Mining, Metallurgy & Exploration (2024) 41:589–605
https://doi.org/10.1007/s42461-023-00906-y

Risk in Ultimate Pit Selection

Edward Holloway[1,2]

Received: 5 June 2023 / Accepted: 15 December 2023 / Published online: 21 February 2024

Abstract
The basis for the selection of the ultimate pit for an open pit mining operation is generally opaque. Some form of value-based analysis will generally guide the final decision; however, the value will often gradually plateau whilst the ultimate pit size continues to increase. There is no established framework to guide specialists in the selection of the ultimate pit, with such decisions regularly made on a somewhat arbitrary basis. The key component that is missing from the analyses typically used to select the size of the ultimate pit is risk. Once risk is quantified within the analysis framework, the decision-making range is significantly reduced, and the process for selecting the ultimate pit becomes more consistent, replicable, and defensible. This paper provides an approach to including risk in the process used to select the ultimate pit.

Keywords Risk · Strategy · Pit optimization · Mine planning

1 Introduction

Pit optimization has become a standard component of the planning process for open pit mining assets since the seminal work of Lerchs and Grossmann [1] nearly 60 years ago. Numerous incremental improvements to the algorithms and processes have been developed; however, two key issues persistently remain:

- Pit optimization remains "loosely coupled" to subsequent planning processes.
- Risk is generally not adequately quantified.

These two points are clearly interrelated.

Pit optimization is considered in this paper as being loosely coupled with subsequent planning processes for several reasons, including:

- Significant mine design work is required to transition from nested shells (or similar) to an operable mining sequence.
- Pit optimization algorithms typically only consider a subset of the complete suite of parameters and constraints.
- The structure of the pit optimization algorithm (referred to as maximum flow in optimization theory [2] and can also be modelled as a maximum graph closure problem) presents limited functionality to consider time, therefore impacting on the assessment of options including blending, stockpiling, and discounted value calculations.

The second point, the issue of risk quantification in the pit optimization process, will be explored in this paper.

For clarity, the ultimate pit is being considered as the excavated landform that will be used as the basis for ongoing planning and decision-making at a given point in time. It is generally, although not always, able to be revised or adjusted as operating conditions change or deposit knowledge evolves; however, each planning process or planning cycle will almost always be based on one selected ultimate pit for all subsequent design and scheduling developed within that program of work. Whilst alternative ultimate pits can be used for differing purposes such as Ore Resource statements, the ultimate pit for detailed design and valuation will almost always be based on one selection.

Importantly, a single ultimate pit is usually selected for use as the basis for decision-making to support any capital investment.

Certain decisions in the mine life cycle will have enduring implications, such as those made in the initial assessment

✉ Edward Holloway
edward.holloway@uq.edu.au;
ed.holloway@quantifiedstrategies.com.au

1 Sustainable Minerals Institute, The University of Queensland, Brisbane, Australia

2 UQ Experimental Mine Site, Quantified Strategies, Indooroopilly, Australia

of a project including the sizing of processing infrastructure, size and location of waste facilities, and environmental management strategies. Again, these can be changed, albeit with varying degrees of complexity depending on the specifics of the operation.

In summary, there are certain investment and planning decisions that will be the basis for less revocable decision-making, and at these times, a more detailed and thorough consideration of the ultimate pit is being advocated as leading practice.

I propose that the consideration of risk in pit optimization should initially be considered as a function of the selection of the ultimate pit shell. This is the most significant risk-impacting output that passes from the pit optimization process to the subsequent planning processes, particularly for greenfield projects. Guidance for stage designs through nested shells also forms an important output of the standard pit optimization process, with an impact on potential financial value generation more so than changing the risk profile in the way that the selection of the ultimate pit does. The ultimate pit selection will directly impact a range of factors (expanded below) including the sizing of the processing infrastructure, the quantity of waste material required for management and rehabilitation, and the life of the asset. These factors drive the scale of capital required, the capital efficiency, and therefore the scale of the impact of any variations in the parameters used as the basis for the underlying planning decisions. The selection of staging pits will impact the way that the scheduled reserve material is available for processing thereby impacting revenue generation, confidence/risk of maintaining the desired production profile, and risks relating to commodity price fluctuations as a function of the grade of ore available. These risks can be more readily managed through ongoing planning processes.

The significance of the selection of the ultimate pit has impacts on strategy even prior to any specific quantification of risk, considering the following:

- Operating life or duration
- The size of the stated Ore Reserve
- Total quantity of commodity extracted and therefore revenue generation potential
- Operation scale or throughput rate selected
- Quantity of waste rock produced
- Quantity of tailings produced
- The exposure of the business to adverse commodity price movements
- Resource to Reserve conversion

The ultimate pit is therefore pivotal in informing subsequent strategic planning decisions and study programs, including the following:

- Concentrator or plant throughput capacities
- Required annual material movements
- Capital investment (and potentially therefore operating unit costs)
- Infrastructure sizing
- Design of and size of waste storage solutions
- Detailed stage designs including the number of stages
- Relocation planning
- Equipment selection
- Ground and surface water modelling
- Environmental approvals
- Closure planning
- Post-mining land use options

Clearly this list is not intended to be exhaustive, but rather serves to illustrate the significance of the decisions that are impacted by the selection of the ultimate pit. Intuitively, the quantification of any risk changing as a function of the ultimate pit size should impact on these decisions. Negative outcomes of such decisions being incorrectly made are not widely publicized; however, they certainly do occur and will often be explained as being caused by adverse price movements. To provide one example, Evans [3] outlines the termination of a significant mine expansion.

Whilst it is possible to revise the ultimate pit selection, and it is in fact good practice for the strategic planning process to revisit the pit optimization during each planning cycle, some decisions cannot be incrementally refined. This would include any of the more significant strategic planning decisions such as the design, sizing or location of the waste storage or tailings storage facilities, the sizing of production infrastructure, and the application for mining leases that are based on an operation of a certain footprint and scale.

Such decisions as outlined in the previous paragraph should be supported by a level of planning and analysis that gives the operation the maximum potential for success under the widest range of potential future operating conditions. The consideration of risk is more critical where there is a wider range of options to be considered, resulting in greater potential for strategy to be varied to subsequently impact on risk and therefore on the resulting value to stakeholders. If the risk varies as a function of a strategy, or a key variable or decision, then using the same discount rate will not support the understanding or quantification of the impact on the risk. If the impact on the outcomes for each scenario can be quantified, then a risk-based decision can be made.

2 Industry and Literature Review

The incorporation of risk in the pit optimization process is considered in the literature. Whittle and Rozman [4] state that "risk is notoriously hard to quantify, and it is sometimes

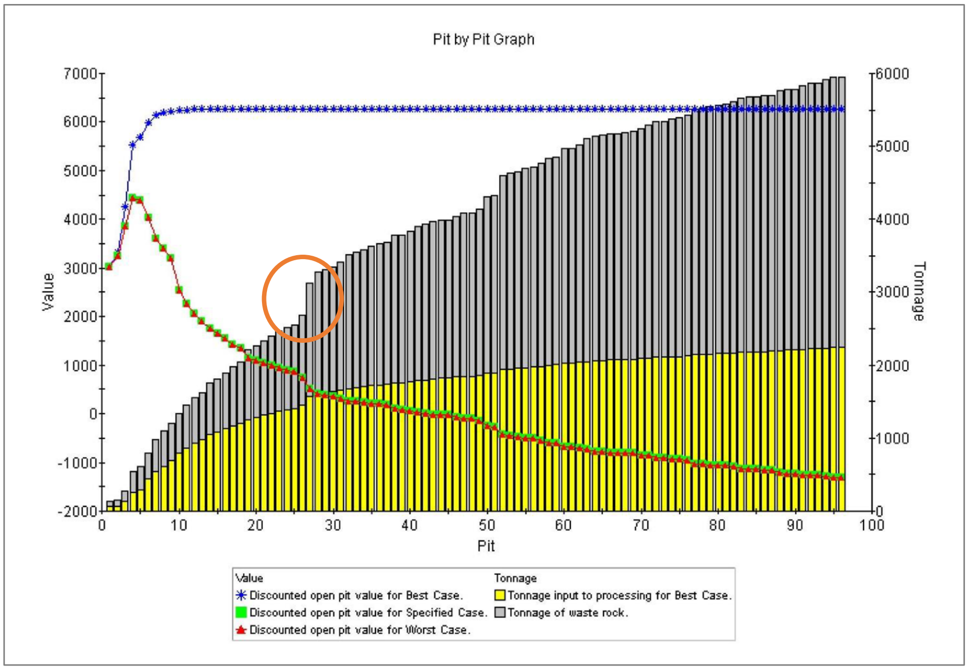

Fig. 1 Example pit optimization chart

better to use a discount rate which covers only economic factors and to present alternatives graphically for direct consideration by management". This aligns with what would often be seen as current leading practice; however, it is not always the case that the second step is included, therefore leaving risk considered only via the discount rate.

In considering risk in the pit optimization process, Richmond [5] utilizes a local search heuristic algorithm to compute an efficiency set or frontier to consider the relationship between value and risk as a function of ore loss, dilution, and geological simulations. Richmond [5] has identified a similar problem to this paper but approached it in a different way. The method of analysis is powerful and well designed; however, for the application of this paper, the level of detail in the schedule algorithm would be the primary concern. Furthermore, the basis for selection of the ultimate pit incorporating the quantification of risk is not specifically addressed.

In further considering risk in pit optimization, Richmond [6] utilizes a heuristic application of a floating cone algorithm to incorporate variability in inputs from geology, commodity price, and cost profiles and start timeframe to demonstrate the potential impacts in pit selection. He concludes that the "uncertainty in future metal prices and operating costs cannot be adequately captured in open pit optimization by simply post-processing a series of nested pit closures with constant values". This is demonstrated using an example. The process utilizes a simple scheduling approach, with the recommendation that "*further experimentation should be undertaken to determine whether this observation holds for more complex mining schedule algorithms*". Whilst this paper presents clearly the impacts of uncertainty on the NPV as a function of ultimate pit size, the basis for selecting the ultimate pit incorporating any risk quantification is again not specifically addressed.

There is no consensus as to what constitutes current leading practice for the selection of the ultimate pit. The process used often relies primarily on the "experience" of the person involved and therefore remains subject to errors and biases. In this instance, the errors can be basic modelling errors or errors made by an operator who believed they were making the correct decisions. These "errors" in ultimate pit selection potentially exist in many operating mines.

Many authors discuss the arbitrary nature of ultimate pit selection including [7–9] amongst numerous others. Hanson and Hodson [10] go so far as to say of ultimate pit selection that it is "usually by use of guesswork, cleverly disguised as experience or rules-of thumb".

The main conclusion is that the processes used for the selection of the ultimate pit are lacking. Empirically, a range of revenue factors from 0.3 to 1.35 have been observed for ultimate pit selection, and almost always these were used without a robust or defensible basis.

Frequently, the selection of the ultimate pit will be based on an interpretation from a graph such as Fig. 1. This may provide some useful context and guidance to planning teams, but it hardly forms a defensible and auditable basis for the making of a decision with wide-ranging implications. Decisions made using charts such as Fig. 1 are often based on visual interpretations, for example, the ultimate pit or a staging pit could be selected based on the step in the size of the pit shells highlighted by the orange circular shape.

Visual interpretations lack any rigorous analyses and provide almost no auditability. The process is not necessarily repeatability, cannot be automated to support any type of variability analysis, and is subject to a range of biases.

One of the most commonly cited approaches for the selection of the ultimate pit is to base the selection on the revenue factor 1.0 shell. If the time value of money is a component of the objective function for the planning process (which it should be and is in NPV), mathematically this approach to the ultimate pit selection will only be appropriate when the revenue generating material in the ultimate pit is mined in the same period as the waste preceding it in the schedule for that shell. The greater the time-based disconnect, the further from correct the selection of a revenue factor 1.0 shell becomes.

The basis for the selection of the revenue factor 1.0 shell is that it returns all material that has an incrementally positive impact on the cashflow, albeit prior to the impact of discounting. This will thereby return the largest incrementally positive undiscounted cashflow reserve and the greatest (undiscounted) value.

Financial value is generally considered to be the appropriate basis for the selection of the ultimate pit. This is logical; however, risk is rarely considered, nor is the associated impact on the expected value quantified. The selection of the ultimate pit based on maximum value is well represented in the literature, including [4, 8, 10–14]. Many of these authors have discussed variations in the approach to making such value-based decisions.

Richmond [6] identifies three significant issues in the standard two-step approach that separates the ultimate pit selection from scheduling and financial discounting: "1. Divorcing the open pit limit delineation from the NPV calculation does not guarantee that an optimal (maximum) NPV open pit solution will be found; 2. NPV calculations are based on a constant commodity price that fails to consider its time-dependant and uncertain nature; and 3. The single 'estimated' orebody model is invariably smoothed, thus it fails to consider short-scale grade variations".

Hanson and Hodson [10] focus on the first point as detailed by Richmond [6] and provide an approach to ultimate pit selection that considers the impact of discounting. They state that "the technique, which is termed 'skin analysis', requires use of engineering judgement and will not itself produce a mathematically exact optimal pit. However, if used properly the technique will give a consistently better result than the more common rules-of-thumb known to the authors". The approach outlined schedules sequentially smaller ultimate pit shells using a shell grouping approach to facilitate the selection of the ultimate pit. Given that high-level schedule constraints are included, this is certainly superior to an analysis based solely on the outputs of a pit optimization algorithm. The approach does not consider risk and can also be expected to produce the same plateauing of value that generally makes ultimate pit selection subjective. In closing Hanson and Hodson [10] state that "the technique does not produce a rigorously optimized pit and it does require proper engineering judgement to produce a reliable, near optimum solution".

Maximizing the financial value is clearly a valid objective and should undoubtedly remain as an initial step in the analysis upon which to base the selection of the ultimate pit. It is not, however, the only relevant consideration.

Abdel Sabour, and Dimitrakopoulos [9] develop a system that is "based on integrating multiple market and geological uncertainties as well as the operating flexibility to revise the ultimate pit limits using a Monte Carlo based real options valuation (ROV) model". This work is insightful and provides some powerful risk-based analyses but is more focussed on the impact on the optimality of the sequence as a function of the uncertainty in geological modelling and mine design than the selection of the ultimate pit.

Baek and Choi [15] directly consider risk in their methodology. Their approach presents some powerful and interesting results, and some of the techniques used herein align closely with those outlined in their paper. Baek and Choi [15] consider price variability, which they use to code a probability of inclusion in the ultimate pit for each block in the block model. This is a clear inclusion of a consideration other than a static value. Whilst this is novel and powerful, there remain further steps that can be taken. Correlations between key price assumption sets (and costs) should be included. The analysis stops at the ultimate pit selection, and so does not incorporate the impact of discounting effectively. The analysis also does not address the basis for the selection of the ultimate pit. A revenue factor 1.0 is used, which is the most logical basis to support the research that the authors presented.

Robins [16] uses a similar approach and produces a probability for each block being mined within the ultimate pit for a range of conditionally simulated geological models. The selection for the ultimate pit again appears to be a positive incremental value basis. Robins [16] successfully demonstrates that the highest probability pits (therefore based on lower geological risk) have a tighter distribution in valuation outcomes when all model outcomes are incorporated. This is an important and logical outcome. Robins [16] also reports the standard deviation in value as a function of the probability-based selection of the ultimate pit. This is an interesting approach to communicating the confidence in the expected outcomes. The approach calculates the probability of outcomes based on the quantification of geological risk but stops short of quantifying the risk of an alternative selection. Nonetheless, this presents a powerful approach to the incorporation of geological risk into pit optimization.

Direct block scheduling (DBS) is an approach that requires consideration. Whilst DBS overcomes some of the shortcomings of maximum flow approaches, with Lerchs-Grossmann (LG) included in this family of algorithms, it still uses a value-based selection in isolation. DBS-based approaches may theoretically maximize the value in a more comprehensive way by considering additional parameters and constraints, and thereby more accurately incorporate discounting. The approach therefore certainly demonstrates potential for the inclusion of a wider range of considerations, including risk in various forms; however, this is not a fundamental feature of the algorithm/process. Smith and Nogueira [17] reported a similar conclusion in comparing DBS to LG, stating that the "… maximum project values are very similar, with DBS reserves producing a slightly higher NPV. The overall production schedules were also similar, but the solution based on DBS resulted in a more aggressive pre-strip allowing for earlier concentrator start-up and greater continuity in oxide processing. The DBS solution provides a better starting point for stage design due to inclusion of constraints on minimum bench width. Overall the time spend on stage design was similar between DBS and LG. We expect that with improvements and better understanding of how best to apply constraints to DBS that there will be significant advantages in terms of man-hours needed to convert a DBS solution into a viable stage design".

Given that risk is not explicitly quantified, DBS-based techniques will be considered to be similar to LG-based techniques for the purposes of this paper.

A range of simple techniques to potentially improve the method by which the discount rate is applied within the pit optimization process are available within commercial software applications. These techniques include, for example, discounting by depth and discounting by sequence. Whilst such approaches do provide mechanisms to vary the application of discounting, they do not quantify the risks associated with any scenario as a specific function of the strategic decisions that should be considered.

Pit optimization is an area where an impressive process has been developed to solve a complex mathematical and mine planning challenge. Given the complexity of the process, the level of detail involved, and the analyses produced, it is common for decision-makers to be satisfied with this and not to push it much further. As in other areas of strategic planning, the consideration of scenario specific risk is routinely ignored, with risk assumed to be adequately incorporated via the discount rate.

Aside from the work of a limited number of researchers, the detailed quantification of risk is not a major consideration in the literature surrounding pit optimization. Pit optimization is an area of strategic planning that has a significant impact on the level of risk, and if this is not a focus of the process and the associated analyses, the result will be to generally increase it, potentially across multiple facets.

3 Concept

The concept being presented in this paper is that including risk in the pit optimization process is both possible and likely to impact the strategic decision-making process. This will be explored using commodity price risk.

The variability in the future price environment has been represented in the analyses by considering discretely two pricing points where there can be expected to be a disconnect:

- The first being the forecast price at which planning decisions must be made (the planning price environment).
- The second being the price at which the operation must then deliver a result (the future operating price environment).

The first dictates the planning decisions, and the second is what eventuates; and clearly these will never be identical. With this as the starting premise, the resulting impacts based on this difference can be quantified. This will then form the basis of the calculation of commodity price risk.

It is recognized that the future price environment will not be static. The use of a series of single pricing assumptions for both the planning price and the future operating price is a deliberate simplification of the problem construct to:

- Simplify the model execution and analysis.
- Support clear and efficient communication of results.

When the purpose of the analysis is to select an ultimate pit as the basis for go-forward decision-making purposes, which is often the case, this simplification is pragmatic. Furthermore, the framework outlined assesses a deliberately wide range of potential price environments and analyses the full matrix of combinations for the planning price and the future operating price.

The synopsis is then to compare the value and the value at risk, in financial terms, as a function of the disconnect between the two price environments. The future price environment remains unknown, and given that the associated impacts can be quantified through this form of analysis, they have been presented as a value at risk. To explain this further by way of an example, if the macroeconomic price environment used as the basis for the selection of the ultimate pit for a copper mine was \$3.00/lb and the price under which the mine must deliver is closer to \$2.50/lb, what would the impact be? Equally if the price was \$4.00/lb how much potential value would have been forfeited due to the disconnect in price assumptions?

Fundamentally, there are two sources of potential value destruction in the selection of the ultimate pit:

- Value is lost by the adoption of a price environment for planning purposes that is too low as compared to the operating price environment, and material is planned to be left in the ground that could have added value and life to the operation. This is the residual resource risk.
- Value is lost by the adoption of a price environment for planning purposes that is too high as compared to the operating price environment, and waste is stripped that does not expose material of significant (discounted) value to pay for the associated mining costs or realize the expected return. This is the excess stripping risk.

Both of the above points are risks and can therefore be calculated using a value at risk approach. Both risks are presented as negatives, including when the value impact is as a lost opportunity. For simplicity, the process will be referred to as the VAR within this paper.

The term value at risk is more commonly associated with portfolio management, where it evolved in response to multiple liquidity crises with the purpose of quantifying the risk exposure of financial services firms [18].

The following definitions/explanations relate to VAR in the context of portfolio management:

"Value at risk is a summary statistic which quantifies the exposure of an asset or portfolio to market risk or the risk that a position declines in value. VAR is the method of measuring the financial risk of an asset, portfolio or exposure over some specified period of time" [19].

"Value-at-risk aims to measure the potential loss on a portfolio that would result if relatively large adverse price movements were to occur. Hence, at its simplest, VaR requires the revaluation of a portfolio using a set of given price shifts. Statistical techniques are used to select the size of those price shifts" [20].

"…VaR is the dollar amount that portfolio losses are not expected to exceed, with a specified degree of statistical confidence, over a pre-specified period of time" [20].

The concept of VAR in this paper has similarities and differences as compared to that used in portfolio management. The following definition is provided for clarity:

Definition (Value at Risk)

Value at risk is a series of summary statistics to quantify financial risk as a function of ultimate pit size selection and probabilistic parameter set(s).

This paper will focus on the commodity price as the probabilistic parameter set; however, the same underlying framework can be used for other risk based probabilistic parameters.

The outputs of the VAR process outlined in this paper are as follows:

- The quantification of the risk (value at risk) as a function of the price-based decisions and the size of the shell selected as the ultimate pit
- To subsequently determine the ultimate pit size and the revenue factor upon which to base the selection of the ultimate pit to provide the best possibility of success for the operation as a function of a probability distribution applied to the future price environment

The selection of the most appropriate revenue factor for an open pit is not simple and is inadequately based on rules of thumb or similar. It is impacted by a range of interrelated factors including the following:

- Underlying geology (dip, relationship between value and depth, sharpness of contacts, size, etc.)
- Stripping ratio
- Geotechnical parameters
- Discount rate
- Starting topography

There is clearly a relationship between the macroeconomic price environments being used and the revenue factor; there is also a key difference. The revenue factor, as the name implies, factors the revenue-based components of the cashflow, however not the costs. Whenever statistically significant relationships exist between the commodity prices that drive revenues and the costs, this must be incorporated in both the pit optimizations and the schedule optimizations for every scenario analysed. Such relationships have been incorporated in the process presented in this paper with further details included in the subsequent section.

The concept will be demonstrated using a case study based on a tier 1 copper–gold porphyry operation. Whilst a VAR approach can be applied whenever planning teams want to make decisions guided by the impacts of risk, the case study is considering the specific scenario of an in-pit dump, which from the point of commencement will essentially sterilize a significant proportion of the remainder of the resource. Conversely, the longer the commencement of the in-pit dump is delayed, the waste storage and approval options become increasing complex and tenuous. The value at risk of this decision will be quantified based on commodity price variability.

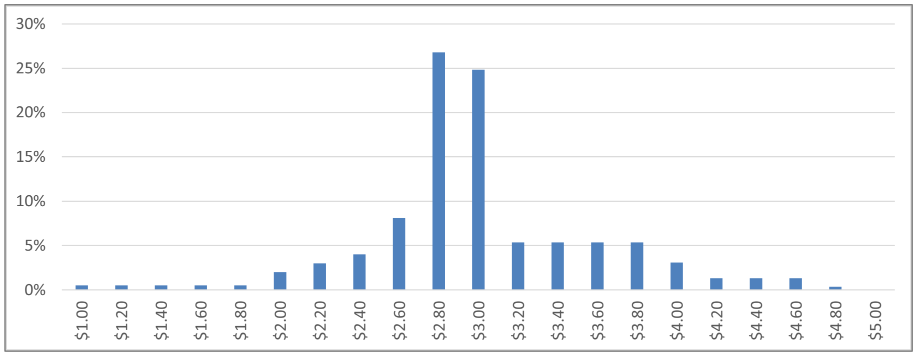

Fig. 2 Histogram of price distribution model

The following summarizes the case study analysis:

- The purpose is to consider the range of value and risk-based outcomes from the inclusion of an in-pit waste dump as a function of probabilistically adjusted future commodity price scenarios.
- The in-pit waste dump also impacts on ex-pit waste management and material destination optimisation (inclusive of the haulage fleet) all of which have been incorporated in the schedule optimization model.
- Additional capacity is not being considered for either the mining fleet or for ore processing; therefore, capital expenses do not feature in the analyses.
- Given that the scale of the operation is being held constant, the constraints used in the schedule optimization are being held constant including processing capacities, total mining capacities, and bench turnover constraints. The base operating costs are also being treated on a consistent base and are only varied as a function of the macroeconomic price environment (further details of the process used are included in the "Inputs" section).
- For context, the year of analysis was 2017.

4 Inputs

"It is change, continuing change, inevitable change, that is the dominant factor in society today. No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be… This, in turn, means that our statesmen, our businessmen, our everyman must take on a science fictional way of thinking" Asimov [21] (1978).

The inputs specific to this paper are based on commodity prices; therefore, to support the analyses, a probabilistic set of copper price assumptions was required and was sourced from a range of commercial providers (for example, this could include Wood Mackenzie, SNL, etc.). These series were then amalgamated into a form of consensus, first-principles price-series dataset, which do not conform specifically with any one dataset, but equally do not materially differ.

Figure 2 presents a histogram of the probabilistic price distribution used. This data is derived from the predicted copper price environment for 2025 (being close to the expected timing for the final stage being mined for the case study operation at the time of analysis, being 2017). Whilst somewhat granular, it can be noted that the distribution is positively skewed, thereby implying some potential for price upside.

Any probabilistic analysis must include any relevant correlations, or otherwise, the analysis will be potentially misleading. Given that the commodity price variation is the parameter set that is being analysed, it must feature as the parameter that is flexed or risk-adjusted in the revised framework. In running analyses that vary this parameter, it is important to consider whether any other parameters could be expected to move as a function of being correlated. The copper price, along with many other commodities, varies due to changes in the supply–demand balance. This is driven to an extent by the level of overall economic activity, which also intuitively impacts on the supply–demand balance and therefore the prices of a range of components in the operating costs, e.g. steel and diesel.

To incorporate these correlations in both the pit optimization and schedule optimization processes, a consistent approach has been used throughout. This is the same process as detailed in the paper Correlated Valuation Methodology [22]. The way that this has been incorporated is by establishing a series of what are referred to as macroeconomic environments. These have as the central parameters the main commodity price, which for this paper is the copper price. All other correlated parameters are then expressed as a function of the copper price. Thus, if the macroeconomic environment is denoted by a \$2.25/lb copper price, the other commodity prices and operating costs are also adjusted using the copper price as the input to the appropriate regression equation developed.

Table 1 Statistics for correlations between relevant variables and the copper price

Response variable	p-value[1]	Interpretation	r-squared
Mining Operating Cost	0.000	Statistically significant	78.4%
Milling Operating Cost	0.000	Statistically significant	78.0%
Gold Price	0.000	Statistically significant	67.7%
Silver Price	0.000	Statistically significant	69.8%

[1]The p-values are not exactly 0.000 but would need significant decimal places to be otherwise. Functionally, the values are 0; they are presented to 3 decimal places as is convention

of inflation on both datasets is not providing an artificial positive impact on the results. The regressions upon which these analyses are based are all first-order linear regressions. More complex and involved modelling could be completed; however for the purposes of demonstrating the process, and given the generally high r-squared values returned, this basis has been deemed acceptable.

Figure 3 has been included as an example of the time-series component of the analysis.

Note: The mining and milling operating cost composites are based on historical proportions of costs for

Fig. 3 Real copper and real mining operating cost composite — time series dataset

Table 2 CVM regression analysis equations

Parameter	Equation
Mining Operating Cost (real)	MineOpex = [64.0403] + [Copper_Price] * [0.0089]
Milling Operating Cost (real)	ProcessOpex = [73.4882] + [Copper_Price] * [0.0054]
Gold Price (real)	Gold_Price = [77.4369] + [Copper_Price] * [0.1502]
Silver Price (real)	Silver_Price = [-0.9796] + [Copper_Price] * [0.0029]

Table 1 presents a summary of the key parameters that were identified as being potentially correlated with the copper price for the case study that this paper is based on. All parameters were statistically significant at the standard α-level of 0.05, and the r-squared values indicate that the models will have a useful predictive power.

To interpret the results in Table 1, if the resulting p-value from the test of significance of the regression analysis is less than the selected α-level (in this case 0.05), the relationship between the parameters can be considered to be statistically significant at the 95% level of confidence. The r-squared values provide a measure of the effectiveness of the model in predicting the response data; the closer to 100%, the more closely the model fits the data.

All datasets are based on real terms (i.e. not nominal), so therefore the potentially common and directional impact the case study operation and comprise a range of inputs including diesel, labour, chemicals, steel, and equipment components. Additional detail could be added at this step; however, for the purposes of illustrating the concept, only high-level adjustments have been incorporated.

Analyses of the identified correlations returned the equations in Table 2 for use in the derivation of the macroeconomic datasets.

The units used in Table 2 are as follows:

Copper_Price	USD$/t
Gold_Price	USD$/troy oz
Silver_Price	USD$/troy oz

MineOpex	Index based on composite operating components for the case study operation
ProcessOpex	Index based on composite operating components for the case study operation

Given that the specific analysis being considered has been designed to quantify the risk-based decision surrounding the ultimate pit selection for the purpose of including an in-pit dump in the subsequent mine planning processes, all capacities have been held constant for the processing infrastructure and the mine equipment fleet. Any capital expense considerations are also therefore omitted from the case study.

All other inputs used can be considered as standard for a pit optimization process.

5 Technologies Used

Whilst it is not fundamental to the VAR process, for clarity, the following technologies have been used for the reasons outlined:

- Hexagon MinePlan (previously MineSight) was used for all block model coding and manipulation. This was the general mine planning (GMP) solution used at the case study operation.
- Geovia Whittle was used for all pit optimization runs, and all runs were executed using the pseudoflow algorithm. Whittle is a well-recognized software for pit optimization and again was the incumbent solution for the case study operation.
- A shell grouping algorithm was used to ensure that the phases (or stages) used in the scheduling process would be achievable for the operation. Whilst it is not fundamental to the process outlined in this paper, the algorithm ensured a sufficient total size to support the scale of equipment being used, resulting mining widths, and also a requirement for a minimum of 3 years ore supply in each set of grouped shells, with ore being considered in a static sense as material with a positive cashflow grade [23].
- Comet was used for all schedule optimizations. Comet is an advanced multi-policy schedule optimization package, with functionality to optimize incorporating multiple complex downstream constraints. The functionality to optimize inclusive of complex downstream system was important at the case study operation for both process and environmental controls in tailings and waste streams inclusive of haul route optimization. As an example of the required complexity, two process streams split with components rejoined subsequently and are then combined with a ROM waste stream. The resultant combined stream must have certain geochemical attributes and remain below a set limit for certain environmentally sensitive grade attributes. This component of the optimization is regularly the constraint for the operation and means that comparatively simple schedule optimization solutions could not be used in this instance. Some further relevant points regarding the schedule configuration are as follows:

  - Comet was configured to include both negative periods and negative phase tails (or stages). This is important to allow the quantification of scenarios that deliver significantly negative results, for example, due to the combination of an overly large ultimate pit and a lower commodity price.
  - Importantly Comet is a "true optimizer" and will defer/adjust the mining profile if it results in a higher NPV (being the objective function).
  - As a general comment, the configuration of the schedule optimisation model is critical to ensure meaningful results are produced, regardless of software selection. The model needs to be able to be deployed across all scenarios for analysis without the introduction of any forms of operator or scenario specific bias.

- Microsoft Excel was used for the VAR calculations and analysis. This did require the development of visual basic scripts given that the VAR formulas were not able to be readily developed using any combinations of standard Excel formulae. Subsequent VAR-based work has been developed using Python code, which is a superior approach based on the data science nature of the VAR analysis and the Python-based libraries available for this purpose.

6 Process

An overview of the process used is included in Table 3.

It could be interpreted that the analysis produced by this process is similar to the well-recognized best-case, specified-case, and worst-case schedules (refer Fig. 1); however, the analysis is fundamentally different:

- The best-case, specified-case, and worst-case schedules consider the impact of vertical to lateral mining, transitioning from mining the shells in order (onion skin mining) through to mining complete benches in sequence (pancake mining).

Table 3 VAR process outline

The base set of macroeconomic price environments were determined, based on copper prices ranging from USD \$1.00/lb to USD \$5.00/lb in increments of USD \$0.20/lb
Historical price and cost series data were analysed and regressed with the copper price
The block model coding was updated based on the above points for the 21 macroeconomic price environments using MinePlan
The 21 variants of the block model representing the 21 macroeconomic price environments were exported and 21 pit optimization models were run using Whittle
The pit optimization shells were exported from each pit optimization model and loaded into the block model
A shell grouping algorithm was used to develop without bias groups of pit shells into mineable phases for each set of shells exported
Schedule optimizations were all run using the same Comet model. The objective function used maximizes standard NPV. Every schedule is optimized for cut-off grade, blending, and mining sequence as a function of the mining areas made available and considers all strategic level constraints specific to the case study
441 schedules were run combining the 21 macroeconomic price environments being used to select the ultimate pit, and then for each ultimate pit selected the same 21 macroeconomic price environments used as the basis for the schedule optimization runs
The results for all schedules were analysed to quantify the value at risk for each schedule analysed

○ The VAR process is quantifying the impacts of the size of the ultimate pit (using consistent and realistic stage sizing) as a function of the probabilistic future commodity price environment.

As mentioned previously, the VAR process is modelling the impact of the disconnect between the planning price environment (pit optimization) and the future operating price environment (schedule optimization). The resulting matrix analysed therefore includes all possible combinations. The intention is to then determine the corresponding ultimate pit size and associated revenue factor that results in the most risk-robust outcome as a function of this disconnect.

Note: The revenue factor is the same as the standard definition used throughout the industry.

Subsequent to the pit optimization and schedule optimization runs being completed for all cases, the VAR analysis was completed. The purpose of the model is quite simple, it needs to identify for each scenario how much value is at risk because of either leaving resources in the ground or because of stripping too much waste relative to the value extracted as a function of the future commodity price distribution modelled. The model therefore also applies the adjustments required to incorporate the price-based probability distribution (Fig. 2).

The challenge comes about from the fact that the model needs to search the entire results range and potentially return a value that does not match either the macroeconomic price-based assumptions used for the pit optimization or the schedule optimization. To explain this using the simple example of identifying the maximum value ultimate pit to compare with for the scenario based on a \$1.60/lb copper price for the pit optimization and a \$3.80/lb copper price for the schedule optimization, the search needs to return the ultimate pit shell that was generated using a \$3.00/lb copper price, ie it does not match either the price used for the pit optimization or the schedule optimization.

Equation 1 (VAR excess stripping) and Eq. 2 (VAR residual resource) provide the base equations used in the VAR model.

$$VAR_{ES} = MIN(NPV_S - NPV_{MSP}, NPV_S - NPV_{SHELL}) \quad (1)$$

$$VAR_{RR} = NPV_S - NPV_{MSP} \quad (2)$$

where:

VAR_{ES}	Value at risk due to excess stripping.
VAR_{RR}	Value at risk due to residual resource.
NPV_S	NPV for the selected case.
NPV_{MSP}	Maximum NPV returned for the schedule price for the selected case. This can return a shell that is smaller or larger than the selected case.
NPV_{SHELL}	NPV for the case based on the schedule being run paired with the price used to select the shell.

As evident from Eq. 1 and Eq. 2, part of the complexity comes from the fact that the maximum value case for the schedule price can result in a shell that is either smaller or larger than the case shell.

Whilst Eq. 2 is relatively intuitive, Eq. 1 is less so; therefore, it will be explained further. When the ultimate pit is too small, the excess stripping risk does not exist; this then transitions through to a risk as a function of the scenario NPV compared with the scenario that delivered the maximum NPV for the schedule price. As the commodity price used for the pit optimization increases further, and the ultimate pit increases in size, implicit within this is the assumption that the operating commodity price will also

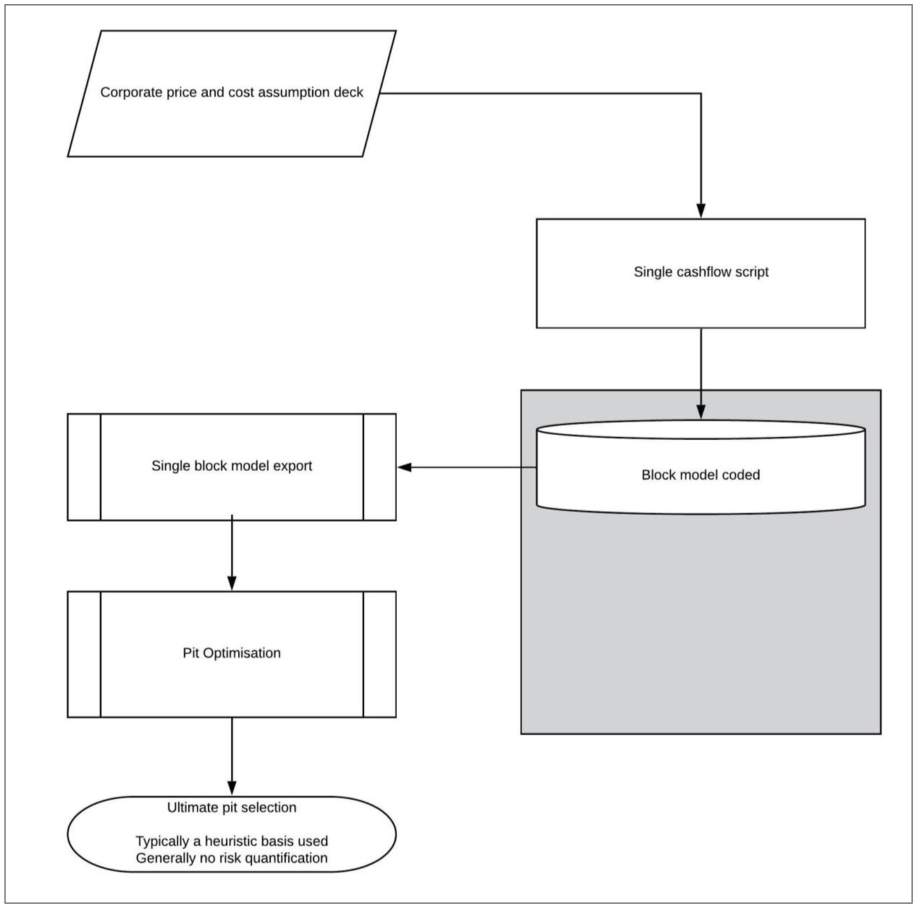

Fig. 4 Schematic of standard pit optimisation process

increase. The second part of Eq. 1 quantifies this by comparing the scenario NPV to the NPV for the schedule paired with the shell price. The component of the VAR calculated Eq. 2 should not be interpreted as a direct reduction in value, although this is one component of the risk calculated, but also includes risk as a function of the price distribution used in the VAR process. The proportion of these two components will vary by deposit and by the price distribution being used.

In a functional sense, Eq. 1 is very effective at constraining the ultimate pit selected from being too large, therefore removing incrementally value destructive sections of the resource from subsequent analyses. Such an outcome results either in value erosion or re-work for planning teams and should therefore be avoided. In some instances, the value destruction is obfuscated and difficult to identify, whilst in other instances an overly large ultimate pit will result in a series of negative value mining stages towards the end of the schedule that subsequently require manual adjustment or removal.

A simplified approach could conceptually be used, with some caution, considering only Eq. 2. Once the residual resource value at risk approaches zero, there should be limited incentive to select a larger shell. This would of course fail to quantify the value at risk if a larger shell were to be selected.

Post solving Eq. 1 and Eq. 2, the results are probability adjusted at the individual scenario level for the schedule price environment only. This is a key connection in the logic required to model the risk as a function of the price distribution being assessed.

The VAR process presented in this paper differs from a standard pit optimization in the following ways:

- It considers the price disconnect between the current (or planning) price and the future price.
- It incorporates static probabilistic commodity price inputs.
- It includes correlations between commodity prices and operating costs.
- It quantifies the value at risk.
- It constitutes a quantified, specific, and repeatable process for the selection of the ultimate pit (assuming underlying model consistency).

The above differences require some fundamental changes to the process and the data flows that are outlined in Fig. 4 and Fig. 5. Figure 4 presents a high-level schematic of the dataflow for a typical pit optimization process, and by comparison, Fig. 5 presents a schematic of the dataflow for the VAR pit optimization process.

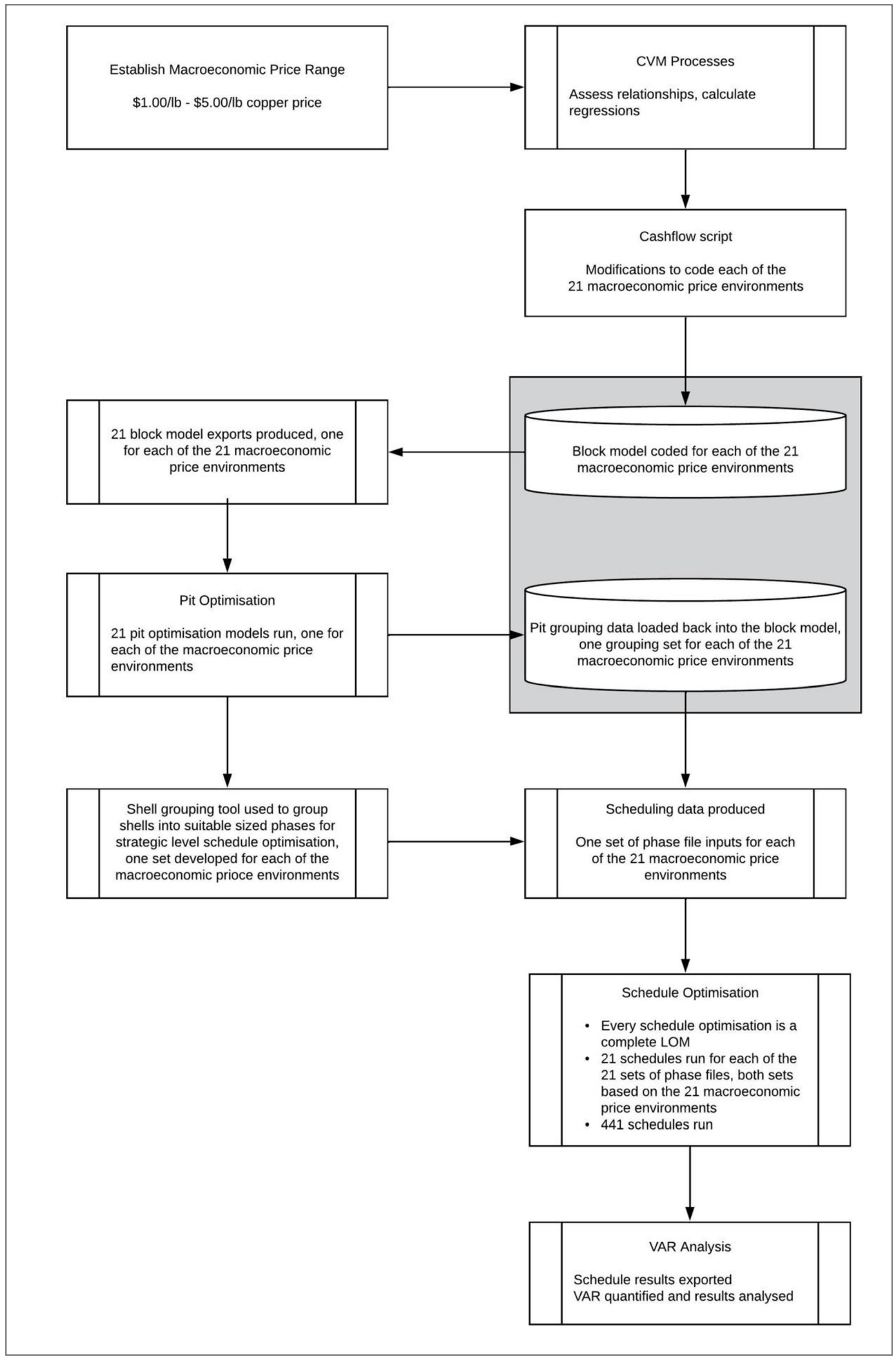

Fig. 5 Schematic of VAR pit optimisation process

7 Results

For the VAR results to be meaningful, it is important that the largest pit shells are too large, and the smallest pit shells are too small as compared to the base commodity price assumptions. Given that the entire analysis is based on a single revenue factor driving the exports from the pit optimization process for input into the schedule optimization process if the first analysis does not deliver a balanced analysis, it may need to be repeated. The range of appropriate revenue factors is specific to each deposit, with rules of thumb often providing little more than a starting position. The revenue factor used to produce the analyses included in this paper is 0.35; to clarify this point, the

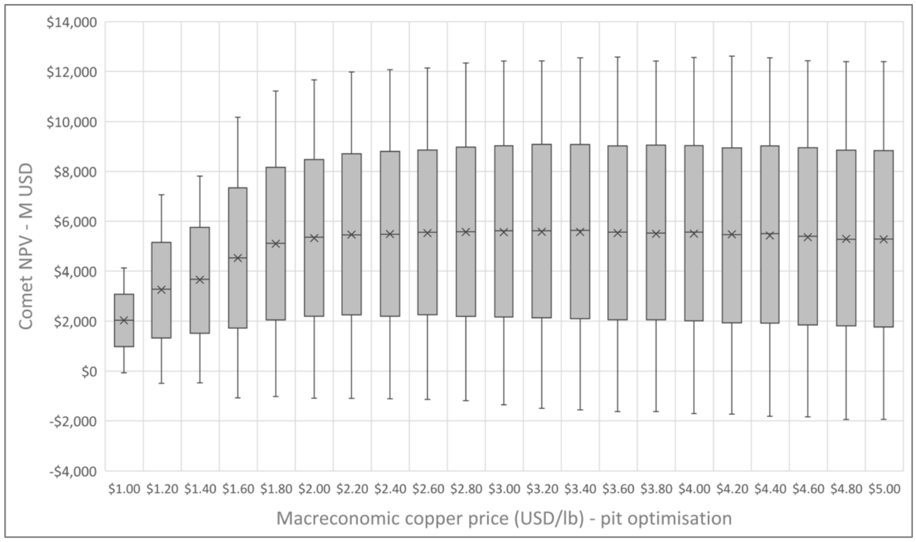

Fig. 6 Comet NPVs by pit optimization price and schedule price (revenue factor 0.35)

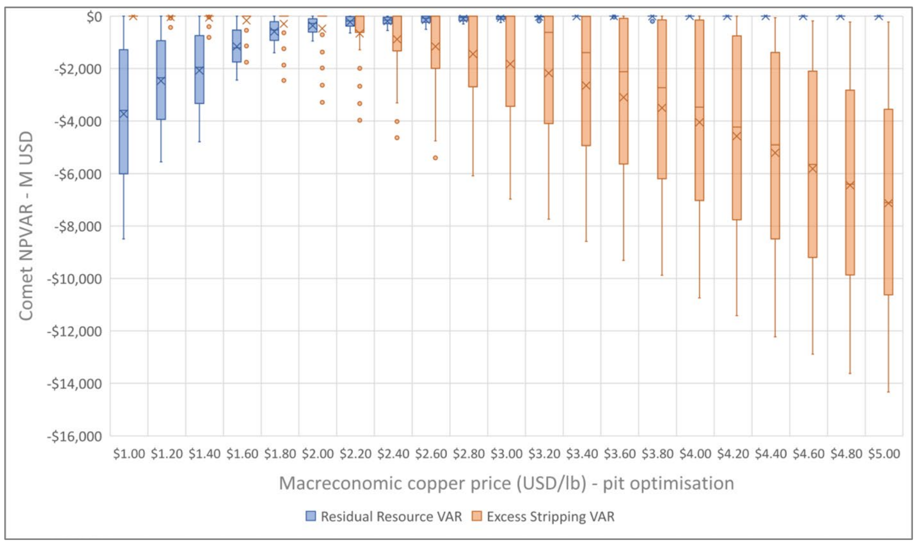

Fig. 7 VAR analysis prior to price probability adjustments

ultimate pit selection made at each of the macroeconomic price environments is 0.35 resulting in ultimate pit sizes ranging from small to large due to the changes in calculated revenues and costs as a function of the macroeconomic price environment varying, allowing the revenue factor to be held constant. To reiterate, whilst one revenue factor is being used for the selection of the ultimate pit, this selection is being made at a range of different macroeconomic price assumptions, therefore driving the 21 different ultimate pit sizes being considered in the analysis.

Figure 6 presents the profile of NPVs output by the analyses with the distribution being based on the range of copper price profiles modelled prior to any probability adjustments.

To explain the data presented in Fig. 6, Fig. 7, and Fig. 8, the *x*-axis is the copper price denoting the macroeconomic price environment used for the pit optimization, with the box and whisker[1] plot presenting on the *y*-axis the associated value or value at risk. For Fig. 6 the *y*-axis presents the NPV results for the 21 schedules run for each of the macroeconomic price environments. The box and whiskers are being used to present the range of outcomes for the schedule results from a $1.00/lb copper price environment through to a $5.00/lb copper price environment for each ultimate pit

[1] The data presented by the Excel box and whisker charts is explained as follows: "The middle line of the box represents the median or middle number. The x in the box represents the mean. The median divides the data set into a bottom half and a top half. The bottom line of the box represents the median of the 1st quartile. The top line of the box represents the median of the 3rd quartile. The whiskers (vertical lines) extend from the ends of the box to the minimum value and maximum value" [24].

The points are explained as "..the outlier points that lie either below the lower whisker line or above the upper whisker line" [25].

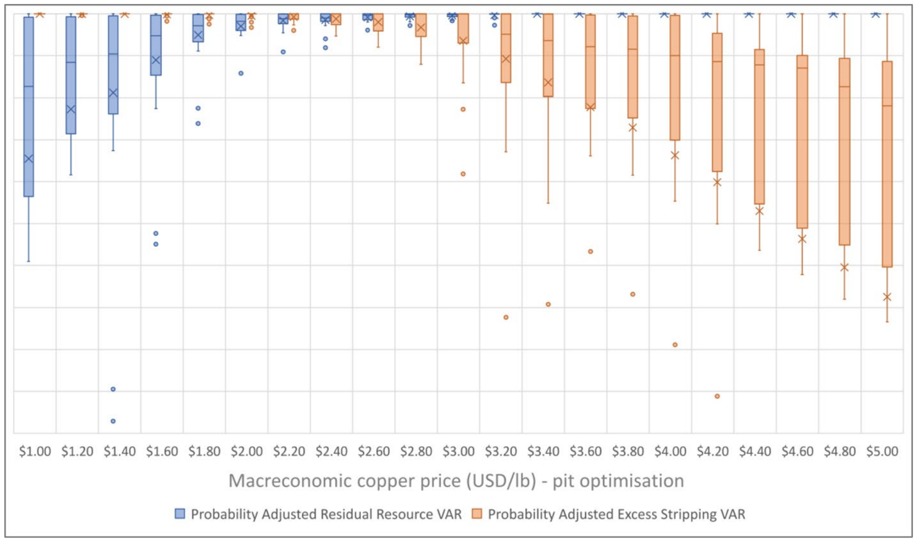

Fig. 8 VAR analysis post price probability adjustments

size. This range in outcome reflects the commodity price risk prior to any probability adjustments.

A potentially more intuitive explanation can be provided in terms of the size of the ultimate pit with the *x*-axis transitioning from smaller ultimate pits on the left through to larger pits on the right, and then for each of the pit sizes, the box and whisker data present the range of schedule results from a low commodity price through to a high commodity price.

The mean NPVs in Fig. 6 can be seen to plateau off from around the $2.00/lb price environment. This is indicating that once scheduled, there is limited incremental value added by the larger pits.

Figure 7 has the same structure as Fig. 6; however, it presents the value at risk, whereas Fig. 6 presents the value. The *x*-axis remains identical between the two, with the *y*-axis now presenting distributions of the VAR calculated using Eq. 1 and Eq. 2.

Figure 7 shows the VAR for residual resource in blue, and for excess stripping in orange prior to being probability adjusted. The pattern displayed for both series is largely as expected. The excess stripping VAR increases from functionally zero in small ultimate pits to being significant as the ultimate pit increases in size. The residual resource VAR displays the opposite trend.

Figure 7 can be interpreted by considering the orange bars (for each price scenario on the *x*-axis) as purchasing an option via waste stripping which serves to reduce the exposure to the risk of leaving value in the ground (the blue bars). Therefore, the initial target would typically be the area where the two bars are closer to being equal.

Figure 8 presents the same data as Fig. 7 after being adjusted for the probability of the schedule price environment, i.e. the VAR for each schedule has been adjusted by the associated price probability *for the schedule only*. This process reduces the impact of the outcomes derived from schedules based on lower probability future price environments and allows a visual interpretation of this as a function of the price environment used to select the ultimate pit. Note that the *y*-axis has been removed from this figure as it now represents a combination of VAR and probability; the chart should be interpreted by comparing the bars.

It should be noted at this point that changes in the price probability distribution used to produce Fig. 8 will materially impact the outputs and therefore the interpretation.

8 VAR-Based Ultimate Pit Selection

The following process has been used to select the ultimate pit in this paper:

- Using the VAR analyses, the preferred price range for the selection of the ultimate pit was made by selecting the shell with the minimum probability adjusted value at risk.
- The selected price environment was then mapped back to the matching macroeconomic pit optimization model.
- The associated revenue factor was then used to identify the size of the target shell.
- The shell size was then mapped back to the base pit optimization model for the selection of the comparable sized shell upon which to base design work.

Table 4 presents the outputs of this process.

9 Discussion

The VAR process provides a more defensible basis for the selection of the ultimate pit, in this instance with a focus on the impact of commodity price risk. Excluding the existence of any modelling errors, biases, or other differences, two

Table 4 VAR results

Output	VAR
Base revenue factor (input)	0.35
Selected price case (output)	\$2.40–\$2.60
Mapped VAR shell size range (output)	750–790Mt
Selected pit shell ranges (base pit optimization model)	10–11
Revenue factor range (output)	0.29–0.31

appropriately skilled operators should deliver the same or very similar results based on mathematics and statistics with minimal subjectivity. The same cannot be said for decisions based on a pit-by-pit graph or similar.

It should be noted at this point that the ultimate pit selection does not have to be the minimum VAR shell. Key strategic decisions such as the ultimate pit selection should be influenced by the risk tolerance of the investors or operating company. For example, if the company is only operating one asset, the potential for periods of negative cashflow may be highly undesirable, whilst the alternative risk of leaving some value in the ground may be significantly less concerning. After all, it could always be considered again later. Conversely, a company with a strong balance sheet may prefer the potential value and life upside of a larger pit and then size the operation and infrastructure to align with this scale from the commencement of the operation. In both instances, the VAR framework provides insight as to the probabilistic value potentially forfeited, or at risk, for smaller or larger ultimate pit selections based on the probabilistic price distributions used.

It is also worth noting that if alternative price distributions were required to be analysed, this is a very rapid process and does not require the re-running of any pit optimizations or schedule optimizations as the price distributions are incorporated in the VAR section of the analysis only.

The VAR approach to the selection of the ultimate pit is clearly different to the standard approach. If it is assumed that the standard approach was to use a single optimization model and base the ultimate pit selection on the revenue factor 1.0 shell,[2] which is one approach that is commonly cited and easy to understand, the following points summarize the differences between the two approaches.

Using a static analysis, the RF1.0 pit delivers the following results by comparison to the VAR selected ultimate pit:

- Is 800Mt larger or increased the size of the ultimate pit by 102%.
- The value is decreased by \$115 M, which is within the accuracy of the model processes used and equates to a decrease of 2%. Effectively, the value is the same.
- The value at risk is increased from \$284 M to \$807 M, an increase of 184%.

Note:

- The environmental risk profile is also significantly reduced (although not quantified herein) based on the VAR approach as compared to the standard approach due to the fact that the quantity of waste and tailings required to be stored has been significantly reduced.
- The analyses presented are all based on standard NPV. This allows the focus to be on the impacts as a result of the change in methodology and to avoid the potential for the outcomes to be obscured by multiple simultaneous changes.

The VAR results indicated that using a revenue factor of 0.35, the ultimate pit should be selected based on a macroeconomic price environment denoted by a copper price of between \$2.40/lb and \$2.60/lb. Given that the base price assumption for the case study was \$3.00/lb for the copper price, this clearly results in a duplication of factors, i.e. there is a revenue factor, and then the selection of a pit shell based on a price assumption that differs from the base price assumption. The issue relates to the fact that the revenue factor selected for the runs is still not quite optimal for the deposit as analysed. Further analyses could improve this alignment; however, the size of the ultimate pit, being the output of focus, would not materially change. The use of two factors is a minimal overhead; however, it does make explanations more protracted with certain audiences; it may therefore be simpler to present only the final selected revenue factor.

10 Conclusions

If risk in the pit optimization process is not quantified, it is possible to select an ultimate pit that is incorrectly sized. This has been demonstrated in this paper using commodity price risk as arguably the most obvious risk to consider; however, this is clearly not the only risk relevant to the selection of the ultimate pit.

The results of the VAR analysis can be effectively used to guide the strategic planning process to select a base shell for the design of an ultimate pit that maximizes the potential NPV and minimizes the VAR as a function of the price probability distribution used. When value is maximized, and risk is either ignored or not quantified, the value is likely lower than the stated value and therefore may be potentially misleading.

[2] This is not being advocated.

Strategic decision-making that does not incorporate risk is not very strategic.

Quantifying the commodity price risk in the ultimate pit selection process has demonstrated that making a risk-based decision can reduce the future exposure to expected commodity price variability. The corollary of this is that if risk is not quantified, it is possible to base the pit optimization process and therefore all subsequent strategic decisions on an ultimate pit that materially increases the business risk and, furthermore, to be completely unaware of this fact.

Concluding points are as follows:

- The selection of the ultimate pit should not be made based on financial value alone.
- Using a revenue factor assumption in isolation ignores risk.
- Considering risk in the selection of the ultimate pit can be expected to impact the outcome.
- Assuming that risk has been adequately modelled using a standard discount rate approach is flawed in this application.
- Risk can be endogenously incorporated into a pit optimization process.

Acknowledgements The author wishes to thank Neville Plint, Roger Higgins, Alice Clark, and Daniel Hastings for useful discussions. The author also wishes to thank the operation and the team involved from the site on which the case study in this paper is based.

Funding Open Access funding enabled and organized by CAUL and its Member Institutions.

Declarations

Conflict of interest The author declares no competing interests.

References

1. Lerchs H, Grossmann L (1964) Optimum design of open-pit mines. Joint CORS and ORSA Conference. The Canadian Mining and Metallurgical Society, Montreal, Canada, pp 17–24
2. Hochbaum DS (2008) The pseudoflow algorithm: a new algorithm for the maximum-flow problem. Oper Res 56(4):992–1009
3. Evans, N (2014) Early finish for Resolute mine expansion, in The West Australian. The West Australian: Internet. https://thewest.com.au/business/finance/early-finish-for-resolute-mine-expansion-ng-ya-381489
4. Whittle J, Rozman L (1991) Open pit design in the 90s, in Mining Industry Optimisation Conference MINOPT 91, AusIMM, Editor. AusIMM: Sydney
5. Richmond A (2007) Integrating multiple simulations and mining dilution in open pit optimisation algorithms. In: Dimitrakopoulos R (Ed) Orebody modelling and strategic mine planning - uncertainty and risk management models. The australasian institute of mining and metallurgy
6. Richmond A (2018) Direct net present value open pit optimisation with probabilistic models. In: Advances in Applied Strategic Mine Planning, Dimitrakopoulos R (Ed) Springer, pp 217–228
7. Elkington T, Durham R (2011) Integrated open pit pushback selection and production capacity optimization. J Min Sci 47(2):177–190
8. Breed MF, van Heerden D (2016) Post-pit optimization strategic alignment. J South Afr Inst Min Metall 116(2):109–114
9. Abdel Sabour SA, Dimitrakopoulos RG, Kumral M (2008) Mine design selection under uncertainty. Mining Technology 117(2):53–64
10. Hanson N, Hodson D, Mullins M (2001) Skin analysis in the selection of final pit limits. In: Strategic Mine Planning, A.I.o.M.a. Metallurgy, (Ed) Australasian Institute of Mining and Metallurgy: Perth, Australia
11. Giannini LM, Caccetta L, Kelsey P, Carras S (1991) PITOPTIM: A new high speed network flow technique for optimum pit design facilitating rapid sensitivity analysis. The AusIMM Procedings, pp 57–62
12. Meagher C, Dimitrakopoulos R, Avis D (2014) Optimized open pit mine design, pushbacks and the gap problem—a review. J Min Sci 50(3):508–526
13. Godoy M, Dimitrakopoulos R (2011) A risk quantification framework for strategic mine planning: Method and application. J Min Sci 47(2):235–246
14. Runge IC (1998) Mining economics and strategy. 1998: Society for Mining Metallurgy and Exploration
15. Baek J, Choi Y, Park H-S (2016) Uncertainty representation method for open pit optimization results due to variation in mineral prices. Minerals 6(1):17
16. Robins S (2018) Using grade uncertainty to quantify risk in the ultimate pit design for the sadiola deep sulfide prefeasibility project, Mali, West Africa. In: Advances in Applied Strategic Mine Planning, Dimitrakopoulos R (Ed) The Australasian Institute of Mining and Metallurgy, pp 487–512
17. Smith ML, Nogueira LM (2017) A comparison of dbs and nested pit stage design as a basis for strategic planning. In: Application of Computers & Operations Research in the Mining Industry (APCOM). Colorado School of Mines and the Society for Mining, Metallurgy and Exploration, Inc. (SME): Golden, Colorado USA
18. Damodaran A (2007) Strategic risk taking. Prentice Hall
19. Kumar R (2016) 1 - Perspectives on strategic finance. In: Strategic Financial Management Casebook, Kumar R (Ed) Academic Press pp 1–29
20. Cassidy C, Gizycki M (1997) Measuring traded market risk: value-at-risk and backtesting techniques. Reserve Bank of Australia: Australia
21. Asimov I (1978) My own view. In: The Encyclopedia of Science Fiction, Holdstock R (Ed) Cathay / Octopus Books
22. Holloway EC (2016) Correlated valuation methodology. In: Project Evaluation 2016, T.A.I.o.M.a. Metallurgy (Ed) The Australasian Institute of Mining and Metallurgy: Adelaide, Australia

23. King B (1999) Cash flow grades - scheduling rocks with different throughput characteristics, in Strategic Mine Planning Conference. 1999, Technological Resources Pty Limited: Perth, Australia.
24. ExcelEasy. *Box and Whisker plot*. 2019 [cited 2019 2019–09–03]; Available from: https://www.excel-easy.com/examples/box-whisker-plot.html.
25. Microsoft. *Create a box and whisker chart*. 2023 [cited 2023 17/04/2023]; Available from: https://support.microsoft.com/en-us/office/create-a-box-and-whisker-chart-62f4219f-db4b-4754-aca8-4743f6190f0d.