Preface to Focused Issue on Discontinuous Galerkin Methods

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The discontinuous Galerkin (DG) method is a class of finite element methods using completely discontinuous piecewise smooth functions (typically polynomials) as basis and test functions. Since its inception in 1973 [10], it has seen a sustained development, both in the computational mathematics community and in many scientific and engineering application communities. The DG methods have several advantages, such as its extreme flexibility in dealing with complex geometry and adaptive computation (both h - and p -adaptivities are easy to implement), extremely high parallel efficiency, good stability properties (energy and entropy stability has been established for DG methods in many situations), nice convergence and superconvergence properties, and capability to solve hyperbolic and convection-dominated problems effectively.

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An early collection of many results in DG schemes is the Springer volume of the conference proceedings for the First International Symposium on Discontinuous Galerkin Methods held at Newport, Rhode Island in 1999, see the survey paper [2] in this volume. Early summaries of the developments in DG methods can also be found in the lecture notes [1] and the review paper [3], as well as the two special issues devoted to the DG methods [4, 5]. There have been several more recent books and lecture notes published on the DG methods [6–9, 11, 12].

In this CAMC focused issue on DG methods, we have over 30 papers on all aspects of DG methods, from algorithm formulation, analysis to implementation and applications. We hope this focused issue will serve as a focused reference source for recent development of DG methods, and will help to stimulate more research and wider applications of DG methods in the future.

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