Aerotecnica Missili & Spazio (2022) 101:309–320
https://doi.org/10.1007/s42496-022-00143-4

Aerotecnica M&S 100 Years Ago: A Study on the Possibility of a Dynamic Gliding Flight

Aldo Frediani[1] · Vittorio Cipolla[1] · Sergio De Rosa[2] · Paolo Gasbarri[3]

Accepted: 7 November 2022 / Published online: 19 December 2022

Dear Reader,

The preface of this issue of Aerotecnica Missili & Spazio (ATMS) is dedicated to the paper "ON THE POSSIBILITY OF DYNAMIC GLIDING" (in Italian "Sulla possibilità del volo a vela dinamico"), published by Prof. Enrico Pistolesi in December 1922.

This paper is an interesting read to understand how, without the means of computing such as the present ones, scientists used the tools of mathematical analysis to bring developments to aeronautical sciences.

1 ON THE POSSIBILITY OF DYNAMIC GLIDING (translation of the manuscript "Sulla possibilità del volo a vela dinamico" by E. Pistolesi, 1922)

It is known that gliding and flight without engines, in its simplest form, does not differ from ordinary hovering flight and that the possibility of maintaining or increasing the altitude is conditioned on the existence of ascending streams.

In fact, if the vertical component of the stream exceeds the minimum descent speed of the aircraft, we can have an ascent speed equal to the difference of the two. In this case, gliding is called static sail flight [1].

Different from this would be the dynamic sail flight in which the wind direction, or at least the average direction of it, is horizontal but the wind itself is pulsating or gusty, such as to give the aircraft appropriate accelerations. These accelerations, if conveniently exploited, would allow the aircraft to maintain altitude and rise above the starting point. The various ways in which dynamic flight could be implemented have been studied by various authors such as Betz, Knoller, Ahlborn, v. Karman, Prandtl and Blume (* MERGEFORMAT [2–7]), etc. and their studies have recently been summarized in a brilliant paper by W. Hoff (* MERGEFORMAT [8]). These studies show that dynamic flight is possible, but of course, certain conditions must be met.

This brief critical study aims at establishing the limits within which the dynamic flight is possible; in this study, we present the above research activities of the authors mentioned before, either in its original form or in another that more easily allows a brief quantitative analysis. Thus, it is easy to verify whether the current construction technologies make it possible not to cross the aforementioned limits and, therefore, if the dynamic flight, at the current state of the today constructions, is an achievable reality or a chimera.

There is a controversy about this point among the experts on the subject: even more so, since in the recent races in Germany, France and England, where surprising results have been obtained, what has actually been done is only to exploit ascending currents. It is not the case, for now, to speak on Dynamic Flight.

The brief notes that follow, without pretending to solve the dispute, would seem to support this opinion, at least for some of the various ways conceived for dynamic flight. More likely, it would seem to present the dynamic spiralling flight, which is observed to be implemented by some sailing birds.

However, this study is intended to be only a modest contribution to the study of flight without an engine, a topic that fascinates both experts and laymen of Aerodynamics today.

1.1 The Karman theory

A way in which the dynamic flight could be implemented is that envisaged by Karman (* MERGEFORMAT [9]), based on the pulsating structure of the wind. For this, Karman

✉ Aldo Frediani
aldo.frediani@unipi.it

1 University of Pisa, Pisa, Italy

2 Università Federico II Napoli, Naples, Italy

3 Università La Sapienza Roma, Rome, Italy

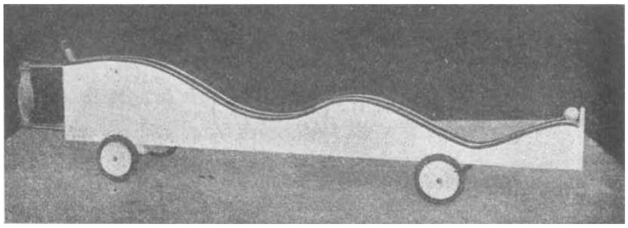

Fig. 1 (from the original paper)

refers to a mechanical model (this model was considered for the first time by A. Bazin in 1890 and, some years later, by Lancester).

A way (achievable with a wooden surface) of sinuous shape (Fig. 1) moves with a motion time-dependent, e.g., with a sinusoidal law.

Let U be the translation speed of this shape; we can put:

$$U = u_0 + u_1 sen(\lambda t)$$

where $\lambda = \frac{2\pi}{T}$, and T is the period.

A sphere, placed on this moving way is subjected, for the accelerations of this, to impulses that, under appropriate conditions, are able to lift it to heights higher than the starting one. Let W be the horizontal component of the speed of the sphere with respect to the guide, where W varies with a sinusoidal law with the same period of the motion of the guide:

$$W = w_0 + w_1 sen(\lambda t + \psi).$$

We can study the motion of the sphere with respect to the guide as if it is fixed, provided to add the accelerations due to the motion of the guide itself. Let us also assume that there is no friction.

That said, if the sphere passes from the height y_0 to y, it is subjected to the increase $mg\left(y - y_0\right)$ of the potential energy, which equals the decrease of the living force and the work of the accelerations impressed by the motion of the guide.

Said φ the angle of the guide with the horizontal at a given point, the speed of the sphere will be:

$$\frac{W}{cos\varphi} = W\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

and, therefore, his living force will be

$$\frac{1}{2}mW^2\left(1 + \left(\frac{dy}{dx}\right)^2\right).$$

Finally, the acceleration due to the motion of the guide will be:

$$u_1 \lambda cos(\lambda t)$$

to which corresponds a horizontal force:

$$-mu_1 \lambda cos(\lambda t)$$

and then the work:

$$-mu_1 \lambda \int_0^t W cos(\lambda t)dt.$$

It follows, therefore:

$$mgy + \frac{1}{2}mW^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = cost - mu_1 \lambda \int_0^t W cos(\lambda t)dt \tag{1}$$

That is:

$$\begin{aligned} y = cost - \frac{1}{2g}\left\{w_0 + w_1 sen(\lambda t + \psi)\right\}^2 \\ \left[1 + \left(\frac{dy}{dx}\right)^2\right] - \frac{1}{g}u_1 w_0 sen(\lambda t) + \frac{u_1 w_1}{4g}cos(2\lambda t)cos\psi \\ + - \frac{u_1 w_1}{4g}sen(2\lambda t)sen\psi - \frac{u_1 w_1}{2g}\lambda t sen\psi. \end{aligned}$$

Thus, the expression of y consists of a periodic part and a last term

$$-\frac{u_1 w_1}{2g}\lambda t sen\psi$$

which, after ony period, undergoes an increase:

$$\Delta y = -\frac{\pi u_1 w_1}{g}sen\psi.$$

Of course, if $sen\psi$ is negative, the increase is positive, that is, the sphere really rises above the starting point.

In first approximation, to calculate the trajectory of the sphere, that is the shape on which the sphere rolls without friction, KARMAN assumes that w_1 and u_1, are small w.r.t w_0 and that $\left(\frac{dy}{dx}\right)^2$ is small w.r.t the unity.

With such assumptions, the expression of y is simplified:

$$y = cost - \frac{1}{2g}{w_0}^2 - \frac{1}{g}w_0 w_1 sen(\lambda t + \psi) - \frac{1}{g}w_0 u_1 sen(\lambda t)$$

that is:

$$y = y_0 - \frac{w_0}{g}\left(u_1 + w_1 \cos\psi\right) sen(\lambda t) - \frac{1}{g} w_0 w_1 sen(\psi)\cos(\lambda t),$$

where y_0 includes all the constants (therefore y_0 is not $y(0)$); or, also:

$$y = y_0 - hsen(\lambda t + \delta),$$

where:
$$h = \frac{v_0}{g}\sqrt{u_1^2 + 2u_1 w_1 cos\psi + w_1^2}$$

and:

$$tg\delta = \frac{w_1 sen\psi}{u_1 + w_1 cos\psi}.$$

So far, the theory of Karman. But we must take the drag into account, which, in the case of the aircraft, consists of friction and cannot be neglected.

1.2 The effect of drag

Assuming, for simplicity's sake, that a constant drag coefficient χ_∞ is constant (it is an average value), it will be necessary to add the following term to the right-hand side of (1):

$$-aS\chi_\infty \int_o^t \left(\frac{W}{cos\varphi}\right)^3 dt.$$

Assuming again, for simplicity, $\cos\varphi = 1$, we have:

$$-aS\chi_\infty \int_o^t \left(w_0 + w_1 sen(\lambda t + \psi)\right)^3 dt.$$

To the expression of y, it is necessary to add the following term, in which we put $mg = Q$ (Q is the weight of the aircraft):

$$-\frac{aS}{Q}\chi_\infty \left\{ w_0^3 t + \frac{3}{2} w_0 w_1^2 t - \frac{3w_0^2 w_1}{\lambda} cos(\lambda t + \psi) - \frac{3}{4} w_0 w_1^2 sen2(\lambda t + \psi) - \frac{w_1^3}{\lambda} cos(\lambda t + \psi) sen^2(\lambda t + \psi) \right\}.$$

Therefore, the increase Δy after each period becomes:

$$\Delta y = -\frac{\pi u_1 w_1}{g} sen\psi - \frac{2\pi}{\lambda}\frac{a\chi_\infty}{q_s}\left(w_0^3 + \frac{3}{2} w_0 w_1^2\right), \tag{2}$$

where q_s indicates the load per square meter.

Finally, the approximate expression of y will be modified as follows:

$$y = y_0 - \frac{w_0}{g}\left(u_1 + w_1 cos\psi + 3\frac{ag\chi_\infty}{q_s\lambda} w_0 w_1 sen\psi\right) sen(\lambda t) - \frac{w_0}{g}\left(w_1 sin\psi - 3\frac{ag\chi_\infty}{q_s\lambda} w_0 w_1 cos\psi\right) cos(\lambda t). \tag{3}$$

Putting

$$y = y_0 - h' sen(\lambda t + \delta) \tag{4}$$

we will obtain the following expression of the amplitude of a half-oscillation:

$$h'^2 = h^2 + \frac{w_0^2}{g^2}\left\{ \frac{9a^2 g^2 \chi_\infty^2}{q_s^2 \lambda^2} w_0^2 w_1^2 + 6\frac{ag\chi_\infty}{q_s\lambda} w_0 w_1 u_1 sen\psi \right\}$$

$$h'^2 = h^2 - \frac{6w_0^3 a\chi_\infty}{q_s\lambda}\left\{ -\frac{u_1 w_1}{g} sen\psi + 6\frac{a\chi_\infty}{q_s\lambda}\frac{3}{2} w_0 w_1^2 \right\} \tag{5}$$

From a comparison with (2), it can be deduced that the quantity subtracted from h^2 is certainly positive, that is, definitely:

$$h' < h.$$

In the expression (2) of Δy, we can omit the term containing w_1^2 and write:

$$q_s = a\chi_\infty w_0^2, \tag{6}$$

where χ_∞ represents the value of χ_y corresponding to the average velocity w_0, that is, approximately, the mean value of χ_y.

It follows:

$$w_0^3 = \frac{q_s^{\frac{3}{2}}}{a^{\frac{3}{2}}\chi_{yo}^{\frac{3}{2}}}$$

and therefore:

$$\Delta y = -\frac{\pi u_1 w_1}{g} sen\psi - \frac{2\pi}{\lambda}\chi_\infty a^{-\frac{1}{2}} {\chi_{y0}}^{-\frac{3}{2}} q_s^{\frac{1}{2}}. \tag{7}$$

The following main consequence can be deduced from (7): for the possibility of a dynamic flight of the type studied so far, low values of the wing load, small drag and high average lift are required and, also, that at high altitude dynamic gliding flight is more difficult than at low altitude. Formula (7) would confirm what the intuition of the practitioners had foreseen: low wing loads and highly lifting profiles are needed.

Within the limits of the first approximation, the wing lift is the sum of the weight, mg, and the force of inertia due to the vertical acceleration, $m\frac{d^2y}{dt^2}$. We obtain:

$$aS\chi_y W^2 = mg + mh'^{\lambda^2} sen\left(\lambda t + \delta'\right). \tag{8}$$

Of course, to calculate χ_y we should put for W its actual value $w_0 + w_1(sen(\lambda t + \psi)$ but, in a first approximation, we can replace the variable speed W with its average value w_0; thus:

$$\chi_y = \chi_{y0} + \chi_{y0}\frac{h'\lambda^2}{g} sen\left(\lambda t + \delta'\right), \tag{9}$$

where:

$$X_Y = {}^{Q}\!/_{aSwo^2}$$

The amplitude of the half-oscillation χ_y is:

$$\chi_{y0}\frac{h'\lambda^2}{g}.$$

In general, it will be convenient that χ_y does not exceed a certain fraction of χ_{y0}, determined by the position that χ_{y0} can assume between its minimum and the maximum values. If we assume zero as a practical minimum, it will be, at most:

$$\frac{h'^2\lambda^2}{g} \leq 1.$$

Since h' < h, the above condition will certainly be verified when:

$$\frac{h\lambda^2}{g} \leq 1,$$

That is, when:

$$\frac{w_0\lambda^2}{g^2}\sqrt{u_1^2 + 2u_1 w_1 cos\psi + w_1^2} \leq 1. \tag{10}$$

Given the value of w_0 (e.g. by fixing $q_s e \chi_{y0}$), Δy results from the condition: $sen\psi = -1, u_1 = w_1 = \frac{g^2}{\sqrt{2}w_0\lambda^2}$ and then:

$$\Delta y = \frac{\pi g^3}{2w_0^2\lambda^4} - \frac{a\chi_\infty}{q_s}\frac{2}{\lambda^3}w_0^3.$$

Putting, e.g.: $\chi_{yo} = 0.25$, $q_s = 10\text{kg/m}^2$ (from which: $v_0 = 18\text{m/sec}$), the maximum value of w_1 will be $w_1 = \frac{96}{25.5\lambda^2}$. With a period of 6.28 s, i.e. $\lambda = 1$, we could obtain:

$$u_1 = w_1 \leq 3.8\frac{m}{sec}.$$

It is easy to remark that, given these orders of magnitude, h' differs only slightly from h, so that, in a rough calculation, it will be possible to identify h' with h.

1.3 The limits of dynamic flight

We will use the previous conclusions to establish the limits within which the dynamic flight is possible.

Assuming again $u_1 = w_1$ and $cos\psi = 0$forsimplicity, it results:

$$h' =\approx h = w_0 w_1\frac{\sqrt{2}}{g}$$

or:

$$h' =\approx \frac{\sqrt{2}}{g} q_s^{\frac{1}{2}} a^{-\frac{1}{2}} \chi_{y0}^{-\frac{1}{2}} w_1,$$

so that:

$$\chi_y = \chi_{y0}\left[1 + \frac{w_1\lambda^2}{g^2}\sqrt{2} q_s^{\frac{1}{2}} a^{-\frac{1}{2}} \chi_{y0}^{-\frac{1}{2}} sen(\lambda t + \delta')\right].$$

It follows that χ_y is between a maximum of:

$$\chi_{y0}\left\{1 + \frac{w_1\lambda^2}{g^2}\sqrt{2} q_s^{\frac{1}{2}} a^{-\frac{1}{2}} \chi_{y0}^{-\frac{1}{2}}\right\}$$

and a minimum of:

$$\chi_{y0}\left\{1 - \frac{w_1\lambda^2}{g^2}\sqrt{2} q_s^{\frac{1}{2}} a^{-\frac{1}{2}} \chi_{y0}^{-\frac{1}{2}}\right\}.$$

If we want that Δ_y is not negative, it must also be, for (7):

$${w_1}^2 \geq \frac{2g}{\lambda}\chi_\infty q_s^{\frac{1}{2}} a^{-\frac{1}{2}} \chi_{y0}^{-\frac{1}{2}} \tag{11}$$

Therefore, the maximum of χ_y will not be less than:

$$\chi_{y0}\left\{1 + 2\lambda^{\frac{3}{2}} q_s^{\frac{3}{4}} a^{-\frac{3}{4}} g^{-\frac{3}{2}} \chi_\infty^{\frac{1}{2}} \chi_{y0}^{-\frac{5}{4}}\right\} \tag{12}$$

and the minimum not higher than:

$$\chi_{y0}\left\{1-2\lambda^{\frac{3}{2}}q_s^{\frac{3}{4}}a^{-\frac{3}{4}}g^{-\frac{3}{2}}\chi_\infty^{\frac{1}{2}}\chi_{y0}^{-\frac{5}{4}}\right\}. \tag{13}$$

If we want, for example, that this minimum is not less than zero, it must be:

$$2(\lambda^2 q_s)^{\frac{3}{4}}g^{-\frac{3}{2}}K_\infty^{\frac{1}{2}}K_{y0}^{-\frac{5}{4}} < 1 \tag{14}$$

That is:

$$\lambda^2 q_s < \frac{g^2}{2^{4/3}}K_\infty^{-\frac{2}{3}}K_{y0}^{\frac{5}{3}} \tag{15}$$

or, also:

$$K_{y0} \geq \sqrt[5]{\frac{16K_\infty^2}{g^6}}\left(\lambda^2 q_s\right)^{\frac{3}{5}}. \tag{16}$$

Putting, to fix ideas, $K_\infty = 0.003$, it follows:

$$K_{y0} \geq 0.011\left(\lambda^2 q_s\right)^{\frac{3}{5}}. \tag{17}$$

Since, when (14) is satisfied the expression (12) grows with K_{y0} (how can be verified easily) we can substitute the expression of K_{y0} in (15) into (12) to determine the lower limit of the maximum value of K_y. This lower limit, which is of course equal to 2 K_{y0}, must not exceed a certain value depending on the airfoil of the aircraft; let it be K_{y*}.

Let's put, to fix the ideas, $K_{y*} = a\,\chi_{y*} = 0.075$; from (14a) it follows:

$$\lambda^2 q_s < \frac{g^2}{8}K_\infty^{-\frac{2}{3}}K_{y0}^{\frac{5}{3}}$$

or:

$$\lambda^2 q_s < 0.161K_\infty^{-\frac{2}{3}}. \tag{18}$$

If, for example, $K_\infty = 0.003$ it results:

$$\lambda^2 q_s < 7.7. \tag{19}$$

We can find another limitation based on the following considerations. In fact, we have already found:

$$w_1 \leq \frac{g^2}{\sqrt{2}w_0\lambda^2} \tag{20}$$

or:

$$\frac{w_1}{w_0} \leq \frac{g^2}{\sqrt{2}{w_0}^2\lambda^2}.$$

Because $w_0^2 = q_s K_{y0}^{-1}$, it follows:

$$\frac{w_1}{w_0} \leq \frac{g^2 K_{y0}}{\sqrt{2}q_s\lambda^2}$$

$$q_s\lambda^2 \leq \frac{g^2 K_{y*}}{2\sqrt{2}\frac{w_1}{w_0}}, \tag{21}$$

and, assuming $K_{y*} = 0.075$:

$$q_s\lambda^2 \leq \frac{2.58}{2\sqrt{2}}. \tag{22}$$

It is not easy to establish within what limits $\frac{w_1}{w_0}$ may vary in practice, even in the absence of precise data on wind fluctuations. Admitted, for example, as a practical lower limit $\frac{w_1}{w_0}$= ¼, we would get:

$$q_s\lambda^2 \leq 10.3.$$

expression (22) may be written as:

$$\frac{w_1}{w_0} \leq \frac{2.58}{q_s\lambda^2}. \tag{23}$$

From (2), we also have:

$$w_1^2 \geq 2g\frac{K_\infty}{q_s}\frac{w_0^3}{\lambda},$$

from which:

$$\frac{w_1^2}{w_0^2} \geq 2g\frac{K_\infty}{q_s}\frac{w_0}{\lambda}$$

$$\left(\frac{w_1}{w_0}\right)^2 \geq 2g\frac{K_\infty}{K_{y0}^{\frac{1}{2}}}\frac{1}{\lambda q_s^{\frac{1}{2}}}$$

and:

$$q_s\lambda^2 \geq 4g^2 \frac{K_\infty{}^2}{K_{y0}} \frac{1}{\left(\frac{w_1}{w_0}\right)^4} \tag{24}$$

because K_∞ cannot be higher than 0.0375:

$$q_s\lambda^2 \geq \frac{10300}{\left(\frac{w_1}{w_0}\right)^4} K_\infty{}^2. \tag{25}$$

It is easy to verify that (23) and (18) are not independent.

From the comparison of (25) and (18), the following condition arises:

$$\frac{10300}{\left(\frac{w_1}{w_0}\right)^4} K_\infty{}^2 < 0.161 K_\infty{}^{-\frac{2}{3}}$$

$$K_\infty{}^{\frac{8}{3}} < \frac{0.161}{10300}\left(\frac{w_1}{w_0}\right)^4$$

$$\frac{w_1}{w_0} > 1.59 K_\infty{}^{\frac{2}{3}}. \tag{26}$$

Thus, if we fix $K_\infty = 0.003$, it follows:

$$\frac{w_1}{w_0} > 0.33, \tag{27}$$

that is, the half-amplitude of the fluctuation of w must be not less than 1/3 of the average value.

From the foregoing, albeit with a criterion of large approximation, it is clear that it is very difficult to carry out in practice a dynamic gliding of the type studied. Assumed for K_{y*} and K_∞ the values 0.075 and 0.003 used previously, the following inequalities must occur simultaneously:

$$\lambda^2 q_s \leq 7.72$$

$$\frac{w_1}{w_0} \geq 0.33$$

$$\frac{w_1}{w_0} \leq \frac{2.58}{\lambda^2 q_s}.$$

The last two inequalities limit the range of choice of the ratio $\frac{w_1}{w_0}$ for which just the only value $\frac{w_1}{w_0} = \frac{1}{3}$ is possible when $\lambda^2 q_s = 7.72$.

The first inequality severely limits the field of the wing loads in relation to the frequency of fluctuations. It appears that high frequencies must necessarily correspond to low wing loads.

Thus, if q_s varies between 5 and 10, the upper limit varies from 1.24 to 0.88, corresponding to periods of 5" and 7"×2, respectively.

Now, let us illustrate the previous results with a practical example:

let be: $q_s = 7\frac{kg}{m^2}\lambda^2 q_s = 7$; then$\lambda = 1$.

From (17), we have:

$$K_{y0} \geq 0.0337.$$

Now, we assume:

$$K_{y0} = 0.035$$

and it follows:

$$w_0 = \sqrt{\frac{q_s}{K_{y0}}} = \sqrt{\frac{7}{0.035}} = 14.1\frac{\text{m}}{\text{sec}}.\left(51\frac{\text{km}}{\text{h}}\right).$$

Assuming $K_\infty = 0.003$, we will have:

$$\left(\frac{w_1}{w_0}\right)^2 > \frac{2g}{\left(\lambda^2 q_s\right)^{\frac{1}{2}}} \frac{K_\infty}{K_{y0}^{\frac{1}{2}}}$$

$$\left(\frac{w_1}{w_0}\right)^2 gt;0.119 \frac{w_1}{w_0} gt;0.345.$$

We put:

$$\frac{w_1}{w_0} = 0.39\, w_1 = 5.5\frac{m}{sec},$$

and, finally:

$$u_1 = 5.5\text{mm}$$

or, also:

$$sen\psi = -1 cos\psi = 0\left(\psi = -\frac{\pi}{2}\right).$$

Thus, from (2) we obtain:

$$\Delta y = 3.09 - 2.97 = 0.12\text{m},$$

and, because the period is 6.28 s, the average Δy per second is only 19 mm.

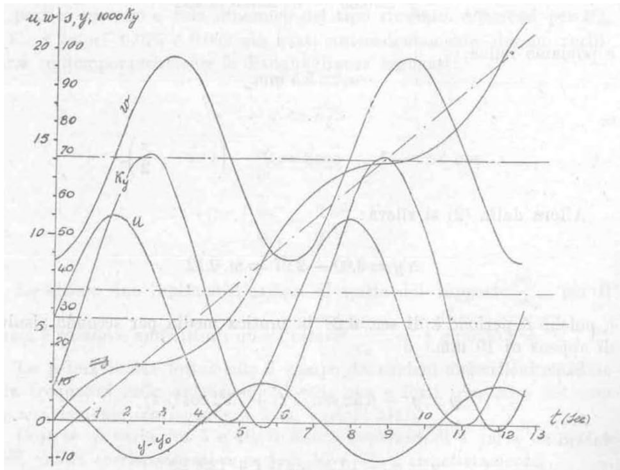

Fig. 2 (from the original paper)

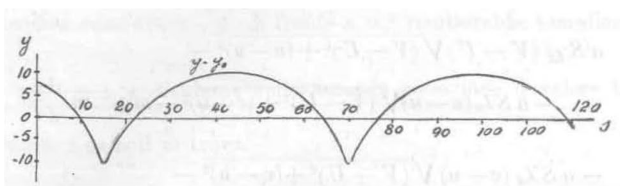

Fig. 3 (from the original paper)

We have, also:

$$y = y_0 - 6.52sen(\lambda t) + 7.92cos(\lambda t)$$

or:

$$y = y_0 + 10.26sen\left(\lambda t + 129°20'\right)$$

$$y = y_0 - 10.26sen\left(\lambda t - 50°40'\right).$$

Therefore, the value of the half-oscillation is 10.26 m. to be compared with the space travelled in one period. In general:

$$w = w_0 + w_1 sen(\lambda t + \psi)$$

$$x = w_0 t - \frac{1}{\lambda} w_1 cos(\lambda t + \psi) + c \tag{28}$$

And, in the case, at hand:

$$w = w_0 - w_1 cost$$

$$x = w_0 t + w_1 sent + c$$

The space travelled with respect to the wind in a period $\left(T = \frac{2\pi}{\lambda}\right)$ is, therefore:

$$w_0 \frac{2\pi}{\lambda}.$$

In our case:

$$w_0 \frac{2\pi}{\lambda} = 88.5m$$

and finally, we have:

$$K_y = 0.035\left[\left(1 - 1.046sen\left(\lambda t - 50°40'\right)\right)\right].$$

Therefore, the extreme values of K_y are -0.0011 and 0.0717; they are acceptable, even though the minimum is negative. This result is not surprising since inequalities (which even in this case are verified), are extreme and necessary, but not sufficient, conditions.

In addition, the research has been conducted with general and not strict criteria.

Figure 2 shows of the various quantities introduced before and Fig. 3 shows the trajectory of the aircraft, in the case of zero wind speed and, therefore, it results:

$$u = 5.5(1 + sent).$$

For the tracking of the trajectory, it is necessary to take as abscissa:

$$s = \left(w_0 - u_0\right)t + w_1 sent + u_1 cost + c \tag{29}$$

that is, in our case:

$$s + 5.5 = 8.6t + 7.78_1 sen(t + 45°)$$

Figure 3 shows the poor likelihood of such a flight (observers of the flight of sailing birds are almost unanimous in ruling out that a type of "Russian mountings" flight, such as that represented in Fig. 3, could be actually practiced by the birds).

1.4 A different procedure

Results, similar in all respects to the previous ones, can be achieved by a rather different procedure, which can serve to control the one followed so far. Therefore, for example, it will serve to show that to exploit the pulsations of the wind in a dynamic gliding flight, it is necessary to be able to vary harmoniously the coefficient of thrust, according to the variation of the wind speed; this can be obtained both by manoeuvring the entire aircraft, if this is a rigid body, and (what would certainly be more ready and effective) by acting on the wing structure or by varying the trim with respect to the body, or by varying the curvature and shape of the lifting surfaces.

We write the equations of motion in a vertical plane. Let U be the horizontal component of wind speed, u its vertical component, V is the horizontal component of the speed of the aircraft, v its vertical component. Given again m the mass of the aircraft and S its surface, we have:

$$\begin{cases} aS\chi_y(V-U)\sqrt{(V-U)^2+(v-u)^2}- \\ -aS\chi_\infty(v-u)\sqrt{(V-U)^2+(v-u)^2}-mg=m\frac{dv}{dt} \\ aS\chi_y(v-u)\sqrt{(V-U)^2+(v-u)^2}- \\ -aS\chi_\infty(V-U)\sqrt{(V-U)^2+(v-u)^2}-mg=m\frac{dV}{dt} \end{cases} \tag{30}$$

Assuming that the wind is horizontal (and therefore $u=0$), that v^2 can be neglected with respect to $(U-V)^2$ (in the first approximation) and neglecting the term in χ_∞ in the first equation, we get:and placing, at first,

$$\begin{cases} aS\chi_y(V-U)^2-mg=m\frac{dv}{dt} \\ -aS\chi_y v(V-U)-aS\chi_\infty(V-U)^2=m\frac{dV}{dt} \end{cases} \tag{31}$$

Putting V-U = W = $w_0 + w_1$ sen(λt), from the first of (25), it follows:

$$mv=-mgt+aS\int\chi_y\left[w_0+w_1 sen(\lambda t)\right]^2dt+C.$$

If we admit that χ_y is constant, and we place K_y in place of $a\chi_y$, we get:

$$mv=-mgt+K_yS\int\left[w_0{}^2+2w_ow_1sen(\lambda t)+w_1{}^2sen(\lambda t)^2\right]dt+C,$$

that is:

$$mv=\left(K_ySw_0{}^2+K_yS\frac{w_1{}^2}{2}-mg\right)t-2\frac{K_yS}{\lambda}w_0w_1cos(\lambda t)-$$

$$-\frac{K_yS}{4\lambda}w_1{}^2sen(2\lambda t)+C.$$

For v to be periodic, the term in t must disappear, that is, we have:

$$K_yS\left(w_0{}^2+\frac{w_1{}^2}{2}\right)=mg,$$

that is:

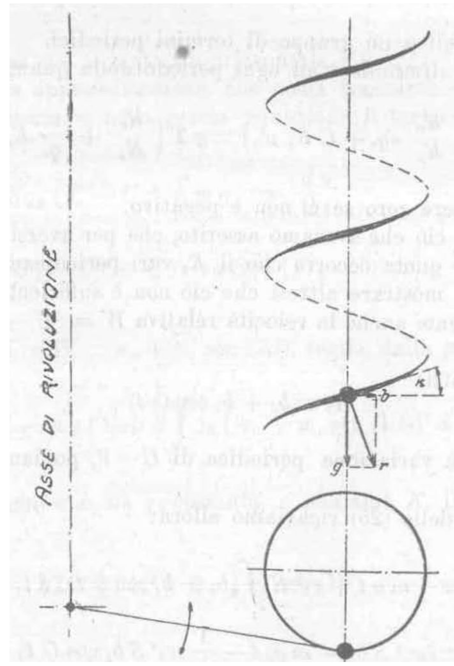

Fig. 4 (from the original paper)

$$q_s=K_y\left(w_0{}^2+\frac{w_1{}^2}{2}\right).$$

Because $\frac{w_1{}^2}{2}$ can be disregarded with respect to $w_0{}^2$, it would result simply $q_s=K_yw_0{}^2$.

Now, we substitute the value of v into the second equation, and, after simple calculations, we find:

$$mV=-St\left(K_\infty w_0{}^2+K_\infty\frac{w_1{}^2}{2}+CK_yw_0\right)+\mathfrak{I},$$

where con $\mathfrak{I}$ indicates a group of periodic terms.

It follows that V, after any period, decreases of the quantity:

$$\frac{gT}{q_s}\left(\frac{K_\infty}{K_y}q_s+CK_yw_0\right)=gT\left(\frac{K_\infty}{K_y}+\frac{c}{q_s}K_yw_0\right)$$

which cannot be zero if C is not negative.

This shows what we had said: that, to have a gliding flight without loss of altitude, it is necessary that K_y depend periodically on time.

But we can also show that this is not sufficient and that the relative speed, W = (U-V), between the wind and the aircraft must vary periodically as well.

In fact, let us put:

$$K_y=b_0+b_1sen(\lambda t)$$

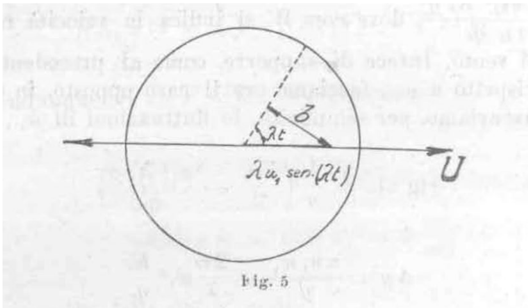

Fig. 5 (from the original paper)

and neglect the periodic variation of U-V, or, simply, put $w_1 = 0$.

Then, from the first of (25), we derive:

$$mv = -mgt + v_0^2 S \int \left[b_0 + b_1 sen(\lambda t)\right] dt$$

$$mv = \left(w_0^2 S b_0 - mg\right)t - \frac{1}{\lambda} w_0^2 S b_1 cos(\lambda t) + C.$$

Arguing as before, we find:

$$w_0^2 S b_0 = mg$$

$$w_0^2 b_0 = q_s,$$

and, from the second of (25) it results:

$$mV = -\left(K_\infty {w_0}^2 + S w_0 b_0 c\right)t + \mathfrak{I}'$$

being $\mathfrak{I}'$ a periodic function. In order not to have a decrease of V after any period, C must be negative, which proves the assertion.

The research could continue in the event that both (V-U) and Ky are harmonically variable and would lead, of course, to results similar to those previously found. But we omit it for the sake of brevity.

1.5 Dynamic spiral flight

A dynamic gliding flight can take place when the aircraft flies spiralling, rather than straight.

This way was also made clear by KÀRMÀN.

The mechanical model, designed by KLEMPERER, consists of a spiral guide, with a double motion: of rotation around its own axis and of revolution around an external axis parallel to the first one (Fig. 4). A sphere moving along the guide is subjected to the tangential acceleration due to the motion of revolution, b, and the acceleration of gravity, g; things can be adjusted so that the resultant, r, is normal to the guide. If we apply a certain speed to the sphere on the guide and a motion of rotation to the guide with peripheral speed opposite to that of the sphere, provided that there is no friction between the sphere and the guide, the sphere will rise. This scheme is difficult to achieve in practice with an aircraft; but it is possible to replace, to the constant centrifugal acceleration, the variable acceleration due to a pulsating wind.

Let be:

$$\lambda u_1 sen(\lambda t)$$

the acceleration due to the periodic wind variations. If the aircraft flies along a circular trajectory so that the direction of motion makes the angle λt with the direction of the wind (Fig. 5), the tangential acceleration will result:

$$b = \lambda u_1 sen^2(\lambda t)$$

That is, always positive. The average acceleration in a complete turn will be:

$$b_m = \frac{2\lambda u_1}{\lambda T} \int_0^{T/2} sen^2(\lambda t) dt = \frac{\lambda u_1}{2}.$$

This acceleration is composed with that of gravity to have the average direction of the resultant r (and then the average slope of the trajectory); we will obtain:

$$(tg\varphi) = \frac{b_m}{g} = \frac{\lambda u_1}{2g}$$

Note that the component of the speed of the aircraft in the direction of the wind is, in general:

$$w_0 + w_1 cos\lambda t,$$

if w_1 indicates the speed with which the aircraft travels the spiral, and with w_0 ia a flight speed added to that.

In addition, the amount of height increase, Δy, in the period $T = \frac{2\pi}{\lambda}$ becomes: $v_1 \left[tg\varphi\right]_m T$, that is:

$$\Delta y = \frac{\pi u_1 v_1}{g};$$

this formula is identical to that already obtained in the paragraph 1, when we put $\psi = -\frac{\pi}{2}$. This result was easily predictable, according to KÀRMÀN.

If we introduce the aerodynamic drag, by way of first approximation, we must subtract the quantity $\frac{W^2 K_\infty S}{m} = \frac{W^2 K_\infty g}{q_s}$ from b, where W indicates speed of the aircraft relevant to the wind. Instead of assuming w_1 small compared to w_0 (as in the previous paragraphs), now make the opposite case, in which $w_0 = 0$ and neglect, for simplicity, the fluctuations of w_1. We obtain:

$$[tg\varphi]_m = \frac{1}{g}\left(\frac{\lambda u_1}{2} - \frac{w_1^2 K_\infty g}{q_s}\right),$$

so that:

$$\Delta y = \frac{\pi u_1 w_1}{g} - \frac{2\pi}{\lambda} w_1^3 \frac{K_\infty}{q_s}.$$

We deduce immediately:

$$\frac{u_1}{g} - \frac{2}{\lambda} w_1^2 \frac{K_\infty}{q_s} > 0$$

or also, indicating with K_{y0} an average value of K_y:

$$\frac{u_1}{g} > \frac{2}{\lambda}\frac{K_\infty}{K_{y0}},$$

$$\lambda u_1 > 19.62\frac{K_\infty}{K_{y0}}.$$

Taken 1/10 as a practical minimum of $\frac{K_\infty}{K_{y0}}$, we get (approximately):

$$\lambda u_1 > 2.$$

This limit can be easily exceeded in practice. Perhaps, other limitations could arise from a more in-depth analysis of the problem: but, at first glance, this kind of gliding seems more easily achievable than the previous one. It is understood that the frequency of wind fluctuations cannot be too strong, since the aircraft must make, in a period, a whole turn of its spiral.

1.6 The dynamic flight of Betz and its limits

Finally, there is another possible way to exploit the energy of the wind for dynamic flight, and it occurs when the wind direction varies with a sinusoidal law.[1]

In this case, we will need (24), which we will simplify with the usual limitations, also neglecting the fluctuations of V-U and placing, at first, v = 0. Thus, we obtain:

$$\begin{cases} SK_y(V-U)^2 - mg = m\frac{dv}{dt} \\ SK_y u(V-U) - SK_\infty (V-U)^2 = m\frac{dV}{dt} \end{cases} \tag{32}$$

and, putting $W = V - U = w_0$, we get

$$\begin{cases} SK_y w_0^2 - mg = m\frac{dv}{dt} \\ SK_y u w_0 - SK_\infty w_0^2 = m\frac{dV}{dt} \end{cases} \tag{33}$$

From the second of (33), putting:

$$u = u_1 sen(\lambda t)$$

We get:

$$SK_y u_1 w_0 sen(\lambda t) - SK_\infty w_0^2 = m\frac{dV}{dt}.$$

We also put:

$$K_y = b_0 + b_1 sen(\lambda t + \beta)$$

to obtain:

$$Su_1 w_0 b_0 sen(\lambda t) + Su_1 w_0 b_1 sen(\lambda t) sen(\lambda t + \beta) - SK_\infty w_0^2 = m\frac{dV}{dt}.$$

Integrating for one period $\left(\frac{2\pi}{\lambda}\right)$, you get:

$$0 = -SK_\infty w_0^2 \frac{2\pi}{\lambda} + Su_1 w_0 b_1 cos\beta \frac{\pi}{2}$$

and it follows:

$$2K_\infty w_0 = u_1 b_1 cos\beta. \tag{34}$$

From the first of (33), integrating for an entire period, we also obtain:

$$Sb_0 w_0^2 - mg = 0,$$

from which:

$$b_0 w_0^2 = q_s. \tag{35}$$

If, instead of a horizontal flight, we have an uphill trajectory with ascending speed v, from the second of (32) we get:

$$SK_y(u - v)w_0 - SK_\infty w_0^2 = m\frac{dV}{dt}, \tag{36}$$

so that:

[1] On the effect of this variability of wind direction on the aerodynamic coefficients of a wing, see the interesting experiences of R. KATZMAYR, *Ueber das Verhalten von Flügelflächen bei periodischen Aenderung der Geschwindigkeitsrichtung*. Z.f.F.u.M. 190. S. 80–95.

$$2\left(K_{\infty}w_0 + b_0 v\right) = u_1 b_1 cos\beta. \tag{37}$$

The most favourable condition is obviously represented by $cos\beta = 1$, for which:

$$K_{\infty}w_0 = \frac{u_1 b_1}{2} - b_0 v. \tag{38}$$

It follows:

$$K_{\infty}w_0 \le \frac{u_1 b_1}{2}, \tag{39}$$

and, for (28):

$$K_{\infty}{}^2 q_s \le \frac{u_1{}^2 b_1{}^2 b_0}{4}. \tag{40}$$

Now, since $b_0 + b_1$ (maximum value of K_y) can not exceed a certain value, which we can still establish, to fix the ideas, in 0.075, it follows that we have the maximum of $b_0 b_1{}^2$ when $b_1 = 2b_0$ and then $b_0 + b_1 = 3b_0$.

It follows $b_0 = 0.025$ and:

$$K_{\infty}{}^2 q_s \le \frac{15.6}{16^6} u_1{}^2.$$

In this case, the lower limit of the K_y is: -0.025.

If assume as zero the lower limit of K_y, we obtain instead:

$$K_{\infty}{}^2 q_s \le \frac{13.2}{10^6} u_1{}^2.$$

Putting $K_{\infty} = 0.003$, it follows:

$$u_1 \ge \left\{ \begin{array}{l} 0.76 \\ 0.83 \end{array} \right. q_s$$

We can remark how the variations of the vertical speed of the wind should be very strong, at least for values q_s between 6 and 12, as it is occurs in general.

The research can be deepened taking the fluctuations in the vertical speed, v, of the aircraft into account.

For this reason, the first of (32) is integrated:

$$mv = -mgt + Sw_0{}^2 b_0 t - \frac{Sw_0{}^2 b_1}{\lambda} cos(\lambda t + \beta) + c.$$

Place, as usual:

$$mg = Sw_0{}^2 b_0,$$

that is:

$$b_0 w_0{}^2 = q_s,$$

it follows:

$$mv = c - \frac{mg}{b_0 \lambda} b_1 cos(\lambda t + \beta),$$

or even:

$$v = c_1 - \frac{gb_1}{\lambda b_0} cos(\lambda t + \beta).$$

Substituting in (26b) gives:

$$Sw_0\left(b_0 + b_1 sen(\lambda t + \beta)\right)\left(u_1 sen\lambda t - c_1 + \frac{gb_1}{\lambda b_0} cos(\lambda t + \beta)\right) -$$

$$-SK_{\infty}w_0{}^2 = m\frac{dV}{dt}.$$

By integrating for one period, you get:

$$2\left(K_{\infty}w_0 + b_0 c_1\right) = u_1 b_1 cos\beta. \tag{41}$$

That is, a relationship perfectly similar to (41). Therefore, the above considerations are also valid in this case.

We conclude that also this type of dynamic flight is difficult to be achieved in practice, requiring very low wing loads and exceptionally favourable wind conditions, perhaps never achievable practically.

It is, therefore, very likely, as mentioned in the beginning, that gliding without decreasing, or even with an increase in altitude, is always a static flight, that is, obtained by exploiting the ascending streams or even a wise combination of static flight and dynamic flight.

Data availability All the data published in this issue are the translation of the original paper published on Aerotecnica in 1922 and, thus, belong to the Journal.

References

1. Dorand 1922 Le volo des avions sans moteur. «L'Aeronatique», N. 32, Janvier
2. BetZ, A.: Ein Beitrag zur Erklärung des Segelflugs. Zeitschrift für Flugtechnik und Motorluftschiffahrt. **3**, 269 (1912)
3. R. Knoller 1913 Zur Theorie des Segelfluges. Z.f.F.u.M. 13
4. F. Ahlborn 1921 Der Segelflug, Erklärung des Segelflugs der Vögel Die Möglichkeit des Fliegens ohne Motor. Berichte und Abhandlungen der Wissenschaftlichen Gesellschaft für Luftfahrt. Heft 5.
5. Th. V. Karman 1921 Mechanische Modelle zum Segelflug. Z.t.F.u.M. 220
6. L. Prandlt 1921 Bemerkungen über den Segelflug. Z.t.F.u.M. 209
7. W. Blume 1921 Das Segelflugzeug der Akademischen Fliegertruppe der Technischen Hochschule Hannover. Z.t.F.u.M. 313

8. Hoff, W.: Der Segelflug und die Rhön- Segelflug-Wettbewerbe. Zeitschrift für angewandte Matematik und Mechanik. **2**, 3 (1922)
9. Th. V. KARMAN 1921 *Mechanische Modelle zum Segelflug*.– Zeitschrift für Flugtechnik und Motorluftschiffahrt. 220