Effects of dual networks on tax strategies: geography and transaction

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Abstract

This study investigates how a revenue-maximizing tax strategy of local and central governments incorporates dual networks, namely, an inter-firm transaction network and an inter-country geography network. We assume a two-stage game in which governments propose discriminatory tax levels for firms, whereas each firm has an incentive to invest in a country near the foreign branch office of its transaction partner. In our model, the centrality index of the Kronecker product of the two networks describes the interplay among the location choices and tax strategies in the equilibrium. A stronger linkage within each network generally increases demand for investment and in turn raises overall tax levels to exploit the high demand. Although more central firms in the inter-firm network are likely to be levied higher taxes because of their high demand for investment, firms in the highest tax bracket differ among countries depending on their geographical location. Finally, we show that a uniform tax in which firms are not discriminated and networks do not matter is the socially optimal tax, which incorporates all inter-country externalities. We also investigate decentralized tax strategies based on the rule of non-discriminatory (uniform) taxation and show, by comparing social welfare under discriminatory and uniform tax regimes, that restricting tax discrimination improves social welfare.

1 Introduction

Transaction networks play an important role in influencing firms' location choice and policy-making. Foreign direct investments involve transactions with local firms in destination markets, sometimes leading to various problems such as the mismatch of product design and quality and delivery systems (e.g., Reid 1995). Hence, some

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firms choose to make joint investments with their trading partners to replicate the current transaction partnership in a new investment destination (Hackett and Srinivasan 1998). Itoh (2014) introduced such transaction network effects in firms' location choice into a two-country tax competition model to show how governments impose different tax rates on firms depending on their position in the network.

Moreover, not all location spillovers remain within the region, because affiliates trade across countries and may even form regional supply networks (e.g., Baldwin and Okubo 2014).[1] At this time, as described by Anderson and van Wincoop (2004), because the trade costs have a large impact on international trade, it is essential for research to consider the geographical structure among regions and countries. In reality, such geographical structures are heterogeneous, and thus, we need a network of matrices to describe them. Many recent studies have investigated the location of economic activities in generalized geography (e.g., Allen and Arcolakis 2014; Redding 2016), and some mention that countries given the central position in the geography are likely to attract a large amount of economic activities (see Barbero et al. 2018 and Behrens et al. 2009). Furthermore, geography is also important in the literature on tax strategy, because policy-making externalities do matter in geographical networks, as some empirical evidence shows (e.g., Wang 2017). However, geographical structures have received only limited attention in the tax competition literature despite their growing importance.

Considering the importance of both network types in tax strategies, this study investigates how governments incorporate the interplay of the dual networks composed of transaction and geography into their revenue-maximizing tax strategies. As with Itoh (2014), we are interested in the relationship between discriminatory taxation across firms and their positions in the transaction network. However, unlike previous studies, we also focus on how the geographical position of each government matters as well as the firm's position in the transaction network. Furthermore, we investigate the distortion due to discriminatory taxation as well as the possibility of improving social welfare by refraining from tax discrimination as a cooperative rule.

To answer the above questions, this study models a situation in which firms in the same home country (e.g., the United States) consider foreign direct investment in rapidly growing markets (e.g., Asia). We assume that these firms have a current transaction relationship in the home country, which is exogenously given, and that the new markets also have an exogenous geographical relationship. Our model considers greenfield investments, with no prior investors in the new markets. The game has two stages: in the first stage, governments propose different tax levels for firms; in the second stage, firms choose to invest in each country with incomplete information.

In this model, the dual networks attempt to diffuse taxation strategies across countries as follows. First, when country A offers a special tax discount to a certain firm i, it directly increases i's investment. Furthermore, it also attracts investments from i's transaction partners, directly by offering colocation incentives and indirectly through the investment decision of i's partners, and so forth. In our model, we describe these processes by expectation diffusion due to incomplete information,

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[1] Baldwin and Okubo (2014) showed that transaction networks among the affiliates of Japanese companies have formed in east Asia.

following Bloch and Quérou ([2013](#)) and Itoh ([2014](#)). Second, the above tax discount also stimulates investment in the geographically neighboring regions of *A* because of the positive spillover caused by the colocation incentive in nearby regions. Therefore, the interplay of the dual networks does matter in firms' location choices and the tax strategies of countries.

The primary finding of this study is that the effects of the dual networks on firms' location choice and governments' tax strategies simply appear as the Kronecker product of these networks to integrate them and allocate a component to each firm–country combination or each individual investment. Our model is identical to Bloch and Quérou's ([2013](#)) framework in a special case with no geographical linkages, where the tax discrimination pattern is independent of the inter-firm network. However, both network structures matter when the geographical network is nonempty. From our results, stronger linkages in both networks increase demand for investment, and then, governments generally impose a higher tax, since it implies a rent for location. Although several numerical examples show that governments are likely to levy higher taxes on firms located in a more central position in the inter-firm network, the discrimination pattern differs by the geographical position of the government. That is, a government with closer relationships to other countries is likely to appreciate the indirect connections between firms, since the positive location externalities among firms are amplified through the geographical network. Finally, it is shown that a non-discriminatory tax across firms is socially optimal. Therefore, cooperation among countries in refraining from tax discrimination could be the second-best rule to increase the overall welfare of countries.

Our study is primarily related to tax competition research. Most initial work on tax competition, such as that of Wilson ([1986](#)) and Zodrow and Mieszkowski ([1986](#)), focuses on how countries and regions set corporate tax levels under fiscal externalities. Several recent studies attribute incentives for tax discrimination to foot-looseness (Haufler et al. [2018](#); Hong and Smart [2010](#)) and productivity (Baldwin and Okubo [2009](#); Haulfer and Stahler [2013](#)). Among them, only Itoh ([2014](#)) considered the role of inter-firm transaction network effects on tax competition between two countries to show that governments offer greater tax incentives to firms with higher degree centrality. Thus, this is the first study that considers both transaction and geography networks in one model. However, we ignore the competitive aspects of tax strategies and only consider the positive externalities of tax discounts, which causes the opposite inefficiency to conventional fiscal externalities and thus makes the taxes inefficiently high. Such a difference leads our result to Itoh's ([2014](#)) in terms of the effect of the transaction network on the tax level.[2] Comparing the results from different settings shows that the externalities influence the tax strategy.

Furthermore, this study is also related to the literature on price competition among companies for consumers in the network if we replace companies,

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[2] In Itoh ([2014](#)), governments impose a lower tax on more central firms to attract investment from other firms, whereas a higher tax is imposed on such firms because of their higher location demand in the present study.

consumers, and price in these studies with countries, firms, and tax, respectively.[3] Indeed, some of Itoh's (2014) basic mathematical structures are almost equivalent to ours, which confirms this study's relevance and contribution to the price competition literature.[4] First, Candogan et al. (2012) and Bloch and Quérou (2013) investigated monopolistic pricing by a single firm. Second, Chen et al. (2018b) examines a duopoly setting in which two firms do competitive pricing to sell two interdependent products to the consumers in a social network. The duopoly case was also examined by Aoyagi (2018) and Itoh's (2014) two-country model. Furthermore, Chen et al. (2020) focused on an oligopoly with multiple firms, leading to a surprising implication about the effect of the number of firms on the equilibrium price. However, they did not consider the network among providers. Furthermore, we assume that the services of providers (countries) are complements, while most previous studies assume that they are substitutes to consider the competition among them.[5] In other words, earlier studies have considered similar externalities to most of the literature on tax competition, and Chen et al. (2018b, 2020) also showed the negative effect of consumers' centrality on price, as in Itoh (2014).

Finally, in regards of the network among providers, Ushchev and Zenou (2018) considered the price competition among providers, given heterogeneous substitutabilities, described in the form of a network of providers. However, in their model, both the provider and the consumer networks are represented by a common geographical network. Therefore, to the best of our knowledge, no research has thus far investigated an independent inter-provider network or the mutual interaction of dual networks in the context of price competition.[6]

The rest of the paper is organized as follows. Our model is presented in Sect. 2. Section 3 investigates firms' location choice based on corporate tax and shows the equilibrium tax strategy. Section 4 considers regimes to improve social welfare. Section 5 concludes.

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[3] To unify the terms of the papers considering different issues, the players who set prices are called providers, and the players who choose providers are called agents.

[4] This study employs an incomplete information game on consumers' behavior, but our results do not significantly differ from those of existing studies. Indeed, the incomplete information model of Bloch and Quérou (2013) and complete information model of Candogan et al. (2012) lead to almost the same reduced form.

[5] Chen et al. 2018a considered two activities (goods) that can be both substitutes and complements, but did not investigate their pricing.

[6] de Marti and Zenou (2015) investigated an incomplete information model with dual networks, which consist of a social network among agents and a network of covariances among choices. Their study thus has some commonalities with this research (e.g., the agent's behavior is dependent on the Kronecker product of two networks).

2 The model

2.1 Inter-firm transaction and inter-regional geography networks

A set of n firms operate in a home country and these firms trade with each other in the domestic market. We assume that the pattern of the domestic transaction is exogenously given and described by an $n \times n$ matrix $\mathbf{G} = \{\phi_{ij}\}$, where the $\{i, j\}$ component of $\mathbf{G}$, denoted as $\phi_{ij} \in [0, 1]$, represents the strength of the transaction between firms i and j. In other words, a higher ϕ_{ij} means that the firms trade either a large amount of or less-substitutable products. We assume that the transaction network is symmetric, in which case $\phi_{ij} = \phi_{ji}$, and that the buyer and seller are not identified. Furthermore, $\phi_{ij} = 0$ holds for the diagonal components of the transaction network.

In addition, m emerging foreign regional markets are embedded in an exogenously given $m \times m$ geography network with no prior investors, denoted by $\mathbf{S} = \{\delta_{qr}\}$. The (q, r) component of $\mathbf{S}$, denoted as $\delta_{qr} \in [0, 1]$, represents the strength of the geographical link or accessibility between q and r. The profit from a transaction between a pair of firms located in regions q and r is discounted by δ_{qr}; the transaction does not yield a profit when $\delta_{qr} = 0$. because firms give up the transaction considering its high cost, while a higher δ_{qr} implies a lower discount for the transaction profit because of low transport costs. In particular, when $q \neq r$, δ_{qr} is defined as the international spatial discount of the transaction profit; δ_{qq} is defined as the domestic discount of the transaction profit of country q. We assume that $\delta_{qq} \geq \delta_{qr}$, because transaction cost is lower in the domestic market than in the international market. We assume that the geography network is also symmetric, so $\delta_{qr} = \delta_{rq}$ holds.

2.2 Firms

Each firm chooses countries for the establishment of new affiliates and invests in m countries at most. The size of an affiliate is exogenously given and the investment involves only a fixed cost. Affiliates of firms trading in the home country can do business with each other and earn profits. If a firm has multiple foreign affiliates, an affiliate of another firm can presumably trade with more than one of them.[7]

Profits depend on the strength of transactions in the domestic market and the spatial discount factor. With Nash bargaining between two sides with equal bargaining power, the firms receive equivalent profits and then assume $\phi_{ij} = \phi_{ji}$.[8] The transaction profit also depends on the spatial discount factor between the locations of the two firms denoted by δ_{qr}. The transaction profit decreases with transportation cost

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[7] We suppose that product of each affiliate is differentiated from the ones of other affiliates, even though they share the same parent company. This is why, we assume that an affiliate receives an independent location externalities from every, not only the nearest, affiliate of its domestic partner. However, the assumption is set just for simplicity of analysis. Profit from colocation with an affiliate will be influenced by transaction with other affiliates in reality.

[8] However, if either the bargaining power or the disagreement profit for the firms involved in the trade is asymmetric, then the network $\mathbf{G}$ is asymmetric, or the transaction network can be described by a directed graph.

between q and r, because it decreases the transaction volume, as verified by most empirical literature in trade (e.g., Anderson and van Wincoop 2004).[9]

Another form of profit accrues from the stand-alone profit from each location. This includes any non-transaction profit in the network, such as investment costs or product demand in the market of the destination country. We suppose that firm i's stand-alone profit from country r consists of two parts, a random term ϵ_r^i and a common fixed term $\overline{\epsilon}_r$. The fixed stand-alone profit $\overline{\epsilon}_r$ represents the location advantage common to all firms, such as lower average wage rate. Country q holds an advantage over r if $\overline{\epsilon}_q > \overline{\epsilon}_r$ holds. On the other hand, the random term depends on the specific properties of each firm, such as special preference for its product in the local market. This is private information for firm i; hence, only firm i can observe its own random stand-alone profit. Therefore, while firm i chooses its location given a specific value of ϵ_r^i, others only have stochastic information of the random term (we assume that ϵ_r^i follows a uniform distribution in the latter analysis).[10] However, from the stochastic information and shape of networks, firms can correctly estimate a firm's probability of investment in each location.

Furthermore, each local government r imposes a location tax, $z_r^i \in (-\infty, +\infty)$, which differs among firms. The vector of z_r^i is common knowledge for all firms; hence, each firm knows the tax levels that others are offered. This explains why taxation of a firm also affects other investment choices; this phenomenon is called the spillover effects of tax.

When firm i chooses country q, firm i's expected profit is denoted by the following:

$$
\hat{u}_q^i = \underbrace{\sum_r^m \sum_{j=1}^n \phi_{ij} \delta_{qr} P_r^j + \overline{\epsilon}_q - z_q^i + \epsilon_q^i}_{\bar{u}_q^i}
$$

$$
= \bar{u}_q^i + \epsilon_q^i,
$$

(1)

where P_r^j is the probability that firm j invests in country r. Moreover, $\bar{u}_q^i$ is called the deterministic part of the expected utility, which is common information for all players. $\sum_{r=1}^m \sum_{j=1}^n \phi_{ij} \delta_{qr}$ is firm i's total profit from transacting with its trading partners. We assume that the foreign affiliates can trade following their domestic relationships. If $q = r$, implying that firms i and j collocate in the same country, they obtain $\phi_{ij} \delta_{qq}$ from the local transactions within country q. Also, even if $q \neq r$, meaning that firms i and j locate in different countries, they earn $\phi_{ij} \delta_{qr}$ from their transactions across the border. We assume that $\delta_{qq} \geq \delta_{qr}$. Furthermore, $\delta_{qr} = 0$ implies that firms i and j give up trading because of too high transaction costs. In that case, the demand

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[9] Firms can give up a transaction if it does not yield a positive profit and seek a new partner in the local destination market. Therefore, $\delta_{qr} = 0$ holds when firms give up a transaction between two regions because of too high costs.

[10] As in Bloch and Quérou (2013) and Itoh (2014), firms play the game with the threshold strategy; that is, a firm decides to invest in region r if ϵ_r^i is higher than a threshold level.

will be fulfilled by transactions with the foreign firm's local affiliate. Finally, we assume that each foreign affiliate provides a different product to fit local demand. Therefore, the presence of an affiliate never affects the profit of other affiliates of the same firm; hence, investment decisions for different locations are independent of each other.

2.3 Governments

We assume that each region is governed by its local government, which imposes discriminatory corporate tax on each firm located in the region. Despite the governments' uncertainty about the actual location of firms, the probability of their location choice can be accurately estimated based on common information: the distribution of random stand-alone profits, tax vector of other regional governments, and structure of the transaction and geographical networks.

Each regional government aims to maximize the expected revenue from tax, which may be budgeted for local public good provision or income redistribution to the population in the country. In the present study, we simply consider that the tax revenue maximization is equivalent to social welfare maximization. Although the effects of foreign direct investment on any market configuration such as wages, employment, and consumption prices are important and have been considered in most tax competition literature, they are intractable in our simplified framework in which households behavior is not explicitly described.

2.4 Procedure of the game

We describe the structure of the game, which consists of the following two stages.

2.4.1 Stage 1: Setting tax by governments

Local governments choose their taxes at the same time to maximize their expected tax revenue. The government of region r chooses a vector of discriminatory taxes, denoted by $\mathbf{z_r} = (z_r^1, \ldots, z_r^n)$. Therefore, the tax revenue function of government r is as follows:

$$\pi_r = \sum_{i=1}^{n} P_r^i z_r^i, \tag{2}$$

where P_r^i depicts the probability that firm i is located in region r, because tax is paid only when the firm actually locates. In this stage, each government knows the structure of the networks, distribution function of random stand-alone profits, and taxation proposals offered to firms by all regions. With this information, P_r^i is correctly estimated.

2.4.2 Stage 2: Investment choices by firms

Before the second stage, a random stand-alone profit is allotted to each firm. Furthermore, each firm knows the tax vectors from the first stage; that is, the information on the discriminatory taxes proposed for everyone is public knowledge. Firm i decides to invest in q if its profit is positive, or $\hat{u}_q^i = \bar{u}_q^i + \epsilon_q^i > 0$ holds. In the first stage, the government and each firm have identical expectations about P_r^i, since they have access to the same common information. Each firm knows that other firms can only forecast its probable decision, but have no way to unveil its final decision.

3 Equilibrium

3.1 Investment

We start with the second stage of the game in which firms determine their investments given the tax levels. We suppose that the random stand-alone profit of firm i in country q, denoted by ϵ_q^i, is uniformly distributed in $[-F, 0]$, where $F > 0$ is the interval of the distribution.

Firms invest in region q if and only if the investment yields positive profits. Therefore, the probability that firm i invests in country q is:

$$
\begin{aligned}
P_q^i &= \text{Prob}\big[\hat{u}_q^i \geq 0\big] \\
&= \bar{u}_q^i / F \\
&= \frac{1}{F}\Big[\sum_{r=1}^{m} \sum_{j=1}^{n} \phi_{ij}\delta_{qr}P_r^j + \bar{\epsilon}_q - z_q^i \Big].
\end{aligned}
\tag{3}
$$

We assume that the internal probability $P_q^i \in (0, 1)$ holds.[11]

Furthermore, we define the vector $\mathbf{P_q} = (P_q^1, P_q^2, \ldots, P_q^n)^{\mathrm{T}}$ to represent all firms' objective probabilities of choosing country q, where the superscript T indicates the transportation of vectors. The vector representation of P_q^i then becomes:

$$
\mathbf{P_q} = \frac{1}{F}\Big[\sum_{r=1}^{m}(\delta_{qr}\mathbf{GP_r}) + \bar{\epsilon}_{\mathbf{q}} - \mathbf{z_q} \Big],
\tag{4}
$$

where the vector $\bar{\epsilon}_{\mathbf{q}}$ equals $(\bar{\epsilon}_q, \bar{\epsilon}_q, \ldots, \bar{\epsilon}_q)^{\mathrm{T}}$; that is, each firm's common fixed stand-alone profit in country q. The vector $\mathbf{z_q}$ equals $(z_q^1, z_q^2, \ldots, z_q^n)^{\mathrm{T}}$, which represents country q's tax level for each firm. We define the vector $\mathbf{P} = (\mathbf{P_1}^{\mathrm{T}}, \mathbf{P_2}^{\mathrm{T}}, \ldots, \mathbf{P_m}^{\mathrm{T}})^{\mathrm{T}}$ to represent all firms' objective probabilities of location choice. We also define the following joint matrix consisting of the geography and transaction matrices as follows:

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[11] The interiority is guaranteed by a sufficiently large a, as shown by Bloch and Quérou (2013) and Itoh (2014).

$$\mathbf{J} \equiv \mathbf{S} \otimes \mathbf{G} = \begin{bmatrix} \delta_{11}\mathbf{G} & \delta_{12}\mathbf{G} & \dots & \delta_{1m}\mathbf{G} \\ \delta_{21}\mathbf{G} & \delta_{22}\mathbf{G} & \dots & \delta_{2m}\mathbf{G} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m1}\mathbf{G} & \delta_{m2}\mathbf{G} & \dots & \delta_{mm}\mathbf{G} \end{bmatrix}. \tag{5}$$

$\mathbf{J}$ is the Kronecker product of $\mathbf{G}$ and $\mathbf{S}$, the inter-firm transaction and inter-regional geography networks, respectively, and is represented by a *joint matrix*. This is a $mn \times mn$ matrix of the values allocated to different country and firm combinations, or investment outcomes. Solving Eq. (4) for vector $\mathbf{P}$ with the joint matrix yields:

$$\mathbf{P} = \frac{1}{F}\left[\mathbf{I} - \frac{1}{F}\mathbf{J}\right]^{-1}(\overline{\epsilon} - \mathbf{z}). \tag{6}$$

In Eq. (6), the vectors $\overline{\epsilon}^{\mathrm{T}} = (\overline{\epsilon}_1^{\mathrm{T}}, \overline{\epsilon}_2^{\mathrm{T}}, \dots, \overline{\epsilon}_m^{\mathrm{T}})$ and $\mathbf{z}^{\mathrm{T}} = (\mathbf{z_1}^{\mathrm{T}}, \mathbf{z_2}^{\mathrm{T}}, \dots, \mathbf{z_m}^{\mathrm{T}})$ represent fixed stand-alone profits and taxes, respectively. Note that $\mathbf{B1} \equiv [\mathbf{I} - \frac{1}{F}\mathbf{J}]^{-1}\mathbf{1}$ is called Katz–Bonacich network centrality $\mathbf{J}$ (Ballester et al. 2006). For convenient referencing, $\mathbf{B}$ is represented by the following $mn \times mn$ matrix:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \dots & \mathbf{B}_{1m} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \dots & \mathbf{B}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{m1} & \mathbf{B}_{m2} & \dots & \mathbf{B}_{mm} \end{bmatrix}, \mathbf{B}_{qr} = \begin{bmatrix} b_{qr}^{11} & \dots & b_{qr}^{1n} \\ \vdots & \ddots & \vdots \\ b_{qr}^{n1} & \dots & b_{qr}^{nn} \end{bmatrix}. \tag{7}$$

It is well known that $\left[\mathbf{I} - \frac{1}{F}\mathbf{J}\right]^{-1} = \sum_{t=0}^{\infty}\left(\frac{1}{F}\right)^t\mathbf{J}^t$ holds. Hence, each component of $\mathbf{B} = \{b_{qr}^{ij}\}$ is described as $b_{qr}^{ij} = \sum_{t=0}^{\infty}(\frac{1}{F})^t\phi_{ij}^{\{t\}}\delta_{qr}^{\{t\}}$, where $\phi_{ij}^{\{t\}}$ and $\delta_{qr}^{\{t\}}$ denote the components of $\mathbf{G}^t$ and $\mathbf{S}^t$, respectively. Therefore, b_{qr}^{ij} indicates how each node in the joint network is accessible to all the other nodes. Itoh (2014) used the Katz–Bonacich centrality measure $\mathbf{B}$ to analyze the effect of the transaction network on the location choice of firms.

However, in our model, we use the Katz–Bonacich centrality measure of the joint matrix $\mathbf{J}$ as in Eq. (5). The $\delta_{qr}\mathbf{G}$ submatrix of the joint matrix $\mathbf{J}$ represents each firm's transaction profit with the spatial discount factor δ_{qr}. Therefore, each component of matrix $\mathbf{J}$, $\delta_{qr}\phi_{ij}$, represents the profit from a transaction between firm i located in country q and firm j located in country r. Therefore, Katz–Bonacich centrality is allocated in the form of $\left[\mathbf{I} - \frac{1}{F}\mathbf{J}\right]^{-1}\mathbf{1}$ not to each i but to each $i - q$, or firm i's investment in country q. This is because the potential of a firm to transact business with others depends on its location as well as the geography and transaction networks.[12]

For analytical convenience in the following sections, we summarize as follows.

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[12] Our joint matrix is also similar to inter-regional input/output tables in which transactions are classified by both sector and region. Isard (1951) introduced a Leontief inverse of these matrices, well known for its equivalence to Katz–Bonacich centrality, to investigate the diffusion of demand across sectors and regions.

Proposition 1 *The equilibrium location probability given $\overline{\epsilon}$ and $\mathbf{z}$ is described by* (6). *Moreover, the probability is proportional to the Katz–Bonacich centrality* **B1** *when all countries and firms are homogeneous in regard to $\overline{\epsilon}$ and $\mathbf{z}$.*

Given the fixed stand-alone profit vector $\overline{\epsilon}$ and tax levels $\mathbf{z}$ in all countries for each firm, the probability that firm i chooses country q is proportional to the Katz–Bonacich centrality in the joint matrix $\mathbf{J}$. Our result shows that Katz–Bonacich centrality matters in equilibrium investment. This result also shows that b_{qr}^{ij} describes the spillover effect of the tax applied to firm j by government r on the investment of firm i in country q. Hence, $\mathbf{B}$ implies how the tax strategies spread all over the economy through the transaction and geography networks. Therefore, we may conclude that the probability depends on the aggregated inflows of the spillover effects from other investment choices.

3.2 Discriminatory tax strategies

This section examines stage 1 of the game in which the local and regional governments propose a different tax level for each firm to maximize their expected tax revenue. The tax revenue of government q is:

$$\pi_q = \sum_{i=1}^{n} P_q^i z_q^i. \tag{8}$$

Government q solves the problem $\max_{\mathbf{z_q}} \pi_q(\mathbf{z_1}, \dots, \mathbf{z_m})$ with the given tax vectors of the other countries. Suppose a tax strategy equilibrium, denoted by $\mathbf{z_1^*} = (z_1^{1^*}, z_1^{2^*}, \dots, z_1^{n^*})^{\mathrm{T}}, \dots, \mathbf{z_m^*} = (z_m^{1^*}, z_m^{2^*}, \dots, z_m^{n^*})^{\mathrm{T}}$. In the equilibrium, the following first-order condition holds for firm k:

$$\frac{\partial \pi_q}{\partial z_q^k} = \underbrace{P_q^k}_{\text{Intensive margin}} + \underbrace{\sum_{i=1}^{n} \left[\frac{\partial P_q^i}{\partial z_q^k} z_q^i \right]}_{\text{Extensive margin}}$$

$$= \frac{1}{F} \sum_{r=1}^{m} \sum_{i=1}^{n} b_{qr}^{ki}(\overline{\epsilon}_r^i - z_r^i) - \frac{1}{F} \sum_{i=1}^{n} (b_{qq}^{ik} z_q^i) \tag{9}$$

$$= 0.$$

The first and second terms are called the intensive margin and extensive margin, respectively. First, a tax increase by the government, given constant investment demand, results in a rise in tax revenue, described as the intensive margin. The greater current demand, the larger is the intensive margin. However, the negative effect on revenue from a decrease in investment demand is captured by the extensive margin, which includes any indirect effects through the network. We find that the intensive and extensive margins depend on the inflow and outflow of spillovers, or b_{qr}^{ki} and b_{rq}^{ik}, respectively. While the intensive margin of each local government

represents all the inflows of externalities into q and k, the extensive margin accounts for only part of the entire outflow, b_{qq}^{ik}, indicating the influence within the country.[13]

Denoting the column vectors of $\mathbf{P}_q^k$, $q \in \{1, \dots, m\}, k \in \{1, \dots, n\}$ and $\mathbf{P}$, we can also write Eq. (9) in matrix form as follows:

$$\mathbf{P} - \frac{1}{F}\mathbf{D}(\mathbf{B})\mathbf{z} = \mathbf{0}. \tag{10}$$

In this equation, $\mathbf{D}(\mathbf{B})$ is a matrix defined as:

$$\mathbf{D}(\mathbf{B}) = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{22} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}_{mm} \end{bmatrix}, \tag{11}$$

where the submatrix $\mathbf{B}_{qq}$, $q \in \{1, \dots, m\}$ is the diagonal submatrix of matrix $\mathbf{B}$, too. As vector $\mathbf{P}$ was expressed in the form of Eqs. (6), (10) becomes:

$$\frac{1}{F}\mathbf{B}(\overline{\epsilon} - \mathbf{z}) - \frac{1}{F}\mathbf{D}(\mathbf{B})\mathbf{z} = \mathbf{0}. \tag{12}$$

We then solve the governments' tax strategy equilibrium as follows:

$$\mathbf{z} = (\mathbf{B} + \mathbf{D}(\mathbf{B}))^{-1}\mathbf{B}\overline{\epsilon}. \tag{13}$$

For isolated countries, or $\mathbf{S} = \mathbf{I}$, our model can confirm $\mathbf{D}(\mathbf{B}) = \mathbf{B}$, where $\mathbf{z} = \frac{1}{2}\overline{\epsilon}$ holds. The result is the same as in an empty inter-firm network or $\mathbf{G} = \mathbf{0}$ since $\mathbf{B} = \mathbf{I}$ holds then. For inter-country linkages, however, dual networks matter, depending on the government's tax policies; hence, we show the following statement.

Proposition 2 *The equilibrium tax rate is described as Eq.* (13), *where dual networks matter. However, if either the geographical network or the inter-firm network is empty, the other network is irrelevant.*

It is not surprising that network effects disappear, considering that the case of isolated countries is similar to the monopolistic supplier's profit maximization rule found by Bloch and Quérou (2013). However, why do (do not) all the network effects vanish without (with) geography?

This is because of the (in)existence of international spillovers. Recall from the above discussion that the tax level is adjusted by two opposite incentives, called the intensive and extensive margins, each of which is related to the network's spillover inflow and outflow, respectively. From Eq. (12), we find that only part of the outflow, $\mathbf{D}(\mathbf{B})$, is accounted for in the extensive margin, while all the inflow is accounted for in the intensive margin. This means that countries do not consider the externalities to other countries, while they do take account of those from all others. However,

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[13] Here, b_{qr}^{ki} represents the inflow spillover into the investment of k in country q, while b_{rq}^{ik} is the outflow.

when countries are isolated, there are no international spillovers, and the intensive and extensive margins are identical, because both only account for intra-national spillovers. Therefore, the opposite two network effects cancel out in that case. However, the network effects remain in the general case with international spillovers, since the two effects are asymmetric.

3.3 How do dual networks matter?

Although the interplay of non-empty dual networks matters for tax strategy, we cannot extract any comprehensive understanding about how they work, because it is hardly tractable under general conditions. Therefore, by limiting some parameters and network structures, we try to supply some implications.

3.3.1 Small discount factor

The first limitation is given by an extreme value of F. We assume the case with $F \to +\infty$, where there is large uncertainty in other players' decisions. That is, $1/F$, or the discount factor of Katz–Bonacich centrality, is strictly positive but sufficiently small to ignore $1/F^t$ for $t \geq 2$. Therefore, $\mathbf{B} = \mathbf{I} + \frac{1}{F}\mathbf{J}$ holds given the assumption; hence, the equilibrium tax satisfies the following.

Proposition 3 *Given* $F \to +\infty$ *and* $\bar{\epsilon} = 1$, *the equilibrium tax is* $z_q^i = \frac{1}{2} + \frac{1}{4F}\deg_q(\mathbf{S}) \times \deg_i(\mathbf{G})$, *where* $\deg_q(\mathbf{S}) \equiv \sum_r \delta_{qr}$ *and* $\deg_i(\mathbf{G}) \equiv \sum_j g_{ij}$ *describe the degree centralities of country q in network* $\mathbf{S}$ *and firm i in network* $\mathbf{G}$, *respectively.*

See Appendix 3 for more details. Since the second-order and higher order indirect effects do not matter, only the first-order direct relationship determines the tax strategy. Therefore, the tax level offered by region *r* to firm *i* is determined by the multiproduct of the degree centralities of *r* and *i* in networks $\mathbf{S}$ and $\mathbf{G}$, and the tax level increases with the number of links to which the country and firm are connected. From Eq. (10), any network effect disappears from the extensive margin under $F \to +\infty$,[14] and then, the (direct) network effect only remains in location demand $\mathbf{P}$ or the intensive margin; hence, a denser network always increases the tax level via the intensive margin.

3.3.2 Homogeneous networks

We next consider a non-negligible discount factor while limiting the network structure. First, for the most tractable case, we assume homogeneous dual networks in which all firms and countries are homogeneously linked to each other; that is,

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[14] Comparing Eq. (9) with Eq. (10), the extensive margin is described by $\mathbf{D}(\mathbf{B})$ in matrix form. Note that $\mathbf{D}(\mathbf{B}) = \mathbf{I}$ holds given $F \to +\infty$, and thus, the network effect disappears there.

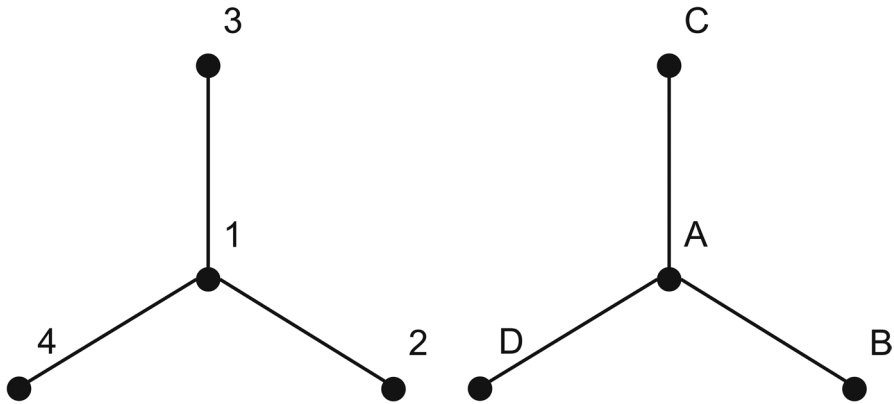**Fig. 1** Star networks

$\phi_{ij} = \overline{\phi}, \forall \ i \neq j, \ \delta_{qr} = \overline{\delta}, \forall \ q \neq r, \ \delta_{qq} = \delta, \forall q \in \{1, \ldots, m\}$. We call $\overline{\phi}$ and $\overline{\delta}$ the strength of transaction linkages and geographical linkages, respectively. We also assume $\bar{\epsilon} = \mathbf{1}$; that is, fixed stand-alone profits are homogeneous. Each country's tax strategies are as follows:

$$z = \frac{1}{2 - \frac{(m-1)\alpha\overline{\delta}}{1-\alpha\delta+\alpha\overline{\delta}}}, \tag{14}$$

where $\alpha = \frac{(n-1)\overline{\phi}}{F}$ holds; see Appendix 4 for more details. We thus have the following proposition.

Proposition 4 *Assuming that both transaction and geography networks are homogeneous, (i) inter-firm and inter-country linkages have positive effects on the tax level; (ii) intra-country accessibility has a negative effect on the tax level; and (iii) the tax level is always higher than one-half.*

The intuition behind result i) is as follows. When $\overline{\phi}$ increases, firms trade larger amounts of products. Then, each firm's total transaction profit increases with $\overline{\phi}$. Therefore, demand for investment increases in each country and the government imposes a higher tax on each firm. This is also the case for $\overline{\delta}$, namely, the strength of geographical linkages, for similar reasons. This spatial discount factor has a positive effect on governments' equilibrium taxes. These results support Proposition 3 under a different assumption. However, from result ii), intra-country accessibility has the opposite effect, because it strengthens the extensive margin, which has a negative effect on tax. Furthermore, as in the previous special case, result iii) shows that the existence of geographical linkages increases the tax level compared with that in isolated countries.

Finally, several studies considering competition among providers show the negative effect of the transaction network on the tax level, which is contrary to our results in point i) of Proposition 4 (Chen et al. 2018b, 2020; Itoh 2014). In their models,
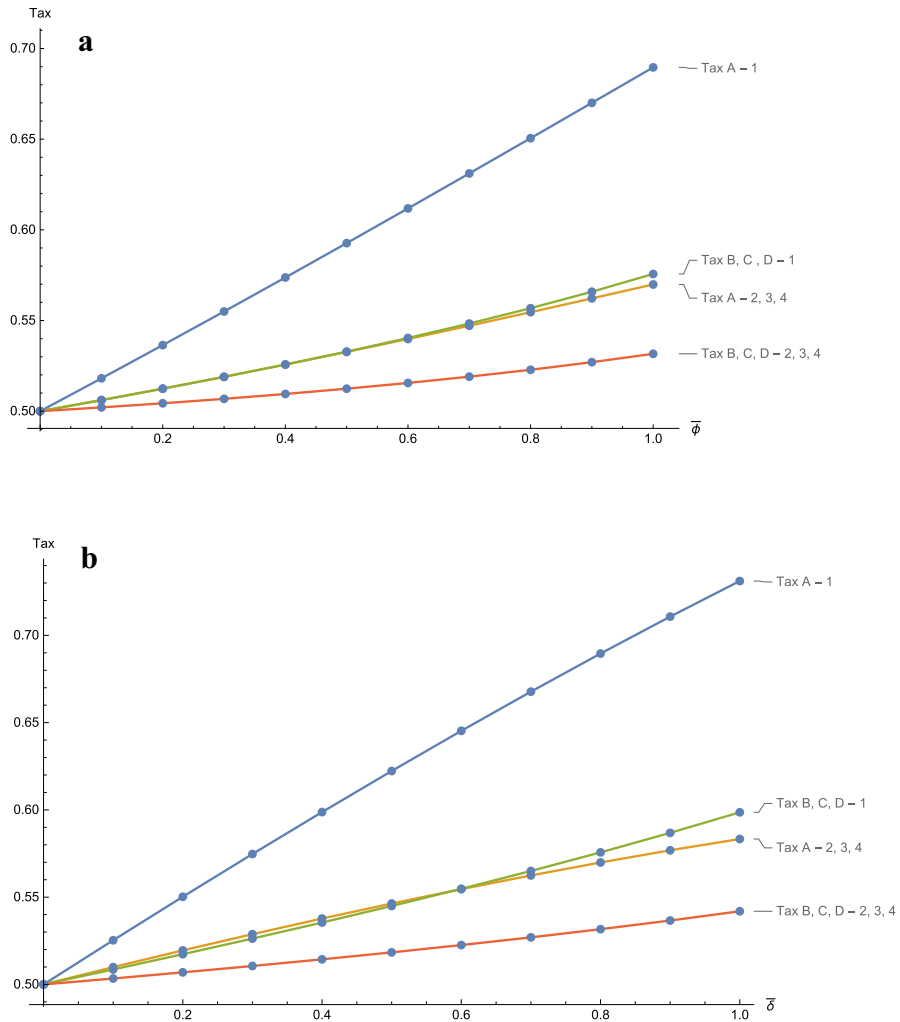
Fig. 2 **a** Effect of $\bar{\phi}$ on taxes. **b** Effect of $\bar{\delta}$ on taxes

products or locations are substitutes, and thus, consumers (or firms in Itoh (2014)) are more sensitive to prices. Hence, the network effects in the extensive margin outweigh those in the intensive margin. However, the intensive margin is dominant in the present model in which competition is not considered.

3.3.3 Dual star networks

Although the above result shows the positive effects of geographical and transaction linkages, it is limited to the special case of homogeneous networks. Therefore, to check the results in more generalized cases, we provide a simple numerical example

of heterogeneous networks. To simplify the example, we consider the star-shaped inter-firm network **G** and inter-country geography network **S**, as described in Fig. 1, each of which has four nodes.

We assume that $\phi_{ii} = 0, i \in \{1, \ldots, 4\}, \phi_{12} = \phi_{13} = \phi_{14} = \bar{\phi}$ in the inter-firm transaction network and $\delta_{qq} = 1, q \in \{A, \ldots, D\}, \delta_{AB} = \delta_{AC} = \delta_{AD} = \bar{\delta}$ in the inter-country geography network. We set $\frac{1}{F} = \frac{1}{10}$, and $\bar{\epsilon}_A = \bar{\epsilon}_B = \bar{\epsilon}_C = \bar{\epsilon}_D = 1$. We then obtain the government's tax level for each firm from Eq. (13). The results are as follows.

First, we set different values of $\bar{\phi}$ for a fixed value of $\bar{\delta} = 0.8$ to examine the effect of the transaction network's linkage strength on taxes. We show the results in Fig. 2a, where the line "Tax A - 1" indicates the tax level of country A for firm 1. Similarly, Fig. 2b presents the effect of geographical linkages $\bar{\delta}$ when the value of $\bar{\phi} = 1$ is fixed.

We find that all the results in this example confirm the validity of the previous analytical results. First, these figures show that the strength of the links of transaction and geography networks has positive effects on taxes. Furthermore, comparing the taxes among firms and countries, the government in the central position proposes higher taxes and the firms in the central position are levied higher taxes. Increasing accessibility in each network leads to higher investment demand, and hence, countries impose higher taxes due to the intensive margin.

3.3.4 Geographical location and tax discrimination pattern

In the previous single network models, only the shape of the social network or transaction network between companies matters for price discrimination among agents (i.e., consumers or companies). In other words, agents in the network, on whom providers impose the higher taxes, were the same for all providers, and this is also the case for our previous results where the geography and other settings are simplified. However, in more general cases, the importance of each agent is determined by a joint matrix of two networks, which assigns components to all provider–agent combinations. In the results, the tax discrimination patterns differ among countries depending on their geographical locations. Let us show a simple numerical example.

The inter-firm network in this example consists of two corporate groups, each with a star-like structure and the most centrally looking hub company (companies). The first group is a perfect star consisting of nodes 1–6, where node 1 is the hub company. The second group consists of nodes 7–12, where nodes 7 and 8 are hub companies, which share four peripheral firms. Intuitively, the structure of the first group is more intensive and dominated by one large company, while that of the second group is flatter. We find that node 1 has a higher degree than firms 7 and 8 to cause (and receive) larger direct externalities, while the second group has a larger number of links, and thus, the hubs may give (and receive) larger indirect externalities. The geography network, on the contrary, is made up of three countries, where we assume $\delta_{AC} = 0$ and $\delta_{AB} = \delta_{BC} = 0.8$; therefore, this is a line (or star) network centered by B.
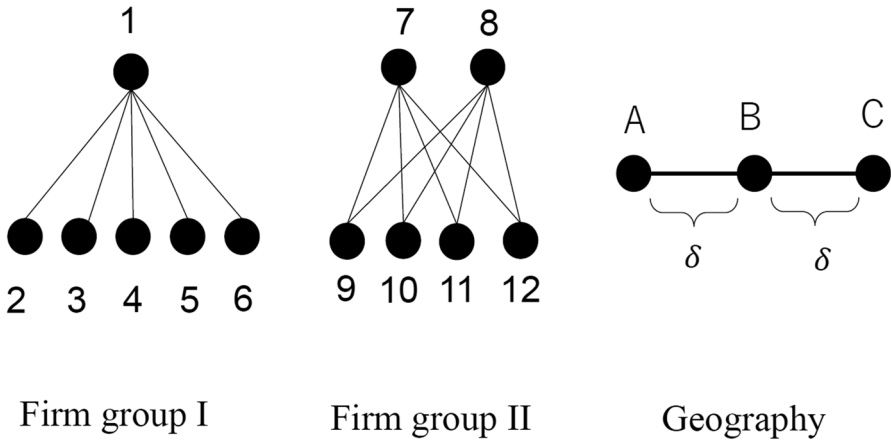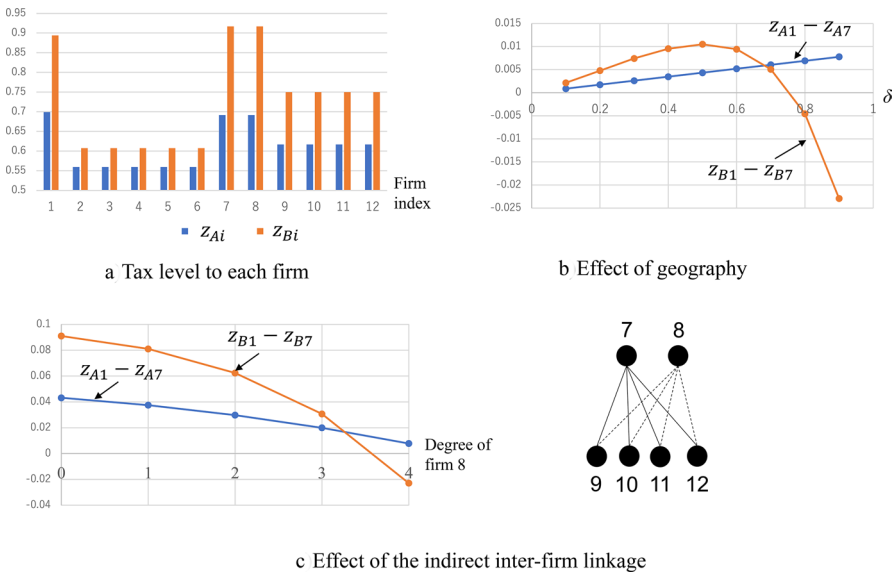
Fig. 3 Networks

a Tax level to each firm

b Effect of geography

c Effect of the indirect inter-firm linkage

Fig. 4 Countries' tax levels on each firm

Figure 4a shows the basic result.[15] As we anticipate from the results presented earlier in this subsection, both countries impose higher taxes on firms 1, 7, and 8, which have central positions in each group because of their higher investment demand. However, note that countries A and C impose a higher tax on the first

---

[15] We assume $\bar{\phi} = 1$ for all the existing transaction links and $1/F = 0.14$. Furthermore, $\bar{\epsilon} = 1$ is also assumed.

group, while country B does so on the second group, because preference for location differs among firms.

To understand this result, remember Proposition 1, which states that investment demand given homogeneous tax depends on the centrality or aggregated externalities in the joint matrix. In this example, compared with firm 1, firms 7 and 8 have fewer direct linkages but more indirect relationships. Therefore, by choosing a more widely accessible location such as country B, they can receive widely spread indirect effects from the investment in other countries. Conclusively, we obtain a tentative implication that a more accessible country is likely to impose a higher tax on those firms with richer indirect relationships.

We also carry out additional numerical calculations to check the validity of the above conjecture. First, to check the role of geography, we change the degree of inter-country linkages, $\delta_{AB} = \delta_{BC} = \delta$. The result in Fig. 4b shows that all the countries impose the highest tax on firm 1 when δ is low and countries are inaccessible, while a pattern as in Fig. 4a appears as δ increases. The inverted U shape of $Z_{B1} - Z_{B7}$ implies that the positive influence of δ on $Z_{B1} - Z_{B7}$ through the direct network effect is more significant when δ is small, while the negative influence through the indirect effect is more significant when δ is large. This is because the indirect effect appears multiplied by a power of delta squared or a higher power of δ, and hence, its marginal value is variable, while the direct effect linearly increases with δ.[16] For the same reason, $Z_{A1} - Z_{A7}$ is monotonically and almost linearly increasing with δ, because the direct effect is always dominant for peripheral countries.

Second, Fig. 4c focuses on the variation of the inter-firm network of the second group to understand how indirect linkages matter. Starting from the situation in which firm 8 has no links, we gradually add the links between firm 8 and firms 9–12, so that the network structure finally reaches the original structure, as shown in Fig. 3. Here, we focus on the taxes on firm 7, which change only because of the indirect effects. When the degree of company 8 is zero, group 2 becomes a star network, but it is smaller than group 1; then, each country imposes a higher tax rate on firm 1 than firm 7, whereas country B chooses larger tax discrimination among the firms than the other countries, because it is more significantly affected by the network. Furthermore, the increase in the indirect effect caused by the new links to firm 8 increases the taxes of all countries on firm 7, but the indirect effect is the most significant for country B. As a result, the two curves intersect and $Z_{B1} - Z_{B7}$ even reaches negative values. These two analyses show that the tax discrimination pattern depends on the indirect linkages in a transaction network.

4 Social welfare and tax coordination

We have thus far considered that local governments have perfect knowledge of the interactions in dual networks. However, decision making by local governments is not socially optimal, because they ignore the externalities influencing other

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[16] The marginal effect of δ on δ^2 is negligible when δ is near zero.

governments. That is, a tax discount offered in a region directly increases the number of firms locating not only in that region but also in all neighboring regions, because the transaction profit increases in those regions.

We now examine social welfare. As the objective function of the local government is given by the tax revenue, it is reasonable that social welfare is also defined by the aggregated tax revenue of all the countries. The wage, price, and consumer surplus are not explicitly described, and thus, they are intractable in this model, as mentioned in Sect. 2.3.[17]

In the following subsection, we compare the social welfare among three tax coordination agreements: the decentralized discriminatory tax examined above, socially optimal tax by the central government, and decentralized uniform tax.

4.1 Central government

We now assume a central government that can govern all emerging regions. The central government is interpreted as a federal government governing all the regions hosting the investment or an authority of an international partnership of several neighboring countries. It aims to maximize total social welfare defined by total tax revenue. Analyzing the central government is a useful benchmark to let us know how decentralized decision-making is distorted and how it should be improved or restricted.

We suppose that the central government, instead of local governments, plays step 1 of the game to maximize social welfare, defined by aggregate tax revenue and described as:

$$\pi = \sum_{q=1}^{m} \pi_q = \sum_{q=1}^{m} \sum_{i=1}^{n} P_q^i z_q^i. \tag{15}$$

The central government solves the problem $\max_{\mathbf{z}} \pi(\mathbf{z_1}, \dots, \mathbf{z_m})$ given the tax vectors of the other countries. In the equilibrium, the following first-order condition holds for country $r \in \{1, \dots, m\}$ and firm $k \in \{1, \dots, n\}$:

$$\frac{\partial \pi}{\partial z_r^k} = P_r^k + \sum_{q=1}^{m} \sum_{i=1}^{n} \frac{\partial P_q^i}{\partial z_r^k} z_q^i$$

$$= \underbrace{\frac{1}{F} \sum_{q=1}^{m} \sum_{i=1}^{n} b_{rq}^{ki}(\bar{\epsilon}_q^i - z_q^i)}_{\text{Intensive margin}} - \underbrace{\frac{1}{F} \sum_{q=1}^{m} \sum_{i=1}^{n} b_{qr}^{ik} z_q^i}_{\text{Extensive margin}} \tag{16}$$

$$= 0,$$

---

[17] The profit of firms is not included in social welfare, since we focus on cooperative taxation among governments as in Itoh (2014). This setting would be reasonable if we consider an inter-regional agreement within a country for investment by foreign companies.

where b_{qr}^{ik} is a component of matrix $\mathbf{B}$. As in the decentralized case, the first and second terms represent the intensive and extensive margins of tax revenue, respectively. We then rewrite Eq. (16) in matrix form as follows:

$$\frac{1}{F}\mathbf{B}(\bar{\epsilon} - \mathbf{z}) - \frac{1}{F}\mathbf{B}^{\mathrm{T}}\mathbf{z} = \mathbf{0}. \tag{17}$$

In our model, we assume that the dual networks are symmetric, so $\mathbf{B} = \mathbf{B}^{\mathrm{T}}$; hence, the network does not matter in Eq. (17). Finally, we obtain the central government tax level for each firm:

$$\mathbf{z} = \frac{1}{2}\bar{\epsilon}. \tag{18}$$

Proposition 5 *The networks do not matter for socially optimal tax strategies as in Eq. (18).*

This result shows that the optimal policy does not consider network information, as with Proposition 2 (and for a similar reason); hence, the policy is easily tractable. From Eq. (16), the central government considers all outflow spillovers to the entire economy in its extensive margin. Therefore, the network effects in the extensive margin are canceled out by those in the intensive margin—the aggregated inflow into the perfectly symmetric joint network.

Finally, as well as Proposition 2, this result is also related to the findings of Bloch and Quérou (2013). With just one region, we can easily see that the central government corresponds to the local government; hence, our result is not surprising. However, our contribution is that this result is sustainable even under the assumption of multiple regions in a geographical network.

4.2 Uniform tax rule

The previous subsection shows that non-discriminatory tax is optimal, even though tax discrimination is feasible, if the heterogeneity of stand-alone profit $\bar{\epsilon}$ is ignored. Therefore, people may expect social welfare to increase if discriminatory taxation is prohibited. Although this is just a conjecture, this idea is worth investigating as a second-best policy when governance by the central government is limited.

We now reconsider local governments, but they must propose a uniform tax level for all firms. The vector of uniform taxes for region q is denoted by $\mathbf{z}_q = (z_q, \ldots, z_q)^{\mathrm{T}}$. Therefore, the revenue function of government q is as follows:

$$\pi_q = \sum_{i=1}^{n} P_q^i z_q. \tag{19}$$

Government q solves the problem $\max_{\mathbf{z_q}} \pi_q(\mathbf{z_1}, \ldots, \mathbf{z_m})$ given the tax vectors of the other countries. Suppose there is a tax strategy equilibrium, denoted by $\mathbf{z_q^*} = (z_q^*, z_q^*, \ldots, z_q^*)^{\mathrm{T}}$. In the equilibrium, the following first-order condition holds:
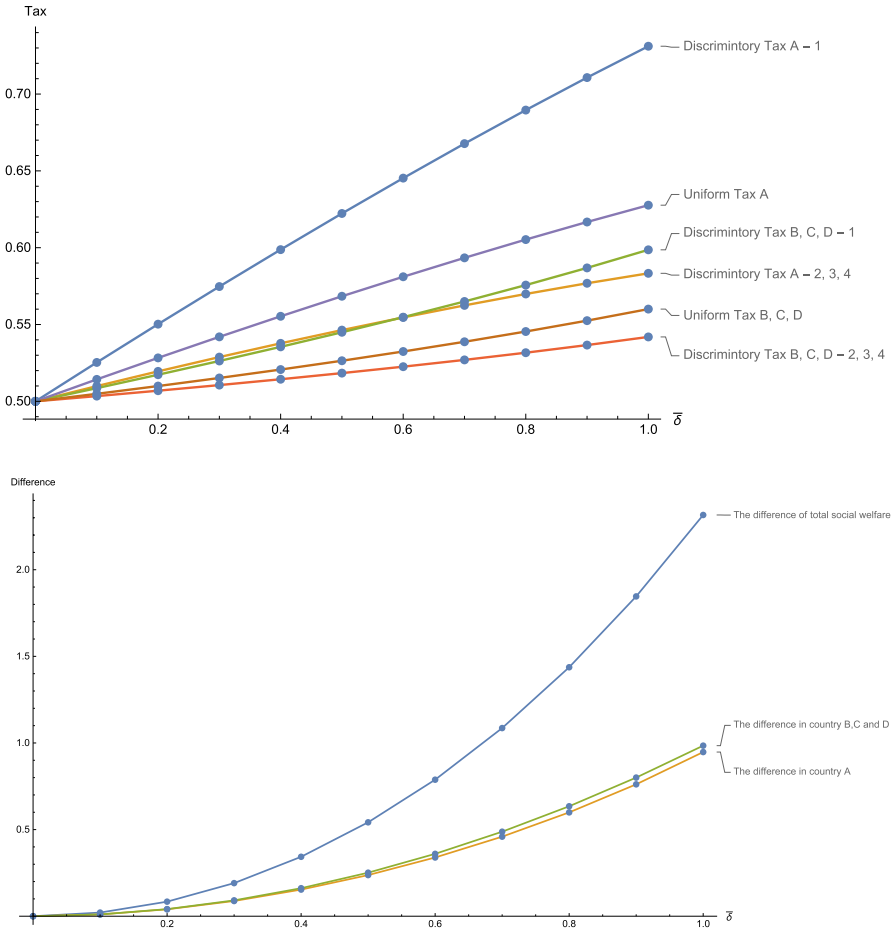
Fig. 5 Effect of $\overline{\delta}$ on taxes and social welfare. **a** Taxes. **b** Difference in social welfare among regimes

$$
\begin{aligned}
\frac{\partial \pi_q}{\partial z_q} &= \sum_{i=1}^{n} \left[P_q^i + \frac{\partial P_q^i}{\partial z_q} z_q^i \right] \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\sum_{r=1}^{m} b_{qr}^{ij}(\overline{\epsilon}_r - z_r) - b_{qq}^{ij} z_q \right] \\
&= 0.
\end{aligned}
\tag{20}
$$

Finally, we obtain the following results:

$$
\mathbf{z} = [\tilde{\mathbf{M}} + \mathbf{D}(\tilde{\mathbf{M}})]^{-1} \tilde{\mathbf{M}} \overline{\epsilon},
\tag{21}
$$

where

$$\tilde{\mathbf{M}} = \begin{bmatrix} \sum_{i=1}^{n}\sum_{j=1}^{n} b_{11}^{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{12}^{ij} & \cdots & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{1m}^{ij} \\ \sum_{i=1}^{n}\sum_{j=1}^{n} b_{21}^{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{22}^{ij} & \cdots & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{2m}^{ij} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n}\sum_{j=1}^{n} b_{m1}^{ij} & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{m2}^{ij} & \cdots & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{mm}^{ij} \end{bmatrix},$$

$$\mathbf{D}(\tilde{\mathbf{M}}) = \begin{bmatrix} \sum_{i=1}^{n}\sum_{j=1}^{n} b_{11}^{ij} & 0 & \cdots & 0 \\ 0 & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{22}^{ij} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{i=1}^{n}\sum_{j=1}^{n} b_{mm}^{ij} \end{bmatrix}.$$

See Appendix 5 for more details about the derivation process. The result is summarized as follows.

Proposition 6 *When local governments set uniform tax levels, these are higher than the socially optimal tax, where networks matter as in Eq.* (21).

Networks matter again. Furthermore, for homogeneous dual networks, we confirm that the decentralized uniform tax is always higher than the socially optimal tax from Proposition 5.[18] This is because the inter-country positive externalities of the tax discount are ignored in decentralized policy-making.

Finally, we conduct a simple simulation to investigate whether imposing a uniform tax can improve social welfare. We use star-shaped transaction and geography networks, as in Sect. 3.3.3, with the same details. Self-evidently, Fig. 5a shows that the uniform tax is set at an intermediate level of discriminatory taxes. Figure 5b compares social welfare with local welfare in two tax regimes, namely, uniform tax and discriminatory tax. This result validates our conjecture; that is, restricting tax discrimination improves social welfare. Moreover, the rule improves welfare in all countries regardless of their position in the network. These results are robust for any $\bar{\delta}$. Therefore, this rule will usually be Pareto improvement, leading to international agreement.

5 Concluding remarks

This study investigated the interplay of inter-firm transaction networks and geographical linkages in the tax strategy of countries. We developed a new model of dual networks, incorporating existing models such as that of Bloch and Quérou

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[18] When firms are homogeneous, the revenue-maximizing tax strategy for each local government is always a uniform tax as in Proposition 4. We also confirm that the tax level is always lower than the socially optimal level, or $z = 1/2$.

(2013) for the special case without geographical structures. The effects of dual networks appear as the centrality index of the Kronecker product, which combines the dual networks to form a large joint network allocating a component to each firm-country combination. Although the role of each individual network is hard to separate from the joint network, some of our results show that the stronger linkages within them increase investment demand and equilibrium tax levels. We also find that tax discrimination patterns differ across countries. This result implies that inter-firm linkages are evaluated according to the geographical position of each country. Countries with rich geographical linkages highly evaluate the indirect linkages of firms, while relatively isolated countries only consider the direct links of firms. This result suggests that geography is an important consideration when countries formulate industrial agglomeration strategies. Furthermore, we suggest a tax coordination policy for dual networks and show that refraining from tax discrimination can improve social welfare.

Finally, an important issue for future studies is how to consider substitutability as well as complementarity among providers in a unified framework. As mentioned in the Introduction, related studies in both tax competition and marketing in networks have considered substitutability rather than complementarity. However, complementarity may also be important in some international situations and with certain types of products, and this study shows that assuming complementarity yields a different result on the effect of networks. Therefore, we must allow a more generalized situation in which some pairs of providers are substitutable, while others are complementary; in other words, the network should be set in a flexible manner.

Appendix 1. Proof of Proposition 1

First, we will show how to derive (4). Combining (3) into matrix form yields:

$$
\mathbf{P_q} = \begin{bmatrix} P_q^1 \\ P_q^2 \\ \vdots \\ P_q^n \end{bmatrix} = \begin{bmatrix} \frac{1}{F}\left[\sum_{r=1}^m \sum_{j=1}^n (\phi_{1j}\delta_{qr}P_r^j) + \overline{\epsilon}_q - z_q^1\right] \\ \frac{1}{F}\left[\sum_{r=1}^m \sum_{j=1}^n (\phi_{2j}\delta_{qr}P_r^j) + \overline{\epsilon}_q - z_q^2\right] \\ \vdots \\ \frac{1}{F}\left[\sum_{r=1}^m \sum_{j=1}^n (\phi_{nj}\delta_{qr}P_r^j) + \overline{\epsilon}_q - z_q^n\right] \end{bmatrix}
$$

$$
= \frac{1}{F}\left[\begin{bmatrix} \sum_{r=1}^m \sum_{j=1}^n (\phi_{1j}\delta_{qr}P_r^j) \\ \sum_{r=1}^m \sum_{j=1}^n (\phi_{2j}\delta_{qr}P_r^j) \\ \vdots \\ \sum_{r=1}^m \sum_{j=1}^n (\phi_{nj}\delta_{qr}P_r^j) \end{bmatrix} + \begin{bmatrix} \overline{\epsilon}_q \\ \overline{\epsilon}_q \\ \vdots \\ \overline{\epsilon}_q \end{bmatrix} + \begin{bmatrix} z_q^1 \\ z_q^2 \\ \vdots \\ z_q^n \end{bmatrix}\right]
$$

$$
= \frac{1}{F}\left[\begin{bmatrix} \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \end{bmatrix}\begin{bmatrix} P_r^1 \\ P_r^2 \\ \vdots \\ P_r^n \end{bmatrix} \\ \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \end{bmatrix}\begin{bmatrix} P_r^1 \\ P_r^2 \\ \vdots \\ P_r^n \end{bmatrix} \\ \vdots \\ \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}\begin{bmatrix} P_r^1 \\ P_r^2 \\ \vdots \\ P_r^n \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \overline{\epsilon}_q \\ \overline{\epsilon}_q \\ \vdots \\ \overline{\epsilon}_q \end{bmatrix} + \begin{bmatrix} z_q^1 \\ z_q^2 \\ \vdots \\ z_q^n \end{bmatrix}\right]
$$

$$
= \frac{1}{F}\left[\begin{bmatrix} \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \end{bmatrix}\mathbf{P}_r \\ \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \end{bmatrix}\mathbf{P}_r \\ \vdots \\ \sum_{r=1}^m \delta_{qr}\begin{bmatrix} \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}\mathbf{P}_r \end{bmatrix} + \overline{\epsilon}_\mathbf{q} + \mathbf{z_q}\right]
$$

$$
= \frac{1}{F}\left[\sum_{r=1}^m \left[\delta_{qr}\begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}\mathbf{P}_r\right] + \overline{\epsilon}_\mathbf{q} + \mathbf{z_q}\right]
$$

$$
= \frac{1}{F}\left[\sum_{r=1}^m (\delta_{qr}\mathbf{GP}_r) + \overline{\epsilon}_\mathbf{q} + \mathbf{z_q}\right].
$$

Then, we obtained (4). We further combine $\mathbf{P_q}$ to derive (6) as follows:

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \vdots \\ \mathbf{P}_m \end{bmatrix} = \begin{bmatrix} \frac{1}{F}\left[\sum_{r=1}^{m}(\delta_{1r}\mathbf{GP}_r) + \overline{\boldsymbol{\epsilon}}_1 - \mathbf{z}_1\right] \\ \frac{1}{F}\left[\sum_{r=1}^{m}(\delta_{2r}\mathbf{GP}_r) + \overline{\boldsymbol{\epsilon}}_2 - \mathbf{z}_2\right] \\ \vdots \\ \frac{1}{F}\left[\sum_{r=1}^{m}(\delta_{mr}\mathbf{GP}_r) + \overline{\boldsymbol{\epsilon}}_m - \mathbf{z}_m\right] \end{bmatrix}$$

$$= \frac{1}{F}\left[\begin{bmatrix} \delta_{11}\mathbf{G} & \delta_{12}\mathbf{G} & \cdots & \delta_{1m}\mathbf{G} \\ \delta_{21}\mathbf{G} & \delta_{22}\mathbf{G} & \cdots & \delta_{2m}\mathbf{G} \\ \vdots & & & \\ \delta_{m1}\mathbf{G} & \delta_{m2}\mathbf{G} & \cdots & \delta_{mm}\mathbf{G} \end{bmatrix}\begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \vdots \\ \mathbf{P}_m \end{bmatrix} + \begin{bmatrix} \overline{\boldsymbol{\epsilon}}_1 \\ \overline{\boldsymbol{\epsilon}}_2 \\ \vdots \\ \overline{\boldsymbol{\epsilon}}_m \end{bmatrix} - \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_m \end{bmatrix}\right]$$

$$= \frac{1}{F}(\mathbf{JP} + \overline{\boldsymbol{\epsilon}} - \overline{\mathbf{z}}).$$

Finally, solving the equation for **P** yields equation (6):

$$\mathbf{P} = \frac{1}{F}[\mathbf{I} - \frac{1}{F}\mathbf{J}]^{-1}(\overline{\boldsymbol{\epsilon}} - \overline{\mathbf{z}}).$$

Appendix 2. Proof of Proposition 2

We first show the followings:

$$P_q^i = \frac{1}{F}\left[\sum_{r=1}^{m}\sum_{j=1}^{n}\left(\sum_{t=0}^{\infty}\left(\frac{1}{F}\right)^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}}\right)(\overline{\epsilon}_r - z_r^j)\right],$$

$$\frac{\partial P_q^i}{\partial z_q^k} = \frac{\partial \frac{1}{F}\left[\sum_{r=1}^{m}\sum_{j=1}^{n}\left(\sum_{t=0}^{\infty}\left(\frac{1}{F}\right)^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}}\right)(\overline{\epsilon}_r - z_r^j)\right]}{\partial z_q^k}$$

$$= -\frac{1}{F}\frac{\partial \left[\sum_{r=1}^{m}\sum_{j=1}^{n}\left(\sum_{t=0}^{\infty}\left(\frac{1}{F}\right)^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}}\right)z_r^j\right]}{\partial z_q^k}.$$

Therefore, the first-order condition is:

$$\frac{\partial \pi_q}{\partial z_q^k} = \sum_{i=1}^n \left[\frac{\partial P_q^i}{\partial z_q^k} z_q^i \right] + P_q^k$$

$$= \sum_{i=1}^n \left[-\frac{1}{F} \frac{\partial \left[\sum_{r=1}^m \sum_{j=1}^n \left(\sum_{t=0}^\infty \left(\frac{1}{F} \right)^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}} \right) z_r^j \right]}{\partial z_q^k} z_q^i \right] + P_q^k \qquad (22)$$

$$= -\frac{1}{F} \sum_{i=1}^n (b_{qq}^{ik} z_q^i) + P_q^k$$

$$= 0.$$

The matrix form of the above expression is described as follows:

$$-\frac{1}{F} \underbrace{\begin{bmatrix} \begin{bmatrix} b_{11}^{11} & \cdots & b_{11}^{n1} \\ \vdots & & \vdots \\ b_{11}^{1n} & \cdots & b_{11}^{nn} \end{bmatrix} & \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} & \cdots & \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} & \begin{bmatrix} b_{22}^{11} & \cdots & b_{22}^{n1} \\ \vdots & & \vdots \\ b_{22}^{1n} & \cdots & b_{22}^{nn} \end{bmatrix} & \cdots & \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \\ \vdots & \vdots & \cdots & \vdots \\ \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} & \cdots & \begin{bmatrix} b_{mm}^{11} & \cdots & b_{mm}^{n1} \\ \vdots & & \vdots \\ b_{mm}^{1n} & \cdots & b_{mm}^{nn} \end{bmatrix} \end{bmatrix}}_{\equiv D(\mathbf{B})} \begin{bmatrix} z_1^1 \\ \vdots \\ z_1^n \\ z_2^1 \\ \vdots \\ z_2^n \\ \vdots \\ z_m^1 \\ \vdots \\ z_m^n \end{bmatrix} + \mathbf{P} = \mathbf{0}. \qquad (23)$$

Using $\mathbf{P} = \frac{1}{F} \mathbf{B}(\overline{\epsilon} - \mathbf{z})$, we derive (13) as follows:

$$\frac{1}{F} \mathbf{D}(\mathbf{B})\mathbf{z} + \frac{1}{F} \mathbf{B}\overline{\epsilon} - \frac{1}{F} \mathbf{B}\mathbf{z} = \mathbf{0}$$

$$\implies \mathbf{z} = (\mathbf{B} + \mathbf{D}(\mathbf{B}))^{-1} \mathbf{B}\overline{\epsilon}.$$

Appendix 3. Proof of Proposition 3

Assume $\psi_{\mathbf{qr}} \equiv \frac{1}{F} \delta_{qr} \mathbf{G}$, and note that $\psi_{\mathbf{qr}}^2 = \mathbf{0}$ holds because of $F \to +\infty$. Therefore, the following holds:

$$\mathbf{B}^{-1}\mathbf{D(B)} = \begin{bmatrix} \mathbf{I} - \psi_{11} & -\psi_{12} & \cdots & -\psi_{1m} \\ -\psi_{21} & \mathbf{I} - \psi_{22} & \cdots & -\psi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ -\psi_{m1} & -\psi_{m2} & \cdots & \mathbf{I} - \psi_{mm} \end{bmatrix} \begin{bmatrix} \mathbf{I} + \psi_{11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} + \psi_{22} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} + \psi_{mm} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\psi_{12} & \cdots & -\psi_{1m} \\ -\psi_{21} & \mathbf{0} & \cdots & -\psi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ -\psi_{m1} & -\psi_{m2} & \cdots & \mathbf{0} \end{bmatrix}$$

$$= \mathbf{I} - \mathbf{\Psi}.$$

Therefore, the following holds given $\overline{\epsilon} = 1$:

$$\mathbf{z} = (\mathbf{I} + \mathbf{B}^{-1}\mathbf{D(B)})^{-1}\mathbf{1}$$

$$= (2\mathbf{I} - \mathbf{\Psi})^{-1}\mathbf{1}$$

$$= \frac{1}{2}(\mathbf{I} - \frac{1}{2}\mathbf{\Psi})^{-1}\mathbf{1}$$

$$= \frac{1}{2}(\mathbf{I} + \frac{1}{2}\mathbf{\Psi})\mathbf{1}$$

$$= \frac{1}{2}\mathbf{1} + \frac{1}{4}\begin{bmatrix} \mathbf{0} & \frac{1}{F}\delta_{12}\mathbf{G} & \cdots & \frac{1}{F}\delta_{1m}\mathbf{G} \\ \frac{1}{F}\delta_{21}\mathbf{G} & \mathbf{0} & \cdots & \frac{1}{F}\delta_{2m}\mathbf{G} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{F}\delta_{m1}\mathbf{G} & \frac{1}{F}\delta_{m2}\mathbf{G} & \cdots & \mathbf{0} \end{bmatrix}\mathbf{1}$$

$$= \frac{1}{2}\mathbf{1} + \frac{1}{4F}\begin{bmatrix} \sum_r \delta_{1r} \times \mathbf{G1} \\ \sum_r \delta_{2r} \times \mathbf{G1} \\ \vdots \\ \sum_r \delta_{mr} \times \mathbf{G1} \end{bmatrix} = \frac{1}{2}\mathbf{1} + \frac{1}{4F}\begin{bmatrix} \sum_r \delta_{1r} \times \sum_j g_{1j} \\ \sum_r \delta_{2r} \times \sum_j g_{2j} \\ \vdots \\ \sum_r \delta_{mr} \times \sum_j g_{nj} \end{bmatrix},$$

where note that $(\mathbf{G1})^{\mathrm{T}} = (\sum_j g_{1j}, \dots, \sum_j g_{nj})$ holds on the sixth line. Also, for the fourth line, remember that $(\mathbf{I} - \frac{1}{2}\mathbf{\Psi})^{-1} = \sum_{t=0}^{\infty}(\frac{1}{2}\mathbf{\Psi})^t = \mathbf{I} + \frac{1}{2}\mathbf{\Psi}$ holds given $F \to +\infty$. Therefore, $z_q^i = \frac{1}{2} + \frac{1}{4F}\deg_q(\mathbf{S}) \times \deg_i(\mathbf{G})$, where $\deg_q(\mathbf{S}) \equiv \sum_r \delta_{qr}$ and $\deg_i(\mathbf{G}) \equiv \sum_j g_{ij}$, and hence, the proposition holds.

Appendix 4. Proof of Proposition 4

We assume that completely homogeneous transaction network $\mathbf{G}$ and geography network $\mathbf{S}$ are:

$$\mathbf{G} = \begin{bmatrix} 0 & \overline{\phi} & \cdots & \overline{\phi} \\ \overline{\phi} & 0 & \cdots & \overline{\phi} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\phi} & \overline{\phi} & \cdots & 0 \end{bmatrix}, \mathbf{S} = \begin{bmatrix} \delta & \overline{\delta} & \cdots & \overline{\delta} \\ \overline{\delta} & \delta & \cdots & \overline{\delta} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\delta} & \overline{\delta} & \cdots & \delta \end{bmatrix}.$$

At this time, matrix $\mathbf{B}$ is:

$$\mathbf{B} = \sum_{t=0}^{\infty} \frac{1}{F^t} \mathbf{S}^t \otimes \mathbf{G}^t = \sum_{t=0}^{\infty} \frac{1}{F^t} \begin{bmatrix} \mathbf{G}^t \cdot \delta^{\{t\}} & \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} & \dots & \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} \\ \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} & \mathbf{G}^t \cdot \delta^{\{t\}} & \dots & \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} & \mathbf{G}^t \cdot \bar{\delta}^{\{t\}} & \dots & \mathbf{G}^t \cdot \delta^{\{t\}} \end{bmatrix}, \quad (24)$$

where $\delta^{\{t\}}$ and $\bar{\delta}^{\{t\}}$ describe the diagonal and non-diagonal components of $\mathbf{S}^t$. Therefore, we can describe as follows:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \dots & \mathbf{B}_2 \\ \mathbf{B}_2 & \mathbf{B}_1 & \dots & \mathbf{B}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_2 & \mathbf{B}_2 & \dots & \mathbf{B}_1 \end{bmatrix}, \quad (25)$$

where:

$$\mathbf{B}_1 = \begin{bmatrix} b_1^d & b_1^n & \dots & b_1^n \\ b_1^n & b_1^d & \dots & b_1^n \\ \vdots & \vdots & \ddots & \vdots \\ b_1^n & b_1^n & \dots & b_1^d \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} b_2^d & b_2^n & \dots & b_2^n \\ b_2^n & b_2^d & \dots & b_2^n \\ \vdots & \vdots & \ddots & \vdots \\ b_2^n & b_2^n & \dots & b_2^d \end{bmatrix}.$$

We also set $\bar{\epsilon} = \mathbf{1}$, and then following holds from (13):

$$\mathbf{z} = (\mathbf{B} + \mathbf{D}(\mathbf{B}))^{-1} \mathbf{B} \bar{\epsilon} = \begin{bmatrix} 2\mathbf{B}_1 & \mathbf{B}_2 & \dots & \mathbf{B}_2 \\ \mathbf{B}_2 & 2\mathbf{B}_1 & \dots & \mathbf{B}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_2 & \mathbf{B}_2 & \dots & 2\mathbf{B}_1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \dots & \mathbf{B}_2 \\ \mathbf{B}_2 & \mathbf{B}_1 & \dots & \mathbf{B}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_2 & \mathbf{B}_2 & \dots & \mathbf{B}_1 \end{bmatrix} \mathbf{1} \quad (26)$$

$$\implies \quad (2\mathbf{B}_1 + (m-1)\mathbf{B}_2)\mathbf{z} = (\mathbf{B}_1 + (m-1)\mathbf{B}_2)\mathbf{1}. \quad (27)$$

Note that $z_q^i = z$ holds for all q, i in the case of homogeneous network, and hence, we obtain:

$$z = \frac{A}{C+A}, \quad (28)$$

where $A \equiv b_1^d + (n-1)b_1^d + (m-1)[b_2^d + (n-1)b_2^n]$ and $C \equiv b_1^d + (n-1)b_1^n$.

Here, each component of matrix $\mathbf{B}$ is derived is as follows. First, we define:

$$\mathbf{B}(t) \equiv \frac{1}{F^t} \begin{bmatrix} \mathbf{G}^t \delta^{\{t\}} & \mathbf{G}^t \overline{\delta}^{\{t\}} & \dots & \mathbf{G}^t \overline{\delta}^{\{t\}} \\ \mathbf{G}^t \overline{\delta}^{\{t\}} & \mathbf{G}^t \delta^{\{t\}} & \dots & \mathbf{G}^t \overline{\delta}^{\{t\}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}^t \overline{\delta}^{\{t\}} & \mathbf{G}^t \overline{\delta}^{\{t\}} & \dots & \mathbf{G}^t \delta^{\{t\}} \end{bmatrix}$$

$$= \frac{1}{F^t} \begin{bmatrix} \begin{bmatrix} b_1^d(t) \dots b_1^n(t) \\ \vdots \\ b_1^n(t) \dots b_1^d(t) \end{bmatrix} & \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} & \dots & \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} \\ \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} & \begin{bmatrix} b_1^d(t) \dots b_1^n(t) \\ \vdots \\ b_1^n(t) \dots b_1^d(t) \end{bmatrix} & \dots & \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} \\ \vdots & \vdots & \dots & \vdots \\ \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} & \begin{bmatrix} b_2^d(t) \dots b_2^n(t) \\ \vdots \\ b_2^n(t) \dots b_2^d(t) \end{bmatrix} & \dots & \begin{bmatrix} b_1^d(t) \dots b_1^n(t) \\ \vdots \\ b_1^n(t) \dots b_1^d(t) \end{bmatrix} \end{bmatrix}.$$

Note that:

$$\mathbf{B}(t)\mathbf{1} = [b_1^d(t) + (n-1)b_1^d(t) + (m-1)[b_2^d(t) + (n-1)b_2^n(t)]]\mathbf{1} \equiv A(t)\mathbf{1}, \quad (29)$$

and $\sum_{t=0}^{\infty} A(t) = A$ holds. Also note that the following holds:

$$\frac{1}{F}\mathbf{G}\mathbf{1} = \frac{1}{F}\begin{bmatrix} (n-1)\overline{\phi} \\ (n-1)\overline{\phi} \\ \vdots \\ (n-1)\overline{\phi} \end{bmatrix} = \frac{1}{F}[(n-1)\overline{\phi}]\mathbf{1}$$

$$\frac{1}{F^2}\mathbf{G}^2\mathbf{1} = \frac{1}{F}\mathbf{G}\frac{1}{F}\mathbf{G}\mathbf{1} = \frac{1}{F^2}[(n-1)\overline{\phi}]^2\mathbf{1} \qquad (30)$$

$$\dots$$

$$\frac{1}{F^t}\mathbf{G}^t\mathbf{1} = \frac{1}{F^{t-1}}\mathbf{G}^t\frac{1}{F}\mathbf{G}\mathbf{1} = \frac{1}{F^t}[(n-1)\overline{\phi}]^t\mathbf{1}.$$

Therefore, using (29) and (30), the following holds:

$$\mathbf{B}(t)\mathbf{1} = A(t)\mathbf{1} = \frac{1}{F^t}(\delta^{\{t\}}\mathbf{G}^t\mathbf{1} + (m-1)\overline{\delta}^{\{t\}}\mathbf{G}^t\mathbf{1})$$

$$= [\frac{(n-1)\overline{\phi}}{F}]^t[\delta^{\{t\}} + (m+1)\overline{\delta}^{\{t\}}]\mathbf{1}$$

$$= [\frac{(n-1)\overline{\phi}}{F}]^t[\delta + (m+1)\overline{\delta}]^t\mathbf{1},$$

where note that $\mathbf{S}^t\mathbf{1} = [\delta^{\{t\}} + (m+1)\overline{\delta}^{\{t\}}]\mathbf{1} = [\delta + (m+1)\overline{\delta}]^t\mathbf{1}$ holds for the homogeneous geographical network. Therefore, we obtain:

$$A = \sum_{t=0}^{\infty} A(t) = \frac{1}{1 - \frac{(n-1)\overline{\phi}}{F}[\delta + (m-1)\overline{\delta}]}. \tag{31}$$

Furthermore, setting $\alpha = \frac{(n-1)\overline{\phi}}{F}$, we obtain the following:

$$C = \sum_{t=0}^{\infty} [b_1^d(t) + (n-1)b_1^n(t)] = \sum_{t=0}^{\infty} \alpha^t \delta^{\{t\}}. \tag{32}$$

Therefore, C is the diagonal component of the following matrix:

$$(\mathbf{I} - \alpha\mathbf{S})^{-1} = \sum_{t=0}^{\infty} \alpha^t \mathbf{S}^t$$

$$= \begin{bmatrix} 1 - \alpha\delta & -\alpha\overline{\delta} & \dots & -\alpha\overline{\delta} \\ -\alpha\overline{\delta} & 1 - \alpha\delta & \dots & -\alpha\overline{\delta} \\ \vdots & \vdots & \vdots & \\ -\alpha\overline{\delta} & -\alpha\overline{\delta} & \dots & 1 - \alpha\delta \end{bmatrix}^{-1}. \tag{33}$$

Therefore, solving the inverse, we obtain:

$$C = \frac{1 - \frac{(m-1)\alpha\overline{\delta}}{1-\alpha\delta_1+\alpha\overline{\delta}}}{1 - \alpha[\delta + (m-1)\overline{\delta}]}.$$

Finally, From (31) and (33), z is derived as follows:

$$z = \frac{A}{C+A}$$

$$= \frac{\frac{1}{1-\alpha[\delta+(m-1)\overline{\delta}]}}{\frac{1-\frac{(m-1)\alpha\overline{\delta}}{1-\alpha\delta+\alpha\overline{\delta}}}{1-\alpha[\delta+(m-1)\overline{\delta}]} + \frac{1}{1-\alpha[\delta+(m-1)\overline{\delta}]}}$$

$$= \frac{1}{2 - \frac{(m-1)\alpha\overline{\delta}}{1-\alpha\delta+\alpha\overline{\delta}}}. \tag{34}$$

From this solution, we can proof that both factors $\overline{\phi}$ and $\overline{\delta}$ have positive effects on governments taxation.

Appendix 5. Proof of Proposition 6

First, the followings hold given the rule of uniform taxation:

$$P_q^i = \frac{1}{F} \sum_{r=1}^{m} \sum_{j=1}^{n} (\sum_{t=0}^{\infty} (\frac{1}{F})^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}})(\overline{\epsilon}_r - z_r) \tag{35}$$

$$
\begin{aligned}
\frac{\partial P_q^i}{\partial z_q} &= \frac{\frac{1}{F} \sum_{r=1}^m \sum_{j=1}^n \left(\sum_{t=0}^\infty \left(\frac{1}{F}\right)^t \delta_{qr}^{\{t\}} \phi_{ij}^{\{t\}} \right)(\bar{\epsilon}_r - z_r)}{\partial z_q} \\
&= -\frac{1}{F} \sum_{j=1}^n \left(\sum_{t=0}^\infty \left(\frac{1}{F}\right)^t \delta_{qq}^{\{t\}} \phi_{ij}^{\{t\}} \right) \\
&= -\frac{1}{F} \sum_{j=1}^n b_{qq}^{ij}.
\end{aligned}
\tag{36}
$$

Therefore, the first-order condition for each government is:

$$
\begin{aligned}
\frac{\partial \pi_q}{\partial z_q} &= \sum_{i=1}^n \left[\frac{\partial P_q^i}{\partial z_q} z_q^i + P_q^i \right] \\
&= \sum_{i=1}^n \left[-\frac{1}{F} \sum_{j=1}^n b_{qq}^{ij} z_q + P_q^i \right] \\
&= \frac{1}{F} \sum_{i=1}^n \sum_{j=1}^n \left[(-b_{qq}^{ij} z_q) + \sum_{r=1}^m b_{qr}^{ij}(\bar{\epsilon}_r - z_r) \right] \\
&= 0.
\end{aligned}
\tag{37}
$$

We can rewrite Eq. (37) into the following matrix form:

$$
\begin{aligned}
& \begin{bmatrix}
\sum_{i=1}^n \sum_{j=1}^n [(-b_{11}^{ij} z_1) + \sum_{r=1}^m b_{1r}^{ij}(\bar{\epsilon}_r - z_r)] \\
\sum_{i=1}^n \sum_{j=1}^n [(-b_{22}^{ij} z_2) + \sum_{r=1}^m b_{2r}^{ij}(\bar{\epsilon}_r - z_r)] \\
\vdots \\
\sum_{i=1}^n \sum_{j=1}^n [(-b_{mm}^{ij} z_m) + \sum_{r=1}^m b_{mr}^{ij}(\bar{\epsilon}_r - z_r)]
\end{bmatrix} \\
& = -\underbrace{\begin{bmatrix}
\sum_{i=1}^n \sum_{j=1}^n b_{11}^{ij} & 0 & \cdots & 0 \\
0 & \sum_{i=1}^n \sum_{j=1}^n b_{22}^{ij} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sum_{i=1}^n \sum_{j=1}^n b_{mm}^{ij}
\end{bmatrix}}_{\equiv \tilde{\mathbf{M}}}
\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} \\
& \quad + \underbrace{\begin{bmatrix}
\sum_{i=1}^n \sum_{j=1}^n b_{11}^{ij} & \sum_{i=1}^n \sum_{j=1}^n b_{12}^{ij} & \cdots & \sum_{i=1}^n \sum_{j=1}^n b_{1m}^{ij} \\
\sum_{i=1}^n \sum_{j=1}^n b_{21}^{ij} & \sum_{i=1}^n \sum_{j=1}^n b_{22}^{ij} & \cdots & \sum_{i=1}^n \sum_{j=1}^n b_{2m}^{ij} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^n \sum_{j=1}^n b_{m1}^{ij} & \sum_{i=1}^n \sum_{j=1}^n b_{m2}^{ij} & \cdots & \sum_{i=1}^n \sum_{j=1}^n b_{mm}^{ij}
\end{bmatrix}}_{\equiv D(\tilde{\mathbf{M}})}
\begin{bmatrix} (\bar{\epsilon}_1 - z_1) \\ (\bar{\epsilon}_2 - z_2) \\ \vdots \\ (\bar{\epsilon}_m - z_m) \end{bmatrix} \\
& = -\mathbf{D}(\tilde{\mathbf{M}})\mathbf{z} + \tilde{\mathbf{M}}(\bar{\epsilon} - \mathbf{z}) \\
& = \mathbf{0}.
\end{aligned}
$$

Finally, we can obtain:

$$\mathbf{z} = [\tilde{\mathbf{M}} + \mathbf{D}(\tilde{\mathbf{M}})]^{-1}\tilde{\mathbf{M}}\bar{\varepsilon}. \tag{38}$$

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