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CASE REPORT

A Stochastic Modelling and Optimization for the Design of an LNG Refuelling System in the Piraeus Port Region

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Abstract
Port activity is assumed to be an integral part of maritime activity. Ships are supplied with water, food, fuel, electricity, spare parts and consumables when they are berthed in ports. Therefore, the purpose of the service system is to reduce the waiting cost, from the time the ship's arrival until its service. Delays in the system, and particularly in the queuing system, could occur due to irregularities in the ships' arrival time causing uncertainty in time of service. On the contrary, a complex waiting system would require an increased commitment of capital for the construction and maintenance of appropriate infrastructure. As such an optimum size and operation of the port service system must be identified. To address this problem, this study presents a method for maximizing port administrator's profit, by modelling and optimizing the waiting system based on Monte Carlo simulation techniques. Port of Piraeus is used to test the method. The case study assumes increased bunkering port calls due to the addition of a liquified natural gas (LNG) refuelling station. The results showed that a range of 3 to 5 stations generate high profit while using too many service stations could lead to deteriorations of the profit.

Keywords LNG · LNG bunkering · Mathematical modelling · Stochastic optimization · Monte Carlo simulation · Queuing theory

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1 Introduction

Oxford Institute for Energy Studies (OIES) highlighted the positive impact of natural gas as transportation fuel in Europe, especially in the marine sector where the environmental restrictions and International Maritime Organisation (IMO) restrictions on fuel oil pose the need for alternative fuel sources [1]. Since then, studies have been focused on the use of alternative solutions, such as liquid natural gas (LNG), marine gas oil (MGO), methanol and heavy fuel oil with scrubber, for fuel in marine sector [2]. From all these solutions, LNG is recognized as the most promising solution in short and medium term due to its environmental advantages compared to conventional petroleum products, such as marine diesel or heavy fuel oil [1]. The majority of the bunkering studies are aiming to shade light to the current gap of legislation and regulations of LNG in marine sector, infrastructures, safety rules and personal training [3–5] to accelerate the transition process from conventional fuel to the continuous increase in the LNG demand.

LNG produces low carbon dioxide (CO_2), nitrogen oxides (NO_x), sulphur oxides (SO_x) and particulate matter (PM) emissions. Even if LNG constitutes a promising fuel solution from an environmental perspective, it is not the only option if economic criteria will be considered. To attract customers and to compete oil prices, LNG suppliers are offering long-term contracts with competitive prices [6]. To this end, more ports are considering the development of LNG bunkering facilities to provide LNG refuelling services to ship owners [7, 8].

Current studies are mainly focusing on the policies and laws that should be imposed in order to support the extension of facilities and services in ports for the wide adoption and use of LNG as ship fuel [2, 9, 10], while others are examining the safety issues by performing risk assessment with respect to safety zones in LNG ports [11–14]. When it comes to the development and installation of LNG systems in ports, some studies are focusing of the design aspects of the network [15–18] and the queuing system [19–21].

In this study, a risk probability assessment is presented for hypothetical scenarios of the development of an LNG bunkering system at the Piraeus Port. Figure 9 illustrates the flow chart of the steps that were followed in this study. Initially, a mathematical modelling based on mathematical programming and specifically

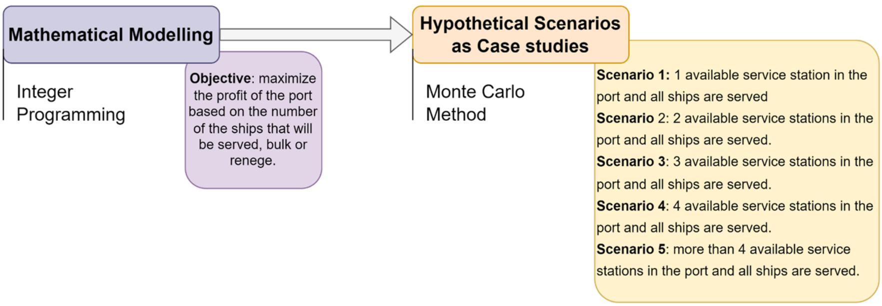

Fig. 1 A flow chart of the methodology adopted in this study

integer programming is proposed for maximizing the profit of the port regarding the number of the ships that will be served, bulk or renege. To integrate the stochastic behavior of this process into our modelling, some parameters that are included in the objective function are expressed as probability distributions. To this end, the Monte Carlo method is employed to estimate the impact of the development of LNG bunkering system for ship refuelling in the Piraeus Port. Five different scenarios regarding the number of available stations in the port were conducted to identify the most profitable case to be implemented for transforming the Piraeus port into a port with LNG stations for ship refuelling.

In Sect. 2, the related work is presented. In Sect. 3, an alternative approach is proposed in order to estimate the risk probability while Sect. 4 analyzes the case study for a hypothetical LNG bunkering system at the Piraeus Port including assumptions, restrictions and a detailed profit function. The results of the optimization model of the selected scenarios are also presented in Sect. 4 while the conclusions and proposals for further research are given in Sect. 5.

2 Related Work

To identify and evaluate the impact of LNG in marine sector, various studies have been conducted comparing LNG to other options under real scenarios. An evaluation of the real option to store LNG at regasification terminal was presented by developing a heuristic approach to decrease the computational demands of existing mathematical formulations [22]. Some studies [2, 10] provided an overview of the use of LNG in the maritime industry, remarks about regulations and insights for the facilities expansion mainly in European ports due to the future growth in LNG demand. A similar study regarding the development of the LNG fleet and bunkering infrastructure limited to the Baltic and North Sea Region was presented [23]. However, studies posing feasibility and commercial considerations were also presented regarding LNG fueled ships [9].

The increasing adoption of LNG as marine fuel and the need for building port infrastructures have raised safety concerns. Thus, numerous studies have been conducted with respect to safety in LNG bunkering stations. An evaluation of safety exclusion zone for LNG bunkering areas on LNG-fueled ships was presented [24]. The study adopts a probabilistic risk assessment approach to determine the safety exclusion zone for two case ships identifying potential risks of LNG bunkering. A similar approach was adopted by [25] to indicate the safety zones in a probabilistic approach for a generic ship-to-ship bunkering case. A study that identified the main factors affecting the safety zone in leaked-gas dispersion ship-to-ship LNG bunkering was conducted [11]. To this end, a parametric analysis and computational fluid dynamics (CFD) simulation for case-specific scenarios were implemented taking into account operational and environmental conditions. A similar study was conducted by [12] to determine the safety exclusion zone for LNG bunkering at fuel-supplying point. The study investigated the influence of heterogenous parameters on the safety of the LNG bunkering proving the effectiveness of the combination of population dependent and independent analyses over the employment of each method alone. A comparative inherent safety assessment was presented to evaluate the alternative technologies for LNG ship bunkering over traditional approaches, such as marine diesel fuel technologies [13]. A ranking system was used based on the overall performance metrics, such as severity of consequences, to give insights on critical units and safety issues. An approach for risk evolution on the LNG bunker operation was proposed based on dynamic theory and catastrophe mathematical models [14]. The methodology aims to predict risk emergences to prevent accidents by early risk assessment and control.

Various studies are focusing on the design of LNG bunkering network. A real case of Busan port was studied where a conceptual design of an offshore LNG bunkering to terminals was proposed [26]. The study incorporates statistical analysis of ship visits, estimation of the required LNG consumption and determination of the hull structure based on design requirements, such as safety, economics and principal functions. The optimization of bunkering network for deep sea container ships was studied in [27]. The authors contributed to the problem of bunkering management in the network design by developing a logico-mathematical model that maximizes the fleet capacity utilization and the supply to transport demands while minimizing the time and cost of bunkering. The LNG bunkering demands were predicted in the context of the floating LNG bunkering terminal in Port of Ulsan [15] and of various ports in Australia [28], while a design for the first LNG bunkering barge system in Korea was proposed demonstrating under numerical simulations the performance evaluation of the system [16]. A port-level LNG demand forecast and a sensitivity analysis were conducted for the real case of the Antwerp port [17]. A fuzzy empirical approach was proposed for LNG bunkering port selection from the shipping companies perspective [18].

To decrease the queuing time in a port for large LNG fueled ships, such as containerships, the simultaneous bunkering and cargo loading/unloading was evaluated with computational fluid dynamics (CFD)–based simulations [29]. The dangerous and safety zones were identified for applying the loading and unloading tasks in parallel. Graph theory was employed for optimizing the location of LNG bunkering stations [20] in order to create an optimal LNG bunkering network. The location problem was also addressed by [21] for various ports in Korea. The problem was formulated as a multiple-criteria ranking problem and a Consistent Fuzzy Preference Relation (CFPR) methodology was employed to evaluate operational factors influencing the location of bunkering, such as average loading speed of LNG, distance of bunkering shuttle or safety parameters.

Studies have also focused on the mooring system of a floating LNG bunkering system. From economic perspective, a study was realized with respect to a proposed pile-guide mooring system for an offshore LNG ship to ship bunkering operation including the bunkering terminal, the bunkering shuttle, the carrier and the receiving ship [30]. A cost–benefit analysis was adopted for proving the economic feasibility of this solution, whole a finite element analysis was employed to design the mooring system and thus to estimate the additional investment. From operation perspective, a numerical study on the optimization of the side-by-side mooring system of floating LNG bunkering terminal with LNG carrier under operational conditions was performed [31]. Concerning bunkering refuelling of

LNG ships on inland waterways, a study was conducted in China proposing the concept of LNG bunkering pontoons as a promising solution [19] to address the problem of the LNG filling difficulty in natural rivers due to the seasonal change of channel width. The design study proves the feasibility of LNG bunkering pontoons assuming the role of floating LNG bunkering station in terms of safety, reliability and management.

When it comes to logistic and supply management in ports, the terms that contributes to the total port cost can be analyzed according to the following norm: 5% in maritime facilities and services, 10% in land facilities and services, 50% cargo handling at shore and ship (equally divided) and 35% ship's service time. Thus, underutilization of berths or inefficient design of port activities could result in higher pricing of port activities and loss of attraction for ships [32]. In order to reduce the waiting, queuing theory is employed to maximize the berths' use, through the optimization of the port system.

The fundamental parameters for the queuing system are identified as follows: (i) the rate of the ships arriving in the system, (ii) the rate of the ship refuelling procedure in the port, and (iii) the number of parallel service stations. A big queuing system means fewer delays, but greater cost of infrastructure. As pointed out by [33], the use of expected (averages) values for project's cost or duration, or point estimations, could be extremely misleading. A commonly used approach to reduce uncertainty is to represent parameters as probability distribution functions. Thus, it is easy to model the financial results by using the Monte Carlo simulation technique. Specifically, Monte Carlo method has been used in various studies, regarding energy efficiency, risk assessment or quantitative analysis among others [14, 34–37], in order to aid business decision in the shipping sector. With respect to LNG adoption to maritime industry, Monte Carlo simulations have been employed for decision-making on containership conversion to use LNG as fuel [38], scheduling optimization for LNG regasification of storage tank at import terminal to maximize the revenue [39], investigation of the fleet replacement decisions under uncertain demand and fuel prices [40, 41], etc.

The scope of this paper is to present a stochastic framework for modelling and optimization of Port of Piraeus queuing system for ships' LNG refuelling. The Port of Piraeus was chosen due to its significance in the Greek maritime sector (largest Greek port). Also based on the recent reports [42, 43], Revythousa terminal has received record 60 loads of LNG imports implying the need for a study regarding the optimum size and optimal operation of the port service system. The problem was formulated as a MILP model where the objective terms are imposed under uncertainties. The objective is to maximize the profit of the administrative with respect to the demand and number of service stations, as it is elaborated further below. To cope with the uncertainties providing in parallel a sensitivity analysis, the Monte Carlo method was employed to solve the aforementioned problem. Different scenarios of LNG demand are studied where 1, 2, 3, 4 or more than 5 LNG refuelling stations were available to identify the optimal number of stations with respect to the profit.

3 Materials and Methods

3.1 A Stochastic Approach for Estimating the Risk Probability

In general, ship, that is in need for LNG refuelling, calls the port administrator and asks for an estimation of the service starting time where the ship will be served. Then, the ship has three options whether to balk, which means to decline the port's service; to renege, which means to initially get in the queue but leave without being served; or to be served. Each state has a different impact on the port's profit. In the first case, if the ship decides to balk, it generates cost for the port administrator. If the ship reneges, this action causes cost for the port administrator equal to cost of balk, and the cost of time spent in the system. In case that the ship is being served, it leads to profit for the port. The profit will be equal to the difference between the payment of the ship and the sum of the variable cost for the administrator and the waiting cost. To formulate the above conditions to express the port's total profit for all the ships that are approaching the port and ask for a refuelling service in a time horizon, we use stochastic integer programming. Integer programming has been widely used for formulating various transportation problems [44, 45]. Below, the mathematical formulation of the problem is given. In Table 1, the sets and variables used in this mathematical modelling are presented.

Decision variables

$$\mathcal{X}_i(t) = \begin{cases} 1, \text{if the ship } i \text{ decided to balk at time } t \\ 0, \text{otherwise} \end{cases}, \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$\mathcal{Y}_i(t) = \begin{cases} 1, \text{if the ship } i \text{ decided to renege at time } t \\ 0, \text{otherwise} \end{cases}, \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

Table 1 The sets and variables used in the mathematical modelling

Sets	**Description**
$\mathcal{I}$	the set of all the ships that will call the port administrator to ask for a refuelling service
$\mathcal{T}$	the time horizon under examination
Variables	**Description**
T_i^{fs}	the time needed for a free service station to be commanded to start serving the $_i$ ship,$\forall i \in \mathcal{I}$
T_i^{Maxfs}	the maximum time that the $_i$ ship is willing to wait until it will be served,$\forall i \in \mathcal{I}$
C_i^b	the port administrative costs due to balk of the i ship,$\forall i \in \mathcal{I}$
C_i^w	the waiting cost of the ship $_i$ per time unit,$\forall i \in \mathcal{I}$
C_i^s	the variable cost for the administrator,$\forall i \in \mathcal{I}$
P_i	the payment of the ship $_i$,$\forall i \in \mathcal{I}$
C^f	the waiting system's fixed cost
s	the maximum number of service stations of the port
$N = \lvert\mathcal{I}\rvert$	the number of the ships that will ask for a refuelling service in the time horizon

$$\mathcal{Z}_i(t) = \begin{cases} 1, \text{if the ship is being served at time } t \\ 0, \text{otherwise} \end{cases}, \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

Objective function: The target is to maximize a port's profit that derives from the number of the ships that balk, renege or are served when they ask for LNG refuelling and an estimation upon the waiting time for a free station in order to be served.

Balk cost term: If the estimation of the service starting time is not acceptable, the ship i will balk, which means that it will leave without entering the system, causing cost for the port administrator, C_i^b. So, the profit from the ship i in this case can be expressed as:

$$B_i = -\sum_{t\in\mathcal{T}} \mathcal{X}_i(t) C_i^b, \forall i \in \mathcal{I} \tag{1}$$

Renege cost term: If the estimation of the service starting time is acceptable, but in practice it is never met, the ship will renege, which means that it will leave the system after having entered without being served, causing cost for the port administrator equal to cost of balk, and the cost of time spent in the system. The ship will renege at moment T_i^{Maxfs} and the additional cost from the unserved ship i can be symbolized as $C_i^w T_i^{Maxfs}$ where C_i^w is the waiting cost of the ship i per time unit. Let T_i^{fs} be the waiting time that the i ship is served, with $T_i^{fs} < T_i^{Maxfs}$. So, the profit from the ship i in this case can be expressed as:

$$R_i = -\sum_{t\in\mathcal{T}} \mathcal{Y}_i(t)\left(C_i^w T_i^{Maxfs} + C_i^b\right), \forall i \in \mathcal{I}, \forall T_i^{fs} > T_i^{Maxfs} \tag{2}$$

Refuelling cost term: The ship can be served in less time than it is originally estimated to wait in the queue. Let T_i^s be the total time needed for the ship i to be served. This case leads to profit for the port. The profit will be equal to the difference between the payment of the ship i, P_i, and the variable cost for the administrator, C_i^s and the waiting cost, $C_i^w T_i^s$. So, the profit from the ship i in this case can be expressed as:

$$F_i = \sum_{t\in\mathcal{T}} \mathcal{Z}_i(t)\left(P_i - C_i^s - C_i^w T_i^s\right), \forall i \in \mathcal{I}, T_i^{fs} \le T_i^{Maxfs} \tag{3}$$

Fixed cost term: In any of the above presented cases, there is a waiting system's fixed cost C^f.

Hence, the objective function based on the above terms can be written as:

$$G = \sum_{i\in\mathcal{I}} \left\{ -C^f - \sum_{t\in\mathcal{T}} \mathcal{X}_i(t) C_i^b - \sum_{t\in\mathcal{T}} \mathcal{Y}_i(t)\left(C_i^w T_i^{Maxfs} + C_i^b\right) + \sum_{t\in\mathcal{T}} \mathcal{Z}_i(t)\left(P_i - C_i^s - C_i^w T_i^s\right) \right\} \tag{4}$$

The constraints of the problem are:

Service stations capacity constraint: In any given moment, the ships served cannot exceed the number of service stations

$$\sum_{i \in \mathcal{I}} \sum_{u=t-(T_i^s - T_i^{fs})}^{t} \mathcal{Z}_i(u) \le s, \forall t \in \mathcal{T} \tag{5}$$

Service decision constraint: For each ship that will ask for refuelling, only one decision can be made, which means it will balk, renege or be served.

$$\sum_{t \in \mathcal{T}} \mathcal{X}_i(t) + \sum_{t \in \mathcal{T}} \mathcal{Y}_i(t) + \sum_{t \in \mathcal{T}} \mathcal{Z}_i(t) = 1, \forall i \in \mathcal{I} \tag{6}$$

Balance ship flow constraint: The total number of the ships should be equal to the number of ships that balk, renege or are served.

$$N = \sum_{i \in \mathcal{I}} \left\{ \sum_{t \in \mathcal{T}} \mathcal{X}_i(t) + \sum_{t \in \mathcal{T}} \mathcal{Y}_i(t) + \sum_{t \in \mathcal{T}} \mathcal{Z}_i(t) \right\} \tag{7}$$

3.2 Assumptions for the Objective Function Parameters

It is important to make some realistic assumptions about the parameters used in the above objective function in order to simplify our model, to integrate the uncertainties and to adjust the mathematical model in the case of hypothetical LNG bunkering system in Piraeus Port.

Assumption 1 for the system's fixed cost: System's fixed cost, C^f, is assumed to consist of two components: the former depends on the number of service station while the latter does not.

$$C^f = C^f_{fixed} + c_s s \tag{8}$$

where C^f_{fixed} is a fixed cost of the system and c_s is the fixed cost of providing each service station, s. The service stations can be bunker ships or berths.

Assumption 2 for service cash flow: Payment P_i, charged to the serviced ship i, can be assumed to be proportionate to the quantity of LNG that the ship i is fueled with.

$$P_i = P_{LNGi} Q_{LNGi}, \forall i \in \mathcal{I} \tag{9}$$

where P_{LNGi} is the selling price of LNG per cubic meter ($\$/m^3$) for the ship i and Q_{LNGi} is the quantity of LNG that the ship i is fuelled with (m^3).

Assumption 3 for the service cost charged to the port: The parameter of the service cost due to the service of the ship is assumed to be proportionate to the quantity of LNG that the ship is fuelled with.

$$C_i^s = c_{sLNGi} Q_{LNGi}, \forall i \in \mathcal{I} \tag{10}$$

where c_{sLNGi} is the cost of selling LNG to the ship i per cubic meter ($\$/m^3$) and Q_{LNGi} is the quantity of LNG that the ship i is fuelled with (m^3).

Assumption 4 for waiting in the queue and leaving the queue cost: Both waiting in the queue and leaving the queue cost are charged to the port and they can be assumed to be different for each ship. The cost of leaving the queue, C_i^b, is assumed to be equal to the payment that was not done because the service was not completed. In the term, we also add the negative impact of the inability of the system to serve a ship in a satisfying waiting time. In fact, this will affect the relationship among the existing costumers but also the attraction of new costumers. Thus, a negative rumour can cause loss of income by other ships' unwillingness to be refuelled in the port.

It is also accepted that the ship owners that are willing to be refuelled in a port will have previously signed contracts with the port administrator. Hence, the failure of serving a ship in a satisfying, or a pre-agreed, waiting time will result in contracts' dissolvement. The appearance of such a scenario is always possible, no matter how large the waiting system is.

Assumption 5 for the waiting time in the system: If the ship $i, i \in \mathcal{S}$, is finally served, the waiting time in the system is assumed to be equal to the time needed a free service station to be found, T_i^{fs}, and the time the service itself to be completed, T_i^{cs}.

$$T_i^s = T_i^{fs} + T_i^{cs}, \forall i \in \mathcal{S} \tag{11}$$

As mentioned above, the service stations can be berths or bunker ships. In both cases, the ship must move and/or manoeuvre its way to the service stations; the ship that needs refuelling must approach a berth or wait to be approached by a bunker ship. Time needed for the movement is symbolized as T_i^t. Then, some time is needed for the refuelling itself, T_i^f. Finally:

$$T_i^s = T_i^{fs} + T_i^t + T_i^f, \forall i \in \mathcal{S} \tag{12}$$

Time needed for refuelling can be assumed to be equal to:

$$T_i^f = \frac{Q_{LNGi}}{u_{LNG,pipe} * A_{pipe}} \tag{13}$$

where Q_{LNGi} is the quantity of LNG that the ship i is fuelled with (m^3), $u_{LNG,pipe}$ is the velocity of LNG in the refuelling pipe and A_{pipe} is the area of the pipe used.

After these assumptions, the system's profit objective function can be written as:

$$\begin{aligned} G = \sum_{i\in\mathcal{I}} \Bigg\{ & -\left(C^w_{fixed} + c_s s\right) - \sum_{t\in\mathcal{T}} \mathcal{X}_i(t) C_i^b - \sum_{t\in\mathcal{T}} \mathcal{Y}_i(t)\left(C_i^w T_i^{Maxfs} + C_i^b\right) \\ & + \sum_{t\in\mathcal{T}} \mathcal{Z}_i(t)\left(P_{LNGi}Q_{LNGi} - c_{aLNGi}Q_{LNGi} - C_i^w\left(T_i^{fs} + T_i^t + T_i^f\right)\right)\Bigg\} \end{aligned} \tag{14}$$

3.3 Computational Algorithm

To solve the presented problem, the Monte Carlo Method, also known as the Monte Carlo Simulation or a multiple probability simulation, is used. Monte Carlo Method is a well-known mathematical technique used to estimate the probability of possible outcomes when the potential for random variables is present. It contributes to a better decision-making under uncertain conditions. Compared to predictive models with fixed inputs, an advantage of Monte Carlo Method is the ability to conduct sensitivity analysis by identifying the impact of individual inputs on a given outcome. Many Monte Carlo methods follow the pattern [46–48]:

- Model a system as a probability density function (PDF) or a series of them
- Repeatedly sample from the PDFs
- Compute the statistics of interest

This pattern is also applied to this study. Specifically, if the components of the function (14) are represented as probability distribution functions, Monte Carlo simulation can provide a probability distribution function for the estimated profit. On the other hand, Monte Carlo optimization could indicate actions able to increase the system's profitability, such as the number of service stations, the time the bunker ships will be called to serve the ships in queue or the exclusion of some clients, as shown in the following case study.

4 Results

4.1 Evaluation Methodology

The present study considers that Piraeus Port region addresses the need for alternative fuel solutions and aligns with the decision of International Maritime Organization by constructing an LNG handling system via which LNG will be imported, stored and finally provided to ships as fuel.

It is assumed that the import and storage of LNG will take place in the Port of Revithousa Island, which is located 16 naval miles away from the central port of Piraeus. Its area is 1.8 square kilometres and it is unoccupied. Due to the distance from the central port of Piraeus and the difficulty of ships' manoeuvring, bunker ships will be used to supply LNG to client ships. In its present state, the terminal station of Revithousa Island has the capacity of serving LNG Carriers of up to 177,000 m^3 cargo capacity and 298 m length. The station's berth can serve ships of up to 10 m draught. There are two reservoirs of 130,000 m^3 LNG total capacity, where LNG can be stored until sent to gasification. Another reservoir of 95,000 m^3 has already begun to be constructed. Gasification capacity will be 1400 m^3/h. Revithousa Island was selected for our study due to its location but also due to the indication by numerous studies of Revithousa Island as a location for a potential future LNG supply network for Greece [43, 49–52].

4.2 Quantification of the Parameters of the Objective Function

Based on the assumptions that have been adopted above, a quantification of the objective function's parameters is attempted.

Table 2 Equipment's buy cost

Equipment	Buy cost $(10^6\$)$
LNG carrier ship of 45,000 m^3 tonnage	40
Bunker ships of 500 m^3 tonnage	1.5

4.2.1 Quantification of the System's Fixed Cost Based on Assumption 1

It is assumed that fixed cost consists of the cost of LNG carrier ships that will import the LNG, the cost of terminal station's infrastructure (construction of berths, piping system construction and reservoirs where the LNG will be stored temporarily), the cost of building the bunker ships that will supply client ships with LNG and other infrastructure construction and maintenance cost.

As far as Revithousa terminal station is concerned, the cost for the infrastructure is assumed zero, as the infrastructure already exists. Based on similar data [43, 53–55], the cost with regard to the LNG Carrier ship that will import the LNG (ship of 45,000 m^3) is assumed 40,000,000$ and cost of each bunker ship (ship of 500 m^3) is 1,500,000$ (Table 2).

All ships are assumed to have a lifecycle of 20 years. The disposal cost for each year can be computed as it is shown in Table 3 [43, 55, 56]:

Disposal cost of the existing infrastructure is supposed to be 400,000 $ per year. Maintenance cost for new built ships is supposed to be additional 10% of their disposal cost.

Finally, it can be written that

$$\begin{aligned} C^f &= (Disposal\ cost\ of\ each\ LNG\ carrier\ ship) \times (Number\ of\ LNG\ carrier\ ships) \\ &\quad + (Disposal\ cost\ of\ each\ LNG\ bunker\ ship) \times (Number\ of\ LNG\ carrier\ ships) \\ &\quad + (Disposal\ cost\ of\ other\ infrastructure\ each\ LNG\ bunker\ ship) + (Maintenance\ cost) \rightarrow \end{aligned}$$

$$C^f = 2{,}000{,}000\$ \times 1 + 75{,}000\$ \times s + 400{,}000\$ + 10 \times \left(2{,}000{,}000\$ \times 1 + 75{,}000\$ \times s\right) \rightarrow$$

$$C^f = 2,600,000\$ + 82,500\$ \times s \tag{15}$$

It is assumed that just one LNG carrier ship can fulfil the import demand.

4.2.2 Quantification of the Service Cash Flow Based on Assumption 2

As mentioned above, payment,P_i, is assumed to be$P_i = P_{LNGi} Q_{LNGi}, \forall i \in \mathcal{I}$. It is estimated that the selling price of LNG will be 13.0$/mmBTU, or 270$/m^3. LNG

Table 3 Equipment's disposal cost

Equipment	Disposal cost per year ($)
LNG carrier ship of 45,000 m^3 tonnage	2,000,000
Bunker ships of 500 m^3 tonnage	75,000

selling price is modelled as a triangular distribution function (Fig. 1), $triang\left(220\$/\mathrm{m}^3, 270\$/\mathrm{m}^3, 320\$/\mathrm{m}^3\right)$ [57–60].

LNG selling price is assumed same for all ships, so $P_i = P_{LNG}Q_{LNGi}, \forall i \in \mathcal{I}$.

4.2.3 Quantification of the Service Cost Based on Assumption 3

As mentioned above, the variable service cost due to the service of the ship is assumed to be:

$$C_i^s = c_{sLNGi}Q_{LNGi}, \forall i \in \mathcal{I} \tag{16}$$

$$C_i^s = c_{sLNGi}Q_{LNGi}, \forall i \in \mathcal{I} \tag{17}$$

In, it is estimated that the import price of LNG will be 4.0\$/mmBTU, or 83 \$/m^3 [57–60]. LNG import price is modelled as a uniform distribution function (Fig. 2),$uniform\left(67\$/\mathrm{m}^3, 83\$/\mathrm{m}^3\right)$.

LNG cost is assumed same for all ships. So, $C_i^s = c_{sLNG}Q_{LNGi}, \forall i \in \mathcal{I}$

4.2.4 Quantification of Waiting in the Queue and Leaving the Queue Cost Based on Assumption 4

Both waiting in the queue and leaving the queue cost can be assumed to be different for each ship. For this case study, in order to simplify the analysis, it was assumed these are different for the different categories of ships [61]. Table 4 shows the waiting in the queue cost per ship category.

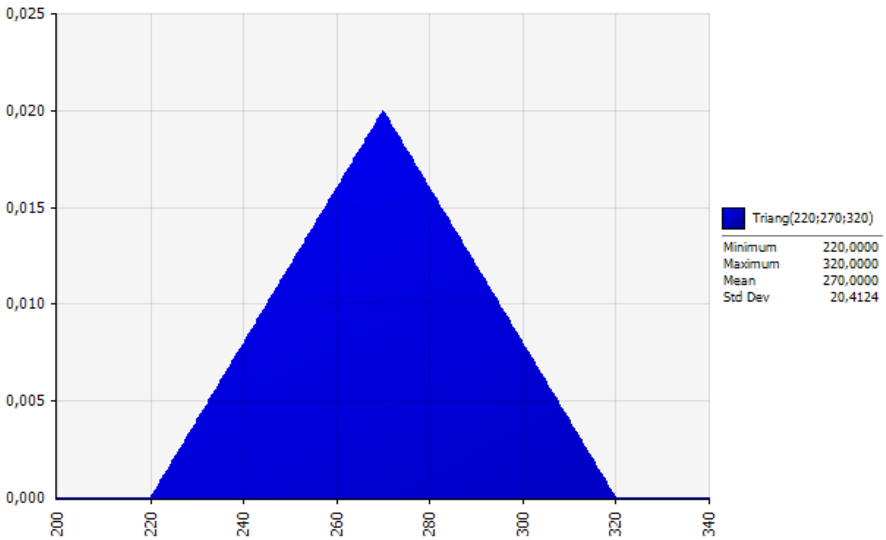

Fig. 2 LNG selling price as a triangular distribution function

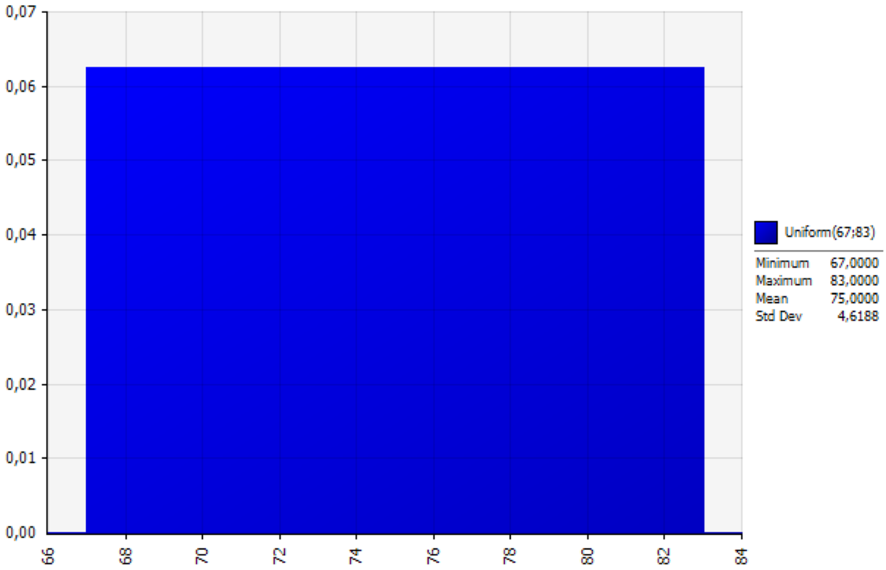

Fig. 3 LNG import price as a uniform distribution function

The cost of leaving the queue, C_i^b, is assumed to be equal to the payment that was not done because the service was not completed, plus the negative rumour of the system because it was not able to serve a ship in a satisfying waiting time, therefore causing loss of ships' refuellings by that or other ships (Table 5).

The cost of leaving the waiting system, for a specific ship category, can be computed as:

$$C_i^b = (Number\ of\ lost\ refuellings) \times (Expected\ refuelling\ profit) \rightarrow$$

Table 4 Waiting in the queue cost per ship category

Ship category	Waiting in the queue cost ($/hr)
Small containerships	5000
Large containerships	10,000
Small cruise ships	5000
Large cruise ships	10,000
Small RoRo ships	5000
Large passenger ships; Destination Crete	2000
Small passenger ships; Destination Cyclades	2000
Large passenger ships; Destination Cyclades	2000
Large passenger ships; Destination North Aegean	2000
Large passenger ships; Destination Dodecanese	2000

Table 5 Number of lost refuelling per ship category

Ship category	Number of lost refuellings
Small containerships	10
Large containerships	10
Small cruise ships	10
Large cruise ships	10
Small RoRo ships	10
Large passenger ships; Destination Crete	2
Small passenger ships; Destination Cyclades	2
Large passenger ships; Destination Cyclades	2
Large passenger ships; Destination North Aegean	2
Large passenger ships; Destination Dodecanese	2

$$C_i^b = (Number\ of\ lost\ refuellings) \times \left(P_i - C_i^s\right) \rightarrow$$

$$C_i^b = (Number\ of\ lost\ refuellings) \times \left(P_{LNGi} - c_{sLNGi}\right) Q_{LNGi} \tag{18}$$

where Q_{LNGi} the mean LNG quantity demanded by the ship category.

Quantification of the waiting time in the system based on Assumption 5.

Piraeus Port Authority S.A. is expected to sign contracts with free service station clause upon ship appearance. So, the maximum time for a ship to wait until a free service station is found will be zero.

$$T_i^{Maxfs} = 0 \tag{19}$$

The waiting time in the system will then be equal to the time needed a free service station, a bunker ship, to move towards the client ship and the time needed to refuel.

$$T_i^s = T_i^{cs} \rightarrow T_i^s = T_i^t + T_i^f, \forall i \in \mathcal{I} \tag{20}$$

or

$$T_i^s = T_i^t + \frac{Q_{LNGi}}{u_{LNG,pipe} \times A_{pipe}}, \forall i \in \mathcal{I} \tag{21}$$

It is supposed that the bunker ship will need half an hour to approach the client ship and start refuelling:

$$T_i^t = 0.5\text{hr} \tag{22}$$

while

$$u_{LNG,pipe} = 15\text{m/s} \tag{23}$$

$$D_{pipe} = 6'' \rightarrow A_{pipe} = \pi \frac{0.1524^2 \mathrm{m}^2}{4} \tag{24}$$

So,

$$T_i^f \cong \frac{Q_{LNGi}}{313.55 \frac{\mathrm{m}^3}{\mathrm{h}}} \tag{25}$$

Finally,

$$T_i^s = 0.5hr + \frac{Q_{LNGi}}{313.55 \frac{\mathrm{m}^3}{\mathrm{h}}} \tag{26}$$

$$T_i^s = 0.5hr + \frac{Q_{LNGi}}{313.55 \frac{\mathrm{m}^3}{\mathrm{h}}} \tag{27}$$

Hence, after the determination of the objective function's parameters, the system's profit in a year can be written as:

$$\begin{aligned} G = \sum_{i \in \mathcal{I}} \Bigg\{ & -(2,600,000\$ + 82,500\$ \times s) - \sum_{t \in \mathcal{T}} \mathcal{X}_i(t) C_i^b \\ & + \sum_{t \in \mathcal{T}} \mathcal{Z}_i(t) \left(\left(P_{LNG} - c_{aLNG}\right) Q_{LNGi} - C_i^w \left(0.5\mathrm{hr} + \frac{Q_{LNGi}}{313.55 \frac{\mathrm{m}^3}{\mathrm{h}}} \right) \right) \Bigg\} \end{aligned} \tag{28}$$

Because there are 10 different categories of ships, each one having an individual waiting and leave cost, let $\mathcal{G}$ be the set of the categories of the ships under consideration, $\}(i)$ be the parameter that returns the category of the ship $i, \forall i \in \mathcal{I}$. Thus, we insert in our model the dependence of ship category in the parameters C_i^b, Q_{LNGi} and C_i^w expressed from now on as $C^b_{i,\}(i)}$,expressed from now on as $C^b_{i,\}(i)}$, and $\mathrm{C}^{\mathrm{w}}_{\mathrm{i,\,g(i)}}$

$$\begin{aligned} G = \sum_{i \in \mathcal{I}} \Bigg\{ & -\left(2{,}600{,}000\$ + 82{,}500\$ \times s\right) - \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I} : \}(i) = j} \mathcal{X}_i(t) C^b_{i,\}(i)} \\ & + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I} : \}(i) = j} \mathcal{Z}_i(t) \left(\left(P_{LNG} - c_{aLNG}\right) Q_{LNGi,\}(i)} - C^w_{i,\}(i)} \left(0.5\mathrm{hr} + \frac{Q_{LNGi,\}(i)}}{313.55 \frac{\mathrm{m}^3}{\mathrm{h}}} \right) \right) \Bigg\} \end{aligned} \tag{29}$$

$$G = \sum_{i \in \mathcal{I}} \left\{ -(2,600,000\$ + 82,500\$ \times s) - \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{X}_i(t) \overline{C}_j^b \right.$$
$$\left. + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{Z}_i(t) \left((P_{LNG} - c_{aLNG}) \overline{Q}_{LNGj} - \overline{C}_j^w \left(0.5\text{hr} + \frac{\overline{Q}_{LNGj}}{313.55 \frac{\text{m}^3}{\text{h}}} \right) \right) \right\} \tag{30}$$

If the mean values of cost $C^b_{i,\}(i)}$ are used for each ship category based on the mathematical expression:

$$\sum_{i=1}^{n} x_i = n\overline{\text{x}} \tag{31}$$

Then, the cost parameter can be expressed with the mean value of the cost for each ship category: $\overline{C}_j^b, \forall j \in \mathcal{G}$. The same can be applied for the other two parameters that are linked with the ship category: $\overline{Q}_{LNGj}$ and $\overline{C}_j^w, \forall j \in \mathcal{G}$.

It can be written that:

$$G = \sum_{i \in \mathcal{I}} \left\{ -(2,600,000\$ + 82,500\$ \times s) - \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{X}_i(t) \overline{C}_j^b \right.$$
$$\left. + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{Z}_i(t) \left((P_{LNG} - c_{aLNG}) \overline{Q}_{LNGj} - \overline{C}_j^w \left(0.5\text{hr} + \frac{\overline{Q}_{LNGj}}{313.55 \frac{\text{m}^3}{\text{h}}} \right) \right) \right\} \tag{32}$$

$$G = \sum_{i \in \mathcal{I}} \left\{ -(2,600,000\$ + 82,500\$ \times s) - \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{X}_i(t) \overline{C}_j^b \right.$$
$$\left. + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{Z}_i(t) \left((P_{LNG} - c_{aLNG}) \overline{Q}_{LNGj} - \overline{C}_j^w \left(0.5\text{hr} + \frac{\overline{Q}_{LNGj}}{313.55 \frac{\text{m}^3}{\text{h}}} \right) \right) \right\} \tag{33}$$

The constraints of the problem are:

All ships asking to be served will find a free service station upon appearance, or they will balk

$$N = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \mathcal{X}_i(t) + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \mathcal{Z}_i(t) \rightarrow$$

$$N = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{X}_i(t) + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{G}} \sum_{i \in \mathcal{I}: \}(i)=j} \mathcal{Z}_i(t) \tag{34}$$

Using the classical definition of probability, let P_{serv} be the probability that a ship will be served:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\mathcal{Z}_i(t) = P_{serv}\times N \tag{35}$$

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\mathcal{X}_i(t) = \left(1-P_{serv}\right)\times N \tag{36}$$

$$\mathrm{N}_{serv,j} = P_{serv}\times N \tag{37}$$

$$\mathrm{N}_{balk,j} = \left(1-P_{serv}\right)\times N \tag{38}$$

However, as we have mentioned above in the constraints of the model, in any given moment, the ships served cannot exceed the number of service stations

$$\sum_{i\in\mathcal{I}}\mathcal{Z}_i(t)\le s, \forall t\in\mathcal{T} \tag{39}$$

Let $\mathrm{N}_{serv}(t)=\sum_{i\in\mathcal{I}}\mathcal{Z}_i(t)$, then the probability that there are up to s clients in the waiting system can be written as:

$$P_{\mathrm{N}_{serv}(t)\le s} = P_0+\cdots+P_s \tag{40}$$

which is the probability of service, as a coming ship must find a free service station, or it will balk. So,

$$P_{serv} = P_0+\cdots+P_s \tag{41}$$

The probability of service is assumed to be the same for all the incoming ships, meaning that there is no distinction among the various ship categories participating in the study.

Hence based on the above, the objective function can be written as:

$$\begin{aligned} G = \sum_{i\in\mathcal{I}}\Bigg\{ & -\left(2,600,000\$ + 82,500\$\times s\right) - \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{G}}\left(1-P_{serv}\right)N\overline{C}_j^b \\ & + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{G}}P_{serv}N\left(\left(P_{LNG}-c_{aLNG}\right)\overline{Q}_{LNGj} - \overline{C}_j^w\left(0.5\mathrm{hr} + \frac{\overline{Q}_{LNGj}}{313.55\frac{\mathrm{m}^3}{\mathrm{h}}}\right)\right)\Bigg\} \end{aligned} \tag{42}$$

Thus,

$$G = G\left(s, P_{serv}, N, \overline{C}_j^b, \overline{C}_j^w, P_{LNG}, c_{aLNG}, \overline{Q}_{LNGj}\right) \tag{43}$$

So, if it is assumed that $\overline{C}_j^b$, $\overline{C}_j^w$, $\overline{P}_{LNG}$, $\overline{c}_{aLNG}$ and $\overline{Q}_{LNGj}$ are external system parameters:

$$G = G(s, P_{serv}, N) \tag{44}$$

Therefore, the optimization problem can be written as:

$$maxG = \max(G(s, P_{serv}, N)) \tag{45}$$

indicating that the parameters sought to be optimized will be the number of service stations, and the categories of ships selected to participate in the system. The probability of service, P_{serv}, is estimated every time using the theory of Poisson models.

4.3 Computational Results

The following case study was coded in Matlab software. Based on the information provided in Table 6, Table 7 shows the refuelling costs and the leaving costs for each ship category taken into account in this study. According to [62], we estimated the demand for refuelling per ship category. For the evaluation, various scenarios (Table 8) are investigated regarding the profit with and without client selection. Table 9 shows the ship categories that are included in the scenarios and the ship categories that are not taken into account.

4.3.1 Scenario 1

In scenario 1, 1 service station is available in the port and all ships that are asking for refuelling are served. Hence, If it is decided all the ships to be served, it will be: $N = 3,624$

Table 6 Refuelling per ship category [62]

Ship category	(N) Number of refuellings in 2020	Q_{LNG}Mean refuelling quantity (m^3)	Annual LNG demand in 2020 (m^3)
Small containerships	314	92	29,018.9
Large containerships	201	202	40,547.35
Small cruise ships	84	143	12,053.3
Large cruise ships	202	450	91,024.25
Small RoRo ships	125	77	9618.18
Large passenger ships; Destination Crete	708	189	133,569.18
Small passenger ships; Destination Cyclades	836	44	36,630.06
Large passenger ships; Destination Cyclades	580	59	34,052.6
Large passenger ships; Destination North Aegean	300	132	39,749.7
Large passenger ships; Destination Dodecanese	274	242	66,237.46

Table 7 Refuelling cost and leaving cost per ship category

Ship category	(N) Number of refuellings in 2025	Q_{LNG}Mean refuelling quantity (m^3)	C^wWaiting cost ($/hr)	C^bLeaving cost (Number of lost refuellings)
Small containerships	314	92	5000	10
Large containerships	201	202	10,000	10
Small cruise ships	84	143	5000	10
Large cruise ships	202	450	10,000	10
Small RoRo ships	125	77	5000	10
Large passenger ships; Destination Crete	708	189	2000	2
Small passenger ships; Destination Cyclades	836	44	2000	2
Large passenger ships; Destination Cyclades	580	59	2000	2
Large passenger ships; Destination North Aegean	300	132	2000	2
Large passenger ships; Destination Dodecanese	274	242	2000	2

Table 8 Evaluation scenarios

Scenario	Description	Number of service stations	Profit under conditions
Scenario 1	All ships are served	1	With and without client selection
Scenario 2	All ships are served	2	With and without client selection
Scenario 3	All ships are served	3	With and without client selection
Scenario 4	All ships are served	4	With and without client selection
Scenario 5	All ships are served	5	With and without client selection

$\overline{Q}_{LNG} = uniform\left(90\% \times 135.97\mathrm{m}^3, 110\% \times 135.97\mathrm{m}^3\right) \rightarrow \overline{Q}_{LNG} = uniform\left(122.37\mathrm{m}^3, 149.57\mathrm{m}^3\right)$
$\overline{C}^b = 131,383.80\$$
$\overline{C}^w = 3,322.57\$/\mathrm{hr}$

It is reminded that:

$P_{LNG} = triang\left(220\$/\mathrm{m}^3, 270\$/\mathrm{m}^3, 320\$/\mathrm{m}^3\right)$
$c_{aLNG} = uniform\left(67\$/\mathrm{m}^3, 83\$/\mathrm{m}^3\right)$

No client selection: For one service station, and if there is no client selection.

$s = 1$
$P_{serv} = P_0 + P_1 = 1 - \rho + (1 - \rho)\rho = 1 - \rho^2$

where

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{\mathrm{N}}{360}}{\frac{24\mathrm{h}}{\overline{T}^t + \frac{\overline{Q}_{LNG}}{313.55\frac{\mathrm{m}^3}{\mathrm{h}}}}} = \frac{N\left[\overline{T}^t + \frac{\overline{Q}_{LNG}}{313.55\frac{\mathrm{m}^3}{\mathrm{h}}}\right]}{8,640\mathrm{h}} \tag{46}$$

It seems that if all ship categories are selected to participate in the system, there is a probability of loss of money, which is equal to 65%. The loss will be equal to about 3,900,000$. This is because $P_{serv} \cong 85\%$, so every seventh ship will balk (Fig. 3).

Table 9 Ship categories included in and excluded from the scenarios

Ship categories included in scenarios	Ship categories excluded from scenarios
• Large cruise ships • Large passenger ships; Destination Crete • Large passenger ships; Destination North Aegean • Large passenger ships; Destination Dodecanese	• Small containerships • Large containerships • Small cruise ships • Small RoRo ships • Small passenger ships; Destination Cyclades • Large passenger ships; Destination Cyclades

Optimized client selection: On the other hand, if the problem is optimized stochastically, so that the mean value of the probability distribution function is maximized, it seems that the system could.

4.3.2 Scenario 2

In scenario 2, 2 service stations are available in the port and all ships that are asking for refuelling are served.

No client selection: For two service stations, and if there is no client selection.

$$s = 2$$

$$P_{serv} = P_0 + P_1 + P_2 = \left(1 + \rho + \frac{\rho^2}{2!}\right) \times \left[\frac{\rho^0}{0!} + \frac{\rho^1}{1!} + \frac{\rho^2}{2!}\frac{1}{1-\frac{\rho}{2}}\right]^{-1}$$

Probability of service is about 98.75% while the mean expected profit is about 75,000,000$ (Fig. 6).

Optimized client selection: Stochastic optimization of the problem indicates all the ship categories to be served.provide profit with mean value 41,600,000$ (Fig. 4).

As above, probability of service is about 98.75% and the mean expected profit is about 75,000,000$ (Fig. 5).

4.3.3 Scenario 3

In the third scenario, 3 service stations are available and still all the ships, that ask for refuelling, will be served.

No client selection: For three service stations, and if there is no client selection.

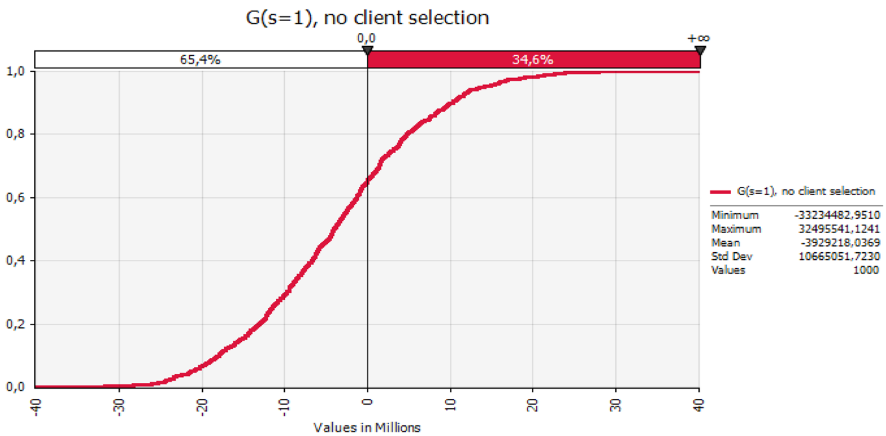

Fig. 4 One service station—profit per probability of service

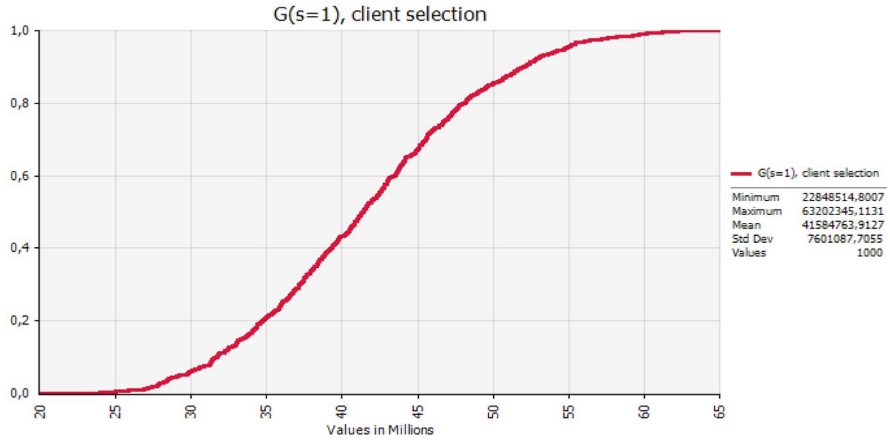

Fig. 5 Optimized client selection for one service station

$s = 3$

$$P_{serv} = P_0 + P_1 + P_2 + P_3 = \left(1 + \rho + \frac{\rho^2}{2!} + \frac{\rho^3}{3!}\right) \times \left[\frac{\rho^0}{0!} + \frac{\rho^1}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!}\frac{1}{1-\frac{\rho}{3}}\right]^{-1}$$

Probability of service is about 99.90% while the mean expected profit is about 81,400,000$ (Fig. 7).

Optimized client selection: Stochastic optimization of the problem indicates all the ship categories to be served.

As above, probability of service is about 99.90% and the mean expected profit is about 81,400,000$ (Fig. 8).

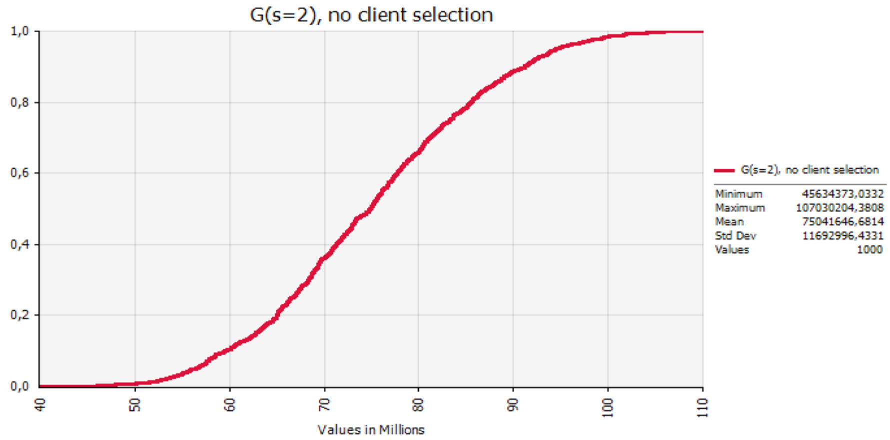

Fig. 6 Two service stations—profit per probability of service

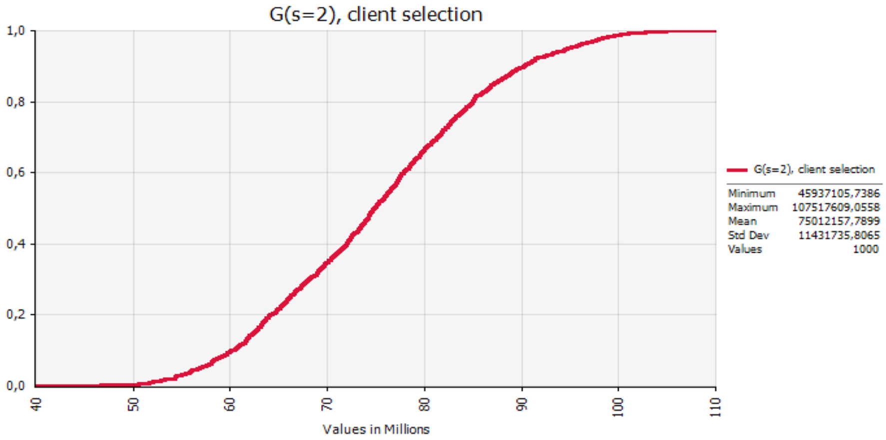

Fig. 7 Optimized client selection for two service stations

4.3.4 Scenario 4

In scenario 4, we have 4 service stations installed in the port and all the ships that are asking for refuelling will be served.

No client selection: For four service stations, and if there is no client selection.

$$s = 4$$
$$P_{serv} = P_0 + P_1 + P_2 + P_3 + P_4 \rightarrow$$
$$P_{serv} = \left(1 + \rho + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4!}\right) \times \left[\frac{\rho^0}{0!} + \frac{\rho^1}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4!}\frac{1}{1-\frac{\rho}{4}}\right]^{-1}$$

Probability of service is about 99.99% while the mean expected profit is about 81,900,000$ (Fig. 10).

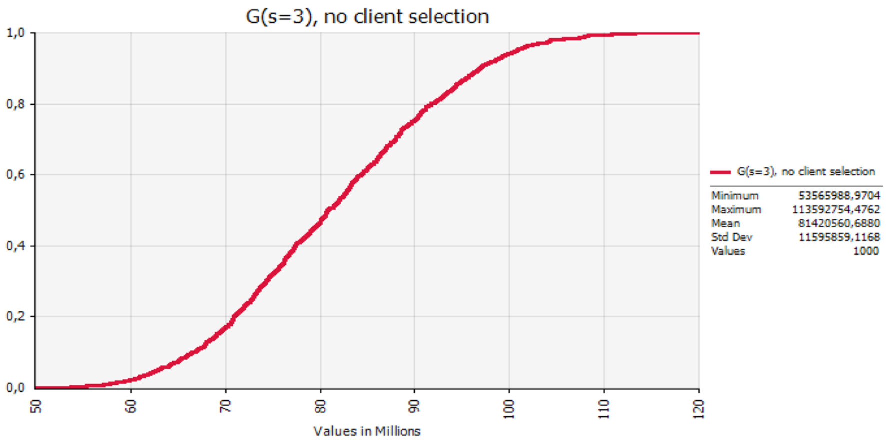

Fig. 8 Three service stations—profit per probability of service

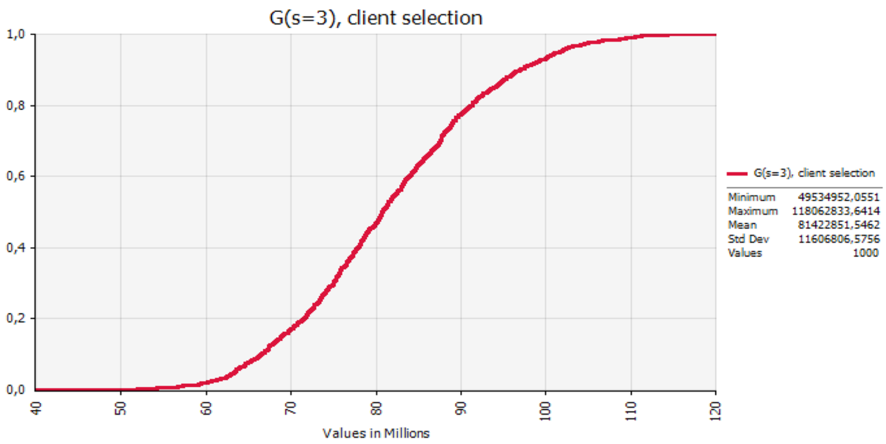

Fig. 9 Optimized client selection for one service station

Optimized client selection: Stochastic optimization of the problem indicates all the ship categories to be served.

As above, probability of service is about 99.99% and the mean expected profit is about 81.900.000$ (Fig. 11).

4.3.5 Scenario 5

If more service stations are added in the waiting system, $s > 4$, the mean time of profit will be reduced due to the increased disposal cost. For example, if $s = 5$ and there is no client selection:

$$P_{serv} = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 \rightarrow$$

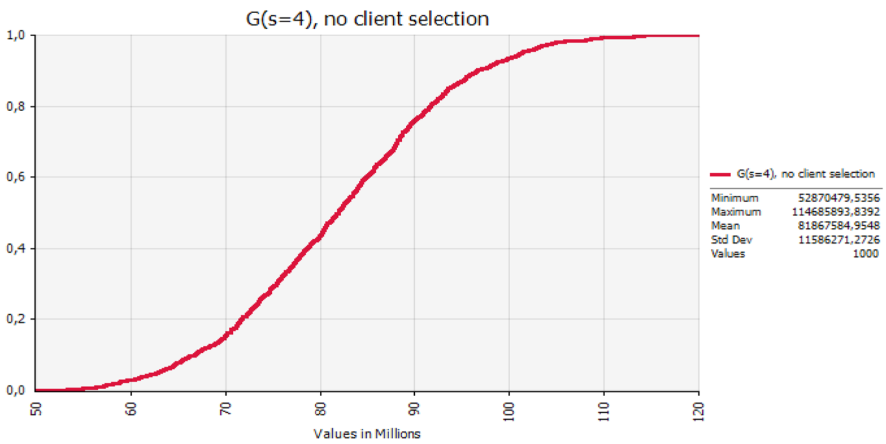

Fig. 10 Four service stations—profit per probability of service

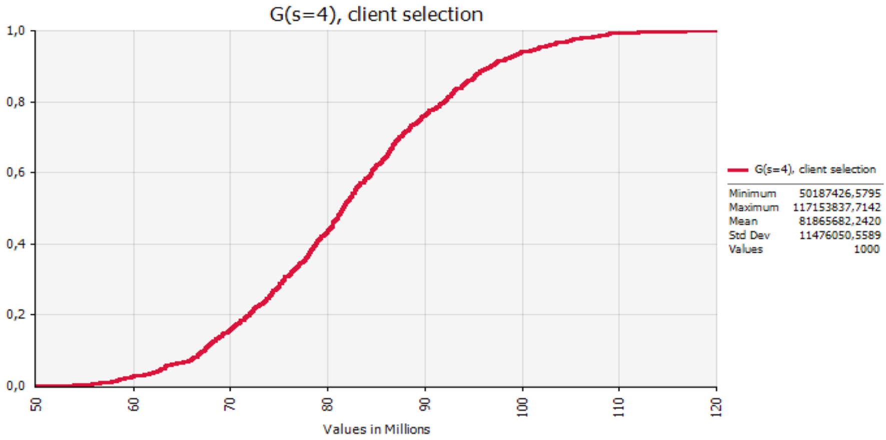

Fig. 11 Optimized client selection for one service station

$$P_{serv} = \left(1 + \rho + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4!} + \frac{\rho^5}{5!}\right) \times \left[\frac{\rho^0}{0!} + \frac{\rho^1}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4!} + \frac{\rho^5}{5!}\frac{1}{1-\frac{\rho}{5}}\right]^{-1} \tag{47}$$

Probability of service is about 99.99% while the mean expected profit is about 81,800,000$ (Fig. 12).

Optimized client selection: Stochastic optimization of the problem indicates all the ship categories to be served.

As above, probability of service is about 99.99% and the mean expected profit is about 81,800,000$ (Fig. 13).

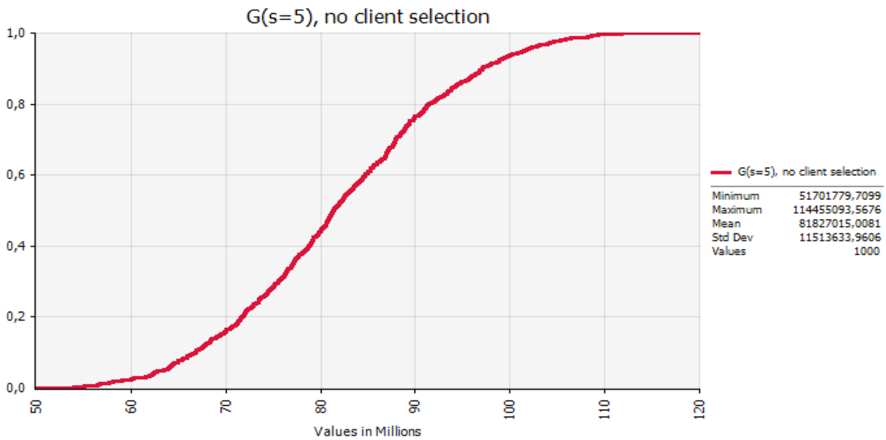

Fig. 12 Five service stations—profit per probability of service

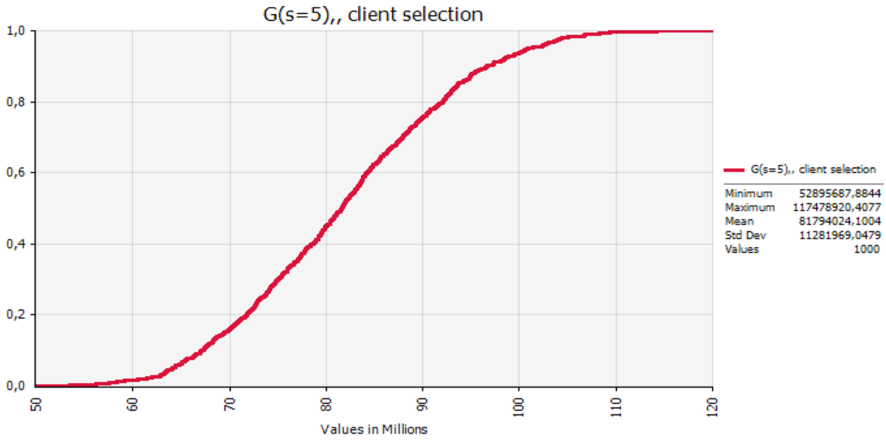

Fig. 13 Optimized client selection for one service station

4.3.6 Summarization of Scenarios and Decision-Making

Based on the above stochastic analysis, we are able to make decision regarding the optimal selection of the number of service stations that should be available in the port and the ships that will be served. In Fig. 14, the best financial result, before or after client selection for one to five service stations, is represented.

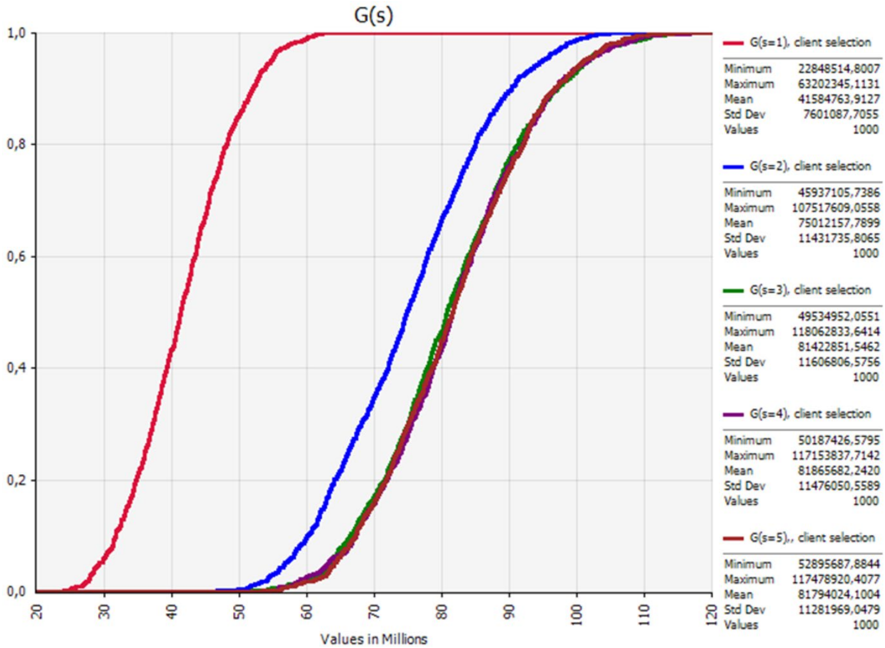

Fig. 14 All service stations—profit per probability of service

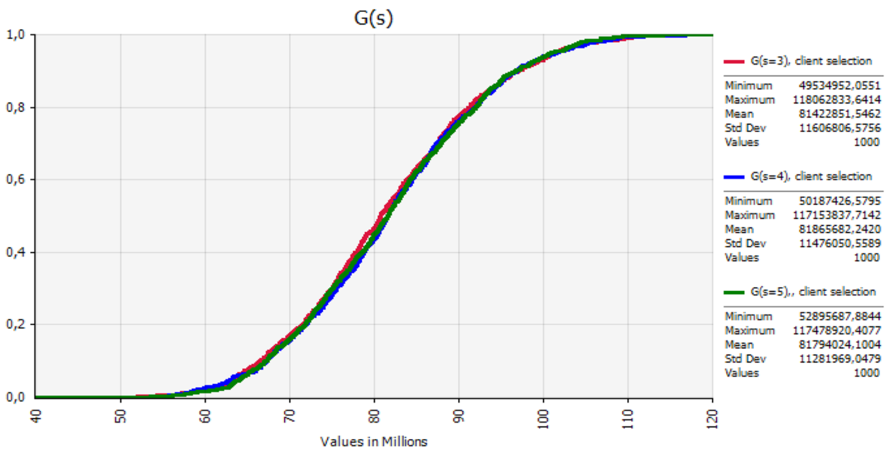

Fig. 15 Cumulative profit distributions for $s=3$, 4 and 5 service stations

It seems that the probability distribution functions for $s = 3$, $s = 4$ and $s = 5$ are not much different. This is also shown in Fig. 15 where the cumulative profit distributions are presented for 3, 4 and 5 service stations:

It is reminded that stochastic optimization's criterion was the maximization of the mean value of the probability distribution functions. So, the initial decision based on the analysis would be to enable four service stations. However, since there is no large difference among the probability distribution functions for $s = 3$, $s = 4$ and $s = 5$, secondary decision criteria could be utilized to reach a decision. So, if the decision maker is also interested in minimizing the service stations' cost, three service stations would be enabled while five service stations would be enabled if she was interested in maximizing the system's reliability. In any of these three cases, all ship categories are indicated to be served.

5 Discussion and Conclusion

The scope of this paper is to suggest a method of maximizing port administrator's profit, by modelling and optimizing the size of the waiting system. The problem is formulated as Mixed Integer Linear Programming while the whole process is being supported by Monte Carlo simulation techniques and applied in the case study of Piraeus Port Region. As all studies come with limitations, the current research, as mentioned above, is based on hypothetical scenarios of Piraeus Port. Also, the modelling and the identification of various parameters of this study are assumed based on expert knowledge or historical data or similar cases as it is mentioned in Sect. 3. Other assumptions but realistic ones are that the service stations are all the same and that the bunker ship needs about half an hour to approach the client ship and start the refuelling process.

A conclusion reached through the study is that, due to high leaving and waiting costs, the probability of service should be large enough, about 99,90%. If there is only one service station, a client selection must occur or the operation of the system could lead to financial loss for the port. If there are only two service stations, the profit is not maximized to a desirable degree. On the other hand, enabling too many service stations increases the expected mean profit initially, which subsequently starts to deteriorate. Thus, the study underlines the importance of the specification of the optimum number of stations in every different scenario.

The use of Poisson models assumes the service stations are all the same; bunker ships of 500 m^3 LNG cargo capacity. Another assumption was the fact that the bunker ship needs about half an hour to approach the client ship and start the refuelling process, as it is believed that this time is adequate for a bunker ship near the port of Piraeus to travel the required distance. If it takes more time for the bunker ship to approach the client ship, the profit of the system will be reduced.

The theoretical modelling and simulation could provide robust results that can support infrastructure owners and/or developers in Port of Piraeus and other ports as well to evaluate potential investments and in LNG refuelling systems. Specifically, the optimization for the number of the refuelling stations could minimize the CAPEX required and maximize the profit of the infrastructure administrator. In addition, ship operators and bunker suppliers could benefit from the model, as they can streamline their operations and provide refuelling on time. The model is versatile in the sense that it could be applied for different types of ships calling different ports. In fact, the results of the presented research and the proposed model can be used for further analysis where more islands can be integrated to cover small-scale LNG needs and therefore create a network of supply chain. Also, current policies impose the need for converting existing conventional power engines with new engines consuming natural gas, and so their supply with liquified natural gas will be imperative need in the near future. The model could also support national decision-makers in Greek maritime and energy sectors regarding an important and crucial problem. All the above conclusions are in accordance with the literature for LNG ports in Greece [43, 63–66].

Future research could provide realistic times based on observation in order to benchmark the assumed time and compare the results. The preferred metric used by the present study was the maximization of the mean value of the probability distribution function. Different metrics would prompt different decision from the port administrator side, regarding client selection and the number of service stations in the system. Further work could investigate the inclusion of other metrics depending on the decision maker's scope and criteria.

The presented approach can be adopted for similar case studies and other ports in order to identify the probable profitability and the optimal design of a hypothetical LNG station in a port.

Appendix

The following table is the table of nomenclature of all the sets, variables and parameters used in the study.

Symbol	Description
$\mathcal{I}$	the set of all the ships that will call the port administrator to ask for a refuelling service
$\mathcal{T}$	the time horizon under examination
T_i^{fs}	the time needed for a free service station to be commanded to start serving the i ship,$\forall i \in \mathcal{I}$
T_i^{Maxfs}	the maximum time that the i ship is willing to wait until it will be served,$\forall i \in \mathcal{I}$
T_i^s	the waiting time in the system
T_i^t	Time needed for the ship to approach a berth
T_i^{cs}	the time the service itself to be completed
C_i^b	the port administrative costs due to balk of the i ship,$\forall i \in \mathcal{I}$
C_i^w	the waiting cost of the ship i per time unit,$\forall i \in \mathcal{I}$
C_i^s	the variable cost for the administrator,$\forall i \in \mathcal{I}$
P_i	the payment of the ship i,$\forall i \in \mathcal{I}$
C^f	the waiting system's fixed cost
C_{fixed}^f	fixed cost of the system
c_s	the fixed cost of providing each service station
c_{sLNGi}	the cost of selling LNG to the ship i per cubic meter $(\$/m^3)$
s	the maximum number of service stations of the port
$N = \lvert\mathcal{I}\rvert$	the number of the ships that will ask for a refuelling service in the time horizon
P_i	The payment charged to the serviced ship i
P_{LNGi}	the selling price of LNG per cubic meter $(\$/m^3)$ for the ship i
Q_{LNGi}	the quantity of LNG that the ship i is fuelled with (m^3)
$u_{LNG,pipe}$	the velocity of LNG in the refuelling pipe
A_{pipe}	the area of the pipe used

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