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ORIGINAL PAPER

Metacognitive strategies in solving mathematical word problems: a case of Rwandan primary school learners

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Abstract
This study aims to understand the use of metacognitive skills by Rwandan learners while solving mathematical word problems. We interviewed and assessed third-, fourth- and fifth-grade learners from a public primary school. The following three points emerged. First, the metacognitive skills of learners with correct answers were considerably higher than that of those with incorrect answers. Second, although there was no considerable difference in metacognitive skills between learners who answered correctly and those who did not at the stage of 'understand the problem', considerable differences were observed in the 'search for solving methods' and 'execute the solving methods' and 'examine the answer' stages. During the 'search for solving methods' and 'execute the solving methods' stage, learners who answered correctly mainly used three metacognitive skills to control their learning—'writing the process by sentences', 'drawing tables' and 'drawing pictograms'. Third, when metacognitive skills were measured and scored, the average scores for fifth and third graders were similar. The interview revealed that the teachers of third graders taught them metacognitive strategies in mathematics lessons. It can be inferred that consequently, the metacognitive skills of third graders were raised to be as high as those of fifth graders. Although this is only a single empirical study in Rwanda, it is a major step towards improving the standard of mathematics education in African countries. In the future, similar research must be conducted in other African countries to accumulate relevant research results.

Keywords Metacognition · Metacognitive skill in mathematics · Mathematical word problem · Monitoring · Rwanda

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Introduction

Metacognition is a cognitive psychological concept, and several physical and practical studies have been conducted about it since the 1970s. Recently, the concept of metacognition has gained attention and has been actively discussed as an indispensable aspect of learning (Stillman and Mevarech 2010; Güner and Erbay 2021). For example, in the 2001 revised edition of the internationally established Bloom's Taxonomy, 'metacognitive knowledge' is established as a new dimension of knowledge and is ranked extremely high. In addition, the Centre for Curriculum Redesign (CCR) states that global frameworks, such as twenty-first-century skills and key competencies, have several commonalities, including metalearning strategies, which are posited as the fourth dimension of education and learning. This concept corresponds to the skill called 'learning to learn' in the European Union's 'Key Competencies for Lifelong Learning' (European Commission 2007) and the ATC21S's '21st Century Skills' (Griffin et al. 2012). Furthermore, the Organisation for Economic Co-operation and Development (OECD) encourages prioritising metacognitive skills. Metacognitive skills are an increasingly critical competency for individuals in a world enduring the effects of globalisation, climate change and technological advances, which will require individuals to acquire new knowledge and skills for jobs fundamentally altered or not yet invented (Horvathova 2019; OECD 2019). In an increasingly volatile and uncertain world, it can be stated with certainty that metacognitive skills are indispensable.

Metacognition began garnering attention in mathematics education through research on the relationship between problem-solving and metacognition (Schoenfeld 1983). When researching problem-solving, the focus has only been on the knowledge and skills that directly influence the process of solving the problem, and little attention has been given to the intellectual functions that regulate them. However, when solving problems, teachers tend to encourage their learners to read and point out known information, predict results, reflect on problem-solving processes and think of other solution methods. This, consciously or unconsciously, enables learners to use metacognition. Regarding the relationship between metacognition and academic abilities, various countries have indicated that children with high mathematical skills also have high cognitive abilities (Okamoto 1992; Chytry 2020). Additionally, mathematical problem-solving can be facilitated and supported using metacognitive activities (Borkowski et al. 2000; Tohir 2019). For example, Desoete and Roeyers (2002) reported that test scores increased significantly in the third grade of elementary school because learners received guidance on metacognitive strategies, such as predicting whether a problem could be solved and self-evaluating answers. Similarly, Dignath and Buettner (2008) reported that encouraging learners to practise metacognition leads to a significant positive effect. However, such studies are predominant in Western countries. William and Maat (2020) surveyed 31 articles published between the year 2017 and 2020 from 2 known databases: ERIC and Scopus. All the articles focused on developed countries' case studies. In developing countries, particularly in Africa, the situation remains largely unclear. As improving the quality of education is an urgent issue among the Sustainable Development Goals and Education 2030, it is important to have a good understanding of metacognitive skills as they are regarded as indispensable in learning in developed and developing countries.

Objectives

There are two main objectives in this study. One is to analyse the metacognitive skills of Rwandan third, fourth and fifth graders solving mathematical word problems. The second is to analyse the influence of metacognitive abilities on problem-solving abilities.

Literature review

Definition of metacognition

The term metacognition was first broadly defined by Flavell (1979) as ‘cognition about cognition’. Brown (1987) classified metacognition into two categories: ‘knowledge of cognition’ and ‘regulation of cognition’. ‘Knowledge of cognition’ is the activity of consciously reflecting upon a cognitive activity and is similar to Flavell’s idea of metacognition. ‘Regulation of cognition’ comprises three processes: ‘planning’, ‘monitoring’ and ‘regulation’. ‘Planning’ refers to devising a plan for how to solve a problem before attempting to do so, and ‘monitoring’ is about examining and observing the solving method while attempting to solve the problem. Based on the ‘monitoring’ results, the ‘regulation’ aspect evaluates and modifies the methods and plans used.

Developmental stages of metacognitive skill

Regarding the relationship between metacognitive development and age, Mevarech (1995) demonstrated that kindergarteners use metacognitive knowledge when solving mathematical problems. Further, Shamir et al. (2009) reported that kindergarteners could recognise the method they used for memorisation tasks and share it with their peers. Whitebread and Coltman (2010) noted that infants (3 years or older) engage in metacognition when performing non-verbal and unconscious activities. Previous studies have demonstrated that as children grow older, their metacognitive skills develop along with their intellectual abilities (Berk 2003; Merchán Garzón et al. 2020). Most adults have metacognitive knowledge, can plan according to situations and attempt to resolve the situation (Schraw et al. 2006). Furthermore, among metacognitive skills, monitoring, which is used in the process of problem-solving, and evaluation, which occurs after solving a problem, comprise skills that develop later than the skill of planning a solving method. This is because children are not involved in such processes at school. Kramarski et al. (2010) reported that 8-year-olds are good at planning when solving problems, but they are ineffective at monitoring during their problem-solving process. Regarding the effectiveness of the educational intervention, Hattie et al. (1996) pointed out that guidance on metacognitive strategies is more effective when given to younger people. Similarly, Dignath and Buettner (2008) provided instructions on metacognitive strategies for learners in grades 1–12 and measured the effect size. The results showed that primary school learners scored 0.61 times above the standard deviation, and secondary school learners scored 0.54 times above the deviation. Thus, they clarified that the effect size of metacognitive intervention was larger for primary school learners than for secondary school ones.

Metacognitive awareness and visualisation

Visualisation, called the representational view of the mind, integrates the mental processes of visual imagery, memory, processing, relationships, attention and imagination (Makina 2010). The use of visualisation supports equity, engagement and learning (Schaffer 2017). Learners can not only plan their own education process, evaluate their results and monitor their progress but also transition to higher levels of cognitive skills, mastering the subject content and the competent use of visualisation methods (David and Sulaiman 2021). Jacobse and Harskamp (2012) indicated that pictorial visualisations show that a learner does not yet know how to explore the problem to arrive at a useful solution, thus indicating low metacognitive regulation. However, drawing steps to solve a problem helps learners reflect on, monitor and evaluate their problem-solving abilities and strategies. This has been shown to increase conceptual understanding and help learners evaluate their learning (Martin et al. 2017). Drawing and writing your thoughts as pictures, diagrams and sentences are considered a metacognitive strategy.

Metacognition and the stages of solving word problems

Mathematical word problems are one of the most difficult types of problems in mathematics, and many reasons have been identified for their challenging nature (Aaron et al. 2022; De Corte et al. 2000; Hegarty et al. 1995; Lewis 1989). One of the greatest difficulties is the process of seeking a solution. A few steps are required. First, the text must be read and understood. Next, a decision must be made regarding which mathematical operations are relevant to formulate an equation. Finally, the learner must solve this mathematical equation to obtain the answer (Boonen et al. 2013; Mevarech 1999; Pimm 1991). How does metacognition function in the process of solving word problems? Pólya (1973) claimed that problem-solving has four stages: understand the problem, search for solving methods, execute the solving methods and examine the answer. Considering the aforementioned knowledge about metacognition, we will consider how metacognition functions in these four stages. The cognitive activity in the first stage, ‘understand the problem’, is to read and understand the word problem. The metacognition used at this stage is to consider whether one has solved a similar problem before or if one’s understanding of a problem is unclear (e.g. ‘I’m not sure that I understand the question, so I’ll read the text again’). Other metacognitive activities include thinking about what is known and unknown in the problem or whether one has understood the problem. These correspond to the ‘task’ in Brown’s ‘knowledge of cognition’.

In the next stage, ‘search for solving methods’, one needs to approach the problem with a deliberate strategy and plan rather than attempting a solution haphazardly. Establishing a path to the solution, such as considering what to find first and then questioning what to solve, and estimating the solution are also possible metacognitive activities at this stage. Moreover, these correspond to ‘planning’ in Brown’s ‘regulation of cognition’. Subsequently, in ‘execute the solving methods’, metacognitive activities, such as checking whether the solving method’s execution is correct or considering whether other solving methods might exist when the current solving method is not working, are performed. This corresponds to ‘monitoring’ in Brown’s regulation of cognition’. In the final stage, ‘examine the answer’, metacognition is required to verify accurately whether the resultant solution is correct. This corresponds to an evaluation of Brown’s ‘regulation of cognition’.

Hence, various metacognitive activities occur in the process of solving a word problem (see Table 1). Thus, metacognitive activity is an especially crucial element at each stage of solving a word problem. Many researchers pointed out that metacognitive abilities influence problem-solving abilities (Chytry 2020). People with strong metacognitive skills can solve word problems efficiently. However, people with poor metacognitive skills cannot perform metacognitive activities and are unsure how to begin. They approach the problem randomly, despite the problem’s complexity. When they find themselves unable to solve a problem, they cannot pause to rethink or return to the previous stage and try a new approach. Therefore, they find it difficult to solve problems efficiently.

Theoretical framework

Figure 1 shows the theoretical framework of this study. As mentioned in the literature review, problem-solving can be divided into four stages namely, ‘understand the problem’, ‘search for solving methods’, ‘execute the solving methods’ and ‘examine the answer’. It would be possible to identify the students’ metacognitive skills by analysing their skills used in each stage and their level of skill. Therefore, in this study, the holistic metacognitive skills of the students solving word problems and their relationship to academic performance are determined through an analysis of each stage.

Research method

Research subjects

Ten learners from each of the third, fourth and fifth grades of a public school in Kayonza district, Eastern Province, Rwanda, were randomly selected from a register of learners’ names. Thus, 30 learners were included in the study.

Table 1 Cognitive skills and metacognitions used in the process of solving a word problem in mathematics

The stages of problem-solving	Cognitive skill	Metacognitive skill	
		Skills	Specific examples
Understand the problem	Understand the problem	Knowledge of cognition	–Do I understand the meaning of the problem? Is the information required to solve the problem available?
Search for solving methods	Formulate a mathematical sentence	Planning	–What steps should I take to solve the problem? Have I seen a similar problem before? What is the answer likely to be, approximately?
Execute the solving methods	Solve the problem	Monitoring	–Am I solving the problem correctly? Is there another more effective solving method? If this does not work, can I solve the problem using a different method?
Examine the answer	Check the answer	Regulation	Did I arrive at the correct answer? Can I explain the solving process I used? Which parts were difficult for me?

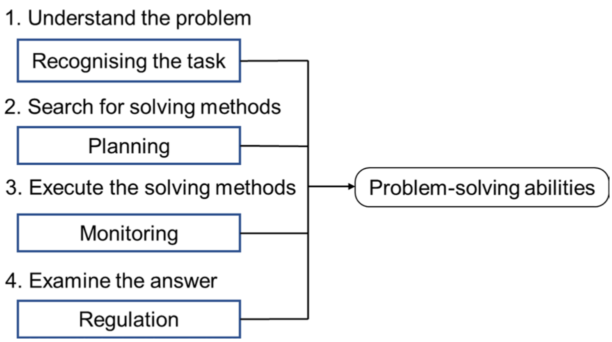


Fig. 1 Theoretical framework of the study

Problem under investigation

In modern society, there is worldwide consensus on the notion that teaching routine problem-solving is insufficient. In this study, we used a problem with elements of a complex, unfamiliar, and non-routine (CUN) task. The only knowledge and skills required to solve the problem are adding and subtracting numbers of 10 or less. However, in this problem, rather than being asked to calculate the final number after a transaction has occurred, learners are asked to determine the original quantity before a transaction takes place. Furthermore, it can be said that this problem is a CUN problem because it cannot be solved by only manipulating the given numerical values.

Methods of data collection

We used the Okamoto (1992) methods in our study. To eliminate differences in the children's writing skills, the problem comprehension and reflection aspects in this study were conducted through interviews instead of asking learners to write their ideas. Additionally, the presentation of the word problem and interviews were conducted in their local language, Kinyarwanda, to eliminate the influence of various levels of language proficiency as much as possible. To measure metacognition and how it works in each of the aforementioned four stages to solve a word problem, we interviewed each learner using the order of questions/instructions (a) to (g) listed below.

(a) Have learners read the question and state their levels of confidence (0–100) to obtain the correct answer and their reason.
(b) Ask what they were mindful of when reading the problem and what they knew after reading the problem.
(c) Tell learners that they are free to write equations and diagrams and ask them to solve the problem.

(d) Have learners explained their method of problem-solving.
(e) Have learners explained what they were mindful of when solving the problem.
(f) Have them explain the parts that were difficult.
(g) Have learners stated their levels of confidence (0–100) about whether they obtained the correct answer and stated their reasons.

Table 2 shows the relationship between the seven interview questions/instructions and the four stages of problem-solving. Notably, (a) and (b) relate to what happens before actually solving the problem and are classified under 'understand the problem'. As (c) corresponds to the process of solving the problem and (d) corresponds to explaining the solving method, we classified them under 'Search for solving methods' and 'Execute the solving methods', respectively. Further, (e), (f), and (g) make the learners reflect on their process of problem-solving and think about what they were mindful of, what was difficult, and whether they obtained the correct answer. We classified these questions under 'examine the answer'.

Method of analysis

This subsection explains the method of assigning scores to the interviews. The study was conducted using a mixed methods approach, incorporating qualitative research to supplement the results of the quantitative analysis. To quantitatively analyse the data, it is necessary to express the extent to which the metacognitions (a) to (g) occur as numerical scores. In this study, we created a rubric showing the criteria to determine the degree to which metacognition occurs, making it possible to assign scores (Table 3). Each item was evaluated using a scale of 0 to 4 (5 levels). Level 0 represents a state in which metacognition has not occurred. Level 1 is the stage where metacognition can be slightly established. Further, Level 2 is when metacognition is established to a certain extent; Level 3 is where metacognition is mostly established. Finally, Level 4 is where metacognition is fully established. For questions/instructions (a) to (g), we developed a criterion corresponding to each level. For example, in the case of question (f), where learners are required to explain the difficult parts, a score of 0 would be given for the learner not providing a response. A score of 1 was given for the learner only to answer which parts were difficult. A score of 2 would be given if, in addition to 1, the learner answered with some reasoning. Further, a score of 3 was given if the learner stated their reason to a certain extent of clarity; a score of 4 was for a learner who aptly and accurately presented their reason. We quantitatively analysed learners' correct answers and qualitatively analysed how their metacognitive skills appeared in their responses, especially the method search and execution aspects.

Additionally, two qualitative analyses were conducted to thoroughly review the data of the quantitative research. First, the metacognitive skills in the 'planning' and 'monitoring' stages were analysed qualitatively, focusing on the learners' drawings and texts. Second, an interview was conducted with the teachers to confirm how they taught the word problems. The author asked the teachers about two points: (1) the instructions given to the students and (2) confirming the answer with the students.

Table 2 Relationship between the seven interview questions/instructions and four stages of problem-solving

Four stages of problem-solving (Metacognition)	Understand the problem (Knowledge of cognition)	Search for solving methods (Planning)	Execute the solving methods (Monitoring)	Examine the answer (Regulation)
Questions	(a), (b)	(c), (d)		(e), (f), (g)

Table 3 Method of assigning scores to the interviews

Level	Criteria	Knowledge of cognition		Planning and Monitoring		Regulation		
		(a)	(b)	(c)	(d)	(e)	(f)	(g)
4	Metacognition is fully established	Aptly and accurately judges whether they can solve the problem, stating their reasons	Aptly and accurately gives responses for both 'solving method' and 'understanding the problem'	Clearly presents a solving method using diagrams, equations, etc	Aptly and accurately explains their solving method in a sequentially logical manner	Aptly and accurately explains the areas they were mindful of, stating their reasons	Aptly and accurately explains which areas were difficult, stating their reasons	Aptly and accurately determines whether they obtained the correct answer, stating their reasons
3	Metacognition is mostly established	Largely, aptly, and accurately judges whether they can solve the problem, stating their reasons	Aptly and accurately gives responses to either the 'solving method' or 'understanding the problem'	Largely and clearly presents solving methods using diagrams, equations, etc	Largely, aptly, and accurately explains their solving method sequentially	Aptly and accurately explains the areas of which they were mindful	Aptly and accurately explains which areas were difficult	Largely, aptly, and accurately determines whether they obtained the correct answer, stating their reasons
2	Metacognition is established to some extent	Judges whether they can solve the problem, providing minimal reasons	Gives responses to both, albeit insufficiently	Presents the solving method using diagrams, equations, etc., although their solving method is incorrect	Explains their solving method, although their solving method is incorrect	Can explain to some extent the areas of which they were mindful	Can explain to some extent the sections that were difficult	Determines to some extent whether they obtained the correct answer, stating their reasons
1	Metacognition is slightly established	Judges whether they can solve the problem using intuition alone	Gives a response to one or the other, to some extent, or can barely respond to both	Attempts, although slightly, to present a solving method using diagrams, formulas, etc	Attempts, although slightly, to explain their solving method	Can explain, although only slightly, the sections of which they were mindful	Can explain, although only slightly, the sections of which they were mindful	Determines whether they obtained the correct answer using intuition alone

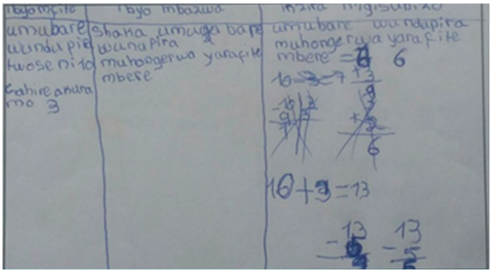
Table 3 (continued)

Level	Criteria	Knowledge of cognition		Planning and Monitoring		Regulation		
		(a)	(b)	(c)	(d)	(e)	(f)	(g)
0	Metacognition is unestablished	No response	No response	No response	No response	No response	No response	No response

Results and discussion

Table 4 shows an example interview (one learner from the third grade). All learners were posed questions similar to those exemplified in Table 4. Using this example, we can explain the scoring. Responding to question (a) in ‘recognition of the task’, this student answered 90% and offered the reason ‘Because we learned it’. As this was a minimal reason, it was judged as Level 2, which is ‘Judges whether they can solve the problem, providing minimal reasons’. In addition, responding to question (d) in ‘planning and monitoring’, the student responded ‘Ten plus three is thirteen, thirteen minus six is six’. He offered an explanation, but it was incorrect. Therefore, it was judged as Level 2, which is ‘Explains their solving method, although

Table 4 Interview example

<table>
<tr><td>Interviewer</td><td>Did you read the question? Do you think you can do it or that you can’t do it?</td></tr>
<tr><td>Learner A</td><td>I can perform it.</td></tr>
<tr><td>Interviewer</td><td>At what percent (how confident) do you think you can do it, and why do you think so?</td></tr>
<tr><td>Learner A</td><td>90%. Because we learned it.</td></tr>
<tr><td>Interviewer</td><td>Did you learn it? When?</td></tr>
<tr><td>Learner A</td><td>In the first term</td></tr>
<tr><td>Interviewer</td><td>When reading that question, what did you care about? What did you understand from the question?</td></tr>
<tr><td>Learner A</td><td>I care what I have, what I am asked about, the procedure, and the answer</td></tr>
<tr><td>Learner A (Solving the problem on a sheet)</td><td>
<table><tr><td>What I have</td><td>What I am asked</td><td>Method and answer</td></tr><tr><td>The total number of balls is 10.
Gahire gave three</td><td>Find the number of balls that Muhongerwa had before</td><td>The number of balls that Muhongerwa had equals six</td></tr></table></td></tr>
<tr><td>Interviewer</td><td>Tell us how you solved that question</td></tr>
<tr><td>Learner A</td><td>The number of balls is ten, Gahire gave three; I searched for how many balls Muhongerwa had before, which was six</td></tr>
<tr><td>Interviewer</td><td>Why is it six?</td></tr>
<tr><td>Learner A</td><td>Ten plus three is thirteen, thirteen minus six is … six</td></tr>
<tr><td>Interviewer</td><td>In solving a question like that, what do you care about?</td></tr>
<tr><td>Learner A</td><td>What I have, what I am asked, resolution and solution</td></tr>
<tr><td>Interviewer</td><td>Ok, what was the difficult part in solving that question?</td></tr>
<tr><td>Learner A</td><td>To find the number of balls he has after</td></tr>
<tr><td>Interviewer</td><td>Why that?</td></tr>
<tr><td>Learner A</td><td>Because, to arrive at it, I used many ways</td></tr>
<tr><td>Interviewer</td><td>Are you confident that you performed it correctly?</td></tr>
<tr><td>Learner</td><td>I don’t think I did it, but I think I did it 75%, perhaps</td></tr>
<tr><td>Interviewer</td><td>Why?</td></tr>
<tr><td>Learner</td><td>Because I don’t know if I did it well</td></tr>
<tr><td>Interviewer</td><td>Ok, alright, thank you!</td></tr>
</table>

their solving method is incorrect'. Finally, responding to question (f) in 'regulation', the answer provided was 'To find the number of balls he has after'. As this was a minimal response, it was judged as Level 1, which is 'Can explain, although only slightly, the sections of which they were mindful'.

Categorisation of the results by grade level

Table 5 presents the results for each grade. The average scores for third, fourth and fifth graders were 13.4, 7.7 and 15.8, respectively.

The rate of correct answers for the third graders was almost equivalent to that of the fifth graders, contradicting a previous study (Berk 2003; Merchán Garzón et al. 2020), stating that metacognitive skills improve as learners' grade levels increase. This is illuminated in the results of our interviews with teachers concerning how they teach word problems. In a third-grade teachers' interview, they stated that, for tasks that involve problem-solving, such as word problems, they instruct their learners daily to be aware of 'what they know', 'what is being asked' and 'how a solution can be reached'. The three aspects they mentioned are metacognitive skills that relate to planning and searching for a solving method. Expressly, it was observed that the third-grade teachers consciously trained their learners to use metacognitive strategies when solving problems. However, teachers of other grades did not provide such guidance. They asked for the mathematical expression and how to solve the expression, which requires cognitive skills. Most third graders, as shown in the previous example, could sequentially respond by dividing the response space into three columns of 'what they know', 'what they are being asked' and 'how to find the answer'. This suggests that their higher metacognitive scores compared to other grades are due to differences in teaching methods. To date, many studies have demonstrated that instruction improves metacognitive skills (Shilo and Kramarski 2019; Zimmerman 2008; Mevarech and Kramarski 1997). As our study does not focus on metacognition instruction, we cannot determine with certainty whether the third-grade teachers actually taught metacognition. However, our study suggests that comparable results can be obtained throughout Rwanda.

Categorisation based on learners with correct answers and learners with incorrect answers

Table 6 shows the number of learners who received the correct answer and those who did not. We investigated by a chi-square examination whether there is any

Table 5 Average score by grade

Grade	Interview questions							
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	Total
3	1.8	2.5	2.1	2.3	1.7	1.5	1.5	13.4
4	1.5	0.9	1.1	1.4	1	0.9	0.9	7.7
5	1.7	1.5	2.5	3.2	2.6	2.3	2.3	15.8

Table 6 Comparison of the learners with correct answers to those with incorrect answers

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	Total
Learners who answered correctly ($n=8$)	2.1 2.1	2.1	3.1 3.5	3.9	3.0 2.8	2.8	2.5	19.5
Learners who answered incorrectly ($n=22$)	1.5 1.5	1.5	1.5 1.6	1.7	1.3 1.2	1.1	1.2	9.8
p-value	$p=0.22$		$p<0.000$		$p<0.000$			$p<0.000$

statistically significant difference for each in the problem-solving process stages, which are 'understand the problem', 'search for a solving method and Execute the plan' and 'Check and extend'. We investigated whether there is a statistically significant difference in the mean scores of metacognitive skills for the two groups or not.

The null hypothesis states that 'to answer correctly or incorrectly does not depend on the metacognitive skills'. Thus, if the *p*-value is high, the null hypothesis is accepted, and if the *p*-value is low, the null hypothesis is rejected. The group of the learners who answered correctly ($n=8$) and the group of the learners who answered incorrectly ($n=22$) are different students. The table shows the arithmetic mean of his/her marks in each question, though the independent samples, *T*-test was conducted by category: 'recognition of the task' (a and b), 'planning and monitoring' (c and d) and 'regulation' (e, f and g)'. The *p*-value was derived from the independent samples *T*-test.

No statistically significant difference was observed at the 'problem comprehension' stage. At this stage, metacognition involved making judgements about what one is mindful of when reading the problem, what one knows, and whether one thinks they can solve the problem.

Both groups of learners could not immediately understand the complex transaction structure in the problem; they judged that the answer could be obtained using only simple addition and subtraction—both groups of learners responded using almost no metacognition. Even among those who answered correctly, their scores on metacognition were the lowest in three of the categories. No statistically significant differences were found between the two groups. Next, for 'method search and execution', a statistically significant difference was found at a significance level of 1%. When searching for a method, learners need to suitably monitor their cognitive processes and make adjustments about whether connections can be successfully made from the various information obtained in the previous process of comprehending the problem. Furthermore, even in the execution process, learners must continue to monitor their thinking and make adjustments about whether the solving method is appropriate to arrive at the correct answer.

According to Glasser, learners with high problem-solving skills can simulate the act of monitoring a problem-solving process while referring to their prior knowledge and correctly predicting the result of their problem-solving. The results of this study also support the findings of existing research, as the gap between the two groups of learners is the most significant in this category when compared to the other two groups. A significant difference was also observed for metacognition, one which occurs during the 'check and extend' stage. In this stage, the learners were asked to explain what they were mindful of about their problem-solving methods and in their process of solving the problem. Additionally, they were asked to state how confident they were in obtaining the correct answer while providing reasons. These situations require a sophisticated level of processing, which involves monitoring and verbalising one's cognitive processes. Theoretically, even if a learner's answer is incorrect, they should still be able to score highly if metacognition occurs during the process. However, a significant difference in metacognition score was observed between those who answered the question correctly and those who did not.

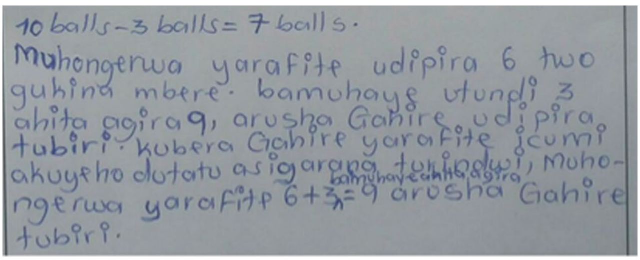
From the above, it can be said that the results we obtained were similar to those from previous studies. Learners who obtained the correct answers generally had high metacognitive skills, and learners with high academic ability also had high metacognitive skills.

Metacognitive skills in 'method search' and 'execution'

The metacognitive skills used in 'method search' and 'execution' were analysed qualitatively based on the learners' responses. We found that learners with correct answers mainly used three methods. All the methods showed that learners used their metacognitive skills to control their learning while solving the problem. The first was to express the interaction between two people using words and mathematical expressions (Fig. 2).

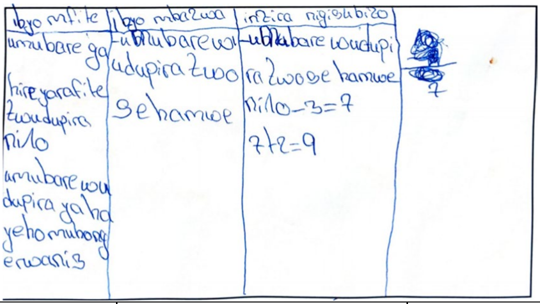
While writing them, the learners organised the movement of the balls between two people to formulate their answers. In other words, they control their learning by writing sentences. The second step was to draw a table about the problem: what they knew, what was being asked, and the solution or answer (Fig. 3).

The learners organised their ideas by filling their ideas Querysystematically in a table. The third method involves drawing two people in the question and drawing the actual direction of the balls' movement in the picture (Fig. 4). Regarding visualisation, drawing increases conceptual understanding and helps learners evaluate



Muhongerwa had six balls before. They gave her three more balls, and now she possesses nine; she has two more balls than Gahire. As Gahire had ten, after giving out three, he remained with seven, while Muhongerwa had six plus three equals nine. She has two more balls than Gahire.

Fig. 2 Writing the interaction between two people in words and mathematical expressions



What I have	What I am asked	Method and answer
The number of balls Gahire had is 10, and that of balls he gave to Muhongerwa is three	What is the sum of all the balls?	The number of all balls is 10-3 = 7 7 + 2 = 9

Fig. 3 Making a table of what they knew, what was being asked, and the solution or answer

Fig. 4 Drawing the actual movement of the balls

information (Martin et al. 2017). In this case, we found that the learners used their metacognitive control in their own pictorial visualisations.

Conclusions

In this study, we conducted tests and interviews with Rwandan third, fourth, and fifth graders in a public elementary school to better understand Rwandan children's metacognitive skills when solving mathematical word problems. For additional data, we conducted interviews with teachers educating the relevant grades on how they teach word problems. The results clarified the following three points about the metacognitive skills used when Rwandan learners solved mathematical word problems.

The first point concerns the difference in metacognitive skills between those who correctly solved the problem and those who did not. The metacognitive skill of those who arrived at the correct answer was observed to be considerably higher than that of those who did not.

The second point is that although a considerable difference in metacognitive skills was not observed at the ‘problem comprehension’ stage, a significant difference was observed in the subsequent stages of ‘search for a solving method and execute the plan’ and ‘check and extend’. In particular, the ‘check and extend’ stage requires sophisticated levels of cognitive processing, such as monitoring and verbalising one’s cognitive processes. It was shown that learners who answered correctly could perform sophisticated processes that use metacognitive skills compared to learners who answered incorrectly.

Third, when metacognitive skills were measured and scored, the average scores for fifth and third graders were similar. However, of the four stages of the problem-solving process, the third graders scored much higher than the fifth graders at the ‘problem comprehension’ stage. At this stage, several of them could provide their responses in a logical sequence, dividing their response space into three columns. When the teachers were interviewed about this, they answered that they had instructed their learners to check each of their steps when solving word problems. This suggests that the third-grade teachers instruct their learners in metacognitive strategies; consequently, the learners’ metacognitive skills increased to the same level as the fifth graders. It has been reported that metacognitive skills are enhanced by instruction. These three points are evident in previous studies; however, as noted in the background, they are significant in this study because the students are from an African country in which very little research has been conducted on metacognition.

In addition, there are two main contributions to the research on metacognition from a theoretical and methodological perspective. The theoretical perspective is that we proposed a link between Brown’s approach to metacognition and Pólya’s problem-solving stages. Metacognitive skills were presented in each of the four stages indicated by Pólya. The methodological perspective is that we developed a rubric to assess students’ metacognitive skills quantitatively, although further improvements are needed to deem the indicator more objective.

To the best of our knowledge, this is the only empirical study in Rwanda. As only one school was surveyed and the number of participants was not sufficient, generalisations cannot be made from this survey alone. However, as the participating school is typical of rural public schools, it is possible to acquire some implications about the metacognitive skills of Rwandan students from the results of this study.

One future task will be to conduct empirical studies regarding this in the context of the Rwandan nation. If this can be demonstrated, it can be suggested that equivalent results can be obtained in other African countries with similar sociocultural contexts. This would be a big step towards improving the quality of mathematics education in African countries.

The datasets generated during and/or analysed during the current study are not publicly available due to protection of students’ rights to privacy but are available from the corresponding author on reasonable request.

Declarations

Conflict of interest We wish to confirm that there are no known conflict of interest associated with this publication and there has been no financial support for this work that could have influenced its outcome.

Ethical approval We further confirm that any aspect of the work covered in this manuscript that has involved students has been conducted with the ethical approval of all relevant bodies and that such approvals are acknowledged within the manuscript.

Concern for publication We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing, we confirm that we have followed the regulations of our institutions concerning intellectual property.

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