inspired meta-heuristic optimization algorithm which mimics jet propulsion and swarm intelligence during the searching for a food source. In this paper, we suggested an enhancement to TSA, named Enhanced Tunicate Swarm Algorithm (ETSA), based on a novel searching strategy to improve the exploration and exploitation abilities. The proposed ETSA is applied to 20 unimodal, multimodal and fixed dimensional benchmark test functions and compared with other algorithms. The statistical measures, error analysis and the Wilcoxon test have affirmed the robustness and effectiveness of the ETSA. Furthermore, the scalability of the ETSA is confirmed using high dimensions and results exhibited that the ETSA is least affected by increasing the dimensions. Additionally, the CPU time of the proposed algorithms are obtained, the ETSA provides less CPU time than the others for most functions. Finally, the proposed algorithm is applied at one of the important electrical applications, Economic Dispatch Problem, and the results affirmed its applicability to deal with practical optimization tasks.

Keywords Optimization · Meta-heuristics · Tunicate Swarm Algorithm (TSA) · Enhanced Tunicate Swarm Algorithm · Economic dispatch problem

1 Introduction

For almost all human problems there is a desire to reach the most with the least, for example, maximizing the strength and efficiency of industrial equipment with minimum cost. The optimization has a great role in human problems which achieve the optimum (maximum or minimum). Optimization is considered the process of obtaining the optimum results under available conditions. Optimization problems become complex with a huge number of decision variables and constraints. There is no definite method available for all different problems so different methods have been suggested to solve these problems. The classical methods of optimization (direct search methods) such as Newton methods, the

steepest method and nonlinear programming are efficient in finding the optimum solution of continuous and differentiable functions only depending on reference starting point, gradient information and failed to deliver the optimum without converging to some local optimum. These problems made the researchers develop new methods of optimization. Deterministic, stochastic and hybrid algorithms are three categories of optimization algorithms. Deterministic algorithms follow the input data and parameters values and produce the same result such as Hill-climbing and downhill simplex algorithms. Stochastic algorithms or meta-heuristics algorithms are introduced based on randomness rules while the hybrid algorithms represent a mixed system among the deterministic and/or stochastic algorithms. For instance, a hybrid PSO-GA algorithm was proposed to deal with the constrained optimization problems [1], where the PSO has been used to explore the search regions while GA has been employed to improve the search direction. A hybrid GA-GSA algorithm for optimizing the performance of an industrial system by utilizing uncertain data was introduced [2], where its ability is proved through maximizing the reliability and

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maintainability in solving a nonlinear optimization problem. A novel TVAC-PSO based mutation strategies algorithm for generation scheduling of pumped storage hydrothermal system incorporating solar units [3] is presented to analyze the impact of renewable energy sources on the distribution of optimal power generation. Meta-heuristic algorithms are considered stochastic which are used with different functions such as discontinuous, non-differentiable, non-smooth functions. The meta-heuristic algorithms have been suggested to avoid the demerits of the classical methods. They follow an iterative generation process to find a high efficient solution near to the optimum and they can reach the promising areas, escape from local optima and solving different and complex problems in a few times [4]. Unless the classical methods, they do not have one initial point and do not depend on the gradient information. According to these merits [5], huge number of meta-heuristics optimization algorithms are proposed over the last few decades [6]. Nature can be represented as the most important inspiration of metaheuristics. Four categories of biologically inspired metaheuristics are discussed as evolution-based, swarm-based, physics-based and human-based algorithms [7]. The biologically algorithms mimic nature evolution such as Biogeography Based Optimizer (BBO) [8] that is established based on biogeography concept for solving optimization problems, Evolutionary Programming (EP) [9] has been introduced to improve the finite-state automata to solve time series forecasting problems and improve learning machines to solve continuous optimization problems, Genetic Algorithm (GA) [10] has been proposed based on the biological improvement for solving constrained and unconstrained problems through some operators including crossover, recombination, mutation and the selection strategy, Evolution Strategies (ES) [11] belongs to the evolutionary algorithms that is developed based on the ideas of the improvement and it is applied to deal with some fields of optimization, and Differential Evolution (DE) [12] is a population-based algorithm for improving the solution using the evolutionary process. Swarm-based algorithms mimic the behavior of living organisms to find the optimum food source such as Particle Swarm Optimization (PSO) [13] that is inspired by the behavior of birds or fish. It is affirmed its ability in dealing with comprehensive optimization fields, however, it suffers from trapping in local optima and premature convergence while solving complex natures, Ant Colony Optimization (ACO) [14] was proposed based on foraging behavior of ant colonies to solve difficult discrete optimization problems, Firefly Algorithm (FFA) [15] was presented by Xin-She Yang based on flashing behavior of fireflies to attract prey or mates. Its efficacy was demonstrated through a large number of test problems, Whale Optimization Algorithm (WOA) [16] was inspired

by the bubble-net hunting to imitate the behavior of hump-back whales, where it is benchmarked on 29 well-known test functions and 6 design problems, Ant Lion Optimizer (ALO) [17] was proposed by Seyedali Mirjalili which imitates the hunting behavior of antlions and it is benchmarked on 19 mathematical functions, 3 engineering problems and two constrained problems, Gray Wolf Optimizer (GWO) [18] mimics the hunting mechanism of grey wolves in nature. It is proposed to deal with optimization tasks including 29 well-known functions and some engineering design problems to confirm its effective performance, Xin-She Yang and Suash Deb developed a nonlinear algorithm called as Cuckoo Search (CS) [19] which is introduced based on the obligate brood parasitism of cuckoo species, the attacking and migration behavior of a seagull in nature are the main inspiration of the Seagull Optimization (SO) [20] that was proposed for solving expensive problems. It was competitive when applied on 44 test functions and 7 large-scale constrained optimization problems, and Moth Flame Optimizer (MFO) [21] is a novel optimization paradigm. The navigation behavior of moths is the main inspiration of this technique which is called transverse direction. It was benchmarked on 29 test functions and 7 engineering problems. Physics-based algorithms are developed based on laws and theories of physics such as Gravitational Search (GSA) [22] was proposed by E. Rashedi et al. based on the gravity law and mass interactions. The performance of GSA is evaluated through 23 test functions. Seyedali Mirjalili proposed a new methodology based on the mathematical model of sine and cosine called Sin Cosine Algorithm (SCA) [23] and applied it on set of test functions and on the cross section of the wing of an aircraft. Human-based algorithms mimic the social behaviors of human such as Exchange Market Algorithm (EMA) [24] that was proposed by N. Ghorbani et al. based on trading the shares on stock market for solving nonlinear continuous problems. It has been applied to 12 test functions with different dimensions. Bus Transportation Algorithm (BTA) [25] was proposed based on the social behavior of humans in transportation and it is applied to solve NP-complete problem. However, the above-mentioned algorithms may suffer from balancing the exploitation and exploration phases that are very important characteristics to achieve a more accurate solution.

TSA is one of the newest swarm-based meta-heuristic algorithms which simulates the swarm behavior of tunicates during its navigation and foraging for food source, it was firstly proposed by (S. Kaur, L.K. Awasthi, G. Dhiman) [26] for non-linear constrained problems. Its performance is proved by applying it on 74 benchmark problems including different types of functions. TSA is compared with other competitive algorithms. Also, its efficiency is confirmed

by solving some constrained and unconstrained engineering design problems such as pressure vessel, welded beam, speed reducer, 25-bar truss, tension/compression spring, displacement of loaded structure and rolling element bearing. However, like the other metaheuristic approaches, TSA may prone to stuck on local optima when dealing with high-dimensional and complicated problems. Therefore, there is still room to enhance the performance of TSA to deal with large-scale and realistic practical problems such as economic dispatch problem.

The main purpose of this work is to introduce an enhanced version of TSA algorithm by adding a new searching equation to the tunicate position. We prove the performance of ETSA by applying it on 20 test functions and the obtained results are compared with traditional TSA, GWO and other algorithms from the literature. The flexibility of the suggested ETSA algorithm is proven by the investigation on the large-scale dimension of functions (i.e., 30-dimension, 50-dimension and 100- dimension). Additionally, the proposed algorithm is realized on the economic dispatch problem as one of the most important electrical problems. The statistical tests and Wilcoxon test have proved the robustness and the efficacy of the proposed algorithm. Furthermore, the computational time of the implemented algorithms has affirmed that the proposed ETSA saves time over the other algorithms for most functions.

We arranged the rest of this paper as preliminaries, motivation of this work, and traditional TSA are presented in Sect. 2. The proposed algorithm is provided in Sect. 3. Experiments and results are in Sect. 4. The investigation on the economic dispatch problem is performed in Sect. 5. Analysis and discussion are exhibited in Sect. 6. Finally, the conclusion and future work are provided in Sect. 7.

2 Preliminaries

This section introduces the mathematical model of the optimization problem, economic dispatch problem, the main motivation, and the traditional TSA.

2.1 Problem Formulation

Optimization problems can be categorized into unconstrained and constrained problems [27]. The mathematical model of them is as follows.

2.1.1 Unconstrained Problem

$$\text{Min } f(u) = f(u_1, u_2, \dots, u_k)$$

Subject to

$$u = (u_1, u_2, \dots, u_k) \in \Phi$$

$$\Phi = \{u \in R^k | LB(u_i) \leq u_i \leq UB(u_i), i = 1, 2, \dots, k\} \quad (1)$$

2.1.2 Constrained Problem

$$\text{Min } f(u) = f(u_1, u_2, \dots, u_k)$$

Subject to

$$\Phi = \{u \in R^k | LB(u_i) \leq u_i \leq UB(u_i), i = 1, 2, \dots, k\}$$

$$\Omega = \{u \in R^k | g_i(u) \leq 0, i = 1, 2, \dots, M; h_i(u) = 0, i = M + 1, \dots, P\} \quad (2)$$

We could expand the constrained problem in one equation as follows.

$$\omega(u) = f(u) + \sum_{i=1}^p \gamma_i \times (\max(0, \beta(u)))^2 \quad (3)$$

$$\beta(u) = \begin{cases} g_i(u), & i = 1, 2, \dots, M \\ |h_i(u)| - \sigma, & i = M + 1, \dots, P \end{cases}$$

where $g_i(u)$ Inequality constraint, $h_i(u)$ Equality constraints, u Vector of decision variables, LB, UB Lower and upper bounds of search space, M Inequality constraints number, P Number of all constraints, k Number of decision variables, R^k k -dimensional rectangular space, Φ Search space that shows the domains of variables, Ω The feasible part of search space, $\omega(u)$ The total expanded cost function, γ_i The penalty factor for all constraints, σ Small tolerance.

Economic dispatch problem is represented as a mathematical optimization problem with non-linear constraints. the traditional optimization techniques such as lamda iteration and gradient methods fail to give accurate results due to nonlinearities in cost function and thus the researchers have been worked on meta-heuristic algorithms to achieve more accurate solutions.

2.1.3 Economic Dispatch: Mathematical Formulation

The mathematical model of the economic dispatch problem can be defined as:

$$\text{Min}(F) = \sum_{i=1}^{ng} F_i(P_{gn_i}) \quad (4)$$

Subject to:

$$\sum_{i=1}^{ng} P_{gn_i} = PD \quad (5)$$

$$P_{gn_i}^{\min} \leq P_{gn_i} \leq P_{gn_i}^{\max} (i = 1, 2, \dots, ng) \quad (6)$$

where PD is power demand for load, P_{gn_i} is real power generation that represents the decision variables, and ng defines the number of generation buses.

Equation (4) can be expressed in two ways:

Without valve point loading effect (smooth quadratic form) and it is considered as follows.

$$F(P_{gn_i}) = \sum_{i=1}^{ng} a_i P_{gn_i}^2 + b_i P_{gn_i} + c_i \quad (7)$$

where a_i, b_i, c_i are coefficients of generation cost and P_{gn_i} the output power of the generating unit in MW for i th unit.

With valve point loading effect (non-smoothing quadratic form), and it is formulated as non-linear, non-smooth and non-convex cost function.

$$F(P_{gn_i}) = \sum_{i=1}^{ng} \left(a_i P_{gn_i}^2 + b_i P_{gn_i} + c_i + |e_i \times \sin\{f_i (P_{gn_i}^{\min} - P_{gn_i})\}| \right) \quad (8)$$

where a_i, b_i, c_i, f_i are coefficients of generation cost with valve point effect for i th unit.

2.2 Motivation of this Work

The researchers are concerned with two concepts: improving the current algorithms or making hybridizing between different techniques to enhance the convergence rate and improve the diversity of solutions. Although these techniques succeed with some problems, they suffer from some demerits of premature convergence and low diversity, especially when dealing with nonlinear problems and high-dimensional problems. For example, the TSA proved its effective performance in comparison with other different meta-heuristic algorithms but it suffers from some problems meanwhile it fails to escape from local optima while optimizing some functions. Therefore, improving its performance and increasing its searching capability present the main motivation. In this context, a novel searching equation is developed to enrich the exploitation phase while dealing with high-dimensional complicated optimization problems. By this improvement, it is planned to prevent the trapping in local optimal points, reach to the global optimum after a few iterations. The proposed algorithm has been applied to

several well-known benchmark test functions and is one of the important applications. The results illustrate that the proposed algorithm improves the solution quality and give an effective performance.

2.3 Traditional Tunicate Swarm Algorithm (TSA)

TSA is one of the latest bio-inspired meta-heuristic algorithm. Tunicates depend on two important behaviors in navigation and foraging for the food source, namely, jet propulsion and swarm intelligence. Jet propulsion is the propulsion of something in one direction, produced by ejecting a jet of fluid in the obverse direction [28]. Swarm intelligence deals with natural and artificial systems. They depend on the collective behavior between individuals of the swarm.

2.3.1 Mathematical Model of Jet Propulsion

To keep the balance of social forces between tunicates and avoid the collision between them, a random vector $\vec{T}$ is represented as follows.

$$\vec{T} = \frac{c_2 + c_3 - 2c_1}{TS_{\min} + c_1 (TS_{\max} - TS_{\min})} \quad (9)$$

where c_1, c_2, c_3 represents random values within range of $[0, 1]$.

$TS_{\min}, TS_{\max}$ are the minimum and maximum speeds of tunicates, respectively, that are responsible for social interaction, where their values are set to 1 and 4. During the navigation search of tunicates for the best one (food source), the movement towards within the search space can be performed as the following equation:

$$\overrightarrow{P_{pop}}(t+1) = \begin{cases} \overrightarrow{FP} + \vec{T} \cdot \overrightarrow{DT} & \rightarrow \text{for random} \geq 0.5 \\ \overrightarrow{FP} - \vec{T} \cdot \overrightarrow{DT} & \rightarrow \text{for random} \leq 0.5 \end{cases} \quad (10)$$

where $\overrightarrow{P_{pop}}(t+1)$ is the update position of tunicates, t denotes the iteration number, $\overrightarrow{FP}$ denotes the food source position, and $\overrightarrow{DT}$ is the distance between each tunicate and food source.

2.3.2 Mathematical Model of Tunicates Intelligence

The two initial best solutions are saved and the positions of other tunicates are updated according to the positions of saved ones. The updated positions of tunicates are represented as follows.

Fig. 1 Traditional TSA flowchart

$$\overrightarrow{P_{pop}}(t+1) = \left(\overrightarrow{P_{pop}}(t) + \overrightarrow{P_{pop}}(t+1) \right) / (2 + c_1) \quad (11)$$

The flowchart of traditional TSA is shown in Fig. 1.

3 The Enhanced Tunicate Swarm Algorithm (ETSA)

In this section, ETSA is introduced based on a novel searching equation to enhance the exploitation ability while dealing with large-scale problems and to avoid the avoid the trapping in the local optima. The main steps of the ETSA are stated as follows.

Step 1: Initialization. In this step, the algorithm' parameters, maximum number of generations (T) and population size. Also, a swarm of tunicates is initialized within the search space at random.

Step 2: Evaluation. In this step, each tunicate is evaluated according to the fitness function.

Step 3: Improvement phase. This step aims to improve the searching process of the TSA. In this sense, a dynamic perturbation is presented to improve the exploitation pattern and explore the neighborhood solutions within the search region. In the searching equation, each position is perturbed with a dynamic step, where this position is survival if it is better than the old one. In this context, the bounds of the search space are updated using a dynamic manner. The new position of ETSA can be expressed as follows.

$$P_{pop}(t+1) = P_{pop}(t) \pm \text{rand}^t \times \frac{\alpha}{2} \quad (12)$$

where α represents a dynamic step that is shrunk with the progress of the optimization process to emphasize the neighborhood searching and thus enables the exploitation ability. It is expressed as follows.

$$\alpha = \theta \times \alpha_1 + (1 - \theta) \times \alpha_2 \quad (13)$$

where θ is a random number follows the uniform distribution ranged from 0 to 1, α_1 and α_2 define the dynamic bounds that are calculated as follows.

$$\alpha_1 = \min(\overrightarrow{P_{pop}}), \alpha_2 = \max(\overrightarrow{P_{pop}}) \quad (14)$$

Step 4: Updating phase. In this step, swarm of tunicates is updated as in classical TSA using Eq. (11). In this sense, the tunicates can explore different regions with search space to perform the exploration ability and afterwards the fitness function of each tunicate is computed to update the best solution so far.

Step 5: The steps 2–4 can be repeated until the termination criterion is reached. The proposed ETSA flowchart is shown in Fig. 2, while the practical procedure of the proposed ETSA is depicted in Fig. 3.

By the means of Equation (11), more areas of the search region can be explored to perform exploration ability. On the other hand, Eq. (12) can enable the algorithm to exploit the promising regions to achieve the exploitation capability can be carried out. By this way, the balance between exploitation and exploration can be reached and thus the trapping into local optima can be avoided (Fig. 4).

Fig. 2 The proposed algorithm flowchart

4 Experiments and Results

The enhanced algorithm is applied on 20 benchmark test functions, two types of functions are used, unimodal and multimodal. The proposed ETSA is compared with the original TSA and GWO. The main details of the functions are mentioned in “Appendix A”. The different parameters for all algorithms are mentioned in Table 1. Additionally, we compared the enhanced algorithm with other methods from the literature taken from Ref. [29, 30]. The proposed algorithms

```

Initialize the population, max number of iteration
Put Best score=Inf
While t < T
  For ii=1:no of tunicates
    Compute the score of each tunicate
    If score < Best score then
      Best score=score
      Best position=position(ii)
    End
    Insert the parameters \theta, \alpha_1, \alpha_2
    Update each position according to Eq (12)
    Compute the score of each tunicate
    If new score < Best score
      Best score=new score
      Best position=new position (ii)
    End
  End
  Update each position and compute the score according to Eq (10)
  Check the boundary the updated tunicates
end
Outputs: Best score, Best position
  
```

Fig. 3 The practical procedure of the ETSA

Fig. 4 The difference between unimodal and multimodal functions

are coded with Matlab R2015b using 64 bit Core i7 processor with 2.60 GHz and 8 GB main memory.

Table 1 Algorithms parameter settings

Algorithms	Used parameters
TSA	Number of tunicates = 30, $TS_{\min} = 1$, $TS_{\max} = 4$
ETSA	Number of tunicates = 30, $TS_{\min} = 1$, $TS_{\max} = 4$
GWO	Number of search agents = 30, control parameter ($\vec{c}$) = [2, 0]
All algorithms	Total number of iterations = 1000, number of runs = 30

Table 2 Results of test functions

Function no	TSA	GWO	ETSA
P1			
Worst	4.2959E-46	6.4478E-58	0
Best	4.4231E-51	7.4651E-63	0
Median	5.1399E-48	2.5105E-59	0
Sd	8.0668E-47	1.2117E-58	0
Mean	3.1731E-47	6.2188E-59	0
P2			
Worst	3.004E-28	7.0021E-34	3.3378E-251
Best	1.5893E-30	8.8644E-36	6.5499E-270
Median	3.3519E-29	7.6231E-35	3.704E-258
Sd	7.2551E-29	1.67E-34	0
Mean	6.2722E-29	1.3597E-34	2.7539E-252
P3			
Worst	3.257E-47	2.3526E-59	0
Best	9.665E-52	1.5449E-62	0
Median	2.2949E-49	1.5478E-60	0
Sd	6.574E-48	5.4361E-60	0
Mean	2.3251E-48	3.7519E-60	0
P4			
Worst	0.04894	1.4161E-13	1.4788E-266
Best	5.2931E-6	8.1526E-16	8.3965E-246
Median	0.0014014	1.011E-14	1.5005E-235
Sd	0.011166	2.6884E-14	0
Mean	0.0068305	1.8642E-14	1.13865E-232
P5			
Worst	0.016814	0.0021579	0.0010039
Best	0.001589	0.00023103	1.2477E-05
Median	0.0054314	0.00071622	0.00017991
Sd	0.0027434	0.00042113	0.00027362
Mean	0.0057277	0.0008169	0.00025589
P6			
Worst	− 4420.1237	− 3885.7917	− 4103.874
Best	− 7398.8698	− 7273.3897	− 11,240.6037
Median	− 6299.4413	− 6087.3934	− 6755.964
Sd	679.9667	800.4601	1470.0439
Mean	− 6179.9911	− 5952.335	− 7052.1924
P7			
Worst	311.2452	0	0
Best	91.4381	0	0
Median	152.8592	0	0
Sd	45.5904	0	0
Mean	165.0582	0	0

4.1 ETSA Results on Unconstrained Benchmark Test Functions

In this subsection, the ETSA is compared with GWO, original TSA. In this sense, 30 independently runs are performed to exhibit the stability of the proposed algorithms under the inherent haphazardness nature. The results are reported by the means of best, worst, median, mean and standard deviation (sd) for each algorithm as in Table 2 (i.e. the results of unimodal functions). Tables 3 and 4 illustrate the best solutions for multimodal functions, where the best results among the compared optimizers are highlighted with bold font. We found that the proposed ETSA outperforms the algorithms for all benchmark test functions. The convergence curves of the proposed algorithm against the comparative algorithms are shown in Fig. 5.

4.2 Comparison Results with Literature Review

To satisfy the validity of the proposed ETSA, its results are compared with other algorithms from the literature. In this context, the ETSA is investigated on unimodal and multimodal functions, where the comparisons have affirmed its capability for achieving the balance between exploitation and exploration.

1) Evaluation on unimodal functions (P1–P5)

The results of unimodal functions are compared with PSO, SCA, FFA, LSCA, OPI-SCA [29] as in Table 5 which illustrates the effective performance of the proposed algorithm against other algorithms. For (P1–P4), ETSA is the best algorithm and is the second optimizer for P5. We can conclude that the proposed ETSA has the capability to exploit the promising regions as it reaches the optimal solutions for most functions of this category.

2) Evaluation of multimodal functions (P6–P20)

Tables 6 and 7 present the results of ETSA on multimodal functions against PSO, FFA, SCA, WOA, ALO, MDWA [30], the obtained results demonstrated that the proposed algorithm gives the best results for most problems whereas

Table 3 Results of test functions

Function no	TSA	GWO	ETSA
P8			
Worst	3.4542	1.8652E-14	4.4409E-15
Best	7.9936E-15	1.1546E-14	8.88E-16
Median	2.2204E-14	1.5099E-14	4.4409E-15
Sd	1.567	1.29773E-15	1.2283E-15
Mean	1.2492	1.5336E-14	3.9672E-15
P9			
Worst	0.029151	0	0
Best	0	0	0
Median	0	0	0
Sd	0.0076303	0	0
Mean	0.0049681	0	0
P10			
Worst	15.1223	0.075225	0.032051
Best	0.66518	0.0065516	0.00074692
Median	5.3015	0.02842	0.0090331
Sd	4.2486	0.016142	0.0088395
Mean	6.3434	0.03175	0.010703
P11			
Worst	4.6114	1.1437	1.7203
Best	1.542	0.16328	0.061856
Median	2.7848	0.50915	0.61481
Sd	0.67965	0.23136	0.43734
Mean	2.8074	0.50522	0.69142
P12			
Worst	0.11067	0.020363	0.022448
Best	0.00030758	0.0003075	0.0003075
Median	0.00064856	0.00030751	0.00030777
Sd	0.022639	0.0069136	0.0040133
Mean	0.010837	0.0030429	0.0013515
P13			
Worst	− 1.0316	− 1.0316	− 1.0316
Best	− 1.0316	− 1.0316	− 1.0316
Median	− 1.0316	− 1.0316	− 1.0316
Sd	0	0	0
Mean	− 1.0316	− 1.0316	− 1.0316
P14			
Worst	0.39812	0.39919	0.39789
Best	0.39789	0.39789	0.39789
Median	0.3979	0.39789	0.39789
Sd	4.4587E-05	0.00023849	9.736E-07
Mean	0.39792	0.39793	0.39789
P15			
Worst	84.0003	84	3
Best	3	3	3
Median	3	3	3
Sd	21.662	14.7885	0
Mean	12.01	5.7	3
P16			
Worst	− 3.86	− 3.86	− 3.86

Table 3 (continued)

Function no	TSA	GWO	ETSA
Best	− 3.86	− 3.86	− 3.86
Median	− 3.86	− 3.86	− 3.86
Sd	0.0014301	0.0015134	5.946E-05
Mean	− 3.86	− 3.86	− 3.86

Table 4 Results of test functions

Function no	TSA	GWO	ETSA
P17			
Worst	− 3.1989	− 3.1371	− 2.4318
Best	− 3.3213	− 3.322	− 3.3213
Median	− 3.3192	− 3.322	− 3.3213
Sd	0.060502	0.071843	0.16685
Mean	− 3.2647	− 3.277	− 3.2435
P18			
Worst	− 2.6029	− 5.0998	− 2.6287
Best	− 10.1532	− 10.1531	− 10.1532
Median	− 5.0255	− 10.1527	− 5.0552
Sd	3.177	2.0556	2.6902
Mean	− 5.7989	− 9.1422	− 6.7612
P19			
Worst	− 1.8348	− 10.4022	− 2.7375
Best	− 10.366	− 10.4029	− 10.4029
Median	− 5.043	− 10.4026	− 10.4027
Sd	3.6181	0.00019298	3.029
Mean	− 6.2549	− 10.4025	− 8.4973
P20			
Worst	− 1.8502	− 2.4217	− 2.8064
Best	− 10.4421	− 10.5364	− 10.5364
Median	− 10.3238	− 10.536	− 10.5317
Sd	3.2211	1.4814	2.7784
Mean	− 8.6184	− 10.2655	− 8.4678

MDWA is better than the ETSA for problem P6, P11 and P14. According to the reported results, it can be revealed that the ETSA is capable to avoid the local optimal points and converge to the global solutions after a few iterations.

4.3 Computational Time Analysis

The CPU-time that is consumed by all implemented techniques is reported in Table 8. We presented these results in Fig. 6. It can be concluded that the ETSA consumed less CPU-time than the classical TSA and GWO for most functions. The main reason for this can be shown as follows. The dynamic perturbation that is presented to improve the

Fig. 5 The convergence curves of the proposed algorithms

Fig. 5 (continued)

Table 5 Results of unimodal functions

Function no	ETSA	SCA	FFA	PSO	LSCA	OPI-SCA
P1						
Worst	0	3399.354	4.949222	10,315.16	1771.946	3.88E-97
Best	0	0.037629	1.103918	40.15349	15.31199	7.86E-104
Sd	0	544.9872	1.026849	3583.89	332.8945	6.12E-98
Mean	0	262.6718	2.815084	1764.488	278.9189	1.13E-98
P2						
Worst	3.3378E-251	2.057532	1.064817	122.5975	0.854306	3.61E-57
Best	6.5499E-270	0.016032	0.358356	1.73E+01	0.011837	1.52E-60
Sd	0	0.398361	0.151224	25.72122	0.181147	7.73E-58
Mean	2.7539E-252	0.320938	0.733864	63.12324	0.205554	5.53E-58
P3						
Worst	0	11,957.57	88.25227	400,422	15,682.34	4.73E-97
Best	0	2.218084	18.12803	5.91E+03	0.836482	3.68E-102
Sd	0	11,957.57	15.73419	79,608.66	4059.022	9.33E-98
Mean	0	1355.54	38.31868	102,184	2507.151	3.85E-98
P4						
Worst	1.4788E-266	41.76243	10.16375	29.36202	38.67541	1.08E-44
Best	8.3965E-246	1.323711	6.723604	9.34E+00	2.395901	7.43E-50
Sd	4.943E-228	14.01386	8.418503	19.02239	15.12672	4.54E-46
Mean	0	9.735961	0.856429	5.60214	9.891748	1.86E-45
P5						
Worst	0.0010039	0.151522	0.551507	5.436821	0.182204	4.29E-04
Best	1.2477E-05	0.002637	0.030315	4.05E-02	0.001541	1.92E-06
Sd	0.00027362	0.031739	0.158497	1.343811	0.040625	9.87E-05
Mean	0.00027074	0.034189	0.242248	0.835089	0.048203	1.24E-04

performance made the ETSA reaches the optimum values after little iterations. On the other hand, the CPU-time of ETSA for some functions is slightly larger than the other techniques. The main explanation for this can be analyzed as follows. ETSA seeks for the promising regions using two searching strategies which cause some delay. This delay can enable ETSA to give concise and more precise results with high performance than the other optimizers. All these results show the superiority and the high quality of the ETSA.

4.4 Performance Assessment

After applying the proposed ETSA on 20 unconstrained test functions and the results showed that it outperforms all other competitive algorithms. The ETSA is further investigated with high-dimensional nature to affirm its scalability and confirm its stable performance. One of the main properties of any optimizer is to solve problems of high dimensions efficiently. In this sense, the ETSA is conducted different dimensions include 30, 50, 100. By applying the proposed algorithm, the obtained results illustrate that the proposed algorithm is running efficiently and is least affected by

increasing the number dimensions, where the obtained results are reported in Table 9.

4.5 Error and Ranking Analyses

In this subsection, the proposed algorithms are further assessed in terms of deviation from the global optimal solution. In this context, the absolute error metric is presented to prove and analyze the performance of the proposed algorithms. The differences between the optimum value and the obtained best values from presented algorithms over the benchmark problems of unimodal and multimodal test functions are calculated and recorded in Tables 10 and 11, respectively. It is clear that the proposed algorithm is more precise than the other competitive algorithms and its results are very close to the optimum value. Furthermore, the ranking of the presented algorithms is performed in terms of the absolute error metric as shown in Tables 12 and 13. Moreover, the ranking level for each algorithm according to its absolute error over the test functions are recorded regarding the statistical frequency or number of appearances as in Tables 14 and 15 (i.e. see

Table 6 Results of multimodal functions

Function no	ETSA	SCA	PSO	WOA	FFA	MDWA	ALO
P6							
Best	− 1.124E+04	− 4.51E+3	− 6.81E3	− 9.02E+3	− 6.45E+3	− 1.14E+04	− 5.42E+3
Sd	1470.0439	1.77E+2	5.84E+02	4.53E+2	1.04E+3	2.56E+02	0
Median	− 6755.964	− 4.20E+3	− 5.95E3	− 8.63E+3	− 4.81E+3	− 1.12E+04	− 5.42E+3
Mean	− 7052.1924	− 4.26E+3	− 6.03E3	− 8.56E+3	− 5.03E+3	− 1.11E+04	− 5.42E+3
P7							
Best	0	9.78E-12	2.74E2	0	3.42E1	0	4.97E+01
Sd	0	4.56E-05	6.09E1	0	2.94E1	0	1.18E+01
Median	0	1.27E-05	3.31E2	0	9.68E1	0	6.96E+01
Mean	0	3.44E-05	3.40E2	0	7.83E1	0	6.71E+01
P8							
Best	8.88E-16	2.92E-12	1.57E1	8.88E-16	3.04E-1	8.88E-16	2.25E-05
Sd	1.2283E-15	8.95E+00	1.87E0	2.97E-15	1.31E0	0	8.83E-01
Median	4.4409E-15	2.01E-11	1.99E1	4.44E-15	1.55E0	8.88E-16	1.16E+00
Mean	3.9672E-15	4.00E+00	1.90E1	3.73E-15	1.64E0	8.88E-16	9.28E-01
P9							
Best	0	1.89E-08	8.93E1	0	5.97E0	0	7.43E-03
Sd	0	2.79E-03	1.20E2	2.32E-02	1.39E0	0	1.18E-02
Median	0	1.62E-07	2.02E2	0.00E+00	7.65E0	0	1.23E-02
Mean	0	1.25E-03	2.38E2	1.04E-02	7.80E0	0	1.67E-02
P10							
Best	7.469E-4	3.13E-1	5.71E7	2.63E-2	4.66E0	1.08E-3	1.27E0
Sd	8.8395E-3	1.15E-1	1.19E+08	1.21E-2	6.20E3	4.62E-2	3.64E0
Median	9.033E-3	4.11E-1	2.05E+08	4.12E-2	2.64E1	3.09E-3	2.411E0
Mean	1.07E-2	4.44E-1	1.91E+08	4.11E-2	2.47E3	2.47E-2	4.11E0
P11							
Best	6.1856E-2	3.66E-3	9.14E-04	2.01E-04	6.22E-2	9.65E-08	2.73E-6
Sd	0.43734	2.60E-3	5.55E-03	3.07E-04	4.75E-2	1.26E-06	4.36E-2
Median	0.61481	5.65E-3	7.95E-03	4.48E-04	8.44E-2	8.05E-07	3.24E-2
Mean	0.69142	5.96E-3	7.33E-03	4.75E-04	9.98E-2	1.12E-06	3.72E-2

Figs. 7 and 8). The proposed algorithm has the first rank for almost functions, therefore it can be concluded that the ETSA gives precise results and emphasis the solution quality.

4.6 Non-parametric Statistical Analysis

Due to the inherent stochastic nature of swarm optimizers, the non-parametric statistical test is vital for confirming confidential comparisons. In this sense, the Wilcoxon sign-rank is presented in this work. Wilcoxon sign-rank computes the difference between two or more algorithms based on hypothesizes. Therefore, the ability of the

proposed ETSA against the other methods are assessed based on the Wilcoxon test and the results are reported in Table 16. From these results, it can be observed that ETSA shows a superior performance over the TSA, GWO, SCA, FFA, PSO, LSCA, and ALO with a level of significance of $\alpha = 0.05$ and its competitors with OPI-SCA, WOA and MDWA.

Finally, the reality of the proposed algorithm to solve more complicated problems has been approved using the practical power economic dispatch problem.

5 Economic Dispatch Problem

The power economic dispatch problem (PED) is one of the most important problems in the power system. In this essence, it is modeled as a mathematical optimization problem along with some operational constraints. The main

Table 7 Results of fixed dimensional multimodal function

Function no	ETSA	SCA	PSO	WOA	FFA	MDWA	ALO
P12							
Best	3.07E-04	3.29E-04	1.49E-3	3.24E-04	1.45E-3	3.08E-04	1.27E-03
Sd	0.004013	3.45E-04	7.82E-03	1.29E-04	3.88E-4	5.07E-07	3.55E-02
Median	3.07E-04	6.52E-04	1.66E-3	5.73E-04	1.49E-3	3.08E-04	9.39E-03
Mean	0.0013515	7.04E-04	6.07E-3	5.75E-04	1.78E-3	3.08E-04	1.96E-02
P13							
Best	− 1.0316	− 1.031627	− 1.031625	− 1.031628	− 1.031567	− 1.031628	− 1.031628
Sd	0	3.65E-06	1.34E-4	9.49E-09	8.33E-3	8.30E-10	2.58E-1
Median	− 1.0316	− 1.031624	− 1.031610	− 1.031628	− 1.030430	− 1.031628	− 1.031628
Mean	− 1.0316	− 1.031623	− 1.031550	− 1.031628	− 1.026843	− 1.031628	− 0.950011
P14							
Best	0.39789	0.397904	0.397887	0.397887	0.397887	0.397887	0.397887
Sd	9.736E-07	6.17E-4	3.61E-06	1.34E-05	4.37E-2	8.50E-06	3.49E-12
Median	0.39789	0.398195	0.397888	0.397889	0.397889	0.397888	0.397887
Mean	0.39789	0.398373	0.397889	0.397895	0.414662	0.397892	0.397887
P15							
Best	3	3	3	3	3	3	3
Sd	0	7.33E-07	1.35E-04	7.91E-06	5.05	3.60E-15	2.91E-12
Median	3	3	3	3	3.09	3	3
Mean	3	3	3	3	6.02	3	3
P16							
Best	− 3.86	− 3.86	− 3.86	− 3.86	− 3.86	− 3.86	− 3.86
Sd	5.946E-05	2.86E-03	3.64E-03	2.30E-02	5.55E-4	3.14E-07	3.20E-02
Median	− 3.86	− 3.85	− 3.86	− 3.86	− 3.86	− 3.86	− 3.85
Mean	− 3.86	− 3.86	− 3.86	− 3.85	− 3.86	− 3.86	− 3.84
P17							
Best	− 3.3213	− 3.182057	− 3.315868	− 3.321823	− 3.163027	− 3.321784	− 3.321995
Sd	0.16685	2.06E-1	8.91E-2	1.62E-1	1.81E-1	3.77E-2	3.75E-2
Median	− 3.3213	− 3.091889	− 3.192964	− 3.304101	− 3.044423	− 3.202312	− 3.321995
Mean	− 3.2435	− 3.002330	− 3.190071	− 3.213210	− 2.983453	− 3.214372	− 3.310105
P18							
Best	− 10.1532	− 7.356572	− 9.513674	− 10.148970	− 10.114176	− 10.147296	− 10.153199
Sd	2.6902	2.28E0	2.14E0	3.69E0	2.65E0	2.48E0	3.02E0
Median	− 5.0552	− 4.721801	− 5.050597	− 5.055135	− 5.425621	− 5.089144	− 7.626985
Mean	− 6.7612	− 4.548113	− 6.563145	− 6.242722	− 6.479194	− 5.619004	− 7.375398
P19							
Best	− 10.4029	− 6.4068310	− 9.587503	− 10.379690	− 10.402683	− 10.402941	− 10.391786
Sd	3.029	5.30E-1	1.82E0	2.84E0	3.28E0	1.68E0	2.94E0
Median	− 10.4027	− 5.023479	− 5.070269	− 5.087390	− 4.816480	− 10.402941	− 7.684956
Mean	− 8.4973	− 5.299324	− 5.794037	− 6.373795	− 5.726286	− 9.871413	− 7.583082
P20							
Best	− 10.5364	− 9.571566	− 10.518471	− 10.527637	− 9.332305	− 10.536409	− 10.530969
Sd	2.7784	2.06E0	2.15E0	3.74E0	2.28E0	1.69E0	2.94E0
Median	− 10.5317	− 5.054679	− 7.398294	− 5.127739	− 6.682649	− 10.536409	− 8.718575
Mean	− 8.4678	− 5.216498	− 7.778021	− 6.225431	− 6.033611	− 10.000333	− 7.745563

Table 8 CPU time for all test functions

Function no	TSA	GWO	ETSA
P1	0.9204	0.65	0.93
P2	0.93	0.48	0.9360
P3	0.9048	0.51	0.9516
P4	0.9516	0.57	0.9672
P5	0.9360	0.53	0.9984
P6	0.1248	0.45	0.078
P7	0.1560	0.64	0.0936
P8	0.956	0.95	0.9360
P9	0.1404	0.65	0.0936
P10	0.3276	1.12	0.1716
P11	0.3120	0.998	0.2028
P12	0.2184	0.36	0.2
P13	0.1248	0.33	0.0936
P14	0.1404	0.33	0.0780
P15	0.2340	0.34	0.12
P16	0.3276	0.499	0.2184
P17	0.296	0.58	0.2496
P18	0.3432	0.577	0.2148
P19	0.5460	0.73	0.3744
P20	0.4056	0.86	0.6396

objective function of the economic dispatch problem is to achieve the optimal generated powers that minimizes the generation fuel cost with satisfying power limits and load demand.

The cost function can be represented in two ways as quadratic polynomial function when neglecting the valve point effect. In large generating units, due to the steam valve effect, any sudden increasing in losses making ripples in the fuel cost function. Therefore, there is a sinusoidal function with the traditional quadratic cost function to simulate the valve point effect of the PED. In this context, the PED is represented as a non-convex and non-smooth optimization problem having equality and inequality constraints, which makes the challenge of finding the global optimum hard. In this work, we applied the proposed algorithm on three test systems to prove its efficiency in solving non-convex and non-smooth PED. Also, the comparisons with other algorithms are presented.

5.1 Test Systems

Four test systems include 3 units, 6 units, 20 units and 118 bus system are applied to prove the capability of the proposed ETSA in solving EDP. The main data of the first three systems are taken from Refs. [31, 32] and the data of 118 bus system are taken from matpower.

5.2 Results of Economic Dispatch

- Case1: The proposed algorithms are conducted on 3 unit system with considering valve point effect, where the demand of load is 850 MW with no transmission losses. We found that the proposed algorithm gives the

Fig. 6 The CPU time for all functions in seconds by all techniques

Table 9 Results of test functions at high dimension

Function	Dimension 30	Dimension 50	Dimension 100
P1			
Worst	0	0	0
Best	0	0	0
Median	0	0	0
Sd	0	0	0
Mean	0	0	0
P2			
Worst	3.3378E-251	5.4667E-246	3.9702E-233
Best	6.5499E-270	1.1096E-265	2.1715E-254
Median	3.704E-258	4.4132E-253	1.4912E-245
Sd	0	0	0
Mean	2.7539E-252	6.1175EE-247	1.3237E-234
P3			
Worst	1.8592E-317	1.633E-266	2.991E-248
Best	0	0	0
Median	0	1.0892E-306	5.0675E-278
Sd	0	0	0
Mean	6.1977E-319	5.4434E-268	1.8351E-249
P5			
Worst	0.0010039	0.0012186	0.0010375
Best	1.2477E-05	2.2057e-05	6.896E-06
Median	0.00017991	0.00019746	0.00021288
Sd	0.00027362	0.00028284	0.00024875
Mean	0.00027074	0.00027457	0.00026761
P7			
Worst	110.3423	179.378	428.5524
Best	0	0	0
Median	0	0	0
Sd	27.4148	53.8666	107.3899
Mean	13.9758	22.5425	28.2164
P8			
Worst	4.4409E-15	4.4409E-15	4.4409E-15
Best	8.8818E-16	8.8818E-16	8.8818E-16
Median	4.4409E-15	4.4409E-15	4.4409E-15
Sd	1.2283E-15	9.0135E-16	9.0135E-16
Mean	3.9672E-15	4.204E-15	4.204E-15
P9			
Worst	0.0092496	0.19155	0.038986
Best	0	0	0
Median	0	0	0
Sd	0.0016887	0.03532	0.0090182
Mean	0.00030832	0.0088475	0.0028134

best result. The results are reported in Table 17 and the convergence curves are shown in Fig. 9.

- Case 2: In this case, 6 unit system is adopted, where the valve point effect is considered, and 283.4 MW for the

load demand is applied. Table 18 shows the results of this case and the convergence curves are shown in Fig. 10. The generation cost obtained by the proposed algorithm is 8234.0717 \$/h which is superior than the other algorithms.

- Case 3: 20 unit system without valve point effect is conducted, where the total demand power is 2500 MW. It is found that the proposed algorithm gives the minimum fuel cost and the results are shown in Table 19. The convergence curves are shown in Fig. 11.
- Case 4: 118 bus system of 54 units is solved, where the load demand is considered as 4242 MW. Table 20 shows the results of this case and the convergence curves are shown in Fig. 12. The generation cost which is obtained by the proposed algorithm is 127205.7 \$/h.

5.3 Comparison Results with Literature Review

To satisfy the robustness of the proposed algorithm in solving different applications, comparisons with other methods are reported. In this line, the proposed ETSA is compared with GWO, TSA, and parallel hurricane optimization algorithm (PHOA) [33] for 118-bus system. The results are reported in Table 20 which shows that the proposed algorithm gives the minimum fuel cost over the others.

6 Analysis and discussion

This paper introduces a new methodology by enhancing the classical TSA. The proposed Algorithm is applied on twenty function, unimodal functions (P1-P5) and multi-modal functions (P6-P20). The results show the superiority of the proposed algorithm with zero percentage error for eight functions and a very low percentage for other functions. It can be concluded from the CPU time that the suggested technique consumed less CPU time than the others for most functions due to the dynamic perturbation and the capability of reaching to the optimum solution after few iterations. The results are implemented at dimensions of 30, 50, and 100, where the ETSA is running efficiently and it is least affected by increasing the dimensions. We have evaluated the rank for each algorithm and the ETSA has the first rank for sixteen functions. We applied the ETSA on one of the major electrical applications (i.e., Economic dispatch problem). We obtained the minimum fuel cost from the four systems (i.e., 3-units,

Table 10 Absolute error for unimodal test functions

Function	TSA	GWO	PSO	FFA	SCA	LSCA	OPI-SCA	ETSA
P1	4.4231E-51	7.465E-63	40.15349	1.103918	0.037629	15.31199	7.86E-104	0
P2	1.5893E-30	8.8644E-36	1.73E+01	0.358356	0.016032	0.011837	1.52E-60	6.5499E-270
P3	9.665E-52	1.5449E-62	5.91E+03	18.12803	2.218084	0.836482	3.68E-102	0
P4	5.293E-6	8.1526E-16	9.34E+00	6.723604	1.323711	2.395901	7.43E-50	8.3965E-246
P5	1.589E-03	2.3103E-04	4.05E-02	0.030315	0.002637	1.541E-03	1.92E-06	1.2477E-5

Table 11 Absolute error for multimodal functions

Function	TSA	GWO	PSO	FFA	SCA	WOA	ALO	MDWA	ETSA
P6	5170.6302	5296.1103	5759.5	6119.5	8059.5	3549.5	7149.5	1169.5	1328.8963
P7	91.4381	0	2.74E2	3.42E1	9.78E-12	0	4.97E+01	0	0
P8	7.9936E-15	1.1546E-14	1.57E01	3.04E-01	2.92E-12	8.88E-16	2.25E-05	8.88E-16	8.88E-16
P9	0	0	8.93E1	5.97E0	1.89E-08	0	7.43E-03	0	0
P10	0.66518	0.0065516	5.71E07	4.66E0	3.13E-1	2.63E-02	1.27E0	1.08E-03	7.4692E-04
P11	1.542	0.16328	9.14E-04	6.22E-02	3.66E-03	2.01E-04	2.73E-06	9.65E-08	0.061856
P12	7.58E-06	7.5E-06	1.19E-03	1.15E-03	2.9E-05	2.4E-05	6.7E-04	8E-06	7.5E-06
P13	0	0	2.5E-05	3.3E-05	2.7E-05	2.8E-05	2.8E-05	2.8E-05	0
P14	1.1E-04	1.1E-04	1.13E-04	1.13E-04	9.6E-05	1.13E-04	1.13E-04	1.13E-04	1.1E-04
P15	0	0	0	0	0	0	0	0	0
P16	0	0	0	0	0	0	0	0	0
P17	1.3E-03	2E-03	4.132E-03	0.156978	0.137943	1.823E-03	1.995E-03	1.784E-03	1.3E-03
P18	0	1E-04	0.639526	0.039024	2.796628	4.23E-03	1E-06	5.904E-03	0
P19	0.0368	1E-04	0.815297	1.17E-04	3.995969	0.02311	0.011014	1.41E-04	1E-04
P20	0.0939	4E-04	0.017829	1.203995	0.964734	8.663E-03	5.331E-03	1.09E-04	1E-04

Table 12 Ranking of absolute error for unimodal functions

Function	TSA	GWO	PSO	FFA	SCA	LSCA	OPI-SC	ETSA
P1	4	3	8	6	5	7	2	1
P2	4	3	8	7	6	5	2	1
P3	4	3	8	7	6	5	2	1
P4	4	3	8	7	5	6	2	1
P5	5	3	7	8	6	4	1	2

6-units, 20-units, and 118-bus system). The ETSA proved its robustness with good performance for dealing with non-constrained optimization problems and economic dispatch problem.

7 Conclusion and Future Work

This paper presents a modification of Tunicate Swarm Algorithm based on a new search equation to overcome the drawbacks of the original TSA. We applied the proposed algorithm on 20 unconstrained test functions of unimodal and multimodal natures. By comparisons with GWO and traditional TSA, it is found that the ETSA performs efficiently and reaches the global optimum after a few iterations for most functions. Also the obtained results by ETSA

Table 13 Ranking of absolute error for multimodal functions

Function	TSA	GWO	PSO	FFA	SCA	WOA	ALO	MDWA	ETSA
P6	4	5	6	7	9	3	8	1	2
P7	6	1	3	4	2	1	5	1	1
P8	2	3	7	6	4	1	5	1	1
P9	1	1	5	4	2	1	3	1	1
P10	6	3	9	8	5	4	7	2	1
P11	9	8	4	7	5	3	2	1	6
P12	2	1	8	7	5	4	6	3	1
P13	1	1	2	5	3	4	4	4	1
P14	2	2	3	3	1	3	3	3	2
P15	1	1	1	1	1	1	1	1	1
P16	1	1	1	1	1	1	1	1	1
P17	1	5	6	8	7	3	4	2	1
P18	1	3	7	6	8	4	2	5	1
P19	6	1	7	2	8	5	4	3	1
P20	7	3	6	9	8	5	4	2	1

Table 14 Statistical frequency of ranking values for unimodal functions

Algorithm	RANK1	RANK2	RANK3	RANK4	RANK5	RANK6	RANK7	RANK8
TSA	0	0	0	4	1	0	0	0
GWO	0	0	5	0	0	0	0	0
PSO	0	0	0	0	0	0	1	4
FFA	0	0	0	0	0	1	3	1
SCA	0	0	0	0	2	3	0	0
LSCA	0	0	0	1	2	1	1	0
OPI-SCA	1	4	0	0	0	0	0	0
ETSA	4	1	0	0	0	0	0	0

Table 15 Statistical frequency of ranking values for unimodal functions

Algorithm	RANK1	RANK2	RANK3	RANK4	RANK5	RANK6	RANK7	RANK8	RANK9
TSA	6	3	0	1	0	3	1	0	1
GWO	7	1	4	0	2	0	0	1	0
PSO	2	1	2	1	1	3	3	1	1
FFA	2	1	1	2	1	2	3	2	1
SCA	3	2	1	1	3	0	1	3	1
WOA	5	0	4	4	2	0	0	0	0
ALO	2	2	2	4	2	1	1	1	0
MDWA	7	3	3	1	1	0	0	0	0
ETSA	12	2	0	0	0	1	0	0	0

Fig. 7 The absolute ratio of each algorithm according to its rank for unimodal functions

Fig. 8 The absolute ratio of each algorithm according to its rank for multimodal functions (P6-P20)

Table 16 Wilcoxon rank-sum test for etsa against other algorithms for unimodal functions

The proposed algorithm	The compared algorithms	<i>p</i> value
ETSA	TSA	0.0038
ETSA	GWO	0.0244
ETSA	SCA	0.002
ETSA	FFA	3.9814E-04
ETSA	PSO	6.292E-04
ETSA	LSCA	0.0079
ETSA	OPI-SCA	0.0625
ETSA	WOA	0.3223
ETSA	MDWA	0.3203
ETSA	ALO	0.0494

Table 17 Results for three units system

Generators	GWO	TSA	ETSA
PG1 (MW)	300.2585	300.265	300.2669
PG2 (MW)	400	400	400
PG3 (MW)	149.7415	149.735	149.7331
Total generation cost (\$/h)	8234.0764	8234.0728	8234.0717

Fig. 9 Convergence curves for three units system

Table 18 Results for six units system

Generators	GWO	TSA	ETSA
PG1 (MW)	199.5995	199.6033	199.5997
PG2 (MW)	20	20	20.00003
PG3 (MW)	20.3362	20.71407	20.74499
PG4 (MW)	19.32794	19.3293	19.64758
PG5 (MW)	12.10688	11.75337	11.40774
PG6 (MW)	12.03202	12	12.00002
Generation cost (\$/h)	863.7497	863.7452	863.7364

Fig. 10 Convergence curves for six units system

Table 19 Results for twenty units system

Generators	GWO	TSA	ETSA
PG1 (MW)	599.9944	598.3855	600
PG2 (MW)	142.7256	164.0541	155.5257
PG3 (MW)	50	50	50
PG4 (MW)	50	52.04896	50
PG5 (MW)	96.76485	85.20117	91.34153
PG6 (MW)	27.94259	23.12971	20
PG7 (MW)	125	125	124.9854
PG8 (MW)	51.65901	50	50
PG9 (MW)	116.9246	122.1554	109.0872
PG10 (MW)	47.98186	57.43934	41.05916
PG11 (MW)	287.4125	282.034	299.9854
PG12 (MW)	436.116	423.0309	434.1859
PG13 (MW)	120.222	120.9308	119.1325
PG14 (MW)	77.83718	71.26416	74.73315
PG15 (MW)	89.83999	95.29038	96.69243
PG16 (MW)	36.22263	36.77249	37.10569
PG17 (MW)	30	30	30
PG18 (MW)	30	30	30
PG19 (MW)	53.35668	53.26299	56.16579
PG20 (MW)	30	30	30
Total cost (\$/h)	60,223.5996	60,225.6139	60,223.1606

Fig. 11 Convergence curves for twenty units system

Table 20 Results for 118 bus system

Genera- tors	GWO	TSA	ETSA	PHOA
PG1 (MW)	5.7418	8.1447	5.9703	63.81
PG2 (MW)	66.1463	6.6296	15.4691	23.66
PG3 (MW)	75.3661	11.5791	7.4427	23.11
PG4 (MW)	8.5575	4.9035	11.1592	27.31
PG5 (MW)	177.3317	405.7800	359.1220	319.25
PG6 (MW)	165.2299	82.6545	63.9639	61.72
PG7 (MW)	24.6556	38.8866	11.7052	27.99
PG8 (MW)	52.1055	18.8635	1.3973	46.61
PG9 (MW)	8.1388	12.3760	20.5173	25.87
PG10 (MW)	16.2598	5.2345	14.5542	52.56
PG11 (MW)	220.9197	204.7956	174.7055	107.16
PG12 (MW)	243.7650	0.3221	285.1440	280.27
PG13 (MW)	73.6086	2.3952	16.1774	40.33
PG14 (MW)	77.7382	1.5246	5.4327	29.88
PG15 (MW)	52.9968	56.7618	24.4425	23.96
PG16 (MW)	7.5339	17.9261	8.9349	36.11
PG17 (MW)	16.4308	3.0260	27.9466	43.85
PG18 (MW)	29.5875	15.5897	7.9534	22.19
PG19 (MW)	36.0759	18.1418	17.8061	33.76
PG20 (MW)	2.6095	10.8503	8.9433	32.77
PG21 (MW)	59.4944	197.8143	172.4137	167.97
PG22 (MW)	48.1300	42.0884	36.9701	47.75
PG23 (MW)	39.3303	13.0539	11.7346	24.39
PG24 (MW)	44.8157	13.8725	6.4124	23.04
PG25 (MW)	48.1299	130.8418	121.5864	98.96
PG26 (MW)	128.4277	150.6685	143.7225	161.50
PG27 (MW)	26.4575	29.9646	10.8037	30.17

Table 20 (continued)

Genera- tors	GWO	TSA	ETSA	PHOA
PG28 (MW)	45.2090	371.0263	345.3932	150.31
PG29 (MW)	20.6235	332.6039	341.2114	315.74
PG30 (MW)	112.8385	473.4537	482.1010	281.08
PG31 (MW)	94.4758	4.6042	3.3158	56.71
PG32 (MW)	71.8983	3.1495	10.9696	38.12
PG33 (MW)	56.5691	2.1618	32.0803	39.42
PG34 (MW)	74.2838	15.9300	1.4451	45.02
PG35 (MW)	67.6994	11.9804	21.8439	23.66
PG36 (MW)	6.8101	13.9748	4.9184	30.74
PG37 (MW)	456.1023	459.1826	435.9206	339.36
PG38 (MW)	11.6792	24.7673	18.7138	23.09
PG39 (MW)	2.1248	4.3066	1.9499	22.89
PG40 (MW)	643.5284	550.6915	554.5390	279.94
PG41 (MW)	66.0258	34.0676	51.0610	47.99
PG42 (MW)	78.2727	10.3449	11.8432	30.55
PG43 (MW)	56.8718	32.9908	1.5107	30.92
PG44 (MW)	29.9453	19.3247	16.8032	22.34
PG45 (MW)	29.9834	256.0731	219.2810	177.04
PG46 (MW)	122.3035	14.4395	36.3473	43.57
PG47 (MW)	45.7775	19.6931	16.8633	28.00
PG48 (MW)	44.6605	24.3242	2.9554	26.95
PG49 (MW)	93.6581	3.1915	5.8472	48.01
PG50 (MW)	48.9372	13.6539	10.8333	27.38
PG51 (MW)	74.7577	27.1870	0.3829	31.33
PG52 (MW)	23.3118	1.0365	4.3768	57.91
PG53 (MW)	36.3151	4.7243	2.2998	47.97
PG54 (MW)	71.7530	8.4273	14.7617	100.00

Table 20 (continued)

Generators	GWO	TSA	ETSA	PHOA
Total cost (\$/h)	152,354.7746	129,734.3095	127,205.7	134,630.3845

Fig. 12 Convergence curves for 118-bus system

are compared other algorithms from the literature and the proposed algorithm have proved its efficiency for most functions. The impacts of large dimensions including 50 and 100 on ETSA conducted, and results affirmed that the ETSA is least affected as it gives stable results. To make sure that the proposed algorithm is qualified to run with different applications, it is applied to one of the important constrained problem, Economic Dispatch problem, through four cases, 3-unit, 6-unit, 20-unit, and 118-units. The results show the capability of the proposed algorithm in solving such problems. The major contributions of this methodology are:

1. The proposed Algorithm improves the convergence and diversity of the traditional TSA by finding the global optimum after few iterations.
2. ETSA proves its efficacy comparing with other different methods in the literature by using large-scale suits and economic dispatch problem.
3. The scalability and quality of ETSA are investigated on large-scale dimensions, and the statistical measures as well as error analysis and the Wilcoxon test have affirmed its robustness.
4. ETSA can deal with realistic applications such as economic dispatch problem.

Although the ETSA proved its robustness and efficacy in different applications, there may elapsd some times for some functions. So in future work, we concerted on developing new mutation mechanism to accelerate its performance. Also, we will focus on solving multi-objective problems, solving very large systems of economic dispatch problem and solving other important applications.

Appendix A

See Tables 21, 22, 23, 24, 25, 26, 27, 28, 29.

Table 21 Unimodal benchmark test functions

Functions details	K	Variable range	Optimum sol
$P_1 = \sum_{i=1}^k u_i^2$	30	$[-100, 100]$	0
$P_2 = \sum_{i=1}^k u_i + \prod_{i=1}^k u_i $	30	$[-10, 10]$	0
$P_3 = \sum_{i=1}^k i u_i^2$	30	$[-100, 100]$	0
$P_4 = \max_i \{ u_i , 1 \leq i \leq k\}$	30	$[-100, 100]$	0
$P_5 = \sum_{i=1}^k i u_i^4 + \text{random}[0, 1)$	30	$[-1.28, 1.28]$	0

Table 22 Multimodal benchmark test functions

Functions details	K	Variable Range	Optimum sol
$P_6 = \sum_{i=1}^k -u_i \sin \left(\sqrt{ u_i } \right)$	30	$[-500, 500]$	$-12,569.5$
$P_7 = \sum_{i=1}^k [u_i^2 - 10 \cos(2\pi u_i) + 10]$	30	$[-5.12, 5.12]$	0
$P_8 = -20 \exp \left(-0.2 \sqrt{\frac{1}{k} \sum_{i=1}^k u_i} \right) - \exp \left(\frac{1}{k} \sum_{i=1}^k \cos(2\pi u_i) \right) + 20 + e$	30	$[-32, 32]$	0
$P_9 = \frac{1}{4000} \sum_{i=1}^k u_i^2 - \prod_{i=1}^k \cos \left(\frac{u_i}{\sqrt{i}} \right) + 1$	30	$[-600, 600]$	0
$P_{10} = \frac{\pi}{k} \{10 \sin(\pi y_i)\} + \sum_{i=1}^k (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1}) + (y_k - 1)^2] + \sum_{i=1}^k s(u_i, 10, 100, 4)$	30	$[-50, 50]$	0
$y_i = 1 + \frac{u_i + 1}{4} S(u_i, a, h, q) = \begin{cases} h(x_i - a)^q & u_i > a \\ 0 & -a < u_i < a \\ h(-x_i - a)^q & u_i < -a \end{cases}$			
$P_{11} = 0.1 \{ \sin^2(3\pi u_1) + \sum_{i=1}^k (u_i - 1)^2 [1 + \sin^2(3\pi u_i + 1)] + (u_k - 1)^2 [1 + \sin^2(2\pi u_k)] \} + \sum_{i=1}^k S(u_i, 5, 100, 4)$		$[-50, 50]$	0

Table 23 Multimodal benchmark test functions with fixed dimension

Functions details	K	Variable range	Optimum sol
$P_{12} = \sum_{i=1}^k \left[A_i - \frac{u_i(B_i^2 + B_i u_2)}{(B_i^2 + B_i u_3 + u_4)} \right]$	4	$[-5, 5]$	0.00030
$P_{13} = 4u_1^2 - 2.1u_1^4 + \frac{1}{3}u_1^6 + u_1u_2 - 4u_2^2 + 4u_2^4$	2	$[-5, 5]$	-1.0316
$P_{14} = \left(u_2 - \frac{5.1}{4\pi^2} u_1^2 + \frac{5}{\pi} u_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos u_1 + 10$	2	$[-5, 10] \times [0, 15]$	0.398
$P_{15} = \left[1 + (u_1 + u_2 + 1)^2 (19 - 14u_1 + 3u_1^2 - 14u_2 + 6u_1u_2 + 3u_2^2) \right] + \left[30 + (2u_1 - 3u_2)^2 (18 - 32u_1 + 12u_1^2 + 48u_2 - 36u_1u_2 + 27u_1^2) \right]$	2	$[-5, 5]$	3
$P_{16} = \sum_{i=1}^4 C_i \exp \left(- \sum_{j=1}^3 A_{ij} (u_j - P_{ij})^2 \right)$	3	$[0, 1]$	-3.86
$P_{17} = \sum_{i=1}^4 C_i \exp \left(- \sum_{j=1}^6 A_{ij} (u_j - P_{ij})^2 \right)$	6	$[0, 1]$	-3.32
$P_{18} = \sum_{i=1}^5 C_i \left[(u - A_i)(u - A_i)^T + C_i \right]^{-1}$	4	$[0, 10]$	-10.1532
$P_{19} = \sum_{i=1}^7 C_i \left[(u - A_i)(u - A_i)^T + C_i \right]^{-1}$	4	$[0, 10]$	-10.4028
$P_{20} = \sum_{i=1}^{10} C_i \left[(u - A_i)(u - A_i)^T + C_i \right]^{-1}$	4	$[0, 10]$	-10.5363

Table 24 A_i, B_i in P_{12}

I	1	2	3	4	5	6	7	8	9	10	11
A_i	0.1957	0.1947	0.1735	0.1600	0.0844	0.0627	0.0456	0.0342	0.0342	0.0235	0.0246
B_i	0.25	0.5	1	2	4	6	8	10	12	14	16

Table 25 A_{ij}, C_i in P_{16}

I	$A_{ij}, j=1, 2, 3$			C_i
1	3	10	30	1
2	0.1	10	35	1.2
3	3	10	30	3
4	0.1	10	30	3.2

Table 26 P_{ij} in P_{16}

I	$P_{ij}, j=1, 2, 3$		
1	0.3689	0.1170	0.2673
2	0.4699	0.4387	0.7470
3	0.1091	0.8732	0.5547
4	0.03815	0.5743	0.8828

Table 27 A_{ij}, C_i in P_{17}

I	$A_{ij}, j=1, 2, 3, 4, 5, 6$						C_i
1	10	3	17	3.5	1.7	8	1
2	0.05	10	17	0.1	8	14	1.2
3	3	3.5	1.7	10	17	8	3
4	17	8	0.05	10	0.1	14	3.2

Table 28 P_{ij} in P_{17}

I	$P_{ij}, j=1, 2, 3, 4, 5, 6$					
1	0.131	0.169	0.556	0.012	0.828	0.588
2	0.232	0.413	0.830	0.373	0.100	0.999
3	0.234	0.141	0.352	0.288	0.304	0.665
4	0.404	0.882	0.873	0.574	0.109	0.038

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Declarations

Conflict of interest There is no conflict of interest.

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Table 29 A_{ij}, C_i in P_{18}, P_{19} and P_{20}

I	$A_{ij}, j=1, 2, 3, 4$				C_i
1	4	4	4	4	0.1
2	1	1	1	1	0.2
3	8	8	8	8	0.2
4	6	6	6	6	0.4
5	3	7	3	7	0.4
6	2	9	2	9	0.6
7	5	5	3	3	0.3
8	8	1	8	1	0.7
9	6	2	6	2	0.5
10	7	3.6	7	3.6	0.5

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