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RESEARCH ARTICLE

Exhaustible resource use under endogenous time preference

Akira Maeda[1] · **Makiko Nagaya**[2]

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Abstract
This paper revisits a classical economic topic of exhaustible resource use on the basis of recent developments in time preference and discount factor models. An analysis of the effects of endogenous time preferences on the dynamic properties of resource use is conducted, contrasting the classical Hotelling results. More specifically, we develop an analytical model that incorporates endogenous time preference into the decision framework of resource consumption. It is expected that the results obtained here not only contribute to the literature of pure economic theory, but also to recent climate policy debates on discounting factors.

Keywords Habit formation · Hotelling's rule · Discounting · Social time preference · Backstop technology

JEL Classification E43 · O13 · Q32

Introduction

Hotelling's well-known rule is that scarcity rents for exhaustible resources increase at the rate of interest. Although there are many variations of the rule that derive similar rules, the implication remains the same. The dynamics of exhaustible resource use is driven by interest rates. Consequently, we face the following question: in what way are these interest rates determined? This question has never been answered in the framework of Hotelling and his successors.

✉ Akira Maeda
akira-maeda@g.ecc.u-tokyo.ac.jp

Makiko Nagaya
makiko.nagaya@gmail.com

[1] Graduate School of Arts and Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro, Tokyo 153-8902, Japan

[2] Faculty of Global Business, Showa Women's University, 1-7-57 Taishido, Setagaya, Tokyo 154-8533, Japan

Unlike in resource economics, debates about interest rates and time preferences have been intense in other fields of economics. In particular, in welfare economics, specifying discount factors falls under the topic of moral philosophy (e.g., [1]). Recent studies within this topic include those on hyperbolic discounting (e.g., [13]).

In macroeconomic theory, many studies have been conducted on models in which time preference depends on endogenous economic variables. Among those, habit formation models in which a history of consumption determines the time preference have recently become popular (e.g., [10]). A classical topic in this category includes the Uzawa–Epstein formulation of time preference [4, 5, 12].

This paper revisits a classical economic topic of exhaustible resource use on the basis of recent developments in time preference and discount factor models. An analysis of the effects of endogenous time preferences on the dynamic properties of resource use is conducted, contrasting the classical Hotelling results. More specifically, we develop an analytical model that incorporates endogenous time preference into the decision framework of resource consumption. The basic model structure consists of three parts: cake-eating economy, availability of a backstop technology, and the Uzawa–Epstein formulation of time preference. As an advanced model, a concept of minimum consumption requirement is incorporated.

There is a caveat before moving on to sections that follow: on the same topic and similar analytical framework, the authors have reported some related results in two papers written in Japanese as Nagaya and Maeda [8] and Maeda and Nagaya [7]. The present paper is different from these previous papers on two points: first, we revised all the calculations and provide a complete set of proofs for propositions that were omitted before. Second, we added a new proposition as Proposition 4.4 and made minor changes in propositions, especially in Proposition 5.1. We admit there are overlaps in descriptions between this paper and previous ones, albeit different languages. However, to reach the above-mentioned new or revised Propositions, it is necessary to provide detailed model explanation and preparatory propositions even if overlaps are significant. Thus, we decided not to skip them.

The rest of the paper is organized as follows. First three sections focus on the basic structure. The next section develops the analytical model. The subsequent section analyzes the dynamic properties followed by which the main results regarding the optimal timing of the switch from exhaustible resource use to a backstop technology are addressed. The penultimate section extends the framework of analysis by introducing a concept of minimum consumption requirement to the model. The final section is the conclusion of the discussion.

Model basis

Consider a closed economy in which an exhaustible resource and a backstop technology are available. Now assume there is no production sector. More specifically, the economic activity of the economy is restricted to consume the resource or utilize the available backstop technology. Further, assume the population is constant. Consequently, we introduce a representative agent who expects to maximize the sum of the discounted instantaneous utilities for the consumption of the exhaustible resource and the equivalent resource consumption supplied by the backstop technology.

We introduce the following notations:

t: Time.

$E(t)$: Exhaustible resource use at t.

$S(t)$: Stock of exhaustible resource at t.

S_0: Initial exhaustible resource stock.

$\Delta(t)$: Cumulated discount rate at t.

$u(*)$: Representative agent's instantaneous utility.

$r(*)$: Instantaneous discount rate.

P_B: The price of the backstop technology.

$\varepsilon(t)$: Resource use supplied by the backstop technology at t.

$q(t)$: Scarcity rent of the exhaustible resource at time t.

T: The time to switch from the exhaustible resource to the backstop technology.

Due to the limited stock of the exhaustible resource, the representative agent is supposed to use up the entire stock of the exhaustible resource by a certain time, after that the agent will begin using a backstop technology. The time to switch from the exhaustible resource to the use of the backstop technology (T) is endogenously determined, as we will see in the next section.

u is an increasing and concave function of $E(t)$ (or $\varepsilon(t)$). We assume that u has the following functional form:

$$\begin{aligned} &u(E) = \frac{E^{1-\eta}}{1-\eta}, \quad \eta > 0, \quad \eta \neq 1 \\ &u'(E) > 0, \quad u''(E) < 0, \end{aligned} \tag{1}$$

where η is the magnitude of the elasticity of the marginal utility and known as the reciprocal of the elasticity of the intertemporal substitution. For large η, consumption is said to be "inelastic" along the time horizon. Conversely, for small η, consumption is "elastic" along the time horizon. We will observe later that this concept plays an important role in extracting insights from our resulting mathematical expressions.

We introduce the Uzawa–Epstein formulation of time preference. That is, we assume that the instantaneous discount rate is a function of resource consumption and that the sum of the discounted instantaneous utilities is described as

$$\max_{\{E(t)\}} \quad \int_0^T u(E(t)) \cdot e^{-\Delta(t)} dt + e^{-\Delta(T)} V, \tag{2}$$

$$\Delta(t) = \int_0^t r(E(s)) ds. \tag{3}$$

Note that Eq. (3) is equivalent to

$$\frac{d\Delta(t)}{dt} = r(E(t)). \tag{4}$$

Following Uzawa [12], Epstein [5], and Epstein and Hynes [4], we assume the following properties[1]:

$$r > 0, \quad r'(E(t)) > 0, \quad r''(E(t)) \leq 0.$$

In the last term of (2), V represents the sum of the discounted instantaneous utilities accruing from the use of the backstop technology that starts being operational at time T. It is a result of the following optimization problem to be solved at time T:

$$V = \max_{\{\varepsilon(\tau)\}} \int_0^\infty \left(u(\varepsilon(\tau)) - P_B\varepsilon(\tau)\right) \cdot e^{-\Delta(\tau)} d\tau.$$

Discounting V back to the initial time (i.e., $t=0$) of exhaustible resource consumption, by the factor $e^{-\Delta(T)}$, we obtain its present value for the initial time expressed as the last term of (2).

The use of the exhaustible resource must adhere to the following dynamics:

$$\frac{dS(t)}{dt} = -E(t). \tag{5}$$

As an integral form, the dynamics can also be written as

$$\int_0^T E(t)dt = S_0 - S(T). \tag{6}$$

A *backstop technology* is that technology whose unlimited reserve is a substitute for the exhaustible resource stock. Although it is physically available, it is too expensive to use currently. It may become economically feasible to use in the future when the price of the backstop technology becomes cheaper than that of the exhaustible resource.[2] The switch from the exhaustible resource use to the use of the backstop technology is intuitively understood as follows: let $q(t)$ denote the scarcity rent of the exhaustible resource at time t. Suppose that the backstop technology becomes economically available at time T. Then, the condition for these prices for the switch at time T is

$$q(T) = P_B. \tag{7}$$

[1] In the literature, some studies investigate alternative settings: Das [3], Chang [2], and Hirose and Ikeda [6] proposed the use of decreasing functions for the instantaneous discount rates. Because such alternative settings are known to create technical difficulties, it is beyond the scope of this paper to deal with them in our model setting.

[2] Scientists and engineers have proposed many advanced power generation technologies. One of the most popular technologies among physicists may be nuclear fusion reactors. Electricity generation in space (e.g., photovoltaic technology) with wireless power transmission (e.g., microwaves) to the Earth (i.e., solar power satellites) is also considered a prospective technology.

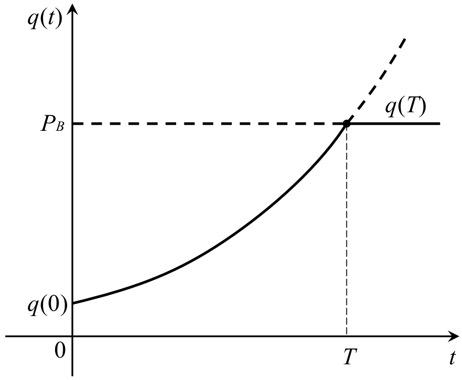


Fig. 1 The time of the switch from the exhaustible resource to the backstop technology

Notice that once the backstop technology becomes economically available at time T, the exhaustible resource will never be used again. If the exhaustible resource remains at the time, it can be used before the time at cheaper prices. This means that the exhaustible resource must be exhausted exactly at the time (i.e., $S(T)=0$). As such, the time T, at which Eq. (7) holds true, is said to be the time of the switch from the exhaustible resource to the backstop technology. Figure 1 depicts the switch.

Dynamics

In this section, we examine the model developed in the previous section. In the model, the maximization of the sum of the discounted instantaneous utilities by the representative agent is formulated as the following optimization problem:

$$\max_{\{E(t)\}} \quad \int_0^T u(E(t)) \cdot e^{-\Delta(t)} dt + e^{-\Delta(T)} V$$

$$\text{s.t.} \frac{dS(t)}{dt} = -E(t) \text{ for } 0 \le t \le T,$$

$$\frac{d\Delta(t)}{dt} = r(E(t)) \text{ for } 0 \le t \le T,$$

$$S(0) = S_0, \text{which is given,}$$

$$V = \max_{\{\varepsilon(\tau)\}} \int_0^\infty \left(u(\varepsilon(\tau)) - P_B \varepsilon(\tau) \right) \cdot e^{-\Delta(\tau)} d\tau$$

$$\text{s.t.} \frac{d\Delta(\tau)}{d\tau} = r(\varepsilon(\tau)) \text{ for } 0 \le \tau < \infty.$$

To solve the problem, let us introduce the present-valued Hamiltonian for $0 \le t \le T$ as follows:

$$\tilde{H} = u(E(t)) \cdot e^{-\Delta(t)} + \tilde{q}(t) \cdot \{-E(t)\} - \tilde{\phi}(t) \cdot r(E(t)). \tag{8}$$

$\tilde{q}$ represents the present-valued shadow prices for the exhaustible resource. $\tilde{\phi}$ represents the present-valued shadow prices for the cumulated discount rate defined as in Eq. (3).

The first order necessary conditions for $0 \le t \le T$ are the following[3]:

$$u'(E(t)) \cdot e^{-\Delta(t)} - \tilde{q}(t) - \tilde{\phi}(t) \cdot r'(E(t)) = 0, \tag{9}$$

$$0 = \dot{\tilde{q}}(t), \text{ and} \tag{10}$$

$$u(E(t)) \cdot e^{-\Delta(t)} = -\dot{\tilde{\phi}}(t). \tag{11}$$

The transversality conditions at T are:

$$S(T) \cdot \tilde{q}(T) = 0. \tag{12}$$

and

$$\Delta(T) \cdot \tilde{\phi}(T) = 0. \tag{13}$$

For analytical purposes, we transform Eqs. (9)–(11) into the current-valued form as follows. Let q and ϕ denote the current-valued shadow prices for the exhaustible resource and the cumulated discount rate, respectively. The following relationships hold true:

$$q(t) = \tilde{q}(t) \cdot e^{\Delta(t)} \text{ and}$$

$$\phi(t) = \tilde{\phi}(t) \cdot e^{\Delta(t)}.$$

With these transformations, Eqs. (9)–(11) are equivalent to the following set of equations:

$$u'(E(t)) - q(t) - \phi(t) \cdot r'(E(t)) = 0, \tag{14}$$

$$\dot{q}(t) = q(t) \cdot r(E(t)), \text{ and} \tag{15}$$

$$\dot{\phi}(t) = r(E(t)) \cdot \phi(t) - u(E(t)). \tag{16}$$

Following a similar procedure, we can estimate the value of V. In particular, an optimal condition for the resource flow is

$$u'(\varepsilon(\tau)) - P_B - \varphi(\tau) \cdot r'(\varepsilon(\tau)) = 0. \tag{17}$$

[3] As is custom in mathematics and related fields, the dot notation denotes derivatives with respect to time, i.e., $\dot{x} \equiv dx(t)/dt$.

In Eq. (17), $\varphi(\tau)$ denotes the current-valued shadow prices for the cumulated discount rate for $0 \le \tau < \infty$. For continuity between these two time regions, $0 \le t \le T$ and $0 \le \tau < \infty$, the following relationships must hold: $E(T) = \varepsilon(0)$ and $\phi(T) = \varphi(0)$. Thus, for Eqs. (14) and (17) to simultaneously hold true at time T, the following condition must hold:

$$q(T) = P_B.$$

Note that we have obtained the same in Eq. (7) of the previous section.

To solve the above set of equations analytically, we assume a specific form of the instantaneous discount rate r as in the following assumption:

Assumption 3.1 The instantaneous discount rate at time t is linearly correlated with exhaustible resource consumption at time t. We call the coefficient β the "time preference coefficient." That is

$$r(E(t)) = \beta E(t), \quad \beta > 0. \tag{18}$$

With Assumption 3.1, Eqs. (14)–(16) for $0 \le t \le T$ are rewritten as

$$u'(E(t)) - \beta\phi(t) = q(t), \tag{19}$$

$$\dot{q}(t) = q(t) \cdot \beta E(t), \text{ and} \tag{20}$$

$$\dot{\phi}(t) = \beta E(t) \cdot \phi(t) - u(E(t)). \tag{21}$$

From the terminal conditions, Eqs. (12) and (13), we have

$$S(T) = 0 \text{ and} \tag{22}$$

$$\phi(T) = 0. \tag{23}$$

To solve for the dynamics of resource consumption, we take the derivatives of both sides of (19) w.r.t. time t. Using Eqs. (20) and (21) and arranging the terms we obtain the following differential equation:

$$\frac{dE}{dt} = \frac{\beta}{1-\eta} E(t)^2. \tag{24}$$

Solving Eq. (24) with the conditions of Eqs. (6) and (22), we obtain the following dynamics for resource consumption:

$$E(t) = \frac{1}{\frac{\beta}{1-\eta} \cdot \left(\frac{e^{\frac{\beta}{1-\eta} S_0}}{e^{\frac{\beta}{1-\eta} S_0} - 1} T - t \right)}. \tag{25}$$

For more detail about the derivation, see Appendix 1.

The dynamics of the shadow price for the exhaustible resource use, $q(t)$, is obtained from Eq. (20) as

$$q(t) = Ae^{\beta \int_0^t E(s)ds} (A : \text{ undetermined constant}).$$

Using Eq. (25) and arranging the terms, we obtain:

$$q(t) = AC^{1-\eta} E(t)^{1-\eta}, \tag{26}$$

where $C = \frac{\beta}{1-\eta} \cdot \frac{e^{\frac{\beta}{1-\eta} S_0}}{e^{\frac{\beta}{1-\eta} S_0} - 1} T$.

At the same time, Eq. (21), with the terminal condition, Eq. (23), can also be specified as

$$\phi(t) = \int_t^T u(E(v)) \cdot e^{-\beta \int_t^v E(s)ds} dv.$$

Using Eq. (25) and arranging the terms, we have

$$\phi(t) = \frac{1}{1-\eta} E(t)^{1-\eta} \cdot (T - t). \tag{27}$$

Equations (19), (26) and (27) yield the following:

$$AC^{1-\eta} = \frac{1}{e^{\frac{\beta}{1-\eta} S_0} - 1} \cdot \frac{\beta}{1-\eta} T.$$

Then, we obtain the dynamics of the shadow price for the exhaustible resource use, $q(t)$, as follows:

$$q(t) = \frac{1}{e^{\frac{\beta}{1-\eta} S_0} - 1} \cdot \frac{\beta}{1-\eta} T \cdot E(t)^{1-\eta}. \tag{28}$$

For more detail of the derivation, see Appendix 2.

Notice that Eq. (28) still contains the undetermined constant T. To fix the value, we employ Eq. (7).[4] That is, due to Eqs. (25) and (28), Eq. (7) leads to:

$$T = P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta} \right)^{-1} \cdot \left(e^{\frac{\beta}{1-\eta} S_0} - 1 \right). \tag{29}$$

Replacing T in Eq. (25) with that of Eq. (29) and arranging the terms, we finally obtain the trajectory of $E(t)$:

[4] Note that T is not assumed to play the role of a control variable in the optimization problem of (2) under Assumption 3.1 because the value of the last term, $e^{-\Delta(T)}V$, in (2), is not affected by the change in T. More specifically, with Assumption 3.1, the value of $e^{-\Delta(T)}$ is proven to be independent of T as follows:

$e^{-\Delta(T)} = e^{-\beta \int_0^T E(t)dt} = e^{-\beta S_0}$.

The value of V does not depend on T either. Thus, we find that the value of $e^{-\Delta(T)}V$ is independent of T.

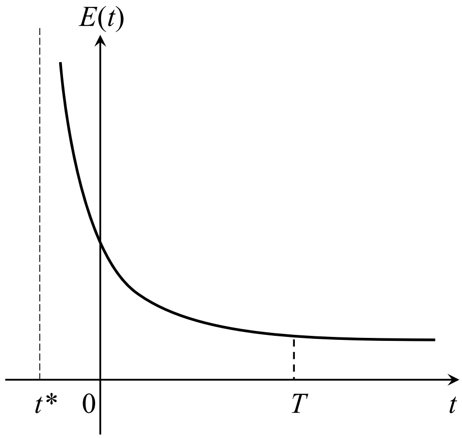


Fig. 2 The trajectory of E and the switching time T for the case of $1 < \eta$

$$E(t) = \frac{1}{P_B^{\frac{1}{\eta}} \cdot e^{\frac{\beta}{1-\eta}S_0} - \frac{\beta}{1-\eta}t}. \tag{30}$$

To examine how the trajectory is shaped, we need to consider two cases for the values of η. In the case of $1 < \eta$, it is easy to see that $E(t)$ is decreasing in t: the consumption of exhaustible resource each time declines as time goes on. In contrast, in the case of $1 > \eta$, $E(t)$ is increasing in t: the consumption of exhaustible resource each time grows as time goes on.

The shapes of the trajectories are closely related to the time of the switch, T. As a hyperbolic function, Eq. (30) has its asymptote:

$$t^* = P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta} \right)^{-1} \cdot e^{\frac{\beta}{1-\eta}S_0}.$$

Thus, by examining the sign of the value, $T - t^* = -P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta} \right)^{-1}$, we determine the following relationships:

(1) for the case of $1 < \eta, T > 0 > t^*$.

(2) for the case of $1 > \eta, 0 < T < t^*$.

The former case indicates that when $1 < \eta$, $E(t)$ declines as time goes on, and stops at the time of the switch T. In contrast, the latter case indicates that when $1 > \eta$, $E(t)$ increases as time goes on, and stops at the time of the switch T that should come before the asymptote. These former and latter cases are depicted as Figs. 2 and 3, respectively.

Finally, let us examine the trajectory of $q(t)$. It is obtained from Eqs. (28) and (29) as

$$q(t) = P_B^{\frac{1}{\eta}} \cdot E(t)^{1-\eta}. \tag{31}$$

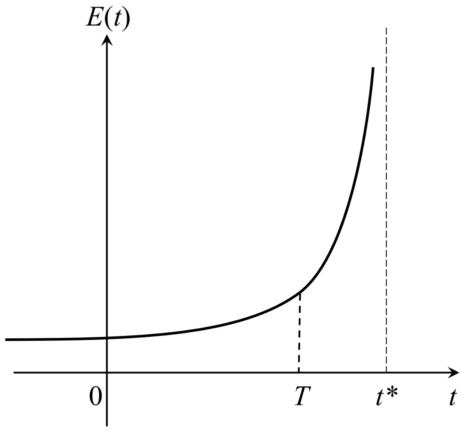


Fig. 3 The trajectory of E and the switching time T for the case of $1 > \eta$

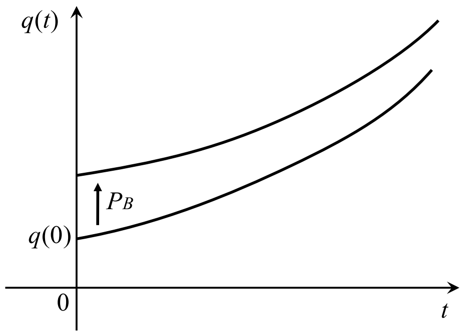


Fig. 4 The shift of the scarcity rent path due to the increase in the backstop price

Notice that q is increasing in t. Namely, $dq/dt > 0$. As the exhaustible resource becomes scarce, its economic value becomes high. This fact is consistent with the idea behind the conventional Hotelling's rule. However, it should be emphasized that the actual trajectory is completely different from that of Hotelling's rule in which it follows hyperbolic functions, rather than exponential functions. In addition, notice that the trajectory of Eq. (31) is dependent on the price of the backstop technology, P_B. To observe this, let us focus our attention on the relationship between $q(0)$ and P_B as

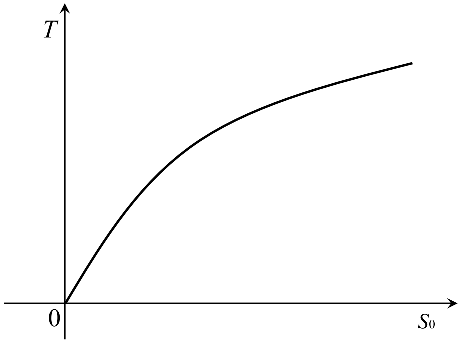
$$q(0) = P_B e^{-\beta S_0}.$$

Thus, we have

$$\frac{dq(0)}{dP_B} = e^{-\beta S_0} > 0.$$

This indicates that these variables are positively correlated. In other words, the increase in the price of the backstop technology raises the initial value of the scarcity rent of the exhaustible resource, which results in a shift of the entire path. This situation is depicted in Fig. 4.

Fig. 5 The time to switch and the initial exhaustible resource stock



These results are summarized in Proposition 3.1.

Proposition 3.1 *In the case of* $1 < \eta$*, exhaustible resource consumption (E) declines as time goes on. In the case of* $1 > \eta$*, E grows as time goes on. The scarcity rent of the exhaustible resource (q) grows in all cases. These trajectories follow the hyperbolic functions in time (t).*

Time to switch

Based on the model and the analysis in the previous sections, we obtain four propositions in this section. The first proposition (Proposition 4.1) addresses the positive correlation between the time to switch and the initial exhaustible resource stock.

Proposition 4.1 *The increase (decline) in the initial exhaustible resource stock,* S_0*, leads to the delay (early arrival) in the time to switch from the exhaustible resource use to the use of the backstop technology, T. The relationship is determined as*

$$\frac{dT}{dS_0} = P_B^{\frac{1}{\eta}} e^{\frac{\beta}{1-\eta} S_0} > 0. \tag{32}$$

The proof is self-evident. Take the derivative of Eq. (29) w.r.t. S_0.

The proposition is depicted in Fig. 5. The intuition is clear: when the stock of the exhaustible resource is large, the economy is allowed to keep consuming that resource, which results in the delay of the time to switch to the backstop technology. This result is consistent with the indication of the conventional Hotelling's rule. Notice that the result is not directly affected by our introduction of the Uzawa–Epstein formulation of time preference. Even if the time preference changes, depending on the current resource consumption, the redundancy of the exhaustible resource always facilitates the delay of the introduction of substitutes.

The next proposition addresses the positive correlation between the time to switch and the price of the backstop technology.

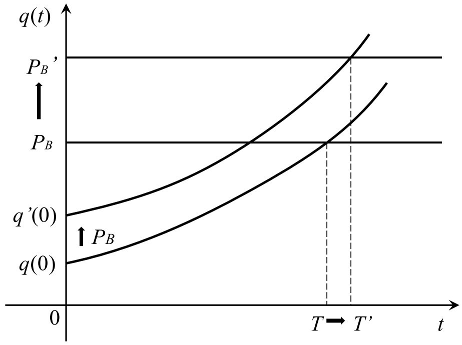


Fig. 6 The time to switch and the price of the backstop technology

Proposition 4.2 *The increase (decline) in the price of the backstop technology, P_B, leads to the delay (early arrival) in the time to switch from the exhaustible resource use to the use of the backstop technology, T. The relationship is determined as*

$$\frac{dT/T}{dP_B/P_B} = \frac{1}{\eta} > 0. \tag{33}$$

The proof is self-evident. Take the derivative of Eq. (29) w.r.t. P_B.

The interpretation of the proposition is clear in that it is consistent with the indications of the conventional Hotelling's rule, where the increase in the price of the backstop technology makes the switch to the technology more difficult. The difficulty appears as the increase in the scarcity rent of the existing exhaustible resource. The higher scarcity rent creates price incentives for the economy to conserve the resource. The result is the delay in resource exhaustion. The situation is depicted in Fig. 6.

Again, notice that the result is unchanged in the presence of the Uzawa–Epstein formulation of time preference. Even with the endogenous discount rate, the increase in the price of the backstop technology always facilitates the conservation of exhaustible resources.

The third proposition (Proposition 4.3) highlights the role of the endogenous discount rates, addressing the bipolar relationships between the time to switch and the time preference coefficient, β.

Proposition 4.3 *The increase in the time preference coefficient, β, facilitates either the earlier introduction or the delay of the use of the backstop technology, depending on the value of the reciprocal of the elasticity of the intertemporal substitution, η. That is, the following relationships hold true:*

(1) in the case of $1 < \eta$, $\frac{dT}{d\beta} < 0$.

(2) in the case of $1 > \eta$, $\frac{dT}{d\beta} > 0$.

Proof: See Appendix 3.

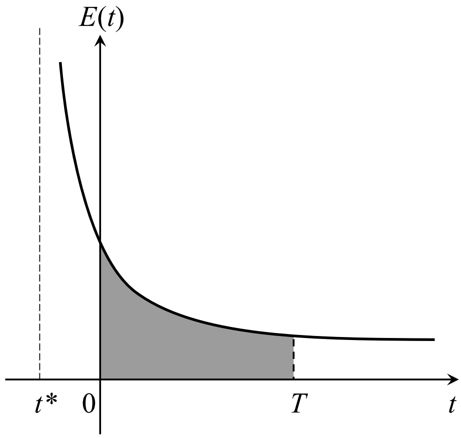


Fig. 7 The integral of E and the cumulated discount rate Δ for the case of $1 < \eta$

The time of the switch to the backstop technology is negatively (positively) correlated with the time preference coefficient, β, when $1 < \eta$ ($1 > \eta$) holds true. To understand this result better, the analysis in Sect. 3 on the trajectory of $E(t)$ is helpful. In the case of $1 < \eta$, $E(t)$ declines as time goes on, while in the case of $1 > \eta$, it grows as time goes on (Proposition 3.1). The property of the trajectory helps to explain the proposition as follows. When $1 < \eta$ holds (the "inelastic" case), because exhaustible resource consumption accumulates faster in an earlier time, the short-sighted consumption habit is formed in an earlier time and remains until the end. The situation is depicted in Fig. 7. Due to Assumption 3.1, the cumulated discount rate $\Delta(t)$ is proportional to the integral of $E(t)$. That is, the following relationships hold true:

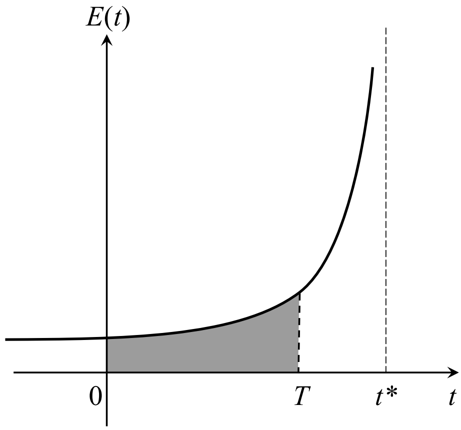
$$\Delta(t) = \beta \int_0^t E(s)ds = -(1-\eta) \cdot \ln\left(1 - \left(1 - e^{\frac{-\beta}{1-\eta}S_0}\right) \cdot \frac{t}{T}\right).$$

The integral of $E(t)$ is depicted by the shaded region in Fig. 7 in which we observe that the area largely expands at the beginning and holds its size until the end although it gains small increments in the middle. This means that the short-sighted consumption habit is fixed from the very beginning.

As a result of such a short-sighted consumption habit formation, resource depletion may occur earlier, which leads to an earlier transition from the exhaustible resource to the backstop technology ($dT/d\beta < 0$).

On the other hand, when $1 > \eta$ holds (the "elastic" case), two effects may exist. First, because of a small η, exhaustible resource consumption (E) is elastic. Thus, it can be easily shifted along the time horizon. This means that the representative agent can make an effort to stay away from resource depletion as long as possible. Second, because of the shape of the resource consumption trajectory, a short-sighted consumption habit is formed gradually, as is shown in Fig. 8, in which we observe that the shaded area grows slowly. This allows the representative agent to

Fig. 8 The integral of E and the cumulated discount rate Δ for the case of $1 > \eta$



try to reserve exhaustible resources. As a result of these two effects, depletion is delayed ($dT/d\beta > 0$).

These interpretations are supported by Proposition 4.4.

Proposition 4.4 *The increase in the time preference coefficient, β, leads to either an appreciation or a depreciation of the values of the exhaustible resource stock at T, $q(T)$, depending on the value of the reciprocal of the elasticity of intertemporal substitution, η. That is, the following relationships hold true:*

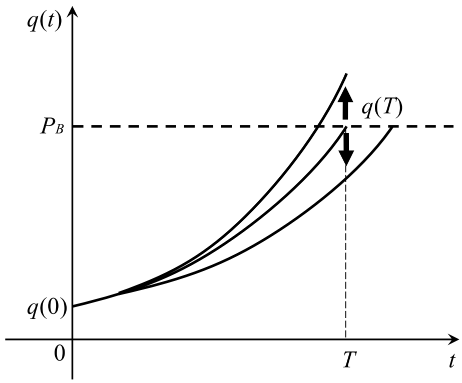
(1) in the case of $1 < \eta$, $\left.\frac{dq(t)}{d\beta}\right|_{t=T} > 0$.

(2) in the case of $1 > \eta$, $\left.\frac{dq(t)}{d\beta}\right|_{t=T} < 0$.

Proof: See Appendix 4.

Let us examine the implication of Proposition 4.4. We have two cases. The first case is that when $1 < \eta$ holds (the "inelastic" case), the derivative of the values of the exhaustible resource stock at T, $q(T)$, w.r.t. the time preference coefficient, β, is positive. This illustrates that the increase in the time preference coefficient, β, leads to an appreciation of the values of the exhaustible resource stock at T, $q(T)$. As a result, the value of the backstop technology is relatively cheap compared to the values of the exhaustible resource stock at T. It means the time to switch comes earlier. The second case is that when $1 > \eta$ holds (the "elastic" case), the derivative of the values of the exhaustible resource stock at T, $q(T)$, w.r.t. the time preference coefficient, β, is negative. This shows that the increase in the time preference coefficient, β, leads to a depreciation in the values of the exhaustible resource stock at T, $q(T)$. As a result, the value of the backstop technology is relatively expensive compared to the values of the exhaustible resource stock at T. This means the time to switch moves forward. These observations are depicted in Fig. 9. This proposition is consistent with Proposition 4.3.

Fig. 9 The time to switch as β changes



Minimum consumption requirement

In this section, we extend our analysis shown in the previous sections by introducing the concept of minimum consumption requirement to our analytical framework.

An implicit, but typical assumption in economic models is that consumption of goods is a non-negative variable. This assumption was naturally presumed in our analytical framework in the previous sections. More specifically, our analysis in previous sections was based on the assumption that the required minimum level of resource consumption is zero. However, this assumption can be considered as unrealistic in that energy and resource use is indispensable in any economic activities. To make our analysis more realistic and general, we need to introduce to our model a positive lower limit for resource consumption.

We hereafter reconsider our analysis of previous sections by replacing the utility function of Eq. (1) with the following form:

$$u(E) = \frac{(E - m)^{1-\eta}}{1 - \eta}, \quad \eta > 0, \quad \eta \neq 1, \quad m \geq 0. \tag{34}$$

This form of utility function indicates the following limit:

$$u'(E) = \frac{1}{(E - m)^{\eta}} \to \infty \text{ as } E \to m.$$

This means that $E > m$ must hold true. Thus, m represents minimum consumption requirement. It should be emphasized that Eq. (1) is a special case of Eq. (34) when $m = 0$.[5]

With the replacement of utility function, we can repeat the same calculation of Sect. 3 to obtain a revised version of Eq. (29) as follows:

[5] Note that this form of utility function (34) can be considered as a simplified version of Stone-Geary utility function that has the following form:

$$u(x_1, x_2, \ldots, x_n) = \prod_{j=1}^{n} (x_j - m_j)^{\gamma_j}.$$

$$\left(\frac{1-\eta}{\eta}\right) \cdot P_B^{\frac{1}{\eta}} \cdot \left(1 - e^{\frac{\beta}{1-\eta}\cdot(S_0 - mT)}\right) = \frac{1 - e^{\frac{\beta mT}{\eta}}}{m}. \tag{35}$$

Notice that the derivation of this equation is elaborated in Appendix 5. Solving this equation for T would yield the optimal time T as an explicit function of P_B, S_0, and m. Unfortunately, it is impossible to obtain an analytical solution for it. In the remainder of this section, we instead explore implications of Eq. (35) in contrast to Eq. (29).

Recall that Eq. (29) is considered as a result of the case that $m = 0$. Thus, when m is set to be zero for Eq. (35), the equation is expected to coincide with Eq. (29). Such coincidence may not be apparent from the current form of equation, but it is verified as follows: Applying the l'Hôpital's rule to the right hand side of Eq. (35), we determine the following:

$$\lim_{m\to 0} \frac{1 - e^{\frac{\beta mT}{\eta}}}{m} = \lim_{m\to 0} \frac{-\frac{\beta T}{\eta} e^{\frac{\beta mT}{\eta}}}{1} = -\frac{\beta T}{\eta}.$$

Replacing m to zero for the left hand side, Eq. (35) reduces to the following:

$$\left(\frac{1-\eta}{\eta}\right) \cdot P_B^{\frac{1}{\eta}} \cdot \left(1 - e^{\frac{\beta}{1-\eta}\cdot S_0}\right) = -\frac{\beta T}{\eta}.$$

Arranging terms, we obtain the following, which is exactly same with Eq. (29).

$$T = P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta}\right)^{-1} \cdot \left(e^{\frac{\beta}{1-\eta}S_0} - 1\right).$$

Having confirmed the coincidence of Eqs. (29) and (35) at $m = 0$, a next question of our interest is to determine the effect of the increase of m from zero to a positive value on T. Comparative statics analysis is a useful tool to deal with it. The result is summarized as the following proposition:

Proposition 5.1 *The increase in the minimum consumption requirement, m, in the neighborhood of zero leads to the early arrival in the time to switch from the exhaustible resource use to the use of the backstop technology, T. The relationship is determined as*

$\frac{dT}{dm} < 0$ *for* $m \geq 0$ *and* $m \approx 0$.

Proof: See Appendix 6.

The above proposition seems natural in that it seems consistent with our intuition. In fact, when the minimum consumption requirement increases from zero to a positive value, instantaneous consumption requirement of resources at each time increases as well. It will naturally lead to the increase of total consumption requirement. However, the available stock of the exhaustible resource is limited. Thus, the time of depletion of the stock comes earlier than before.

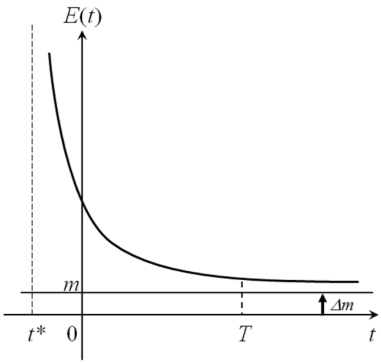


Fig. 10 Small increase in the lower limit, m, for Fig. 2

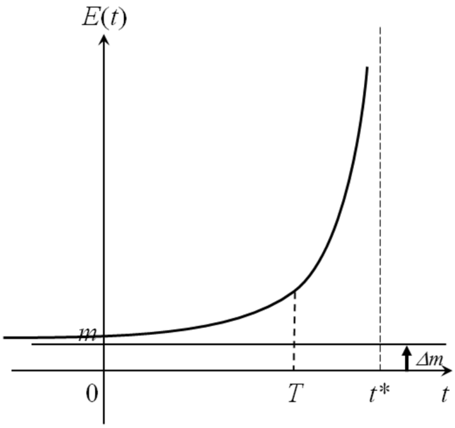


Fig. 11 Small increase in the lower limit, m, for Fig. 3

This interpretation of the earlier time to switch is plausible at a glance, but is not correct. To see this, we need to recall the shape of consumption path in the case of $m = 0$. It is described by Eq. (30) and is depicted as Figs. 2 and 3. As is apparent from these figures, instantaneous resource consumption at each time ($E(t)$) keeps away from the level of zero until the end of resource use. That is, $E(t) > 0$ always holds true. This indicates that even if the lower limit of the resource consumption, which is zero in this case, slightly increases with a very small amount, the increase does not impose an additional constraint to the value of $E(t)$. Such situation is shown in Figs. 10 and 11.

These figures naturally allow us to surmise that the following equation holds true:

$$\frac{dT}{dm} = 0 \text{ for } m \approx 0.$$

In this respect, Proposition 5.1 addresses an interesting result that declines our initial expectation.

The claim of Proposition 5.1 is not natural or self-explanatory. Rather, it is difficult to prepare a comprehensive explanation for the claim. A possible explanation will be that once we introduce the concept of minimum consumption requirement to the framework of analysis, the optimal behavior of resource use changes completely. Setting a lower limit to consumption level influences the dynamics of exhaustible resource use as a whole, and thus changes the optimal plan that identifies the time to switch from the exhaustible resource use to the use of the backstop technology,

Conclusions

In this paper, we investigated a classical model of the optimal use of an exhaustible resource with the availability of a backstop technology. The new feature added to our model is an endogenous time preference determined by the history of resource consumption. The dependence of the time preference on consumption is interpreted as a form of habit formation, and is known as the Uzawa–Epstein formulation of time preference. By investigating the optimal trajectory of exhaustible resource consumption, we found that the reciprocal of the elasticity of the intertemporal substitution (η) in the instantaneous utilities plays a significant role in identifying the shapes of the trajectories. As addressed in Proposition 3.1, when $1 < \eta$ ($1 > \eta$), the consumption of the exhaustible resource at each time is declining (growing) in time t. This affects the property of the time to switch from the exhaustible resource use to the utilization of the backstop technology.

Our primary results regarding the time of the switch are addressed as four propositions. Proposition 4.1 shows that the increase in the initial exhaustible resource stock leads to a delay in the time of the switch. The result is consistent with our intuition as well as the indication of the classical Hotelling's rule. It is interesting that the result is unaffected by the parameters that represent the intertemporal substitutions as well as the endogenous discount rates. Proposition 4.2 shows that the increase in the price of the backstop technology leads to a delay in the time of the switch. Moreover, this result is consistent with our intuition and interestingly unaffected by other parameters.

The indication of Proposition 4.3 is novel in that the increase in the time preference coefficient, β, influences the time of the switch in two directions, depending on the value of the reciprocal of the elasticity of intertemporal substitution, η: The increase in β leads to an incentive for consuming the exhaustible resource slower (faster) and switching to the backstop technology later (earlier) as much as possible in the case of $1 > \eta$ ($1 < \eta$). Proposition 4.4 illustrates that the sensitivity of the values of the exhaustible resource stock at the time of the switch w.r.t. β depends on the sign of $1 - \eta$, which supports Proposition 4.3.

Finally, to extend the above basic model, we introduced a concept of minimum consumption requirement to the framework of analysis. It is based on an idea that energy and resource use is indispensable in any economic activities, and thus it is not allowed to fall to zero. As a modeling technique, the extension is just to modify the instantaneous utility function. However, this modification turned out to dramatically change the shape of exhaustible resource consumption path as well as the optimal time to switch to the backstop technology. Proposition 5.1 points out this result.

In this paper, we focused only on a theoretical foundation of the endogenous time preference, leaving its policy implications aside. However, our analysis is not irrelevant to the economic policies. The effects of the time preference and discounting factors on the properties of the economic dynamics have been a central issue in economic policy debates. In particular, in policies for climate change, discounting has been one of most important issues. For example, in the reports prepared by Working Group 3 of the Intergovernmental Panel on Climate Change (IPCC) of the United Nations; published in 1995, 2001, and 2007; certain chapters are devoted to the discussion on discounting in economic models.

The book published by Stern [11], known as the "Stern Review," triggered a controversy about the economic assessment of climate change policy, advocating that prompt action for climate change is the need of the hour. In his Summary of Conclusions, Stern wrote:

> "In contrast, the costs of action—reducing greenhouse gas emissions to avoid the worst impacts of climate change—can be limited to around 1 % of global GDP each year."
> "Using the results from formal economic models, the Review estimates that if we don't act, the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP each year, now and forever. If a wider range of risks and impacts is taken into account, the estimates of damage could rise to 20% of GDP or more."
> "So prompt and strong action is clearly warranted."

The proposals addressed in the Stern Review created strong pros and cons not only in the policy arena but also in academia. One of the most debatable issues was the treatment of discount factors. Nordhaus [9], for example, criticized the Stern Review, stating that it was assuming very low discount rates and that the setting helps to explain most of its unusual conclusions.

Such debate between Stern and Nordhaus illustrated that discounting is a long-standing issue, not only in economic theory, but also in policy-making, and that the theory and practice are currently getting much closer to each other. That is why we believe that the results obtained in this theoretical paper contribute to such recent policy debates on discounting factors.

Appendices

Appendix 1: The dynamics of the exhaustible resource consumption

Taking the derivative of Eq. (19) w.r.t. t, we obtain:

$$-\eta E(t)^{-\eta-1}\frac{dE(t)}{dt} = \beta\dot{\phi}(t) + \dot{q}(t).$$

Using Eqs. (20) and (21), the right side of above equation leads to

$$\beta^2 E(t)\phi(t) - \frac{\beta}{1-\eta}E(t)^{1-\eta} + \beta q(t)E(t)$$
$$= \beta E(t)\cdot(q(t)+\beta\phi(t)) - \frac{\beta}{1-\eta}E(t)^{1-\eta}$$
$$= \beta E(t)\cdot u'(E(t)) - \frac{\beta}{1-\eta}E(t)^{1-\eta}$$
$$= \frac{-\eta}{1-\eta}\beta E(t)^{1-\eta}.$$

Then,

$$\frac{dE(t)}{dt} = \frac{\beta}{1-\eta}E(t)^2. \tag{24}$$

Solving Eq. (24), we obtain the following dynamics for resource consumption with an undetermined constant, C:

$$E(t) = \frac{1}{\frac{-\beta}{1-\eta}t + C}. \tag{A1.1}$$

Taking the integrals of both sides of Equation (A1.1), we obtain:

$$\int_0^T E(t)dt = -\frac{1-\eta}{\beta}\ln\left(\frac{-\beta}{1-\eta}\cdot\frac{T}{C}+1\right).$$

From $\int_0^T E(t)dt = S_0 - S_T$ (Eq. (6)) and $S_T = 0$ (Eq. (22)), we obtain:

$$S_0 = -\frac{1-\eta}{\beta}\ln\left(\frac{-\beta}{1-\eta}\cdot\frac{T}{C}+1\right)$$

.

Then, we obtain C as

$$C = \frac{\beta}{1-\eta}\cdot\frac{e^{\frac{\beta}{1-\eta}S_0}}{e^{\frac{\beta}{1-\eta}S_0}-1}T.$$

Appendix 2: The dynamics of the scarcity rent of the exhaustible resource

From Eq. (20), we obtain the following equation with the undetermined constant, A:

$$q(t) = Ae^{\beta\int_0^t E(s)ds}.$$

Integrating both sides of Equation (A1.1), we obtain:

$$\int_0^t E(s)ds = \int_0^t \left(\frac{-\beta}{1-\eta}s + C\right)^{-1} ds$$
$$= -\frac{1-\eta}{\beta} \ln\left(\frac{-\beta}{1-\eta}\frac{t}{C} + 1\right)$$
$$= \frac{1-\eta}{\beta} \ln(C \cdot E(t)).$$

Thus, we have:

$$q(t) = AC^{1-\eta}E(t)^{1-\eta}. \tag{26}$$

From Eq. (21), with the condition of Eq. (23), we have:

$$\phi(t) = \int_t^T u(E(v)) \cdot e^{-\beta \int_t^v E(s)ds} dv.$$

From Equation (A1.1), we also have:

$$e^{-\beta \int_t^v E(s)ds} = \left(\frac{\frac{-\beta}{1-\eta}v + C}{\frac{-\beta}{1-\eta}t + C}\right)^{1-\eta} = \left(\frac{E(t)}{E(v)}\right)^{1-\eta}.$$

Thus, we obtain:

$$\phi(t) = \frac{1}{1-\eta}\int_t^T E(v)^{1-\eta}\left(\frac{E(t)}{E(v)}\right)^{1-\eta} dv = \frac{1}{1-\eta}E(t)^{1-\eta}\int_t^T dv,$$

that is,

$$\phi(t) = \frac{1}{1-\eta}E(t)^{1-\eta} \cdot (T-t). \tag{27}$$

Using Eqs. (26) and (27), Eq. (19) leads to the following:

$$E(t)^{-\eta} = \frac{\beta}{1-\eta}E(t)^{1-\eta} \cdot (T-t) + AC^{1-\eta}E(t)^{1-\eta},$$

or

$$E(t)^{-1} = \frac{\beta}{1-\eta}(T-t) + AC^{1-\eta},$$

or

$$C = \frac{\beta}{1-\eta}T + AC^{1-\eta},$$

or

$$AC^{1-\eta} = \frac{1}{e^{\frac{\beta}{1-\eta}S_0} - 1} \cdot \frac{\beta}{1-\eta}T.$$

Thus, we obtain:

$$q(t) = \frac{1}{e^{\frac{\beta}{1-\eta}S_0} - 1} \cdot \frac{\beta}{1-\eta}T \cdot E(t)^{1-\eta}. \tag{28}$$

Appendix 3: Proof of Proposition 4.3

Taking the derivative of Eq. (29) w.r.t. β, we obtain the following:

$$\frac{dT}{d\beta} = -\beta^{-2}P_B^{\frac{1}{\eta}} \cdot (1-\eta) \cdot \left(e^{\frac{\beta}{1-\eta}S_0} - 1\right) + P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta}\right)^{-1} \cdot e^{\frac{\beta}{1-\eta}S_0} \cdot \frac{S_0}{1-\eta},$$
$$= \beta^{-1}P_B^{\frac{1}{\eta}} \cdot \left(\frac{\beta}{1-\eta}\right)^{-1} \cdot e^{\frac{\beta}{1-\eta}S_0} \cdot \left(-1 + e^{\frac{-\beta}{1-\eta}S_0} + \frac{\beta}{1-\eta}S_0\right).$$

From Lemma A1, $-1 + e^{\frac{-\beta}{1-\eta}S_0} + \frac{\beta}{1-\eta}S_0 > 0$ holds true for any η. Thus, the sign of $\frac{dT}{d\beta}$ is the same as it is with the sign of $1-\eta$. This completes the proof.

Lemma A1: The following inequality holds:

$$x - e^x + 1 \le 0, \quad \forall x$$

Proof:

Define the following function:

$$f(x) = x - e^x + 1, \quad x \in R.$$

Then,

$$\frac{df}{dx} = 1 - e^x \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0, \quad for\ x \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0,$$

$$\frac{d^2f}{dx^2} = -e^x < 0, \quad \forall x.$$

This means that $f(x)$ is a continuous, concave function attaining its maximum $f(0) = 0$ at x=0. Thus, $f(x) < 0 \ \ \forall x (\neq 0)$ and $f(0) = 0$.

Appendix 4: Proof of Proposition 4.4

Taking the derivative of Eq. (31) w.r.t. β, we obtain the following:

$$\frac{dq(t)}{d\beta} = P_B^{\frac{1}{\eta}} \cdot (1-\eta) \cdot E(t)^{-\eta} \frac{dE(t)}{d\beta}.$$

From Eq. (30), we have

$$\frac{dE(t)}{d\beta} = E(t)^2 \cdot \left(-\frac{S_0}{1-\eta} P_B^{\frac{1}{\eta}} \cdot e^{\frac{\beta}{1-\eta}S_0} + \frac{1}{1-\eta} t \right).$$

Then, we obtain

$$\left.\frac{dq(t)}{d\beta}\right|_{t=T} = P_B^{\frac{1}{\eta}} \cdot E(T)^{2-\eta} \cdot \left(-S_0 P_B^{\frac{1}{\eta}} \cdot e^{\frac{\beta}{1-\eta}S_0} + T \right).$$

Using Eq. (29), the above equation leads to

$$\left.\frac{dq(t)}{d\beta}\right|_{t=T} = P_B^{\frac{2}{\eta}} \cdot E(T)^{2-\eta} \cdot \left(\frac{\beta}{1-\eta} \right)^{-1} e^{\frac{\beta}{1-\eta}S_0} \left(1 - \frac{\beta}{1-\eta} S_0 - e^{\frac{-\beta}{1-\eta}S_0} \right).$$

From Lemma A1 in Appendix 3, we know $1 - \frac{\beta}{1-\eta} S_0 - e^{\frac{-\beta}{1-\eta}S_0} < 0$ for any η. Thus, the sign of $\left.\frac{dq(t)}{d\beta}\right|_{t=T}$ is the opposite of the sign of $1-\eta$. This completes the proof.

Appendix 5: The derivation of Eq. (35)

With the replacement of utility function from (1) to (34), we can repeat the same calculation of Sect. 3 from Eqs. (8) to (23). Then, Eq. (24) is revised to the following:

$$\frac{dE(t)}{dt} = \frac{\beta}{1-\eta} (E(t) - m) \left(E(t) - \frac{m}{\eta} \right). \tag{A5.1}$$

This differential equation can be solved for the revision of Eq. (25) as follows:

$$E(t) = \frac{m}{\eta} \cdot \left\{ 1 - \frac{1-\eta}{1 - De^{\frac{-\beta m}{\eta} t}} \right\}, \tag{A5.2}$$

where D is a constant of the following:

$$D = \frac{1 - e^{\frac{\beta}{1-\eta}(S_0 - mT)}}{1 - e^{\frac{\beta}{1-\eta}\left(S_0 - \frac{m}{\eta}T\right)}}. \tag{A5.3}$$

Using Equation (A5.2), Eqs. (26) and (27) are revised to the following:

$$q(t) = Ae^{\beta mt} \cdot \left(\frac{1-D}{1-\eta} \right)^{1-\eta} \cdot \left(1 - \frac{\eta}{m} E(t) \right)^{1-\eta}, \tag{A5.4}$$

$$\phi(t) = \frac{\eta}{(1-\eta)\beta m} \cdot e^{\beta m t} \cdot \left(E(t) - \frac{m}{\eta}\right)^{1-\eta} \cdot D^{1-\eta} \cdot \left(e^{\frac{-\beta m}{\eta}t} - e^{\frac{-\beta m}{\eta}T}\right). \quad \text{(A5.5)}$$

Applying Equations (A5.4) and (A5.5) to Eq. (19) leads to the following equations:

$$\left\{\frac{(1-\eta)m}{\eta}\right\}^{-\eta} \cdot D^{-\eta} \cdot \left(De^{\frac{-\beta m}{\eta}T} - 1\right) = A \cdot (D-1)^{1-\eta}, \quad \text{(A5.6)}$$

$$q(t) = Ae^{\beta m t} \frac{(D-1)^{1-\eta}}{\left(De^{\frac{-\beta m}{\eta}t} - 1\right)^{1-\eta}}. \quad \text{(A5.7)}$$

An endogenous variable T is determined by the use of Eq. (7): Replacing t with T for Equation (A5.7), and using Eqs. (7) and (A5.6), the following equation is obtained:

$$\frac{(1-\eta)m}{\eta} P_B^{\frac{1}{\eta}} = 1 - e^{\frac{\beta m T}{\eta}} D^{-1}.$$

Using Eq. (A5.3), this is arranged to the following:

$$\left(\frac{1-\eta}{\eta}\right) \cdot P_B^{\frac{1}{\eta}} \cdot \left(1 - e^{\frac{\beta}{1-\eta}\cdot(S_0 - mT)}\right) = \frac{1 - e^{\frac{\beta m T}{\eta}}}{m}. \quad (35)$$

Appendix 6: Proof of Proposition 5.1

We consider an approximation of Eq. (35). Exponential functions are approximated as follows:

$$e^{\frac{\beta}{1-\eta}\cdot(S_0 - mT)} \cong e^{\frac{\beta}{1-\eta}\cdot S_0} \cdot \left(1 - \frac{\beta m T}{1-\eta}\right),$$

$$e^{\frac{\beta m T}{\eta}} \cong 1 + \frac{\beta m T}{\eta}.$$

Using these relations, Eq. (35) is approximated to the following:

$$\left(\frac{1-\eta}{\eta}\right) \cdot P_B^{\frac{1}{\eta}} \cdot \left\{1 - e^{\frac{\beta}{1-\eta}\cdot S_0} \cdot \left(1 - \frac{\beta m T}{1-\eta}\right)\right\} \cong -\frac{\beta m T}{\eta m}.$$

Arranging terms, we obtain the following:

$$(1-\eta)\cdot P_B^{\frac{1}{\eta}}\cdot\left(1-e^{\frac{\beta}{1-\eta}\cdot S_0}+e^{\frac{\beta}{1-\eta}\cdot S_0}\frac{\beta}{1-\eta}mT\right)\cong-\beta T.$$

Taking derivatives of T with respect to m, we obtain the following:

$$(1-\eta)\cdot P_B^{\frac{1}{\eta}}\cdot e^{\frac{\beta}{1-\eta}S_0}\cdot\frac{\beta}{1-\eta}\cdot\left(T+m\frac{dT}{dm}\right)\cong-\beta\frac{dT}{dm}.$$

This is arranged to the following, which completes the proof:

$$\frac{dT}{dm}\cong-\frac{P_B^{\frac{1}{\eta}}\cdot e^{\frac{\beta}{1-\eta}S_0}}{m\cdot P_B^{\frac{1}{\eta}}\cdot e^{\frac{\beta}{1-\eta}S_0}+1}T<0\text{ for }m\geq 0.$$

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Data availability All data generated or analyzed during this study are included in this published article.

Declarations

Conflict of interest The first author, Akira Maeda, is the Editor-in-Chief of the International Journal of Economic Policy Studies. To avoid the competing interests, the whole reviewing process to the submission has been handled by other editorial board members excluding the EiC.

References

1. Arrow, K. J., & Kurz, M. (1970). *Public Investment, the Rate of Return, and Optimal Fiscal Policy*. Johns Hopkins Press for Resources for the Future.
2. Chang, F. (2009). Optimal growth and impatience: A phase diagram analysis. *International Journal of Economic Theory, 5*, 245–255.
3. Das, M. (2003). Optimal growth with decreasing marginal impatience. *Journal of Economic Dynamics and Control, 27*, 1881–1898.
4. Epstein, L. G., & Hynes, J. A. (1983). The rate of time preference and dynamic economic analysis. *The Journal of Political Economy, 91*(4), 611–635.
5. Epstein, L. G. (1987). A simple dynamic general equilibrium model. *Journal of Economic Theory, 41*, 68–95.

6. Hirose, K., & Ikeda, S. (2008). On decreasing marginal impatience. *The Japanese Economic Review, 59*(3), 259–274.
7. Maeda, A., & Nagaya, M. (2016). Hitsuyousaiteishohi To Shukankeisei Wo Koryoshita Kokatsusei Shigen No Dogaku (Exhaustible resource use with habit formation and minimum consumption requirement). *Keizai Seisaku Janaru, 12*(1), 40–43. in Japanese.
8. Nagaya, M., & Maeda, A. (2013). Kokatsusei Shigen Shohi To Shukankeisei No Dogaku (Dynamic properties of exhaustible resource consumption and habit formation). *Keizai Seisaku Janaru, 10*(1), 17–30. in Japanese.
9. Nordhaus, W. D. (2007). A review of the stern review on the economics of climate change. *Journal of Economic Literature, 45*(3), 686–702.
10. Obstfeld, M. (1990). Intertemporal dependence, impatience, and dynamics. *Journal of Monetary Economics, 26*, 45–75.
11. Stern, N. (2007). *The economics of climate change: The Stern review*. Cambridge University Press.
12. Uzawa, H. (1968). Time preference, the consumption function, and optimum asset holdings. In J. N. Wolfe (Ed.), *Capital and Growth: Papers in Honor of Sir John Hicks.* Aldine.
13. Weitzman, M. L. (2001). Gamma discounting. *American Economic Review, 91*, 260–271.