Original Article

Supplementary span-to-depth ratio expressions for one-way slab complying with ACI-318 and SBC-304 deflection limits

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Abstract

ACI-318 provides two levels for incremental deflection control depending on the damage by large deflections of the nonstructural elements supported to one-way slabs. For these slabs, the provided deflection limit is relatively low and taken as span/480. Otherwise, the ACI-318 code provides span-to-depth ratio limitations as an alternative approach for deflection control. For the case of damage control by large deflection of nonstructural elements, the ACI-318 code does not provide limitations for the minimum slab thickness. The deflection for these slabs should be checked against ACI limits. For conceptual design, designers prefer to use the tabulated minimum thickness for slabs not supporting non-structural elements and consequently they must check for deflection. Design limitations for the minimum thickness of such slab are essential to facilitate the conceptual prediction of the slab thickness with safely expected deflection. This paper aims to establish span-to-depth ratio expressions for one-way slabs not provided by ACI-318. A parametric study is performed on one-way slabs supporting non-structural elements to study the effect of design variables on the calculated thickness. The deflection limit considered in this study is (L/480). New expressions for span-to-depth ratio incorporating design variables were developed based on the outcomes of the parametric study and ACI-318 Code deflection limits. The results obtained with the proposed expressions have been verified with the deflection limits given by the code. The predicted deflection values are below the code limits in all considered cases.

Keywords One-way slabs · Building codes · Structural concrete · Deflection control · Reinforced concrete · Serviceability

List of symbols

h_s	Slab overall thickness, mm
d	The effective depth of section, mm
L	Span length of the one-way slab, m
f_c'	Specified compressive strength of concrete, MPa
Ec	Modulus of elasticity for concrete, MPa
W_{sw}	The self-weight of the member
W_{ed}	Superimposed dead load kN/m^2
W_D	Total dead load
$W_{L.L.}$	Live load
$W_{D+L.L}$	Total applied dead and live loads
I_g	Section gross moment of inertia
I_e	Effective moment of inertia
$I_{e(D)}$	Effective moment of inertia corresponding to dead service load
$I_{e(D+L.L)}$	Effective moment of inertia corresponding to the total (dead + live) service load
λ	Long-term deflection multiplier for sustained loads
δ	Immediate deflection
δ_D	Deflection due to total dead load
$\delta_{L.L}$	Instantaneous deflection due to applied live load

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$\delta_{D+L.L}$ Deflection due to total (dead + live) load
δ_{LT} Incremental (long-term) deflection

1 Introduction

According to the damage control of the nonstructural elements supported or attached to one-way slabs by large deflections, ACI-318 [1] provides two levels for incremental deflection (long-term) limitations. The first level (without damage control of nonstructural elements), ACI-318 presents minimum thickness limits as a straightforward method for deflection control. Further, a limit of L/240 is provided as an upper bound of incremental deflection for this case. In the second level (with damage control of nonstructural elements), ACI-318 does not provide a minimum thickness limit, while provides a limit for incremental deflection of L/480.

The study described in this paper aimed to develop supplemental formulae for the span-to-depth ratio that are not provided in ACI-318 for the case of solid non-prestressed one-way slabs with damage control of the nonstructural elements. The formulae for the minimum thickness values will be consistent with the specified deflection limits and cover a broader range of design conditions than the current values. The current study is based on the ACI-318 Code's basic approach to deflection calculations. To account for cracking, the effective moment of inertia approach is used. Also, the long-time multiplier is considered to account for the time-dependent effects of creep and shrinkage.

The ACI limits of deformation were set decades ago when the design and construction technology were very different from what they are today. These limits are based on historical precedent rather than well-documented objective measures of serviceability for deflection. Warwaruk [2] provides an account of the historical development of deflection limit criteria.

The ACI-318 provisions have not changed essentially since 1971 and are impressive in simplicity. To answer these questions, many authors have raised concerns about the validity of the current provisions of ACI-318 under design conditions (Rangan [3]; Gilbert [4]; Scanlon and Choi [5]; Scanlon et al. [6]; Gardner [7]).

Scanlon and Lee [6], recommended a unified span-to-depth ratio function for deflection control in one- and two-way concrete construction. The proposed equation can be applied to one-way slabs, beams, flat plates, flat slabs, and edge-supported two-way slabs. They performed a parametric study to investigate the effects of design variables on the calculated span-to-depth ratios in comparison with the ACI-318 values. The considered span length ranged from 3.0 to 12.0 m, while live load (including partition load) ranged from 2.87 to 9.6 kN/m^2. These two ranges are wider than the common practical range as per Hasson [8]. The effect of the concrete compressive strength and steel yield strength had not been investigated. The right side of the proposed equation contains two quantities that are only known after selecting the thickness and determining the required reinforcement. This formula cannot, therefore, be used directly to calculate the minimum thickness. These two quantities are the self-weight of the element and the effective moment of inertia, depending on the dimensions and values of the reinforcements. They concluded that the proposed design equation provides slightly conservative values of the span-to-depth ratio.

Bischoff and Scanlon [9], studied the effects of the effective moment of inertia, shrinkage constraints, construction loads, sustained live loads, long-term deflection multipliers, support conditions, and deflection limits on the resulting span-to-depth ratio. They also compared the minimum thickness values specified in ACI318, which were only a function of the span and support conditions. The study concludes with some limitations of the minimum thickness requirement provided by ACI 318.

Lee et al. [10] evaluated the effect of design parameters such as span length, support condition, and applied load value on the deflection calculation. The results indicated that ACI-318 (SBC-304 [11]) provisions should be revised to account for the variety of design parameters that are common in today's practices. The parametric study discovered that while these minimum thickness values are simple to apply, the current Codes' applicability should be limited Elgohary et al. [12], compared the provisions of ACI-318 (SBC-304) to the Egyptian Code ECP-203 [13]. They concluded that the span-to-depth ratio in ACI-318 is more conservative and always results in overdesigned slab thickness. Using a larger span-to-depth ratio, as in ECP-203, results in a more efficient design with a 10% reduction in the overall self-weight of the skeleton.

Alamri [14], discussed the span-to-depth ratio provided in the SBC-304 code for solid non-prestressed one-way slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections. They considered the practical range of one-way solid slabs defined by Hasson [8]. It was observed that the provided span-to-depth ratio in SBC-304 is very conservative and not considering the effect of the main factors affecting deflection. A simple modified equation has been proposed in which the effect of concrete compressive strength and live load is included. It was also concluded that the steel yield strength does not affect the span-to-depth ratio.

El-Abbasy (2003) [15] proposed direct equations for minimum thickness of one-way solid slab design according to Egyptian Code ECP-203 [13], incorporating the code deflection limits. The proposed equations take into consideration the effect of live load, superimposed load, concrete strength, reinforcing steel yield strength, and the amount of compression steel. The results obtained using the proposed equations are more accurate compared with the conservative results obtained using the code limits.

Many researchers carried out similar studies for one-way slab span-to-depth presented in EC-2 [16]. Pérez Caldentey et al., [17] proposed a new formula of slenderness limits for concrete members based on EC-2 provisions. The proposed formula considers all design variables affecting the deflection calculation. The formula also is applicable for any deflection limit.

Marí et al. [18] derived span-to-depth formulae for deflection control considering the effect of main parameters influencing the serviceability of reinforced concrete members according to EC-2.

Pecić et al. [19], proposed an allowable span-to-depth expression for deflection control. To formulate the proposed expression, the numerical integration of curvature according to EC-2 and MC 2010 [20], has been used. The parameters considered in the proposed expression are the size of the cross-section, area, and position of both tensile and compressive reinforcement, the stress in tensile steel, concrete modulus of elasticity, tensile strength, long-term properties of concrete, and deflection limit provided in EC-2.

2 Deflection equations incorporating code limit

ACI-318 Code does not provide a minimum thickness limitation for one-way solid non-prestressed slabs supporting or attached to non-structural elements likely to be damaged by large deflections as a portion of the span. The values presented in Table 7.3.1.1, are provided for slabs without damage control of the non-structural elements. As concluded by many researchers, the slab thickness obtained using the expressions presented in Table 7.3.1.1 is very conservative [5–7, 9–11]. The thickness obtained using these relations is always very large and consequently results in an inefficient design. At the same time, the deflections predicted when using these relations are very small compared with the code limit. However, many designers prefer to use the tabulated limits for the initial determination of slab thickness for both cases of damage control.

The supplemental span-to-depth ratio for one-way non-prestressed solid slabs with damage control of non-structural elements by large deflections will be studied and new expressions will be proposed. The incremental deflection limit of (L/480) will be considered in the present work.

The deflection of a one-way slab under uniform load has the form

$$\delta = \frac{KwL^4}{384E_cI_e} \tag{1}$$

k is the deflection coefficient depending on support condition (K = 5 for simply supported, K = 1.4 for both ends continuous, K = 2 for one end continuous and K = 48 for fixed end cantilever).

For the case of the simply supported slab, the following condition for incremental deflection must be satisfied

$$\delta_{LT} = \lambda\delta_D + \delta_{L.I} \le \frac{L}{480} \tag{2}$$

In which

$$\delta_D = \frac{5w_DL^4}{384E_cI_e} \text{ and } \delta_{L.I} = \delta_{D+L.I} - \delta_D = \frac{5w_{D+L.I}L^4}{384E_cI_{e(D+L.L)}} - \frac{5w_DL^4}{384E_cI_{e(D)}}$$

Substituting into Eq. (2), considering $\lambda = 2$, the following expression is obtained:

$$\delta_{LT} = \delta_D + \delta_{D+L.I} \le \frac{L}{480} \tag{3}$$

$$\frac{5w_{D+L.I}L^4}{384E_cI_{e(D+L.L)}} + \frac{5w_DL^4}{384E_cI_{e(D)}} \le \frac{L}{480} \tag{4}$$

Substituting for $E_c = 4700\sqrt{f'_c}; I_g = \frac{bh_s^3}{12} = \frac{1000 \times h_s^3}{12}$ and $I_e = \alpha I_g$

Rearranging and solving for (L/h_s), Eq. (4) will take the following form:

$$\frac{L}{h_s} \le 39.7 \times (f'_c)^{1/6}\left(\frac{\alpha_D}{w_D} + \frac{\alpha_{D+L.I}}{w_{D+L.L}}\right)^{\frac{1}{3}} \tag{5}$$

In which α_D is the ratio between the effective moment of inertia ($I_{e(D)}$), under dead load only, and the concrete section gross moment of inertia (I_g), and $\alpha_{D+L.I}$ for the case of total load (dead + live). The dead load on the right-hand side is unknown since it's a function of the undetermined slab thickness (hs). As defined by Hasson (2020) [8], one-way slabs are most suitable for spans of 10 to 20 ft (3.0–6.0 m) (and a live load of 60 to 100 psf) (2.87 kN/m^2 to 4.79 kN/m^2). Two parametric studies have been performed on Eq. (5) using design variables covering the practical range. The span ranges from 2.0 to 7.0 m, the live

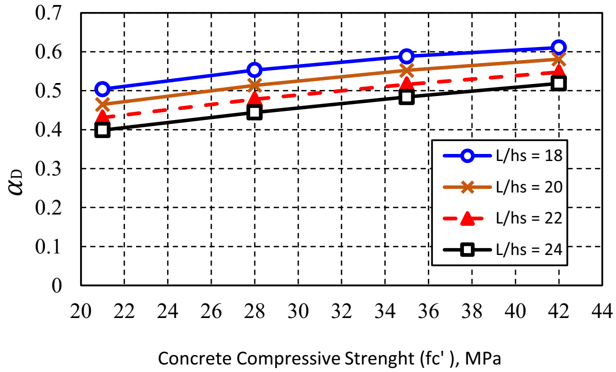

Fig. 1 Values of ratio αD

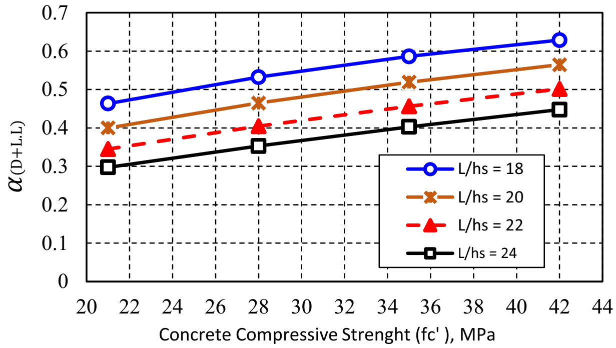

Fig. 2 Values of Ratio αD + L.L

load ranges from 2.0 kN/m^2 to 5.0 kN/m^2, and concrete compressive strength fc′ = 21, 28, 35, 42 MPa.

The first parametric study has been performed to solve for the two ratios of the effective moment for inertia, αD for dead load only, and αD + L.L for the case of total load (dead + live). Span-to-depth ratios considered in this study L/hs = 18, 20, 22, and 24. Figure 1 shows the relationship between the span-to-depth ratio and the ratio αD. The values presented are the average for the different cases of live load. The minimum value of αD = 0.4 and the maximum value = 0.64 with an average value = 0.56. Values of the ratio αD + L are presented in Fig. 2. The ratio αD + L has a minimum value of 0.3, and the maximum is 0.62 (average αD + L = 0.46).

Substituting into Eq. (5) for $\alpha_D = 0.52$, $\alpha_{D+L.L} = 0.46$; superimposed dead load = 1.5 kN/m^2; and slab-self weight $= h_s \times w_c$, the following expression can be obtained:

$$\frac{L}{h_s} \le 39.7(f_c')^{1/6}\left(\frac{0.52}{w_c\cdot L/(L/h_s)+1.5}+\frac{0.46}{w_c\cdot L/(L/h_s)+1.5+L.L}\right)^{\frac{1}{3}} \tag{6}$$

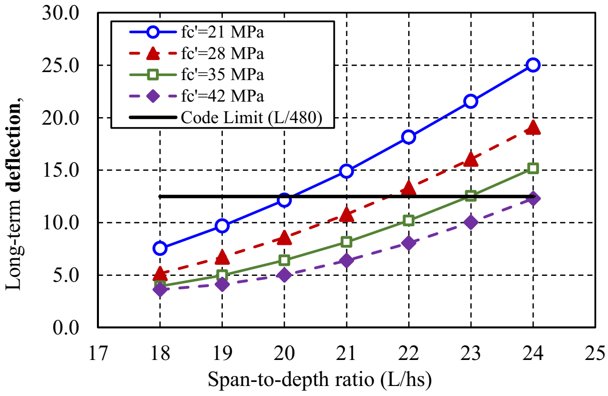

Fig. 3 Span-to-depth ratio-Deflection relationship for the case of live load = 5 kN/m^2 and span = 6 m

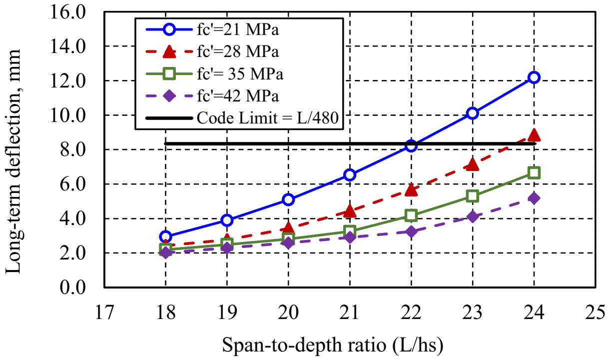

Fig. 4 Span-to-depth ratio-Deflection relationship for the case of live load = 5 kN/m^2 and span = 4 m

3 Effect of design variables on the span-to-depth ratio

To simplify the right-hand side of Eq. (6), a further parametric study has been performed to study the effect of different design variables on the span-to-depth ratio. For more details, the span-to-depth ratio has been considered as 18; 19; 20; 21; 22; 23; and 24, in this study. According to

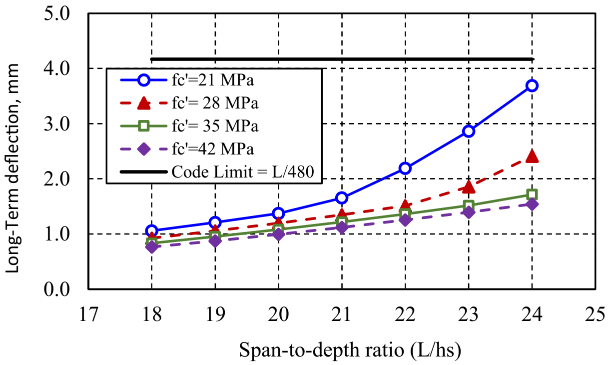

Fig. 5 Span-to-depth ratio-Deflection relationship for the case of live load = 5 kN/m^2 and span = 2 m

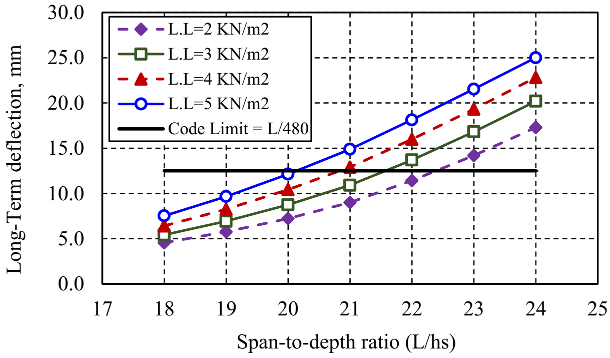

Fig. 6 Span-to-depth ratio-Deflection relationship for the case of fc′ = 21 MPa and span = 6 m

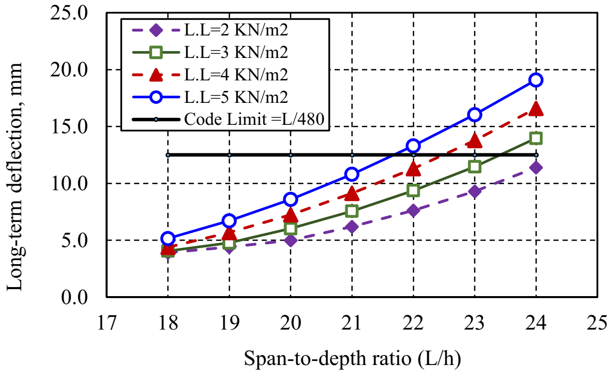

Fig. 7 Span-to-depth ratio-Deflection relationship for the case of fc′ = 28 MPa and span = 6 m

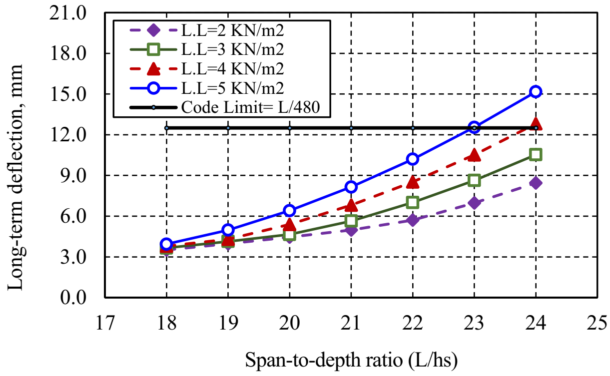

Fig. 8 Span-to-depth ratio-Deflection relationship for the case of fc′ = 35 MPa and span = 6 m

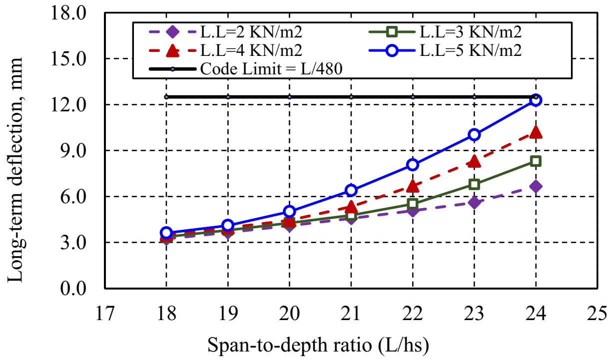

Fig. 9 Span-to-depth ratio-Deflection relationship for the case of fc′ = 42 MPa and span = 6 m

Eq. (6), the span-to-depth ratio decreases as the concrete compressive strength (fc′) decreases and the span and live load values increase.

Figures 3, 4, and 5 present relationships between incremental deflection and span-to-depth ratio for the case of live load = 5 kN/m^2, concrete compressive strength fc′ = 21, 28, 35, and 42 MPa, and span = 6, 4, and 2 m, respectively. All figures also show the ACI-318 Code's limit of deflection (L/480). These Figures enable the investigation of the effect of concrete compressive strength and slab spans on the span-to-depth ratio. The intersections of the code limit with the curves represent the accurate safe values of the span-to-depth ratio.

According to Fig. 3 (span = 6 m), for the case of concrete compressive strength fc′ = 42 MPa, the safe span-to-depth ratio is 24. While for concrete compressive strength fc′ = 35, 28, and 21 MPa, the safe span-to-depth ratio decreases to 23; 21.6, and 20, respectively. It can be concluded that with the increase of the concrete compressive strength, the safe span-to-depth ratio increases. The safe ratio in most cases is larger than 20 (case provided in ACI-318 Code for one-way slabs with deflection limit (L/240)). The efficient span-to-depth ratio for one-way slabs with damage control for nonstructural elements by large deflection, in most cases of practical design range, is larger than 20. It is obvious that with the decrease of the span length the safe span-to-depth increases (intersections of code limit with the curves in Figs. 4 and 5).

For the investigation of the live load effect on the span-to-depth ratio, Figs. 6, 7, 8, 9) are presented the relationships of span-to-depth ratio for slab span of 6.0 m, different values of the live load, and the cases of concrete compressive strength fc′ = 21, 28, 35 and 42 MPa, respectively. The safe span-to-depth ratio decreases with the increase of the live load. Also, in all cases, the span-to-depth ratio is greater than 20.

The relationships between the safe span-to-depth ratio and the effective design parameters considered in the current study are presented in Figs. 10, 11, 12. According to Fig. 10 the safe span-to-depth ratio increase with the increase of the concrete compressive strength, while it decreases with the increase of both the span and the live load as demonstrated in Figs. 11 and 12, respectively. The safe span-to-depth ratio is directly proportional to the

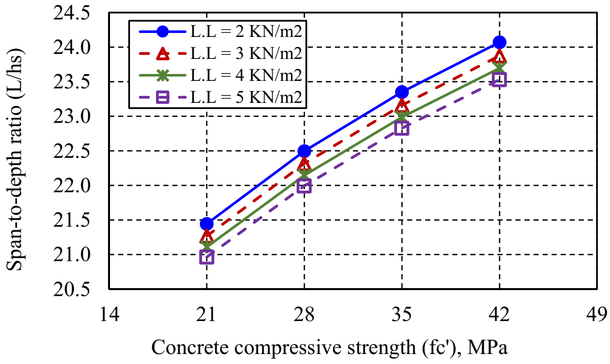

Fig. 10 Effect of concrete compressive strength on the safe span-to-depth ratio

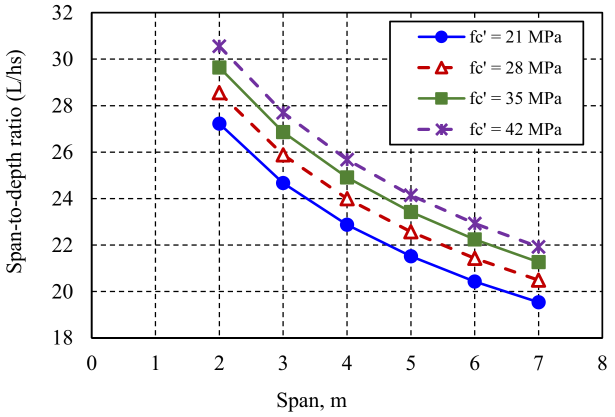

Fig. 11 Effect of span length on the safe span-to-depth ratio

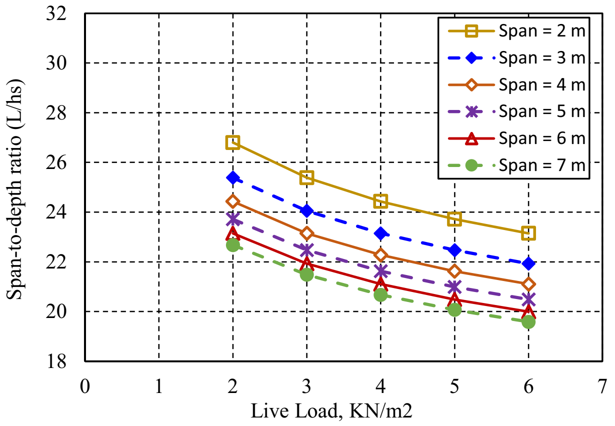

Fig. 12 Effect of live load on the safe span-to-depth ratio

concrete compressive strength and inversely proportional to the span length and the live load in a nonlinear order.

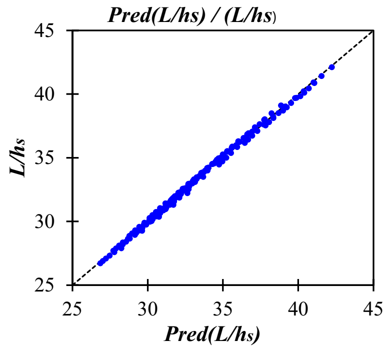

Fig. 13 Predicted span-to-depth ratio by Eq. 7 to the exact value

4 Proposed Expressions for Span-to-depth ratio

The nonlinear regression analysis [21] has been applied to the parametric study findings and the following span-to-depth ratio expression has been obtained for the case of simply supported one-way nonprestressed slabs with a deflection limit (L/480):

$$\frac{L}{h_s} = 18.5 \times \frac{\left(f_c'\right)^{1/6}}{L^{2/15} \times L.L^{2/15}} \tag{7}$$

Considering the deflection coefficient (K) depending on the supporting condition in Eq. (1), modified expressions can be obtained for different cases of supports.

For slabs continuous from one-end

$$\frac{L}{h_s} = 25 \times \frac{\left(f_c'\right)^{1/6}}{L^{2/15} \times L.L^{2/15}} \tag{8}$$

For slabs continuous from both ends

$$\frac{L}{h_s} = 28 \times \frac{\left(f_c'\right)^{1/6}}{L^{2/15} \times L.L^{2/15}} \tag{9}$$

For cantilever slabs

$$\frac{L}{h_s} = 8.5 \times \frac{\left(f_c'\right)^{1/6}}{L^{2/15} \times L.L^{2/15}} \tag{10}$$

where fc′ concrete compressive strength in MPa, L span in meters, and L.L live load in kN/m^2. The ratio of the predicted values from Eq. (7) to the outcomes of the parametric study is presented in Fig. 13, with the coefficient of determination R2 = 0.997.

5 Verification of the proposed equation results with code limit

One-way simply supported solid slabs with different span ranges from 2 to 7 m, under live load = 5 kN/m^2 with additional superimposed dead load 1.5 kN/m^2, have been designed using the proposed Eq. (7) considering four different grades of concrete compressive strength. The predicted incremental deflections (long-term deflection) are presented in Fig. 14 with the comparison with the ACI-318 code limit (L/480) for the case of one-way nonprestressed slabs supporting or attached to non-structural elements likely to be damaged by large deflections. All predicted deflections are within the code limit with sufficient safety factor (deflection limit/predicted deflection) ranging from 1.03 to 2.28 (average safety factor = 1.6).

The minimum values of the span-to-depth ratio can be received when concrete compressive strength fc′ = 21 MPa, span 6.0 m and live load = 5.0 kN/m^2. These values can be recommended to be added to ACI-318 Table 7.3.1.1 for the case of slabs supporting or attached to non-structural elements likely to be damaged by large deflections. The recommended values are presented in Table 1.

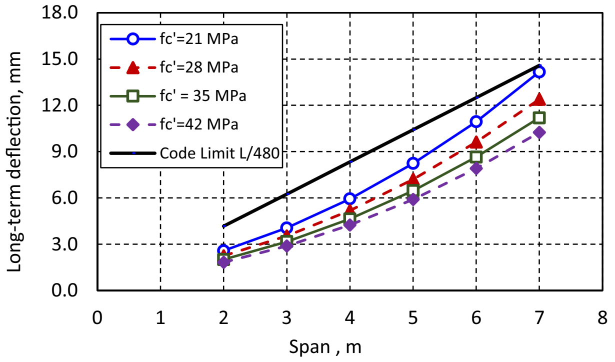

Fig. 14 Verification of Results

Table 1 Minimum thickness of solid nonprestressed one-way slabs supporting or attached to non-structural elements likely to be damaged by large deflections

Support condition	Minimum hs
Simply supported	L/20
One end continuous	L/26
Both ends continuous	L/30
Cantilever	L/9.5

6 Conclusions

The span-to-depth ratio for one-way solid nonprestressed slabs supporting or attached to non-structural elements likely to be damaged by large deflections is not provided in ACI-318 Code. The current study investigates the effect of different design variables on the span-to-depth ratio considering the deflection limit (L/480). New direct simple formulae have been recommended for the calculation of the span-to-depth ratio for these slabs, considering the different boundary conditions and the main factors affecting the design of the slabs. Verification of design using the recommended formulae, shows that the predicted deflections are within the ACI-318 code limits. Tabulated values for the span-to-depth ratio are also recommended for the design of the considered type of one-way slab.

7 Research significance

The research results in this article can be directly transferred to engineering practices. Simple equations are presented as supplementary limitations for one-way slabs not covered by ACI tabulated cases. The new minimum thickness values provided by the proposed equations have been evaluated with the deflection limits specified by the ACI-318 code and show good agreement.

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Availability of data and materials Data beyond what was provided in the article can, on a case-by-case basis, be made available to others on request to the corresponding author.

Declarations

Conflict of interest The author declares no conflict of interest.

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