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RESEARCH ARTICLE

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Negative weights in network time model

Abstract Time does not go backward. A negative duration, such as "time period" at first sight is difficult to interpret. Previous network techniques (CPM/PERT/PDM) did not support negative parameters and/or loops (potentially necessitating recursive calculations) in the model because of the limited computing and data storage capabilities of early computers. Monsieur Roy and John Fondahl implicitly introduced negative weights into network techniques to represent activities with fixed or estimated durations (MPM/PDM). Subsequently, the introduction of negative lead and/or lag times by software developers (IBM) apparently overcome the limitation of not allowing negative time parameters in time model. Referring to general digraph (Event on Node) representation where activities are represented by pairs of nodes and pairwise relative time restrictions are represented by weighted arrows, we can release most restraints in constructing the graph structure (incorporating the dynamic model of the inner logic of time plan), and a surprisingly flexible and handy network model can be developed that provides all the advantages of the abovementioned techniques. This paper aims to review the theoretical possibilities and technical interpretations (and use) of negative weights in network time models and discuss approximately 20 types of time-based restrictions among the activities of construction projects. We focus on pure relative time models, without considering other restrictions (such as calendar data, time-cost trade-off, resource allocation or other constraints).

Keywords graph technique, network technique, construction management, scheduling

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1 Introduction

The graphic analogy of mathematical problem was limited to a graph of special structure referred to as network since Kelley Jr and Walker (1959) published their famous algorithm for the first high capacity digital computer (Univac-I) to calculate the time schedules of work on the basis of estimated duration of activities and set of direct precedence. The network was defined as a connected weighted directed graph with a single starting node (origin or source), a single ending node (terminal node or sink), and without circular references (loop or circle) and negative weights (lengths) along the edges (arcs or arrows).

The mathematical analogy was to find the longest path(s) from the starting node to all other nodes for calculating the relative time positions of activities represented by weighted directed edges of the graph (Activity on Arrow (AOA), correspondence of graph elements). The nodes (and so-called dummy activities (edges with zero weight)) represented direct precedence. The arrows entering to a starting node of an activity represented its so-called predecessor activities, and the arrows leaving from an ending node of an activity represented its successor activities.

The analogy is clear. The chain of preceding activities (series of arrows) determines the possible start of any activity, and the duration of any activity is rarely a negative value. Aside from solving the scheduling problem, a secondary target function of minimizing the associating cost to the schedule of works is involved to establish the optimal time policy of an executing company known as the Critical Path Method (CPM). The proper duration of activities with their associating direct cost could be defined within the set of positive ranges for each activity in advance as input values in their method.

Shocked by the Sputnik Crisis in 1957 with significant military efforts (Polaris Project) to develop America's First Hit Nuclear Power to overcome the missile gap against the Soviets, Malcolm et al. (1959) published a computer-aided method called Program Evaluation and Review Technique (PERT) to schedule the works of manifold development.

The likely duration of the necessary developments (subprojects, activities) were estimated on a probabilistic basis assuming beta distribution defined by triplexes of positive values (optimistic–realistic–pessimistic time estimates) to fit the research and development nature of the project.

Subsequently, Fondahl (1962) published his method for non-computer applications and proposed a reversed correspondence of graph elements (Activity on Node (AON)), where nodes represent activities with preset duration and arrows represent direct precedence. Although he did not overcome the direct precedence (no weights were assigned to the arrows), he eliminated the necessity of dummy activities, and his graphs were easy to read and to draw. The basis of current widespread technique known as Precedence Diagramming Method (PDM) is recognized as the fruit of his theorem.

Representing simple precedence in a CPM time model rapidly proved to be insufficient when scheduling production processes either in manufacturing or in construction projects. The need of timing successions (setting parametric precedence relationships) is unavoidable when modeling interrelations of overlapped activities in time. To overcome this deficiency in the early 1960s, a group of British researchers proposed to apply a special arrangement and decompose the graph elements (i.e., sectioning activities and establishing links between the sections by either parametric dummies). Their proposal was called "Ladder" convention because of its appearance (Weaver, 2014).

Independent from Fondahl's works, Roy (1964) elaborated a method to determine the potential starts of related activities with fixed durations. In the graphic representation of his method, AON correspondence of graph elements would be applied, where the time parameters of directed edges serve as minimum delays between the starts of related activities. His method was later improved on the basis of the assumptions of fixed durations and linear progression of activities involved in the time model, and further tools, such as Finish-to-Start (FS), Finish-to-Finish (FF), and Start-to-Finish (SF) were added to the initial Start-to-Start (SS) interpretation of the weighted directed edges, representing either lower or upper bounds on the relative time positions of the activities (Kerbosch and Schell, 1975). Roy's method is mainly acknowledged as Potential's Method or METRA Potential's Method (MPM) in European countries (Roy and Sussmann, 1964). In the USA and other British speaking countries, Fondahl's PDM is a popular reference for these types of network techniques.

Offering its computers to market enterprises, IBM popularized the PDM technique in the Users' Guides of its 1400 series products. In the 1966 issue (IBM, 1966), the sign of lag times to be set between activities to overlap them in time could be either positive or negative. Although the negative "delay" times generate heavy disputes among some practitioners (Douglas III et al., 2006), the integration of activities with fixed duration entrains the acceptance of negative parameters along the edges (shown in later sections). The unusual characteristics or behavior of critical activities highlighted by Wiest (1981) are consequences of implicated negative parameters of activities with fixed durations, and the involvement of negative weighted edges along the critical (longest) path is the ultimate basis of classifying critical activities by Hajdu et al. (2016), as shown by Vattai (2017).

Hajdu (2015a) proposed a new approach for the activities involved in network time models and defined the so-called "continuous" relation between the progression of activities that are not necessarily linear (constant) in time. In the same year, he introduced the "point-to-point" relationship between the activities at any level of their progression (Hajdu, 2015b), embracing the classic FF, FS, SF, and SS relations. Some similar thoughts can be recognized in the proposal of Kim (2018), who emphasized the inner milestones of related activities when introducing the Beeline method.

To reduce the restrictions on the graph structure, Vattai (2016) published a modified Floyd–Warshall algorithm to schedule open networks, where all the restrictions set for the early graph structures can be eliminated, and indicated that only one single expectation against computable time (graph) models is essential: No positive loops would be accepted. In this study, we focus on pure relative time models without considering other restrictions, such as calendar data, time-cost trade-off, resource allocation, and other constraints.

2 Facing the all-pair longest path problem

The analogy of network scheduling techniques is a special problem in graph theory, that is, the problem of finding the longest path(s) between two nodes (usually between the start/finish node and other nodes) in a weighted directed graph. It is rarely mentioned that the later problem has its pair as a "dual" problem known as the minimal potentials' problem interpreted on a set of potentials having pairwise relative restrictions (lower bounds on their differences) amongst the potentials (Kelley Jr, 1961).

Usual algorithms focus on the minimum potential's problem and execute a type of implicit labeling technique, such as Dijkstra (1959)'s greedy algorithm developed to find the shortest path(s) on a weighted graph. These techniques of determining feasible solution(s) are usually based on a kind of roll-on-type calculation of early and late times (time potentials) through a series of consecutive steps starting from a base point (from start/finish) and increasing the set of examined elements of the graph step by step in an appropriate order (forward/backward pass), thereby actually solving the dual problem. After solving the dual problem, the solution to the primal problem, which is the set of longest path(s) between the originating and terminal nodes of the graph commonly referred to as the "critical path", can be identified.

Since the length of any path in a weighted graph is defined as sum of weights of the arrows constituting the given path, it is irrelevant in what an order the numbers are added together. So, reversing the logic of solution, that is, finding the longest path(s) first and subsequently assigning the time potentials to the nodes along it (them) support us in discarding all unnecessary restrictions on the graph structure. Here is the main advantage of the "blind" logic of the well-known Floyd-Warshall algorithm (Floyd, 1962; Warshall, 1962): Adjusting it to the scheduling problem through a slight modification so that all-pairs longest path(s) can be determined and identified, while all difficulties of and restrictions on composing the logical time model can be eliminated except of the only thing — exclusion of positive loops in the weighted directed graph (Vattai, 2016).

To keep the longest path analogy of the scheduling problem, it is essential to have minimum-typed restrictions in the model only. But, according to the rules of elementary algebra, multiplying inequality representing a bound on the difference of a given pair of potentials (π_i, π_j) by minus one, any upper bound (Eq. (1)) can be transformed by a lower bound limitation (reversing the direction of subtraction, that is the direction of the edge in the graph, and changing the sign of the limit value τ_{ij}, Eq. (2)). Thus, any mixed bounding system can be transformed to a homogeneous one having lower bounds only.

$$\pi_j - \pi_i \leqslant \tau_{ij} \text{ (upper bound)}, \tag{1}$$

$$\pi_i - \pi_j \geqslant -\tau_{ij}$$

$$\text{(upper bound transformed to lower bound)}. \tag{2}$$

Analogically, any fixed duration of a task can be set using a pair of a lower and an upper bound having the same limit value (τ_{sf}, as duration) between its start (s) and finish (f):

$$(\pi_{\mathrm{f}} - \pi_{\mathrm{s}} = \tau_{\mathrm{sf}}) \equiv (\pi_{\mathrm{f}} - \pi_{\mathrm{s}} \geqslant \tau_{\mathrm{sf}}) \cup (\pi_{\mathrm{s}} - \pi_{\mathrm{f}} \geqslant -\tau_{\mathrm{sf}}). \tag{3}$$

Thus, the loops of directed edges (between the starting and finishing nodes of tasks) and negative weights are given (upper bounding for fixed durations), the analogy of the longest path problem is still ongoing, and the model remains calculable.

In the following sections we discuss practical problems, where lower and upper bounds should be set on the related time potentials — either simultaneously or in combination.

3 Activities of flexible duration

In this section, we review the theoretical means of setting relative limitations on the differences of ending and starting time potentials, that is, on the duration of activities. For the references of individual cases, we illustrate the proposed representations and interpretations of limitations in small figures as follow:

(a) proposed α-numerical notation of limitation(s) in network models;

(b) representation of limitation(s) in the structure matrix of the graph;

(c) proposed graphic representation of limitation(s) in the network models;

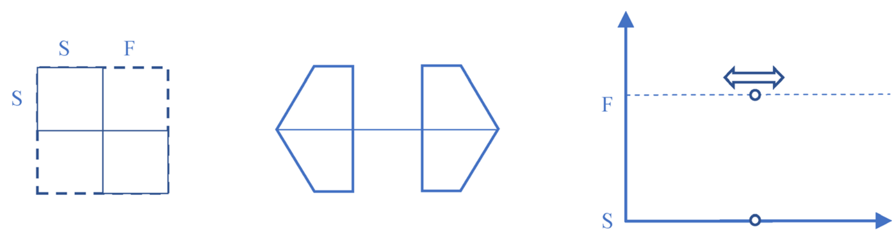
(d) interpretation of limitation(s) in linear schedules ("cyclograms").

3.1 Neither lower nor upper bound on the duration: Hammock activity

Hammock activities are typical in situations of providing accessibility of some special resources, such as pumps (dewatering), cranes (vertical transport), scaffolding (falsework), etc., where expected start and finish of the given job are technologically well identifiable, but the actual duration of it depends on numerous other processes and activities of the project. In cases of this kind, we indicate necessity of the activity without predicting the duration of it (Fig. 1).

3.2 Lower and upper bounds are equal to zero: Event

An item with zero duration (event) identified in this



(a) (b) (c) (d)

Fig. 1 Hammock activity.

manner has a position but no extent in time (fiction). It is typical at timing so called milestones, interim deadlines or expected states (representing either the targeted or the basic conditions) (Fig. 2).

3.3 Lower bound only: Pauseable activity

Activities with lower bound on their durations are typical in cases when considering available resources, appropriate technologies, and actual conditions. The shortest time of their performance can be fairly estimated, but their continuous performance is not expected (Fig. 3). Such kind of activities can be some external or internal finishes (painting, furniture, rendering, landscaping, etc.) or other activities with low needs of resource capacities (e.g., refilling, proofing, etc.).

3.4 Upper bound only: Conditional activity

Conditional activities may occur when modeling non-basic auxiliary works that are highly dependent on expendable time (that is, we have time to perform them). The upper limit as maximum expendable time can be defined on the basis of economic calculations, weather forecasts, and availability considerations, etc. (Fig. 4). For example, demolished materials will be stored on site and can be sold for recycling or for reusing purposes at the limited period determined by time analysis; the upper bound represents the maximum time of onsite sale, storage and/or guarding.

3.5 Differing lower and upper bounds: Stretchable activity

Most of the activities behave as stretchable activity during execution (Fig. 5). The lower bound value can be estimated on the basis of available technologies, resource capacities or known conditions, while the upper bound can be set in accordance with other processes, economic analyses, and general availability features of the project. Applying this kind of activities is useful in situations when the determination of favored "master intensity" (general rate of progression or would be daily progression) is the subject of time analysis (synchronization, elimination of paradox situations).

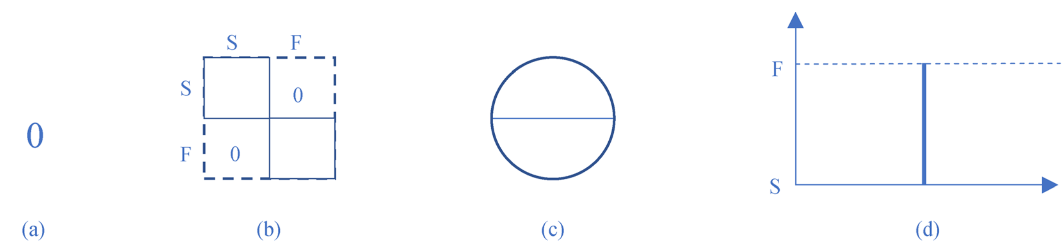


Fig. 2 Event.

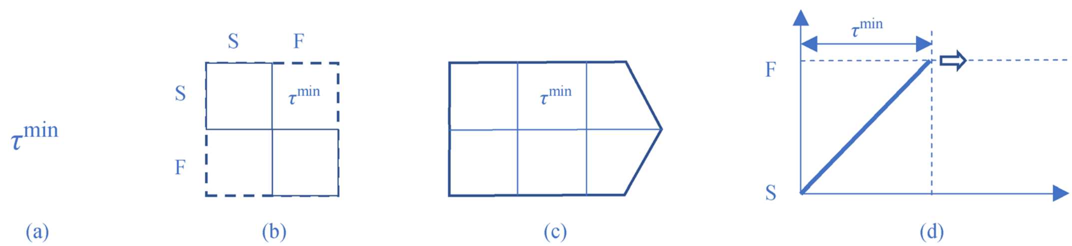


Fig. 3 Pauseable activity.

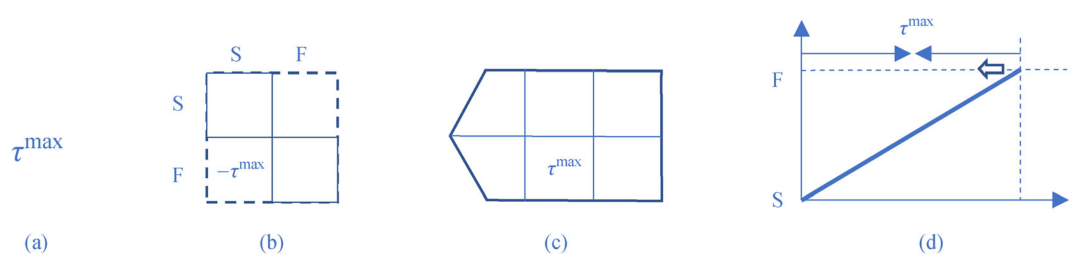


Fig. 4 Conditional activity.

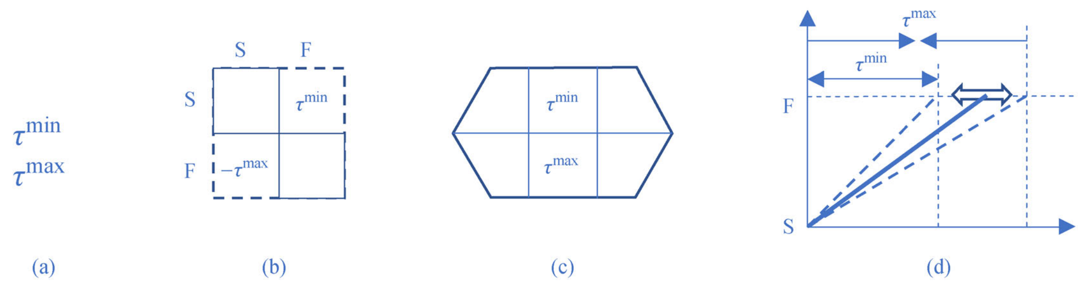


Fig. 5 Stretchable activity.

3.6 Equal lower and upper bounds: Activity with fixed duration

Activities with fixed duration are the most frequent in the practice of network scheduling techniques. Applying them is expedient in situations when we have firm ideas about the processes of performance and the actual contracts of contributors combined with associated processes strictly confine expendable time of realization. They have potentials at complex technical projects of many contributors and at computable mechanized or automated processes of manufacturing (Fig. 6).

Remark: When involving "hammock" and/or "conditional" activities in the model it may occur that the time analysis results in a—mathematically correct, but—technically not or hardly interpretable figures, such as, an activity of these kinds may be finished before it has been started. Such a situation may typically emerge because of improperly constructed network model or failures in the logic of the model. We may also think the scheduling activities of building, such as linear structures (tunnels, roads, transmission lines, etc.), where the given activity is a subtask of the entire construction project. In that case, we can interpret the time turnover as an indication that the given activity would proceed in the opposite direction than its original assumption. In this case, the upper bound on the duration of conditional activity becomes absolutely ineffective.

4 Relations

In this section, we review the theoretical means of setting relative limitations on the differences of ending or starting time potentials of related activities. For the references of individual cases, we illustrate the proposed representations and interpretations of limitations in small figures as follows:

(a) proposed α-numerical notation of limitation(s) in network models;

(b) representation of limitation(s) in the structure matrix of the graph;

(c) interpretation of limitation(s) in a bar chart ("Gantt chart");

(d) interpretation of limitation(s) in linear schedules ("cyclograms").

4.1 Single lower bound-type limitations: Minimum-type relations

Four basic types of relations are available in most well-known computer applications (such as MS Project or Primavera) (Fig. 7), and the one of the four theoretical pairs of time potentials applied by the scheduler mainly depends on the technical environment of the modeled processes. For processes with known fixed durations, the four types are mutually interchangeable combined with proper modification of bound values.

FF$\tau^{\min}$: This relation is mostly applicable in modeling "retroactive" time conditions. Such a situation may emerge when surveying substructures before the earth refill or when reviewing and approving reinforcement before casting the concrete of the monolithic reinforced concrete structures. This relation can also be a useful means in scheduling activities with fixed duration when an activity (or a technological process) has evidently shorter duration

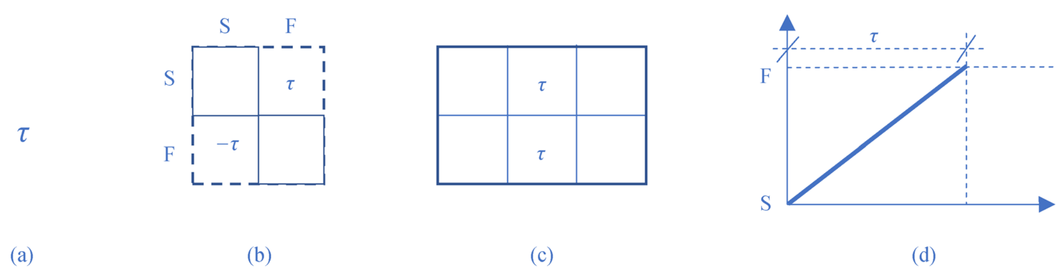


Fig. 6 Activity with fixed duration.

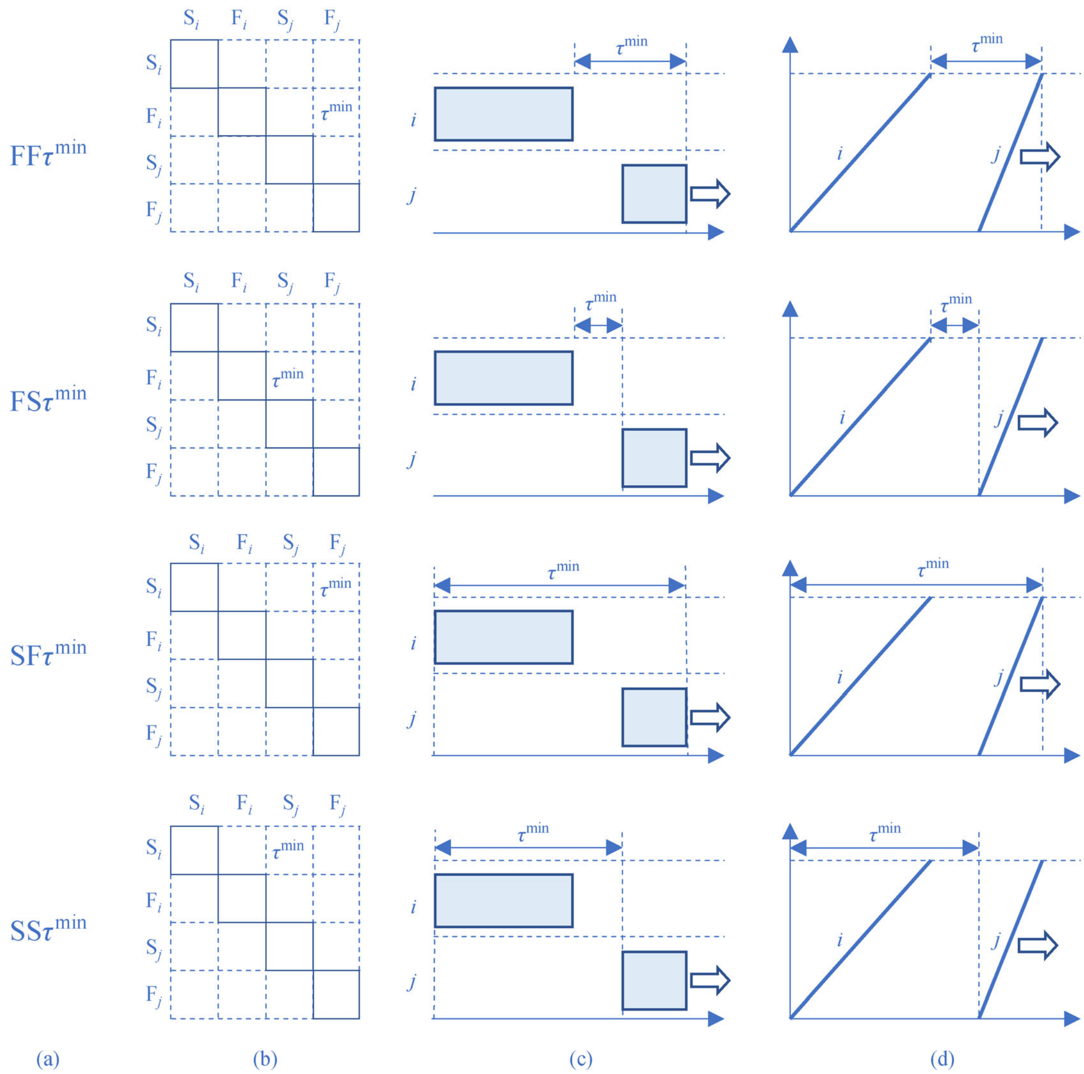


Fig. 7 Minimum-type relations.

than its preceding activity;

$FS\tau^{\min}$: This relation is basically used to schedule activities with strict technological dependencies or activities to be performed using the same resources with limited availabilities. Such activities may include erecting a prefabricated reinforced concrete structure where the columns are erected first before the beams are placed on them to receive the floor panels. The concrete of a monolithic structure needs some time to harden before the molds are demolished. The pile driving unit must move away from the pile to provide access for the team demolishing the upper segment. A crane must place the lifted unit before taking the succeeding one;

$SF\tau^{\min}$: We can appropriately set the minimum overlapping of related activities by applying this type of relation. This relation may emerge when scheduling activities or services substituting or replacing each other. In case of building a house with extensive substructural works, temporary dewatering system of the worksite must be operated until the permanent dewatering system of the new building is confirmed to be fully operational. In accordance with the actual legal regulations, the maintenance or repair services of a product on the market must be provided by the manufacturer or by the vendors years after the manufacturing of the given product is ceased and/or a new (substituting) product enters the market (guarantee, warranty);

$SS\tau^{\min}$: This relation serves well in the initial scheduling of synchronized (or quasi-synchronized, mutually adjusting) processes of construction. Such relation may emerge in works of linear structures, such as sewer lines or pipelines to be performed in trenches (excavation, timbering, bedding, pipe laying, testing, refill, etc.).

4.2 Single upper bound-type limitations: Maximum-type relations

These types of relation are usually unavailable in most widespread computer applications (such as MS Project or Primavera) on the market (Fig. 8), and the one of the four theoretical pairs of time potentials applied by the scheduler mainly depends on the technical environment of the modeled processes. For processes with known fixed durations, the four types are mutually interchangeable combined with the proper modification of bound values.

–FF$\tau^{\max}$: Contemporary or parallel (technologically independent) processes can be scheduled relative to each other by applying this relation. Most frequent samples of these kinds include adjusting the finishes and removal of temporary structures (and/or cleaning up the site, rendering the environment, landscaping, etc.) at the terminal phase of the project;

–FS$\tau^{\max}$: This relation can be used to set time restrictions or technological specifications in cases with sensitive conditions, especially on spot-type structures. After rendering/leveling the earth structure, the blind concrete or other bedding layer atop of it should be immediately spread to protect the surface of the relatively loose earth structure. Demolishing the upper segment of a monolithic reinforced concrete pile (to free the rebars for joining them to the joint structural units) should follow the casting of the concrete within a limited time before the concrete hardens too much;

–SF$\tau^{\max}$: We can appropriately specify the maximum acceptable "overlapping" of succeeding activities using this type of relation. Similar to SF$\tau^{\min}$ relation, an example of its application can be the related timing of activities or services substituting or replacing each other, but with the opposite effect. Following the launch or introduction of a new service or system, the old ones should be terminated or removed within a limited time. The technical units and facilities of a workshop should be moved to the new hall within a limited time before the old workshop is totally or partially demolished;

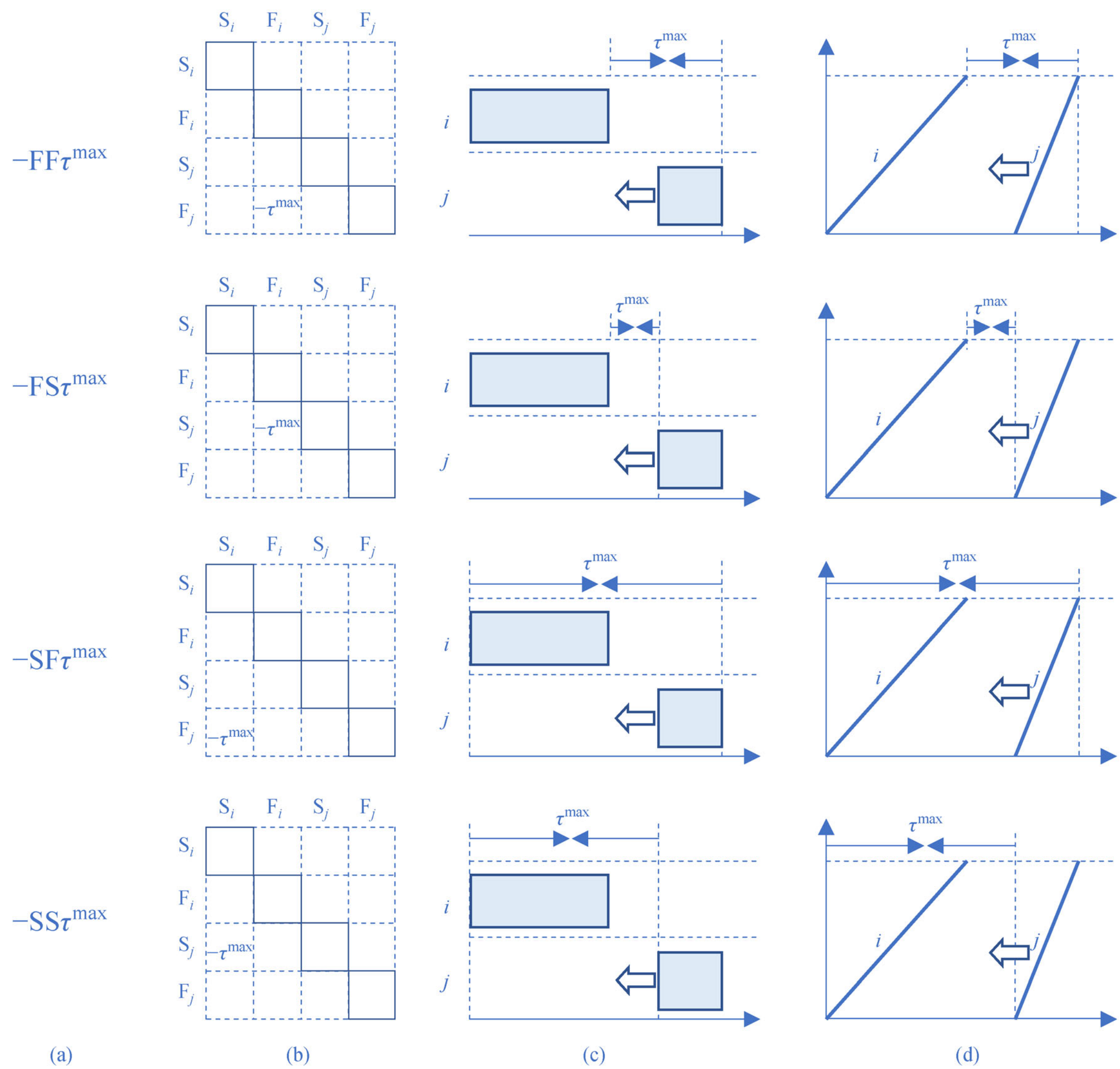


Fig. 8 Maximum-type relations.

–SSτ^{max}: This relation can be used as a proper tool to specify sensitivity-based restrictions in case of well-synchronized or mutually adjusting processes. Such relation may emerge in timbering the earth walls of a trench within a limited time after excavating a given length or depth before losing humidity (dummy cohesion) and/or supporting power of the surrounding soil the earth walls collapse.

4.3 Combined limitations with non-equal lower and upper bounds: Relations of limited codomains

Relative timing of activities usually can be guided (limited) by upper and lower bounds simultaneously. The above-mentioned sample can be used as reference. Excavation of a trench must be followed by supporting the earth walls. On one hand, the team performing the timber works should wait while the excavator proceeds to a safe distance; on the other hand, the timbering should be installed as soon as possible.

These types of time restrictions with “limited codomains” can be constructed by any or through combinations of the four basic types of relations (Fig. 9). Limited set (codomains) of relative time positions (lead or lag times) are available to fit these restrictions.

4.4 Combined limitations with equal lower and upper bounds: Fixed (Forced) succession

To find the proper function of these relations, we should consider the mass amount long-distance transportation of materials and/or prefabricated structural elements characterizing many construction projects. In case of oversized or complex units to be transported to the site from the preassembling or manufacturing plant associating the strictly limited capacities of on-site storage or deposition,

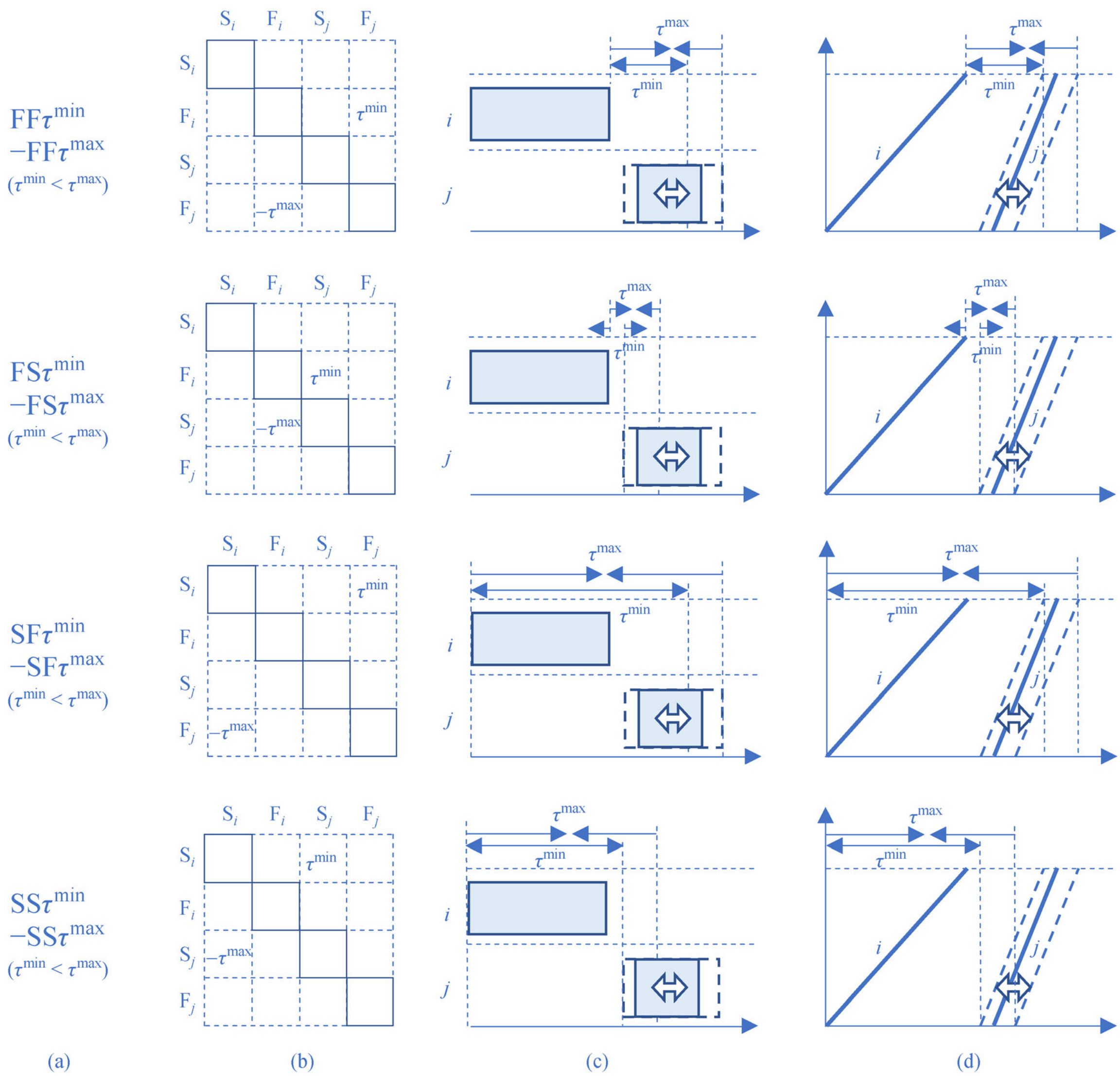


Fig. 9 Relations of limited codomains.

the transportation of the units must be timed in accordance with a synchronized tough schedule (“top time deliveries”).

These types of “single chance” time restrictions can be constructed by any or through combinations of the four basic types of relations (Fig. 10). One single solution is available (lead or lag time) to schedule the related activities in time (relative to each other) and to fit these restrictions (The “fixed” or “forced” succession is a subtype of relations with “limited co-domains”).

4.5 Single and combined limitations specifying technological succession times: Technological break-type relations

4.5.1 Specifying immediate succession

The activity of successor j must be immediately started after finishing predecessor i without break or delay. Typical samples for this requirement are the continuous use or application of resources with high operating/renting costs or with great importance (main equipment). Timing performance may include a heavy crane, a Tunnel Boring Machine (TBM) or a finisher during road construction. We exclude any overlapping of activities assigned to the given resource using the lower bound, that is, the same resource can perform them, and we eliminate expensive idle times using the associated upper bound.

These types of “single chance, no overlap, no break” time restrictions can be constructed using other combinations of the four basic types of relations. “Immediate succession” is a subtype of “fixed” or “forced” succession (Fig. 11).

4.5.2 Upper limit on overlapped activities in time

We can set a maximum acceptable rate of overlapping for succeeding activities in time and space using this restriction. An applicable solution is an $FS\tau^{min}$-type

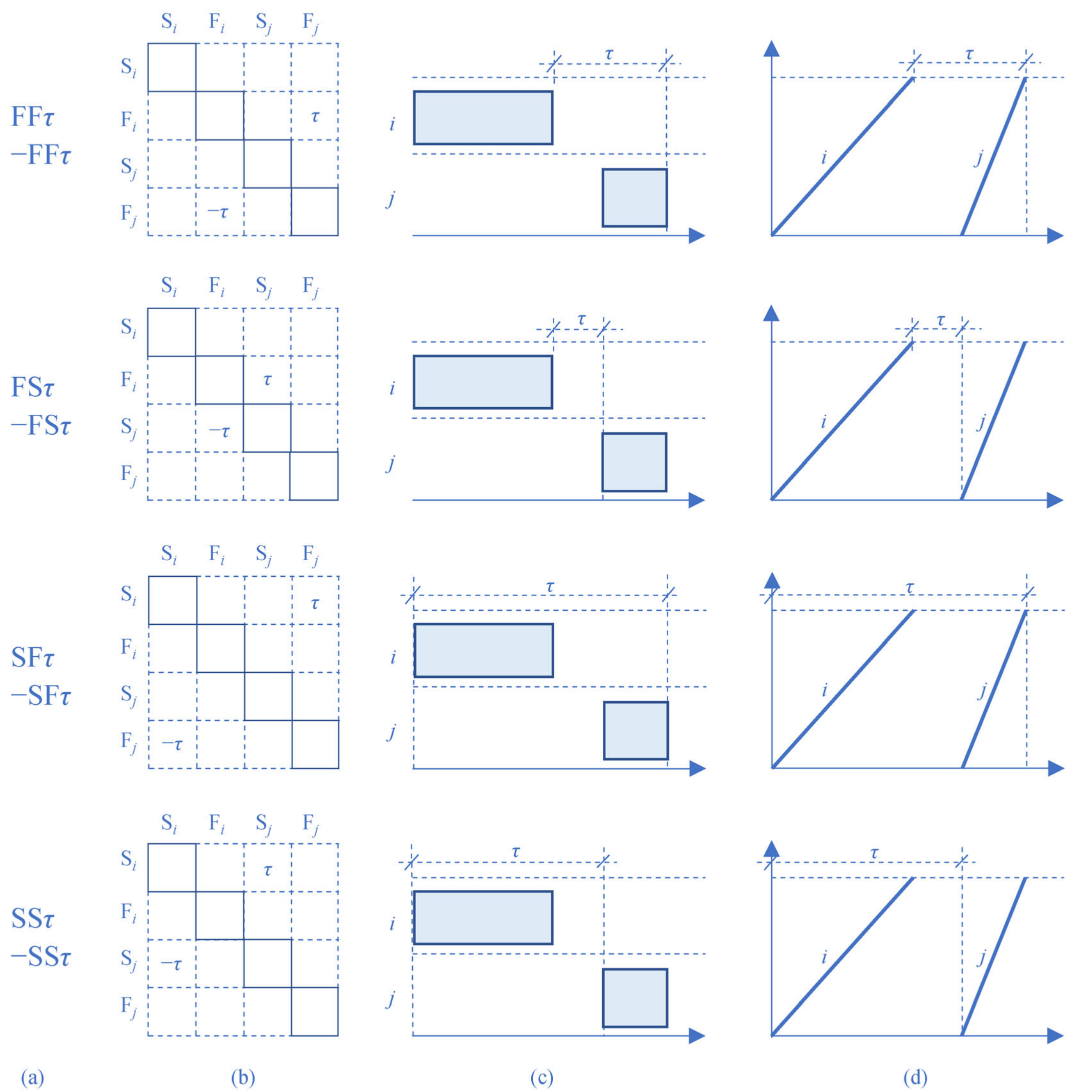


Fig. 10 Fixed (Forced) succession.

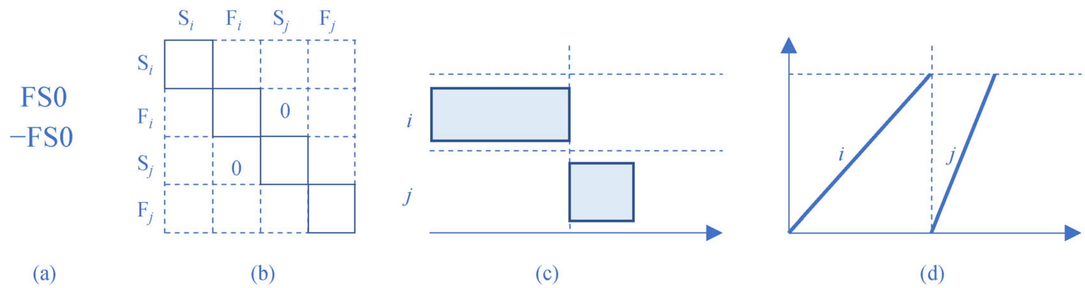


Fig. 11 Immediate succession.

relation with a negative delay value (lead time), which has the same effect as a $-SF\tau^{max}$-type relation (Fig. 12). Similarly, an example for the application of the latter can be the related timing of activities or services substituting or replacing each other. Following the launch or introduction of a new service or system, the old ones should be terminated or removed within a limited time. The technical units and facilities of a workshop should be moved to the new hall within a limited time before the old workshop is partially or entirely demolished. The capacities of resources commanded to the same work area should be limited to provide smooth undisturbed performance. Well-known samples of these activities are the finishing works of an office building scheduled to the final months of construction.

4.5.3 Setting the minimum of technological break between overlapped activities in time

With the proposed combination of relations, we can set a minimum time as the technical break between overlapped succeeding activities with long duration independent of their actual duration (Fig. 13). Their typical samples are the concrete layers (base layer, strip foundation, paving), drying of coating, and consolidation of earth embankment.

4.5.4 Setting the maximum of technological break between overlapped activities in time

These restrictions may earn great importance in constructing the so-called "stable" technological time models when the technological expectations and/or restrictions are well-known (Fig. 14). However, the actual durations of activities are not set because of the lack of actual technical drawings, known time restrictions, and assigned resources. They may also play key roles when applying stretchable activities in the network model, that is, when defining the proper duration of activities is expected from the time analysis of the entire model itself.

4.5.5 Limiting technological break between overlapped activities in time

This relation can be mainly used to set time restrictions or technological specifications in cases with sensitive conditions between overlapped succeeding activities with long duration independent of their actual duration (Fig. 15). A good sample is the protection of rendered earth surface at foundation works or immediately closing the "open" surface of the binding layer (by spreading the top layer on it) in case of an asphalt road construction.

4.5.6 Fixing technological break between overlapped activities in time

These limitations can perform well in case of more or less accurately known activity durations (Fig. 16). However, improper application may casually result in an unintentional disturbance in the technological order of activities. The abovementioned combinations may play key roles in applying stretchable activities in the network model, that is, when defining the proper duration of activities

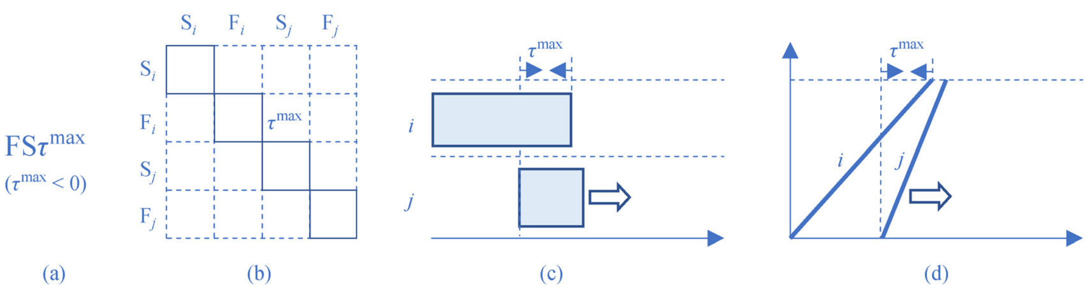


Fig. 12 Upper limit on overlapped activities in time.

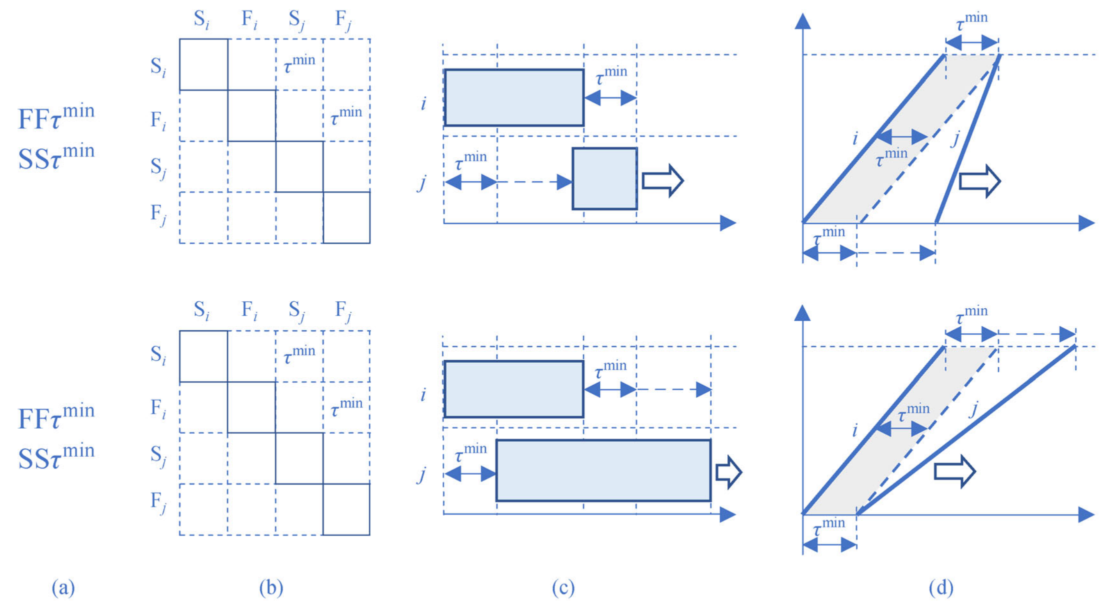


Fig. 13 Setting the minimum of technological break between overlapped activities in time.

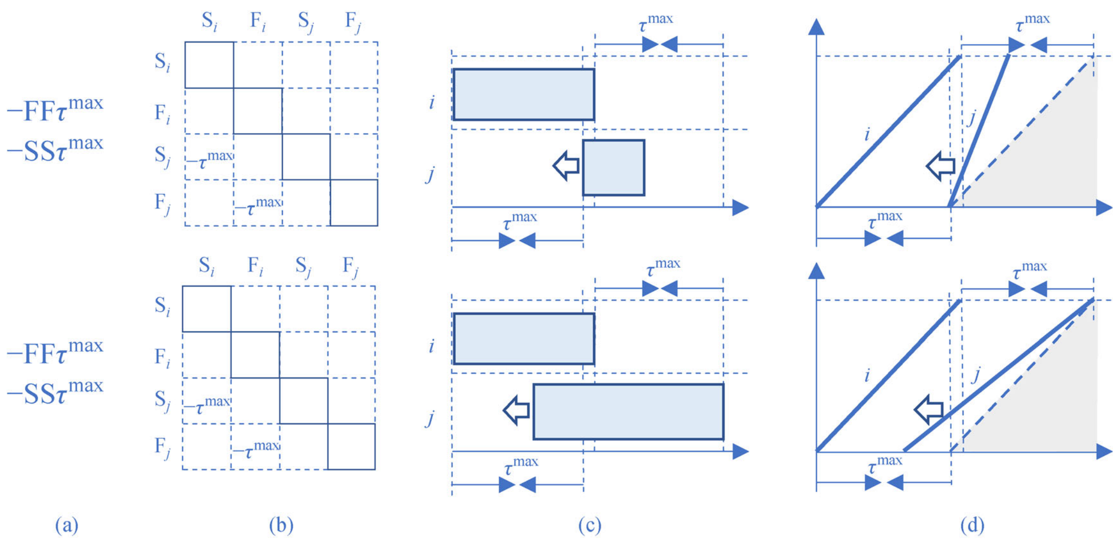


Fig. 14 Setting the maximum technological break between overlapped activities in time.

is expected from the time analysis of the entire model itself.

5 Conclusions

We introduced the development of network scheduling techniques with the main focus on the acceptance and integration of negative time parameters in time models through a short historical review of technical literature. We proved the validity and feasibility of their role and use in practical and technical managerial environment by using some graphical and literal terms as references and citing technological situations. First, we discussed the limitations on the duration of activities. We then systematically surveyed the theoretical and practical limitations on the relative time positions of related activities. We highlighted the primal-dual relation between the minimum potentials' problem and the longest path problem by focusing on the latter, thereby enabling the preparation of handy and stable logical networks with time dependencies of related activities of (construction) projects without any restrictions on the graph structures and on the weights along its edges, although no positive loop is acceptable. The contribution to the body of knowledge shows that using negative time parameters in a network model in up-to-date computing (scheduling) technology should not be considered a necessity rather than a mistake, problem or source of any

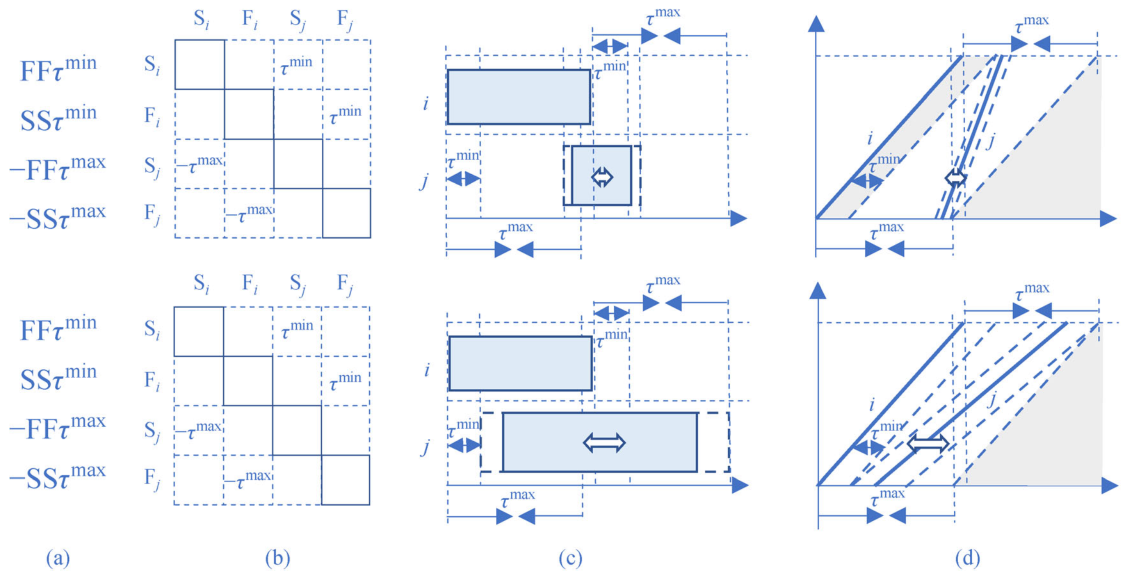


Fig. 15 Limiting technological break between overlapped activities in time.

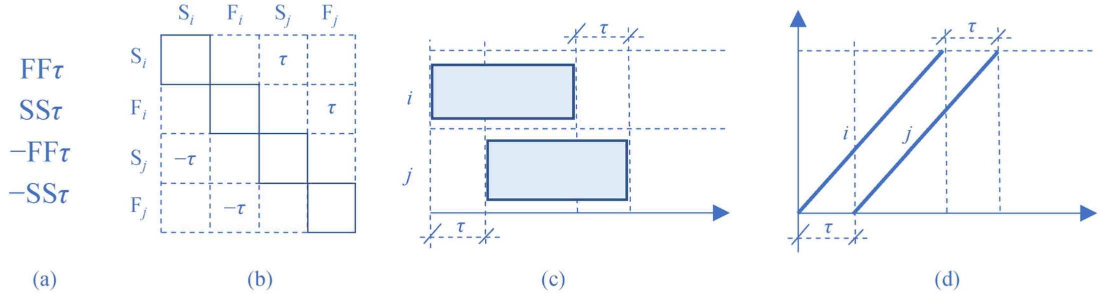


Fig. 16 Fixing technological break between overlapped activities in time.

contradiction. To better understand (overview) the occurrences and applications of positive and negative weights in a network time model, readers can refer to the "radiographic view" of a typical MPM/PDM network published in Vattai (2016).

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