An Adoptable Multi-Criteria Decision-Making Analysis to Select a Best Hair Mask Product-Extended Weighted Aggregated Sum Product Assessment Method

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Abstract

Hair masks (HMs) act as one of the solutions for most of the hair problems like dandruff, frizziness, breakage, premature greying and so on. Due to its various benefits, HM products are acquiring more popularity among the individuals. As there are different varieties of HM products available in the market, the confusion arises in choosing a HM which suits the individual's hair profile and causes less side effects. Here, we have employed multi-criteria decision-making (MCDM) combined with fuzzy set theory to obtain better results. We used the extended Weighted Aggregated Sum Product Assessment (WASPAS) method based on trapezoidal interval type-2 fuzzy set (TIT2FS) in this research paper to handle vagueness and complexity in real-world problems. For determining the objective weights of the criteria, we used the entropy method of weight finding. An example of selecting a hair mask product (HMP) among four alternatives based on five criteria is provided to illustrate the applicability of the proposed method. In comparison to other MCDM methods, the approach yielded more practical results. By doing a sensitive study, the method's stability is also assessed.

Keywords MCDM · Type-2 fuzzy sets · Trapezoidal interval type-2 fuzzy set · Extended WASPAS method · Hair masks

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1 Introduction

Hair is one of the best natural gifts that everyone has. It is important to make the hair look beautiful and presentable. Human hair necessitates special attention and maintenance. Exposure to hot and cold climates, improper use of heating equipment, such as blow dryers, hair straighteners, and curlers, exposure to harmful ultraviolet sun rays, excessive washing with harsh alcohols and soaps, use of chemical treatments, excessive styling, and stress are some of the causes for hair damage. As a result, hair becomes dry and dull. Split ends, a rough texture, and an itchy scalp are also symptoms of damaged hair. Due to these factors, the demand for hair mask products (HMPs) has risen over time. Hair mask (HM) is a nourishing hair treatment for both men and women that makes hair shiner, smoother, and more nourished and moisturized. It appears mostly in paste form and strengthens the hair roots, hydrates the scalp, repairs damage and provides good results for dandruff free hair. For those with dry, impaired, frizzy, or extremely long or thin hair, HM is essential. Grey hair is now prevalent among teenagers as

well as elderly. This is due to the pollution levels, which are toxic to human hair and cause damage to the scalp and hair. HM is different from conditioners as they provide reparative treatments by penetrating deep into the hair shaft and fixing the issue whereas the latter one makes the hair softer and easier to manage. A HM should be applied after applying the shampoo and before using the conditioner and be used once in a week or twice in a week based on the condition of the hair. HM can be used for various purposes like hair growth, dandruff, dry and frizzy hair, hair fall, damaged hair and so on. These features make the HMP popular all over the world. Thus, it is important to choose a HM which suits an individual's hair profile and causes minimal side effects to the skin, the nomenclature is given in Table 1.

MCDM often entails ambiguity, which can be addressed using Zadeh's fuzzy set theory [1]. Type-2 fuzzy sets (T2FS) [2] which are the extension of type-1 fuzzy sets (T1FS) served this purpose whose membership function (MF) is given by a fuzzy set whereas in the case of T1FS, the MF is a certain value lying in $[0, 1]$. As the T2FS are three-dimensional, they are more flexible and efficient in handling uncertainty when compared to T1FS. In spite of its advantages over T1FS, the computations involving T2FS were becoming difficult to handle. So, the researchers implemented a special case of T2FS called interval type-2 fuzzy sets (IT2FS) with some modifications [3]. The MF of a IT2FS is defined in terms of crisp intervals [4, 5] and are useful for theoretical and computational studies of higher order fuzzy sets due to

their relative simplicity. Hence, IT2FS were used to solve real-world MCDM problems under uncertain conditions.

Abdullah and Zulkifli et al. [6] have employed trapezoidal interval type-2 fuzzy set (TIT2FS)-based DEMATEL method for evaluating the knowledge management criteria. Mehdi Keshavarz Ghorabae [7] have developed VIKOR method using IT2FS for selecting the industrial robots. Spearman correlation coefficient is used for analyzing the stability of the method. Hasan Dinçer et al. [8] proposed a new assessment method based on a hybrid MCDM method that integrates DEMATEL-ANP (DANP) and MOORA method and is capable of dealing with IT2FS. Keshavarz Ghorabae et al. [9] have proposed IT2FS-based COPRAS method for supplier selection problem and have used centroid method for ranking the alternatives. Zhiming Zhang and Shouhua Zhang [10] have employed trapezoidal interval type-2 soft sets in MCDM problems for selecting robots for a car company in manufacturing sector. Zhen-Song Chen et al. [11] introduced the concept of a proportional IT2FS, which can be regarded as a computational surrogate for a proportional hesitant fuzzy linguistic term sets, PHFLTSS and IT2FS have played a prominent role in the development of type-2 fuzzy logic and systems in the application of linguistic approximation transformations. Based on IT2FS, Chen and Lee [12] presented an interval type-2 fuzzy TOPSIS method for dealing with fuzzy multiple attributes group decision-making problems. Mehdi Keshavarz Ghorabae et al. [13] proposed the extended VIKOR method with IT2FS for the project selection problem. Zhou Xu et al. [14] proposed the AHPSort II sorting method in a fuzzy environment IT2FS and a new method of identifying relevant points for inferring supplier priorities, which will improve the effectiveness of vague class assignments.

Zavadskas et al. [15] proposed a method for optimizing weighted aggregated functions and estimation accuracy using WSM, WPM, and WASPAS. Shankar Chakraborty et al. [16] have employed WASPAS method in the application of choosing various tools and methods involved in manufacturing process. The stability of the method is also discussed by varying the λ values. Edmundas Zavadskas et al. [17] have employed WASPAS technique for multiple criteria assessment of alternative building designs. The robustness of the method is verified by comparing with other MCDM methods like MOORA and MULTIMOORA methods. Mehdi Keshavarz-Ghorabae et al. [18] have proposed an interval type-2 fuzzy MCDM method based on WASPAS and CRITIC for evaluating and selecting an appropriate third-party logistic provider. Edmundas Kazimieras Zavadskas et al. [19] have extended the WASPAS method for MCDM in intuitionistic fuzzy environment. The applications of ranking derelict buildings redevelopment decisions and investment alternatives are presented for evaluating the efficiency of the proposed method. Sarfaraz Hashemkhani

Table 1 Nomenclature

T1FS	Type 1 fuzzy set
T2FS	Type 2 fuzzy set
IT2FS	Interval type-2 fuzzy sets
TIT2FS	Trapezoidal interval type-2 fuzzy set
TFN	Trapezoidal fuzzy numbers
IT2FN	Interval type-2 fuzzy number
TIT2FN	Trapezoidal interval type-2 fuzzy number
MF	Membership function
UMF	Upper membership function
LMF	Lower membership function
FOU	Footprint of uncertainty
MCDM	Multi-criteria decision making
MCGDM	Multi-criteria group decision making
DM	Decision matrix
HM	Hair mask
HMP	Hair mask product
WSM	Weighted sum model
WPM	Weighted product model
WASPAS	Weighted aggregated sum product assessment
VIKOR	Vlsekriterijumska Optimizacija I Kompromisno Resenje
TOPSIS	Technique for order preference by similarity to an ideal solution

Zolfani et al. [20] have proposed SWARA and WASPAS, a hybrid MCDM model in the site selection of building shopping mall in Tehran. A hybrid method involving Best Worst Method (BWM), WASPAS and TOPSIS have been developed based on the intuitionistic fuzzy environment for resilient green supplier selection problem by Lei Xiong et al. [21]. Shima Lashgari et al. [22] have employed QSPM and WASPAS method in the development of outsourcing of healthcare services in Tehran. Darjan Karabasevic et al. [23] have developed SWARA-WASPAS-based MCDM approach for recruiting employees for a firm. Mehdi Keshavarz Ghorabae et al. [24] have employed IT2FS-based extended WASPAS method for green supplier selection problem. Hepu Deng et al. [25] have developed entropy-based objective weight finding method and have followed the modified TOPSIS method for inter-company comparison problem. Yanwei Li et al. [26] employed interval valued intuitionistic fuzzy set-based TODIM method in site selection of airport terminals. Mahdi Bitarafan et al. [27] have developed the SWARA-WASPAS method for evaluating sensors for the real-time health monitoring system of Iran bridges. An MCDM approach based on the WASPAS framework for selecting and ranking the feasible location areas for wind farms and evaluating the types of wind generators was proposed by Bagocius et al. [28]. Shankar et al. [29] employed the WASPAS method for manufacturing-based decision-making problem. Staniunas et al. [30] have used different multi-criteria decision-making methods like TOPSIS, WASPAS and COPRAS methods for finding the optimal solution for multi-dwelling modernization problem.

In the literature, various researchers have discussed about the different hair cosmetics and hair treatments done by the people and the excessive use of hair cosmetics and the side effects caused by them. Draelos [31] have discussed about the hair cleansing products particularly the variety of shampoos available in the markets, the different hair types, the chemicals used in preparing the products and the side effects caused by them. Madnani and Khan [32] have discussed about the various hair cosmetics products and their after effects. Sinclair [33] gives the definition of a healthy hair, the excessive use of heating tools and the treatments that affect the hair. The physiology, structure, and chemical composition of hair, as well as the agents used for smoothing, colouring, grooming, and cosmetic care, and their side effects have been discussed by Abraham et al. [34]. Maria Fernanda [35] have discussed about the different types of hair cosmetics used by the people, their side effects and the better treatment according to different hair types and ethnicity. Satheesha Nayak et al. [36] have focused on establishing the awareness among the Malaysian medical students about factors that affect the hair and scalp and the importance of hair care. Marek et al. [37] have discussed about the use of heavy metals, namely lead, mercury, arsenic and other toxic

metals, involved in the preparation of ayurvedic products and the side effects caused by using these products.

The novelty of the paper is that, the extended WASPAS method is employed in choosing a hair mask product which is efficient in all ways. Nowadays hair mask products are used by different age groups for nourishing and maintaining the texture of the hair. As it acts as one of the solutions for different hair problems, TIT2FS-based WASPAS method is used in choosing a suitable hair mask product which suits different hair profile and causes minimal side effects as it provides more realistic results in handling MCDM problems. TIT2FS are involved as they are more flexible in handling uncertain data when compared to T1FS.

1.1 Objective of the Research Work

The main objective of this study is to choose a HM which is suitable for various hair profiles and causes minimal damage. As there are different products available in the market, it is important to choose HM which suits an individual's hair type and contains fewer preservatives and chemicals as possible. In this research paper, we have considered extended WASPAS method under TIT2F environment. Here, we have found HMP which meets an individual's expectations and provides better results. The proposed method provides better results when compared to other MCDM methods.

1.2 Motivation of the Research Work

From the reviewed literature, the researchers have discussed about the importance of hair care, different hair cosmetics used by the people, their side effects and the various hair treatments available for different hair types. Some of them have focused on establishing the factors affecting the hair and the scalp and the importance of hair care among the students. HMs are gaining popularity among the individuals as it acts as one of the solutions for some of the major hair problems like hairfall, frizziness, dandruff, hair breakage, premature greying and so on. The difficulty lies in choosing HMP which suits the hair profile and causing minimal damage to the skin. Taking this problem into consideration, we have employed fuzzy MCGDM approach in finding a HMP which is efficient in all ways.

1.3 Contribution of the Research Work

The contributions of the research work are as follows:

- The MCDM method namely WASPAS method, which is the aggregation of two methods namely WPM and WSM, is employed in this study to provide more accuracy in decision-making process when compared to the individual methods.

- The defined arithmetic operations, defuzzification method of TIT2FS and some modifications in normalization are performed to extend the proposed method.
- An application of choosing a HMP which suits different hair profile and causes minimal damage to the skin is illustrated to show the applicability of the method.
- A sensitive analysis is performed by varying the method's parameters to demonstrate the method's reliability, and the suggested method's consistency is demonstrated by comparing the results to those of other type-2 MCDM methods.

The paper is further organized as follows: In Sect. 2, we discuss about the IT2FS and in Sect. 3, TIT2FS and its properties are discussed. In Sect. 4, the entropy method of weight finding and in Sect. 5, we introduce the extended type 2 WASPAS method for dealing with MCDM problem. In Sect. 6, we apply the proposed method in the real-life application in choosing a best hair mask product. In Sect. 7, we have made a comparison analysis and in Sect. 8, sensitive analysis of the proposed method. The conclusions are discussed in Sect. 9.

2 Preliminaries

In this section, we introduce the concept of T2FS, IT2FS, and then discuss the arithmetic operations involved in dealing with TIT2FS.

Definition 1 (*Type-2 fuzzy set*) [7] A type-2 MF, expressed as follows, describes a T2FS $\tilde{P}$ where

$$\tilde{P} = \int_{k \in K} \int_{t \in J_K} \mu_{\tilde{P}}(k, t) / (k, t), \quad (1)$$

where the domain of $\tilde{P}$ is represented by K , $\mu_{\tilde{P}}$ refers to the secondary MF of $\tilde{P}$ and $J_K \subseteq [0, 1]$ denotes the primary MF and $\int \int$ denotes the union over all k and t that are admissible.

Definition 2 (*Interval type-2 fuzzy set*) [3] A special case of T2FS where all $\mu_{\tilde{P}}(k, t) = 1$ is called a IT2FS and is represented as follows:

$$\tilde{P} = \int_{k \in K} \int_{t \in J_K} 1 / (k, t), \quad (2)$$

where $J_k \subseteq [0, 1]$.

Definition 3 (*Footprint of uncertainty*) [3] The primary membership function, which is the union of all primary memberships, has an uncertain bounded region called the

footprint of uncertainty (FOU). Upper (UMF) and lower membership functions (LMF) describes FOU and are represented in terms of T1FS.

Definition 4 (*Trapezoidal fuzzy numbers*) [11] A trapezoidal fuzzy number (TFN) $\tilde{d}$ can be defined as (d_1, d_2, d_3, d_4) has the membership function as follows:

$$\mu_{\tilde{d}}(x) = \begin{cases} 0, & x < d_1, \\ \frac{x-d_1}{d_2-d_1}, & d_1 \leq x \leq d_2, \\ 1, & d_2 \leq x \leq d_3, \\ \frac{x-d_4}{d_3-d_4}, & d_3 \leq x \leq d_4, \\ 0, & x > d_4. \end{cases} \quad (3)$$

The TFNs are represented in Fig. 1

3 Trapezoidal Interval Type-2 Fuzzy Sets

If an IT2FN's UMF and LMF are both TFN, it is referred to as a trapezoidal interval type-2 fuzzy number (TIT2FN) [12]. A $\tilde{\tilde{P}}$ TIT2FS is of the form:

$$\tilde{\tilde{P}} = (\tilde{P}^t : t \in \{u, l\}) \\ = (p_i^t; H_1(\tilde{P}^t), H_2(\tilde{P}^t) : t \in \{u, l\}, i = 1, 2, 3, 4), \quad (4)$$

where $\tilde{P}^u$ and $\tilde{P}^l$ represent the UMF and LMF of $\tilde{\tilde{P}}$ an IT2FS, $H_j(\tilde{P}^u)$ and $H_j(\tilde{P}^l)$ lies in $[0, 1]$ and $j = 1, 2$ denote the membership values of the corresponding elements p_{j+1}^u and p_{j+1}^l , respectively, as shown in Fig. 2.

3.1 Operations on Trapezoidal Interval type-2 Fuzzy Sets

Let $\tilde{\tilde{M}}$ and $\tilde{\tilde{N}}$ be two TIT2FS which are of the form,

Fig. 1 Trapezoidal fuzzy number

Fig. 2 Trapezoidal interval type-2 fuzzy number

$$\tilde{M} = \tilde{M}^t = (m_k^t; H_1(\tilde{M}^t), H_2(\tilde{M}^t)), \quad (5)$$

$$\tilde{N} = \tilde{N}^t = (n_k^t; H_1(\tilde{N}^t), H_2(\tilde{N}^t)), \quad (6)$$

where $t \in \{u, l\}, k = 1, 2, 3, 4$.

Then, operations involving TIT2FS are defined as follows [13, 24]:

$$\tilde{M} \oplus \tilde{N} = (m_1^t + n_1^t, m_2^t + n_2^t, m_3^t + n_3^t, m_4^t + n_4^t; \min(H_1(\tilde{M}^t), H_1(\tilde{N}^t)), \min(H_2(\tilde{M}^t), H_2(\tilde{N}^t))) \quad (7)$$

$$\tilde{M} \ominus \tilde{N} = (m_1^t - n_4^t, m_2^t - n_3^t, m_3^t - n_2^t, m_4^t - n_1^t; \min(H_1(\tilde{M}^t), H_1(\tilde{N}^t)), \min(H_2(\tilde{M}^t), H_2(\tilde{N}^t))) \quad (8)$$

$$\tilde{M} \otimes \tilde{N} = (X_k^t; \min(H_1(\tilde{M}^t), H_1(\tilde{N}^t)), \min(H_2(\tilde{M}^t), H_2(\tilde{N}^t))), \quad k = 1, 2, 3, 4, \quad (9)$$

$$\text{where } X_k^t = \begin{cases} \min(m_k^t n_k^t, m_k^t n_{5-k}^t, m_{5-k}^t n_k^t, m_{5-k}^t n_{5-k}^t) & \text{if } k = 1, 2, \\ \max(m_k^t n_k^t, n_k^t n_{5-k}^t, m_{5-k}^t n_k^t, m_{5-k}^t n_{5-k}^t) & \text{if } k = 3, 4 \end{cases} \quad (10)$$

$${}_v \tilde{M} = \begin{cases} (v.m_k^t; H_1(\tilde{M}^t), H_2(\tilde{M}^t), k = 1, 2, 3, 4) & \text{if } v \geq 0, \\ (v.m_{5-k}^t; H_1(\tilde{M}^t), H_2(\tilde{M}^t), k = 1, 2, 3, 4) & \text{if } v \leq 0, \end{cases} \quad (11)$$

$$\frac{\tilde{M}}{w} = \begin{cases} \left(\frac{m_k^t}{w}; H_1(\tilde{M}^t), H_2(\tilde{M}^t), k = 1, 2, 3, 4 \right) & \text{if } w > 0, \\ \left(\frac{m_{5-k}^t}{w}; H_1(\tilde{M}^t), H_2(\tilde{M}^t), k = 1, 2, 3, 4 \right) & \text{if } w < 0, \end{cases} \quad (12)$$

$$\tilde{N}^p = ((n_k^t)^p; H_1(\tilde{N}^t), H_2(\tilde{N}^t)), k = 1, 2, 3, 4, \quad (13)$$

where $t \in \{u, l\}$, v, w are crisp values and p represents the p th power of a TIT2FS.

• The **defuzzified value**(κ) of a TIT2FN is defined as follows: [13]

$$\kappa(\tilde{N}) = \frac{1}{2} \left(\sum_{t \in \{u, l\}} \frac{n_1^t + (1 + H_1(\tilde{N}^t))n_2^t + (1 + H_2(\tilde{N}^t))n_3^t + n_4^t}{4 + H_1(\tilde{N}^t) + H_2(\tilde{N}^t)} \right). \quad (14)$$

4 Algorithm of Trapezoidal Interval Type-2 Fuzzy Set-Based Entropy Method

In probability theory, the measure of uncertainty is formulated in terms of entropy method. This concept is employed in decision-making process in the determination of objective weights [25]. Let y_w denote the weight of w th criterion, B_{vw} denote the performance rating of an v th alternative on w th criteria $1 \leq v \leq b, 1 \leq w \leq c$. The procedure involved in finding the objective weight is summarized as follows:

Step 1. Normalize the performance value of each alternative as follows:

$$x_{vw} = \frac{\kappa(B_{vw})}{\sum_{v=1}^b \kappa(B_{vw})}. \quad (15)$$

Step 2. For each criterion y_w , compute the entropy values using the equation,

$$e_w = -s \sum_{v=1}^b x_{vw} \ln x_{vw}, \quad (16)$$

where $s = \frac{1}{\ln b}$ is a constant value which enables e_w to lie in

the interval $[0, 1]$

Step 3. Determine the degree of divergence of each criteria as follows:

$$d_w = 1 - e_w. \quad (17)$$

Step 4. The weight of each criterion is calculated as follows:

$$y_w = \frac{d_w}{\sum_{w=1}^c d_w}. \quad (18)$$

5 Proposed Method: Extended WASPAS Method

WASPAS (The Weighted Aggregated Sum Product Assessment) is one of the recently developed MCDM techniques introduced by Zavadskas et al [15] in 2012 which is widely used in solving problems of different nature. This method is a unique combination of two MCDM approaches, namely the Weighted Sum Model (WSM) and Weighted Product Model (WPM). Hence, this method is more efficient and gives more accuracy when compared to various other ranking methods.

In the proposed method, the criteria weights are obtained by combining the weights given by the experts and the one obtained by following the entropy method. This procedure of weight finding gives more realistic data for criteria weights in the decision-making process. As IT2FS are more flexible in handling the uncertain environment, we extended the proposed method to TIT2FS. The hierarchical structure of the proposed method is given in Fig. 3.

A MCGDM problem involving TIT2FS is described as follows: Consider a set of b alternatives, namely $G_1, G_2, \dots$,

G_b , c criteria $F_1, F_2, \dots, F_c$ and p decision-makers $E_1, E_2, \dots, E_p$. The proposed WASPAS method involves the following steps:

Step 1. The decision matrix (DM) A_q obtained from q th expert is of the form,

$$A_q = [\tilde{A}_{vwq}]_{b \times c} = \begin{bmatrix} \tilde{A}_{11q} & \tilde{A}_{12q} & \dots & \tilde{A}_{1cq} \\ \tilde{A}_{21q} & \tilde{A}_{22q} & \dots & \tilde{A}_{2cq} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{b1q} & \tilde{A}_{b2q} & \dots & \tilde{A}_{bcq} \end{bmatrix}, \quad (19)$$

where $\tilde{A}_{vwq}$ denotes the performance ratings of v th alternative on w th criterion assigned by the q th expert, $1 \leq v \leq b$, $1 \leq w \leq c$ and $1 \leq q \leq p$.

Step 2. The average DM $\bar{A}$ is computed using the expression,

$$\tilde{A}_{vw} = \left((\tilde{A}_{vw1} \oplus \tilde{A}_{vw2} \oplus \dots \oplus \tilde{A}_{vw p}) / p \right) \quad (20)$$

$$\bar{A} = [\tilde{A}_{vw}]_{b \times c}, \quad (21)$$

Fig. 3 Hierarchical structure of the extended WASPAS method

Fig. 4 Different hair problems

Hair fall

Dandruff

Premature greying

Frizzy hair

Damaged hair

where $\tilde{A}_{vw}$ denotes the average performance ratings of the v th alternative on w th criteria.

Step 3. Compute the crisp weight values (y_w^o) for each criteria using the Eqs. (15)–(18).

Step 4. Calculate the subjective weighting matrix (Y_q^s) of the q th DM, as follows:

$$Y_q^s = \left[\tilde{Y}_{wq}^s \right]_{c \times 1} = \begin{bmatrix} \tilde{Y}_{1q}^s \\ \tilde{Y}_{2q}^s \\ \vdots \\ \tilde{Y}_{cq}^s \end{bmatrix}, \quad (22)$$

where $\tilde{Y}_{wq}^s$ represents the subjective weight of w th criterion given by q th DM, $1 \leq w \leq c$ and $1 \leq q \leq p$.

Step 5. Calculate the average subjective weight $\tilde{Y}_w^s$ using the expression:

$$\tilde{Y}_w^s = \left(\left(\tilde{Y}_{w1}^s \oplus \tilde{Y}_{w2}^s \oplus \dots \oplus \tilde{Y}_{wp}^s \right) / p \right) \quad (23)$$

Step 6. Combine the subjective and crisp weight values and calculate the aggregated weight of each criteria $\tilde{Y}_w$ where,

$$\tilde{Y}_w = \beta \tilde{Y}_w^s + (1 - \beta) y_w^o \quad (24)$$

where β is the coefficient of aggregation and it takes values between 0 to 1

Step 7. The average DM is normalized following the below equations:

$$\tilde{A}_{nw} = \begin{cases} \frac{\tilde{A}_{nw}}{\max_v \kappa(\tilde{A}_{nw})} & \text{if } w \in C_1 \\ 1 - \frac{\tilde{A}_{nw}}{\max_v \kappa(\tilde{A}_{nw})} & \text{if } w \in C_2 \end{cases} \quad (25)$$

where C_1 and C_2 denotes the set of benefit and cost criteria, respectively.

Step 8. Compute the WSM ($\tilde{R}_v^{(1)}$) and WPM ($\tilde{R}_v^{(2)}$) measures for each alternative as follows:

$$\tilde{R}_v^{(1)} = \left(\tilde{Y}_1 \otimes \tilde{A}_{bv1} \right) \oplus \left(\tilde{Y}_2 \otimes \tilde{A}_{bv2} \right) \oplus \dots \oplus \left(\tilde{Y}_c \otimes \tilde{A}_{bvc} \right) \quad (26)$$

$$\tilde{R}_v^{(2)} = \left(\left(1 + \tilde{A}_{bv1} \right)^{\kappa(\tilde{Y}_1)} \right) \otimes \left(\left(1 + \tilde{A}_{bv2} \right)^{\kappa(\tilde{Y}_2)} \right) \otimes \dots \otimes \left(\left(1 + \tilde{A}_{bvc} \right)^{\kappa(\tilde{Y}_c)} \right). \quad (27)$$

Step 9. The normalized values of $\tilde{R}_v^{(1)}$ and $\tilde{R}_v^{(2)}$ are calculated using the following equations,

$$\tilde{R}_v^{(1s)} = \frac{\tilde{R}_v^{(1)}}{\max_v \kappa(\tilde{R}_v^{(1)})} \quad (28)$$

$$\tilde{R}_v^{(2s)} = \frac{\tilde{R}_v^{(2)}}{\max_v \kappa(\tilde{R}_v^{(2)})}. \quad (29)$$

Step 10. Compute the WASPAS measure by incorporating $\tilde{R}_v^{(1s)}$ and $\tilde{R}_v^{(2s)}$ as follows:

$$\tilde{R}_v = \lambda \tilde{R}_v^{(1s)} \oplus (1 - \lambda) \tilde{R}_v^{(2s)}. \quad (30)$$

Step 11. The alternatives are ranked based on the decreasing defuzzified values of $\tilde{R}_v$

6 Numerical Example

HMs are used for various purposes like treating dandruff, grey hair, damaged hair, frizzy hair and so on. The different hair problems faced by the people are represented in Fig. 4. Hair mask helps in deep nourishing, moisturizing, prevents breakage and makes the hair soft and shiny. The different hair types of an individual are presented in Fig. 5. Due its various benefits, the demand for HMP is gradually increasing. Due to its availability in different forms, care must be taken in choosing a HM which suits the specific hair type and causes less side effects. For this purpose, we consider the problem of four alternatives, namely Ayurvedic, Herbal, Natural and Organic HMP. A couple of decision-makers are consulted and they suggest five criteria for evaluating HMP. These criteria are listed as follows:

- *Hair type* (F_1) The HM should suit the different hair types, namely dry hair, greasy or oily hair, normal hair and combination hair.
- *Cost* (F_2) The cost of the HM must be affordable for all people.
- *Availability* (F_3) The HM must be available in the market
- *Eco-friendly* (F_4). The tubes, packets, tins, etc. which are used in the packing of hair masks must be decomposable and recyclable and should not affect the environment.
- *Side effects* (F_5) The compounds used in making the HM should not affect the skin.

Now let us briefly list out the alternatives considered as follows:

- *Ayurvedic Hair masks* (G_1) In terms of Ayurveda, Pitta imbalance is the main reason for hair fall and damaged hair. The ingredients used are the mixture of herbal and natural products like herbs, flowers, fruits, cooling oil and nuts. The herbs used in making these masks are good for hair health, brain health, memory, etc. Bhringraj, amala, neem are the main ingredients used in these masks as they promote hair growth, prevents hair lice and dandruff, respectively.
- *Herbal hair masks* (G_2) Herbal hair masks are made with different combinations of natural materials to incorporate properties suitable to every individual's hair profile. Herbal hair packs consist of natural ingredients like Amla, Shikakai, Bhringraj, Neem, Tulsi, Hibiscus and Brahmi. These ingredients deeply nourish and strengthen the hair, improve hair growth, control dandruff and give natural shine to the hair.
- *Natural hair masks* (G_3) Natural hair masks are made out of flowers, fruits, plants, honey, herbs, etc. These HMs can also be prepared in our homes as only natural ingredients are involved. Natural hair masks are preferred by most of the individuals as they provide better results and cause minimal damage to the skin. These masks are available according to different hair profiles of the individuals. A natural hair mask helps in softening, hydrating, shining, hair growth and fighting off infections.

Fig. 5 Hair types

Greasy hair

Dry hair

Normal Hair

Combination Hair

Fig. 6 Framework of selecting the hair mask products

Table 2 Linguistic terms and their corresponding interval type-2 fuzzy sets

Linguistic terms	Interval Type-2 fuzzy sets
Bad (B)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Poor (P)	((0,0.1,0.15,0.3;1,1),(0.05,0.1,0.15,0.2;0.9,0.9))
Fair (F)	((0.3,0.5,0.55,0.7;1,1),(0.4,0.5,0.55,0.6;0.9,0.9))
Good (G)	((0.7,0.85,0.9,1;1,1),(0.8,0.85,0.9,0.95;0.9,0.9))
Excellent (E)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

Table 3 Weight of the criteria evaluated by the decision-maker

Criteria	Decision-makers	
	E_1	E_2
F_1	E	E
F_2	G	G
F_3	G	F
F_4	G	E
F_5	F	F

Avocado, eggs, bananas, and coconut oil are some of the natural ingredients used as they provide shine, essential protein, vitamins, and protects the hair from excessive heat.

- **Organic hair masks (G_4)** These products are made from natural ingredients and are free from fertilizers, pesticides, herbicides, etc. The costs of these products are little expensive when compared to other products and

the products are not available in all markets. In recent years, these products are acquiring more popularity as these are free from chemicals and are made out of natural ingredients.

The framework of selecting a HMP is represented in Fig. 6. In the problem considered, the criteria cost and side effects are taken to be non-beneficial and the rest are considered as beneficial. The linguistic scale is given in Table 2. The weight of the criteria given by the experts E_1 and E_2 is given in Table 3. The calculative procedure is presented as follows:

Step 1. DMs A_1 and A_2 of the alternatives are given as follows:

Fig. 7 Objective weights of the criteria

Table 4 The average decision matrix

	$\tilde{A}_{vw}^u$						$\tilde{A}_{vw}^l$					
	a_{1vw}^u	a_{2vw}^u	a_{3vw}^u	a_{4vw}^u	$H_1(\tilde{A}_{vw}^u)$	$H_2(\tilde{A}_{vw}^u)$	a_{1vw}^l	a_{2vw}^l	a_{3vw}^l	a_{4vw}^l	$H_1(\tilde{A}_{vw}^l)$	$H_2(\tilde{A}_{vw}^l)$
$\tilde{A}_{11}$	0.1500	0.3000	0.3500	0.5000	1	1	0.2250	0.3000	0.3500	0.4000	0.9	0.9
$\tilde{A}_{21}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{31}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{41}$	0.3500	0.4250	0.4500	0.5500	1	1	0.4000	0.4250	0.4500	0.5000	0.9	0.9
$\tilde{A}_{12}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{22}$	0.4500	0.5000	0.5000	0.5500	1	1	0.4750	0.5000	0.5000	0.5250	0.9	0.9
$\tilde{A}_{32}$	0.3000	0.5000	0.5500	0.7000	1	1	0.4000	0.5000	0.5500	0.6000	0.9	0.9
$\tilde{A}_{42}$	0.3500	0.4250	0.4500	0.5500	1	1	0.4000	0.4250	0.4500	0.5000	0.9	0.9
$\tilde{A}_{13}$	0	0.0500	0.0750	0.2000	1	1	0.0250	0.0500	0.0750	0.1250	0.9	0.9
$\tilde{A}_{23}$	0.1500	0.3000	0.3500	0.5000	1	1	0.2250	0.3000	0.3500	0.4000	0.9	0.9
$\tilde{A}_{33}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{43}$	0	0.0500	0.0750	0.2000	1	1	0.0250	0.0500	0.0750	0.1250	0.9	0.9
$\tilde{A}_{14}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{24}$	0.5000	0.6750	0.7250	0.8500	1	1	0.6000	0.6750	0.7250	0.7750	0.9	0.9
$\tilde{A}_{34}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{44}$	0.5000	0.6750	0.7250	0.8500	1	1	0.6000	0.6750	0.7250	0.7750	0.9	0.9
$\tilde{A}_{15}$	0.8000	0.9250	0.9500	1.0000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{A}_{25}$	0.3500	0.4750	0.5250	0.6500	1	1	0.4250	0.4750	0.5250	0.5750	0.9	0.9
$\tilde{A}_{35}$	0.3000	0.5000	0.5500	0.7000	1	1	0.4000	0.5000	0.5500	0.6000	0.9	0.9
$\tilde{A}_{45}$	0.5000	0.6750	0.7250	0.8500	1	1	0.6000	0.6750	0.7250	0.7750	0.9	0.9

Table 5 The average weighting matrix

	$\tilde{y}_w^{su}$						$\tilde{y}_w^{sl}$					
	y_{1w}^{su}	y_{2w}^{su}	y_{3w}^{su}	y_{4w}^{su}	$H_1(\tilde{y}_w^{su})$	$H_2(\tilde{y}_w^{su})$	y_{1w}^{sl}	y_{2w}^{sl}	y_{3w}^{sl}	y_{4w}^{sl}	$H_1(\tilde{y}_w^{sl})$	$H_2(\tilde{y}_w^{sl})$
$\tilde{y}_1^s$	0.9000	1.0000	1.0000	1.0000	1	1	0.9500	1.0000	1.0000	1.0000	0.9	0.9
$\tilde{y}_2^s$	0.7000	0.8500	0.9000	1.000	1	1	0.8000	0.8500	0.9000	0.9500	0.9	0.9
$\tilde{y}_3^s$	0.5000	0.6750	0.7250	0.8500	1	1	0.6000	0.6750	0.7250	0.7750	0.9	0.9
$\tilde{y}_4^s$	0.8000	0.9250	0.9500	1.000	1	1	0.8750	0.9250	0.9500	0.9750	0.9	0.9
$\tilde{y}_5^s$	0.3000	0.5000	0.5500	0.7000	1	1	0.4000	0.5000	0.5500	0.6000	0.9	0.9

$$A_1 = \begin{bmatrix} P & G & B & E & G \\ G & E & F & G & P \\ E & F & G & G & F \\ G & B & P & F & G \end{bmatrix}, A_2 = \begin{bmatrix} F & E & P & G & E \\ E & B & P & F & G \\ G & F & E & E & F \\ B & G & B & G & F \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} & \tilde{A}_{14} & \tilde{A}_{15} \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{A}_{23} & \tilde{A}_{24} & \tilde{A}_{25} \\ \tilde{A}_{31} & \tilde{A}_{32} & \tilde{A}_{33} & \tilde{A}_{34} & \tilde{A}_{35} \\ \tilde{A}_{41} & \tilde{A}_{42} & \tilde{A}_{43} & \tilde{A}_{44} & \tilde{A}_{45} \end{bmatrix}$$

Step 2. The average DM $\bar{A}$ is computed following expressions (20), (21) and the results are shown in Table 4 and the matrix representation is given as follows:

Step 3: Using the Table 4 and Eqs. (15)–(18), we compute the objective weight (y_w^o) for each criterion. The weights calculated are given as follows and are represented

Table 6 The aggregated weights of the criteria

	$\tilde{y}_w^u$						$\tilde{y}_j^w$					
	y_{1w}^u	y_{2w}^u	y_{3w}^u	y_{4w}^u	$H_1(\tilde{y}_w^u)$	$H_2(\tilde{y}_w^u)$	y_{1w}^l	y_{2w}^l	y_{3w}^l	y_{4w}^l	$H_1(\tilde{y}_w^l)$	$H_2(\tilde{y}_w^l)$
$\tilde{y}_1$	0.5207	0.5706	0.5706	0.5706	1	1	0.5456	0.5706	0.5706	0.5706	0.9	0.9
$\tilde{y}_2$	0.3866	0.4616	0.4866	0.5366	1	1	0.4366	0.4616	0.4866	0.5116	0.9	0.9
$\tilde{y}_3$	0.6095	0.6970	0.7220	0.7845	1	1	0.6595	0.6970	0.7220	0.7470	0.9	0.9
$\tilde{y}_4$	0.4080	0.4705	0.4830	0.5080	1	1	0.4455	0.4705	0.4830	0.4955	0.9	0.9
$\tilde{y}_5$	0.1752	0.2752	0.3003	0.3752	1	1	0.2253	0.2752	0.3003	0.3252	0.9	0.9

Table 7 The normalized decision matrix

	$\tilde{A}_{nvw}^u$						$\tilde{A}_{nvw}^l$					
	a_{1nvw}^u	a_{2nvw}^u	a_{3nvw}^u	a_{4nvw}^u	$H_1(\tilde{A}_{nvw}^u)$	$H_2(\tilde{A}_{nvw}^u)$	a_{1nvw}^l	a_{2nvw}^l	a_{3nvw}^l	a_{4nvw}^l	$H_1(\tilde{A}_{nvw}^l)$	$H_2(\tilde{A}_{nvw}^l)$
$\tilde{A}_{n11}$	0.1614	0.3229	0.3767	0.5382	1	1	0.2422	0.3229	0.3767	0.4305	0.9	0.9
$\tilde{A}_{n21}$	0.8610	0.9956	1.0225	1.0763	1	1	0.9418	0.9956	1.0225	1.0494	0.9	0.9
$\tilde{A}_{n31}$	0.8610	0.9956	1.0225	1.0763	1	1	0.9418	0.9956	1.0225	1.0494	0.9	0.9
$\tilde{A}_{n41}$	0.3767	0.4574	0.4843	0.5920	1	1	0.4305	0.4575	0.4843	0.5382	0.9	0.9
$\tilde{A}_{n12}$	-0.0763	-0.0225	0	0.0044	1	1	-0.0494	-0.0225	0.0044	0.0582	0.9	0.9
$\tilde{A}_{n22}$	0.4080	0.4618	0.4608	0.5157	1	1	0.4349	0.4618	0.4618	0.4888	0.9	0.9
$\tilde{A}_{n32}$	0.2466	0.4080	0.4618	0.6771	1	1	0.3542	0.4080	0.4618	0.5695	0.9	0.9
$\tilde{A}_{n42}$	0.4080	0.5157	0.5426	0.6233	1	1	0.4618	0.5157	0.5426	0.5695	0.9	0.9
$\tilde{A}_{n13}$	0	0.0538	0.0807	0.2153	1	1	0.0269	0.0538	0.0807	0.1347	0.9	0.9
$\tilde{A}_{n23}$	0.1614	0.3229	0.3767	0.5382	1	1	0.2422	0.3229	0.3767	0.4305	0.9	0.9
$\tilde{A}_{n33}$	0.8610	0.9956	1.0225	1.0763	1	1	0.9418	0.9956	1.0225	1.0494	0.9	0.9
$\tilde{A}_{n43}$	0	0.0538	0.0807	0.2153	1	1	0.0269	0.0538	0.0807	0.1345	0.9	0.9
$\tilde{A}_{n14}$	0.8610	0.9956	1.0225	1.0763	1	1	0.9418	0.9956	1.0225	1.0494	0.9	0.9
$\tilde{A}_{n24}$	0.5382	0.7265	0.7803	0.9149	1	1	0.6458	0.7265	0.7803	0.8341	0.9	0.9
$\tilde{A}_{n34}$	0.8610	0.9956	1.0225	1.0763	1	1	0.9418	0.9956	1.0225	1.0494	0.9	0.9
$\tilde{A}_{n44}$	0.5382	0.7265	0.7803	0.9149	1	1	0.6458	0.7265	0.7803	0.8341	0.9	0.9
$\tilde{A}_{n15}$	-0.0763	-0.0225	0.0044	0.1390	1	1	-0.0494	-0.0225	0.0044	0.0582	0.9	0.9
$\tilde{A}_{n25}$	0.3004	0.4349	0.4888	0.6233	1	1	0.3811	0.4349	0.4888	0.5426	0.9	0.9
$\tilde{A}_{n35}$	0.2466	0.4080	0.4618	0.6771	1	1	0.3542	0.4080	0.4618	0.5695	0.9	0.9
$\tilde{A}_{n45}$	0.0851	0.2197	0.2735	0.4618	1	1	0.1659	0.2197	0.2735	0.3542	0.9	0.9

Table 8 The WSM measure of each alternative

	$\tilde{R}_v^{(1)u}$						$\tilde{R}_v^{(1)l}$					
	$r_{v1}^{(1)u}$	$r_{v2}^{(1)u}$	$r_{v3}^{(1)u}$	$r_{v4}^{(1)u}$	$H_1(\tilde{R}_v^{(1)u})$	$H_2(\tilde{R}_v^{(1)u})$	$r_{v1}^{(1)l}$	$r_{v2}^{(1)l}$	$r_{v3}^{(1)l}$	$r_{v4}^{(1)l}$	$H_1(\tilde{R}_v^{(1)l})$	$H_2(\tilde{R}_v^{(1)l})$
$\tilde{R}_1^{(1)}$	0.3658	0.6724	0.7705	1.1496	1	1	0.5280	0.6724	0.7705	0.9148	0.9	0.9
$\tilde{R}_2^{(1)}$	0.9766	1.4679	1.6038	2.0117	1	1	1.2370	1.4679	1.6038	1.7603	0.9	0.9
$\tilde{R}_3^{(1)}$	1.4629	2.0310	2.1789	2.6226	1	1	1.7889	2.0310	2.1789	2.3793	0.9	0.9
$\tilde{R}_4^{(1)}$	0.5883	0.9388	1.0576	1.4793	1	1	0.7793	0.9388	1.0576	1.2275	0.9	0.9

Table 9 The WPM measure of each alternative

	$\tilde{R}_v^{(2)u}$						$\tilde{R}_v^{(2)l}$					
	$r_{v1}^{(2)u}$	$r_{v2}^{(2)u}$	$r_{v3}^{(2)u}$	$r_{v4}^{(2)u}$	$H_1(\tilde{R}_v^{(2)u})$	$H_2(\tilde{R}_v^{(2)u})$	$r_{v1}^{(2)l}$	$r_{v2}^{(2)l}$	$r_{v3}^{(2)l}$	$r_{v4}^{(2)l}$	$H_1(\tilde{R}_v^{(2)l})$	$H_2(\tilde{R}_v^{(2)l})$
$\tilde{R}_1^{(2)}$	1.3744	1.6558	1.7706	2.2804	1	1	1.5165	1.6558	1.7706	1.9602	0.9	0.9
$\tilde{R}_2^{(2)}$	2.4489	3.0869	3.2798	3.8846	1	1	2.7871	3.0869	3.2798	3.5082	0.9	0.9
$\tilde{R}_3^{(2)}$	3.4876	4.3187	4.5476	5.2808	1	1	3.9983	4.3187	4.5476	4.9104	0.9	0.9
$\tilde{R}_4^{(2)}$	1.7657	2.1387	2.2780	2.8387	1	1	1.9719	2.1387	2.2780	2.5029	0.9	0.9

Table 10 The scaled values of WSM measure of each alternative

	$\tilde{R}_v^{(1s)u}$						$\tilde{R}_v^{(1s)l}$					
	$r_{v1}^{(1s)u}$	$r_{v2}^{(1s)u}$	$r_{v3}^{(1s)u}$	$r_{v4}^{(1s)u}$	$H_1(\tilde{R}_v^{(1s)u})$	$H_2(\tilde{R}_v^{(1s)u})$	$r_{v1}^{(1s)l}$	$r_{v2}^{(1s)l}$	$r_{v3}^{(1s)l}$	$r_{v4}^{(1s)l}$	$H_1(\tilde{R}_v^{(1s)l})$	$H_2(\tilde{R}_v^{(1s)l})$
$\tilde{R}_1^{(1s)}$	0.1749	0.3216	0.3685	0.5498	1	1	0.2525	0.3216	0.3685	0.4375	0.9	0.9
$\tilde{R}_2^{(1s)}$	0.4670	0.7020	0.7670	0.9621	1	1	0.5916	0.7020	0.7670	0.8418	0.9	0.9
$\tilde{R}_3^{(1s)}$	0.6996	0.9713	1.0420	1.2542	1	1	0.8555	0.9713	1.0420	1.1379	0.9	0.9
$\tilde{R}_4^{(1s)}$	0.2813	0.4490	0.5058	0.7075	1	1	0.3727	0.4490	0.5058	0.5870	0.9	0.9

Table 11 The scaled values of WPM measure of each alternative

	$\tilde{R}_v^{(2s)u}$						$\tilde{R}_v^{(2s)l}$					
	$r_{v1}^{(2s)u}$	$r_{v2}^{(2s)u}$	$r_{v3}^{(2s)u}$	$r_{v4}^{(2s)u}$	$H_1(\tilde{R}_v^{(2s)u})$	$H_2(\tilde{R}_v^{(2s)u})$	$r_{v1}^{(2s)l}$	$r_{v2}^{(2s)l}$	$r_{v3}^{(2s)l}$	$r_{v4}^{(2s)l}$	$H_1(\tilde{R}_v^{(2s)l})$	$H_2(\tilde{R}_v^{(2s)l})$
$\tilde{R}_1^{(2s)}$	0.3103	0.3739	0.3998	0.5149	1	1	0.3424	0.3739	0.3998	0.4426	0.9	0.9
$\tilde{R}_2^{(2s)}$	0.5530	0.6970	0.7406	0.8772	1	1	0.6293	0.6970	0.7406	0.7922	0.9	0.9
$\tilde{R}_3^{(2s)}$	0.7875	0.9752	1.0269	1.1924	1	1	0.9028	0.9752	1.0269	1.1088	0.9	0.9
$\tilde{R}_4^{(2s)}$	0.3987	0.4829	0.5144	0.6410	1	1	0.4453	0.4829	0.5144	0.5652	0.9	0.9

Table 12 The WASPAS measure of each alternative

	$\tilde{R}_v^u$						$\tilde{R}_v^l$					
	r_{v1}^u	r_{v2}^u	r_{v3}^u	r_{v4}^u	$H_1(\tilde{R}_v^u)$	$H_2(\tilde{R}_v^u)$	r_{v1}^l	r_{v2}^l	r_{v3}^l	r_{v4}^l	$H_1(\tilde{R}_v^l)$	$H_2(\tilde{R}_v^l)$
$\tilde{R}_1$	0.2426	0.3478	0.3841	0.5323	1	1	0.2974	0.3478	0.3841	0.4401	0.9	0.9
$\tilde{R}_2$	0.5100	0.6995	0.7538	0.9196	1	1	0.6104	0.6994	0.7538	0.8170	0.9	0.9
$\tilde{R}_3$	0.7435	0.9732	1.0345	1.2233	1	1	0.8792	0.9732	1.0345	1.1233	0.9	0.9
$\tilde{R}_4$	0.3400	0.4659	0.5101	0.6743	1	1	0.4090	0.4659	0.5101	0.5761	0.9	0.9

graphically in Fig. 7. $y_1^o=0.1413$, $y_2^o=0.0732$, $y_3^o=0.7190$, $y_4^o=0.0160$ and $y_5^o=0.0505$

Step 4. From the Table 3, the subjective weighting matrices Y_1^s and Y_2^s are of the form:

$$Y_1^s = \begin{bmatrix} E \\ G \\ G \\ G \\ F \end{bmatrix}, Y_2^s = \begin{bmatrix} E \\ G \\ F \\ E \\ F \end{bmatrix}$$

Table 13 The score values of $\tilde{R}_v$

i	$\kappa(\tilde{R}_v)$
1	0.3700
2	0.7224
3	1.0000
4	0.4920

Fig. 8 Graphical representation of the values based on WASPAS method

Step 5. Following Eq. (23) of the proposed method, the average subjective weights are obtained and are presented in Table 5.

Step 6. The aggregated weights of the criteria with $\beta=0.5$ are calculated using the Eq. (24) and the results obtained are shown in Table 6.

Step 7. For the Table 4, using Eq. (25) the normalized DM is calculated and are shown in Table 7.

Step 8. Following Eqs. (26), (27) the WSM and WPM measures for each alternative are calculated and the Tables 8 and 9 show the obtained results:

Step 9. Using Eqs. (28) and (29) of the proposed method, the normalized values of WSM and WPM measures are calculated and are presented in Tables 10 and 11.

Step 10. Considering $\lambda=0.5$, the WASPAS measure for each alternative is calculated based on Tables 10 and 11 using Eq. (30) of the proposed method and the results are shown in Table 12.

Step 11. The score values of each $\tilde{R}_v$ are calculated using the Eq. (14) and the ranking values are listed in the Table 13. From the obtained values, the ranking order of the alternatives is $G_3 > G_2 > G_4 > G_1$ and by the proposed method, G_3 is chosen to be the best product.

The graphical representations of these values are shown in Fig. 8. Thus, by the proposed method, G_3 is chosen as the best product as these products are made out of natural ingredients like coconut oil which helps to keep the hair moist, eggs which give the hair, a shine, lemon helps to prevent dandruff as it has anti-fungal properties and so on. These products are free from synthetic fragrance which causes irritation in the scalp and other additives. Natural hair

Table 14 Comparison results

Methods	Ranking values	Ranking order	Ranks
VIKOR	0.4527, 0.3580, 0.0001, 0.9515	$G_4 > G_1 > G_2 > G_3$	G_3
TOPSIS	0.1329, 0.5034, 0.9585, 0.2684	$G_3 > G_2 > G_4 > G_1$	G_3
COPRAS	0.4282, 0.5170, 0.5948, 0.4599	$G_3 > G_2 > G_4 > G_1$	G_3
Proposed Method	0.3700, 0.7224, 1.0000, 0.4920	$G_3 > G_2 > G_4 > G_1$	G_3

Fig. 9 Comparison analysis results

Table 15 Sensitivity analysis results

β	λ	Ranking values	Ranking order	Ranks
0.3	0.3	0.2271, 0.3970, 0.6000, 0.3241	$G_3 > G_2 > G_4 > G_1$	G_3
0.5	0.5	0.3856, 0.7889, 0.9883, 0.6145	$G_3 > G_2 > G_4 > G_1$	G_3
0.7	0.7	0.5667, 1.2715, 1.4000, 0.9831	$G_3 > G_2 > G_4 > G_1$	G_3

mask products are available in the market and can also be easily prepared in our homes using the natural ingredients and so causes minimal side effects when compared to other products.

7 Comparison Analysis

The proposed method's validity is determined by comparing the obtained results to those of the IT2FS-based extended TOPSIS [12], VIKOR [13] and COPRAS [9], Chen et al. [12] methods for the same data considered. The results obtained show that the VIKOR method shows a change in the preference order of the alternatives but the TOPSIS and COPRAS methods show no change in the ranking order when compared to the proposed method. The WASPAS method provided more realistic results and is consistent when compared to other MCDM methods. The obtained values are tabulated in Table 14 and are represented graphically in Fig. 9.

Fig. 10 Results of sensitivity analysis

8 Sensitivity Analysis

For the sensitivity analysis, we change the importance of weights of the benefit and cost criteria and also the values of the parameters β and λ to 0.3, 0.5 and 0.7. (Poor, Excellent, Bad, Fair, Good) and (Bad, Fair, Bad, Poor, Good) are the set of weights of the criteria considered for the analysis. The change in the subjective weights of the criteria and aggregation coefficients show a change in the ranking values of the alternatives and the preference order remains unaffected when compared to the results from the proposed method. The obtained results are represented in Table 15 and the graphical representations of these values are given in Fig. 10.

9 Limitations, Conclusion and Future Extensions

Dealing with complexity is one of the most difficult problems confronting DM. IT2FS has more versatility than T1FS when it comes to modelling the uncertainties of real-world problems. When compared to T2FS, the ambiguous data are represented in terms of IT2FS, which provides more flexibility in managing vagueness.

Because of its ability to improve ranking accuracy, the WASPAS approach is favoured over a number of other methods. However, the traditional WASPAS approach was unable to effectively manage uncertainty and imprecision. As a result, the IT2FS-based extended WASPAS approach was developed to deal with the uncertainty. The method's robustness is demonstrated by evaluating an application for selecting an HM product. The obtained results are more realistic and are in line with other MCDM methods. The method's stability has been improved by incorporating subjective and crisp weights of criteria. Some of its advantages include computational simplicity, a consistent and agreed logic of simultaneously considering weighted sum average and weighted product average, and compatibility with other methods.

The method can be extended to interval valued neutrosophic, hesitant, intuitionistic and pythagorean fuzzy sets. Other weight finding methods like Analytic Hierarchy Process (AHP), Best Worst Method (BWM) and Level-Based Weight Assessment (LBWA) can be involved for future research.

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Declarations

Conflict of Interest The authors declare that there is no conflict of interest regarding this research work.

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