



Research Article

Reliability analysis of an earth dam in operating conditions using direct coupling

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Abstract

In this study, we present a numerical investigation of the direct coupling between the deterministic GeoStudio package (Seepage/W and Slope/W software) and the StRANd reliability package for normal operating conditions of an existing old earth dam over time. Direct coupling avoids errors associated to point estimate methods and to using response surface surrogates. One relevant feature of the study is the use of realistic pore water pressures for each equilibrium analysis, accomplished by considering a long-term steady-state analysis as an initial condition, followed by four years of equilibrium analysis, before each transient seepage analysis. All reliability analysis were performed using the first-order reliability method. The most important random parameters found in sensitivity analyses are four dam body parameters (saturated hydraulic conductivity, k_s ; specific weight, γ ; effective cohesion, c' ; and friction angle, ϕ') and two filter parameters (k_f and ϕ_f). A range of values of the relationship between the reliability index (β) and the factor of safety (FS) was found for all probabilistic and deterministic results. Finally, a large difference in terms of critical deterministic and probabilistic slip surfaces is identified for the same time of analysis.

Article highlights

- Realistic pore water pressures used in dam equilibrium analysis, by considering random seepage analysis in previous 5 to 30 days.
- Direct coupling of deterministic and reliability softwares avoids errors associated to point estimate and response surface surrogate methods.
- Existing old earth dam studied in Normal Operating Condition.
- For same analysis time, large differences are observed between critical deterministic and probabilistic slip surfaces.

Keywords Earth dam reliability · Reliability analysis · Normal operating case · Transient analysis · Equilibrium analysis

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1 Introduction

A country's economic growth is directly related to the construction of dams. The intended purposes of large dams usually include providing water for irrigation, water supply to cities, improving navigation, generating hydroelectric power and flood control [1]. Few dams [2] serve all these purposes (e.g., Peixotos, Taiaçupeba, Capivari-Cachoeira, Euclides da Cunha Hydroelectric Power Plants in Brazil), but some multi-purpose dams serve more than one (e.g., Itaipu, Belo Monte, Tucuruí, and Santo Antônio Hydroelectric Power Plant in Brazil).

The failure of large dams is a concern in many countries due to the high economic and social consequences associated with it. Zhang et al. [3] present a study of failures in dams from over 50 countries; additionally, Menescal [4] lists 166 cases of accidents and incidents that occurred in Brazil from 1954 to 2009. In a study of a new dam or during the construction, the geotechnical engineer must be able to make reliable predictions about the behavior of the dam, under every anticipated operating condition (e.g., end of construction, first filling of the reservoir, rapid draw-down and normal operating conditions). The predictions usually involve both judgments and quantitative analyses, based on data provided by the site investigation team [2]. All the information about the dam must be available (e.g., topography, hydrology, geology, laboratory and field test, ..., etc.), and additional laboratory or field tests (e.g., geophysics, additional perforations, ..., etc.) may be required to perform a complete study.

The safety evaluation of aging dams is an essential but complex undertaking, especially when the original foundation investigation, dam design and construction details are not known or are associated with significant uncertainty [3, 5]. When information is limited, the real condition of the dam can often only be estimated. Continuous monitoring of the dam by the variation in the water reservoir is an important activity of dam safety management during normal operating conditions (NOC).

In a deterministic approach, the safety condition of a dam is usually verified by Factors of Safety (FS). During the last four decades, different authors [6, 7] have worked to quantify the variability in geotechnical materials and in developing methodologies to perform reliability analysis of slopes and embankments in different conditions [8–11]. Kumar et al. [12] performed reliability analysis in a zoned dam using a deterministic GeoStudio package (Seepage/W and Slope/W software) with two different methods specifically multivariate adaptive regression splines (MARS) and relevance vector machine (RVM). Reliability analysis of earth dams using response surface methodology, in combination with first order reliability method and numerical

analysis was presented by Sivakumar Badu e Srivastava [13]. Additionally, the random field theory is emerged in the field of geotechnical during the last years in contrast to the random variables. Guo et al. [14] performed different types of random fields in a real earth dam. An advanced machine learning algorithm called XGBoost to evaluate the earth dam slope failure probability in spatially variable soils was developed by Wang [15]. Other different studies [16–18] use seepage and stability analysis in dams to perform stochastic analysis with random finite elements. These approaches are different in mechanical terms and it takes more computational effort.

In new dams or dams with less than 5 years of operation, the dam is analyzed from empty to the actual state of the dam. This is possible using available information, and the computational cost is admissible. However, in old dams or dams with more than 5 years of operation (in our case, more than 50 years of operation), the dam is analyzed with a different approach. If monitoring information is available, the best approach is performing a long-term steady-state analysis as the initial condition, followed by some years of equilibrium analysis (in our case, 4 years as a recommendation). The numerical pore water pressures are calibrated based on dam instrumentation readings. The numerical calibration gives reliable results.

In this paper, an existing old earth dam is used as study object in a reliability analysis of NOC. A deterministic software (GeoStudio 2019, [19, 20]) and a structural reliability software (StRANd [21]) are combined to perform reliability analysis of dams. This approach is known as Direct Coupling (DC) in the literature [22, 23].

A relevant feature of the study is the use of realistic pore water pressures for each equilibrium analysis, which is accomplished by considering a long-term steady-state analysis as the initial condition, followed by four years of equilibrium analysis, before each transient seepage analysis.

Recent dam failures in Brazil forced the government to change the laws and make them stricter. Different laws were passed in the last 10 years to regulate different types of dams [24]. These laws require dam owners to perform periodic evaluations of dam stability to check the current general safety state of the dam. The aim here is to provide a tool for monitoring old earth dams, considering deterministic and probabilistic approaches.

Dam safety management using a probabilistic approach represents an enhancement over traditional dam safety practices, through an integrated process that involves quantifying the uncertainties in geotechnical and operational parameters, and yielding a quantitative measure of dam safety: the probability of failure, as a function of time. An existing old earth dam in NOC is the object of study herein.

The remainder of this paper is organized as follows. The problem setting is presented in Sect. 2, which also briefly describes the performance function and reliability analysis techniques employed herein. The complete direct coupling technique for transient analysis and the characteristic of the studied dam are described in Sect. 3. The results of the transient numerical dam equilibrium analysis are presented and discussed in Sect. 4. The concluding remarks are presented in Sect. 5.

2 Problem setting

2.1 Numerical soil conditions

The Richards equation [25] represents the movement of water in unsaturated soils. This equation was obtained by the modification of Darcy's law. The computational software SEEP/W [19] uses this formulation for solving a transient and two-dimensional seepage analysis. In the case of anisotropy, the anisotropy coefficient $r_k = k_x/k_z$ is defined in the function of the horizontal k_x and vertical k_z permeabilities. In the hypothesis of completely saturated soil, the permeability at saturation is assumed to be constant [19, 26].

The Soil Water Retention Curve (SWRC) is fundamental for analyzing the behavior of unsaturated soil in a variety of geotechnical applications [27, 28]. The empirical equation proposed by Van Genuchten [29] was used.

The Mohr–Coulomb criterion was extended by Fredlund et al. [30] which is available in the software Slope/W. Two independent stress state variables, namely net normal stress and soil suction are commonly used to represent the shear strength of unsaturated soils.

2.2 Random seepage analysis and random transient pore water pressures

In transient problems, the equilibrium condition changes as a function of time, mainly due to the behavior of the water reservoir (variation of the water reservoir level). These conditions produce changes in pore water pressures (PWP) which affect the equilibrium condition. In a deterministic study, the safety factor (FS) is understood to change over time, following the variation in the water reservoir level. In a similar way, in a probabilistic study, the probability of failure (P_f) and the reliability index (β) describe the safety of the dam over time. However, the point in time is unknown when the minimum FS , P_f and β occurs. Thus, the deterministic and probabilistic analyses have to be performed over time using a discretized time step (t_k , $k = 0, 1, \dots, K$).

The reliability analysis for limit equilibrium at any time t_k depends on the pore water pressures at that time. However, because some seepage parameters are random, the PWPs at time t_k are random as well. Describing randomness in PWPs is difficult, because of their distribution in space. Alternatively, the correct random PWPs at time t_k can be found by running the random seepage analyses since time zero [31]. This also means running the full random seepage analysis again, from time zero, for the PWPs at time t_{k+1} . This has a significant computational cost, when numerical solutions with many degrees of freedom are computed.

In the Normal Operation Conditions (NOC) studied herein, the reservoir water level changes constantly, going up and down, attenuating large deviations in PWPs, which could result from extreme realizations of volumetric water content or hydraulic conductivity. A practical consequence is that analyzing the whole seepage time-history is not necessary. This is very relevant in the context of this study, as the finite element models employed are very large, and the life of the studied dam is very long.

In this paper, the reliability analyses for limit equilibrium were performed considering different time intervals for the random seepage analysis ($sp = 1, 5, 10$ or 30 days). Hence, the reliability analysis at time t_k starts at time t_{k-sp} . In all cases, the initial condition of the dam is the mean PWP found in the deterministic (mean value) analysis. For $sp = 5$ days or more, no differences were observed in the calculated reliability indexes. Hence, $sp = 5$ days was employed in all the reliability analyses discussed herein. It means that random seepage analysis will be begun 5 days before to the desired day.

2.3 Performance function

To perform structural reliability analysis, it is convenient to describe failure events in terms of performance functions $g(\mathbf{X})$, where $\mathbf{X} = \{X_1, X_2, \dots, X_n\}$ denotes the vector of random input parameters. The limit state function, $g(\mathbf{x}) = 0$, sets the boundary between failure ($\mathbf{x}|g(\mathbf{x}) < 0$) and non-failure ($\mathbf{x}|g(\mathbf{x}) > 0$) states. In slope stability problems, the limit state function is expressed by the following equation [6]:

$$g(\mathbf{X}) = g(X_1, X_2, \dots, X_n) = FS(X_1, X_2, \dots, X_n) - 1.0 \quad (1)$$

where FS is the factor of safety with respect to stability, a dependent variable [32]. The critical surface and FS are calculated for each realization \mathbf{x} during the search for the design point. The probability of failure (P_f) is given by:

$$P_f = P[g(\mathbf{X}) \leq 0] = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

where $f_{\mathbf{X}}(\mathbf{x})$ represents the joint probability density function of the random variable vector \mathbf{X} and the integral is carried out over the failure domain. Analytical solution of Eq. (2) only exists in particular cases of limited practical interest. In most practical applications, approximate methods must be employed, as described in reference works [33–35].

2.4 The first-order reliability method

The First-order Reliability Method (FORM) linearizes $g(\mathbf{X})$ at the so-called design point \mathbf{y}^* in the transformed standard Gaussian \mathbf{Y} -space, by means of a transformation $\mathbf{y} = \mathbf{T}(\mathbf{x})$. The \mathbf{y}^* point is found by solving the following constrained optimization problem: Find \mathbf{y}^* which minimizes $\mathbf{y}^T \mathbf{y}$, subjected to $g(\mathbf{y}) = 0$, where $g(\mathbf{y})$ is the limit-state function in the \mathbf{Y} -space. The smallest distance between \mathbf{y}^* and the origin of the standard Gaussian space is the reliability index: $\beta = \sqrt{(\mathbf{y}^*)^T (\mathbf{y}^*)}$. In FORM, the failure probability is obtained by approximating the limit state by a hyperplane, centered at the design point:

$$P_f \approx \Phi(-\beta) \tag{3}$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function [33].

The Hasofer-Lind Rackwitz-Fiesler (HLRF) algorithm is a very popular algorithm for solving the constrained optimization problem. The HLRF algorithm is employed in this study, but no convergence problems were encountered [33–35]. The gradient vector of the limit-state function is evaluated numerically via progressive finite differences method.

In this study, a coupling between the deterministic and the reliability software was developed. Each time the reliability software requires the evaluation of the limit state function, the deterministic software is called with the current set of input parameters. Then the limit state function is evaluated. This technique is called direct (DC) in the literature.

The vector that indicates the direction of the design point \mathbf{y}^* , called vector of direction cosines, is given by:

$$\alpha = \{ \alpha_i \}_{i=1, \dots, n}^t = \frac{\nabla g_{\mathbf{y}}(\mathbf{y}_i^*)}{\| \nabla g_{\mathbf{y}}(\mathbf{y}_i^*) \|} \tag{4}$$

where $\nabla g_{\mathbf{y}}$ is the gradient of the limit state function with respect to the random variables. From Eq. (4), sensitivity factors α_i^2 are computed: these factors reveal the relative

contribution of each random variable to calculated failure probabilities [33–35].

2.5 Transient analysis using FORM

In transient analysis, the importance of the random variables (factors α_i^2) changes over time. The FORM solution depends on the cumulative effect of uncertain seepage variables up to analysis time t_k . As mentioned before, random seepage analyses were performed for $sp = 5$ days, starting at time t_{k-5} .

In contrast to FORM, in the Monte Carlo Simulation (MCS) each simulation is computed from the initial time up to final time t_k , and the final solution show all the history of PWP. The probabilistic limit equilibrium analysis at time t_k can be performed by sampling stability variables, and by sampling one of the PWPs curves generated in the random seepage analysis at time t_k . This is an advantage over FORM. However, the FORM analysis is still competitive if the preceding seepage analysis is limited to five days. FORM would lose competitiveness if the seepage analyses had to be performed from time zero.

2.6 Review of target reliability index standards

The target reliability indices (β_{target}) for different civil structures (building, bridges, etc.) are defined in various structural design codes, but this is not the case for dams. As an example, evaluation of dam safety in Brazil does not use target safety index by two major arguments: (i) the safety format used is based on safety factors; (ii) authorities do not specify the required or minimum safety levels.

Reliability indices provide a qualitative estimation of a dam performance taking into account uncertainty in loads and materials. Dams are classified by the expected performance level [36]. The target reliability values shown in Table 1 should be used in general.

Table 1 Target reliability indexes for dam stability analysis [36]

Expected performance level	Reliability index (β)	Probability of failure (P_f)
High	5.0	3.0×10^{-7}
Good	4.0	3.0×10^{-5}
Above average	3.0	1.0×10^{-3}
Below average	2.5	6.0×10^{-3}
Poor	2.0	2.3×10^{-2}
Unsatisfactory	1.5	0.07
Hazardous	1.0	0.16

The β_{target} for structural design is determined by calibrating to existing practice [37]. The target safety values of different structures cannot be used for dams. The reasons are as follows: (i) the reliability index is nominal and comparison between other structures (especially by the difference in failure modes and loading conditions) may not be correct (e.g., dams, slopes or foundations with buildings, bridges or power towers), (ii) consequences might not be comparative; (iii) Generally, dam safety risk management have large scale consequences, and this affects societal judgment.

A dam is usually designed abiding to national or international norms. Generally, a numerical model is used with information available from laboratory or field tests in critical and normal conditions of load. Computational results prove whether the modeled structure meets the safety requirements. The code requirement is:

$$\beta_{prior} \geq \beta_{target} \quad (5)$$

For existing dams, the following requirement should be met:

$$\beta_{posterior} \geq \beta_{target} \quad (6)$$

To validate these two statements, different actions may be applied in the existing dam. Ditlevsen and Madsen [38] define three different categories of actions: (i) let the structure without changes, (ii) strengthen the structure; (iii) demolish the structure.

2.7 Relationship between reliability index (β) and factor of safety (FS) for specific dams

Usual safety analysis, monitoring and inspection of dams is based on safety factors. Structural reliability analysis provides a more comprehensive quantitative estimate of dam safety, but it is more complex to evaluate. Hence, it is helpful to have a measure of the relationship between reliability index and safety factors, even if this measure is valid only for very specific conditions. If such a measure can be found, the safety factor can be used as a substitute for the reliability index to monitor the safety of a specific dam.

In this paper, we propose measuring the ratio $R = \beta/FS$ for different operational conditions over time, with other problem parameters remaining unchanged (such as nominal values and probability distributions of geotechnical parameters). After ratio R is computed at a minimum number of points in time, a confidence interval is evaluated for this ratio:

$$R_{lower} \leq E(R) \leq R_{upper} \quad (7)$$

where $E()$ is the expected value operator. The lower and upper bounds are evaluated as:

$$R_{lower} = E(R) - k\sqrt{Var(R)}$$

$$R_{upper} = E(R) + k\sqrt{Var(R)} \quad (8)$$

where $Var()$ is the variance operator, and parameter k yields the desired confidence interval. For $k=2$, for instance, one has a confidence of 95.5% that the actual ratio is contained within the bounds. Now assume that the mean ratio $E(R)$ and the bounds in Eq. (8) have been evaluated, for some normal operational conditions, and the safety factor $FS_{current}$ is evaluated for another normal operational condition. The bounds on the estimated reliability index and the mean estimated reliability index are obtained as:

$$\beta_{lower} = FS_{current} \times R_{lower}$$

$$\beta_{mean} = FS_{current} \times E(R)$$

$$\beta_{upper} = FS_{current} \times R_{upper} \quad (9)$$

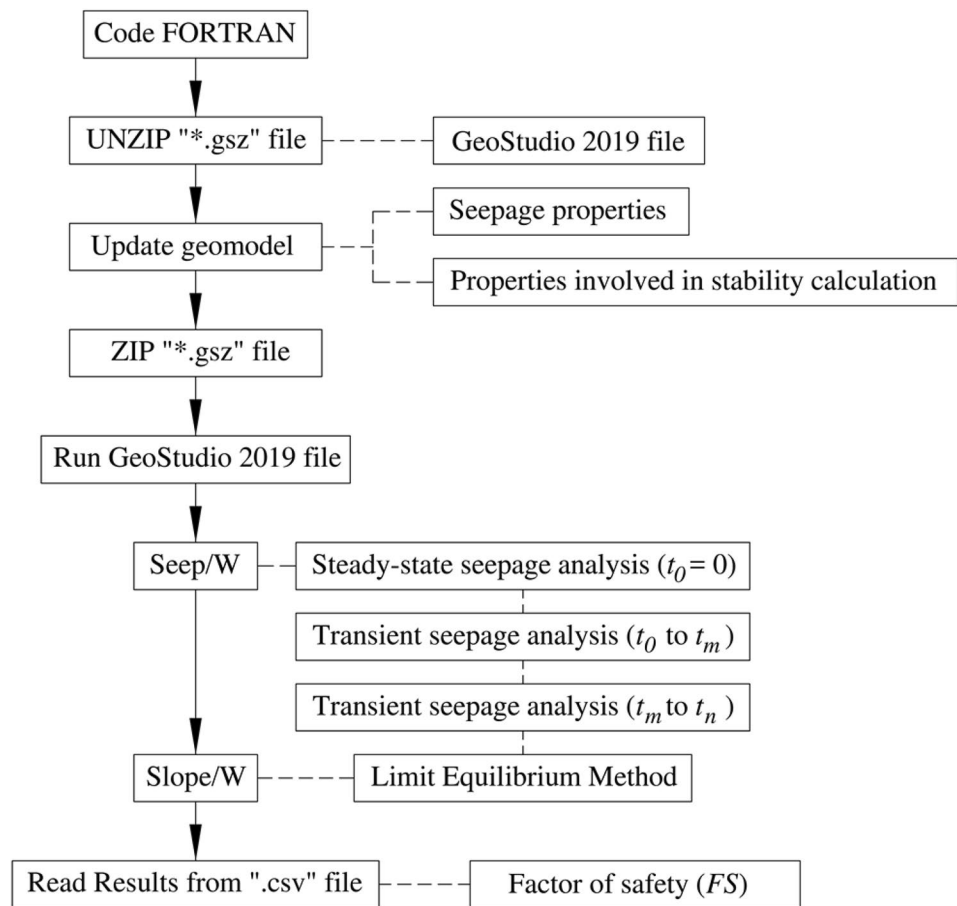
Surely, the ratios and bounds in Eqs. (7) and (8) change for different dams, and for different probabilistic characterization of the seepage and stability properties of the dam. The relations above are assumed to be valid for a single dam in "stationary" normal operating conditions (NOC). Surely, the relations above are not valid in the presence of extreme loading events, such as rapid drawdown, extreme rainfall, degradation/failure of the drains or earthquakes.

3 Methodology

3.1 Probabilistic modeling

The deterministic GeoStudio 2019 [19, 20] software and the probabilistic StRAnD 1.07 [21] software were implemented using direct coupling to perform deterministic and probabilistic calculations. An earth dam in long-term steady-state and during a rapid drawdown was discussed by Siacara et al. [31, 39]. In this research, an earth dam in normal operating conditions (NOC) was evaluated. The deterministic limit equilibrium method (LEM) (e.g. Morgenstern and Price) was used to find the factor of safety (FS). The reliability index (β) is evaluated in a specific time or in a time frame using structural reliability methods (e.g. FORM). The procedure implemented is described in this section. The flowchart shown by Siacara et al. [31, 39], and the FORTRAN code were modified as shown in Fig. 1:

Fig. 1 Flowchart for the transient reliability analysis in NOC by coupling GeoStudio/StRANd software packages (based on [31, 39])



- (1) Initial data of the problem: studies of topography, geology, geotechnical (laboratory and field test), hydraulic and hydrological.
- (2) Initial conditions of the model: A two-dimensional (2D) model in the Seep/W software (GeoStudio 2019 package) is used. The initial boundary conditions (e.g., analysis with the maximum water level of the dam) are defining to perform seepage analysis in a long-term steady-state. Then, the pore water pressure (PWP) of the computational model is defined (Fig. 2).
- (3) Transient conditions of the model—Equilibrium time: in the second step in Seep/W, an equilibrium time interval is defined to perform transient analysis. The boundary conditions are in function of time (e.g. long time of the reservoir in normal operating conditions). The main purpose of these steps is to find the correct PWP (equilibrium of the PWP) by the transient analysis taking into account the variation in the reservoir (measurements in the field). The equilibrium time of the analysis (t_m) are defined, and the number of steps (m) and time increment (Δt) are configured from t_m (Fig. 2).
- (4) Transient conditions of the model—Analysis time: The third step in Seep/W is defining an interval of time, which is a sequence of the last step. The boundary conditions are in function of the time (e.g. normal operating conditions). In every time increment (Δt), the phreatic surface and PWP of the model are found. The time of the analysis (t_n) is defined, and the number of steps (n) and Δt are configured from t_n (Fig. 2).
- (5) Stability conditions of the model: A 2D model in the Slope/W software (GeoStudio 2019 package) is used. The Slope/W employs the LEM to perform slope stability analysis. The factor of safety (FS) is found for every Δt of the seepage analysis or at a specific time of the NOC. The deterministic critical condition or minimum safety of the structure is represented by the critical factor of safety (FS_{cr}) during the critical time (t_{cr-d}) (Fig. 2).
- (6) Define the probabilistic conditions of the model: the STRAND_INPUT.txt file of the StRANd 2.00 software is used to define reliability methods, random materials and correlations when it is necessary.

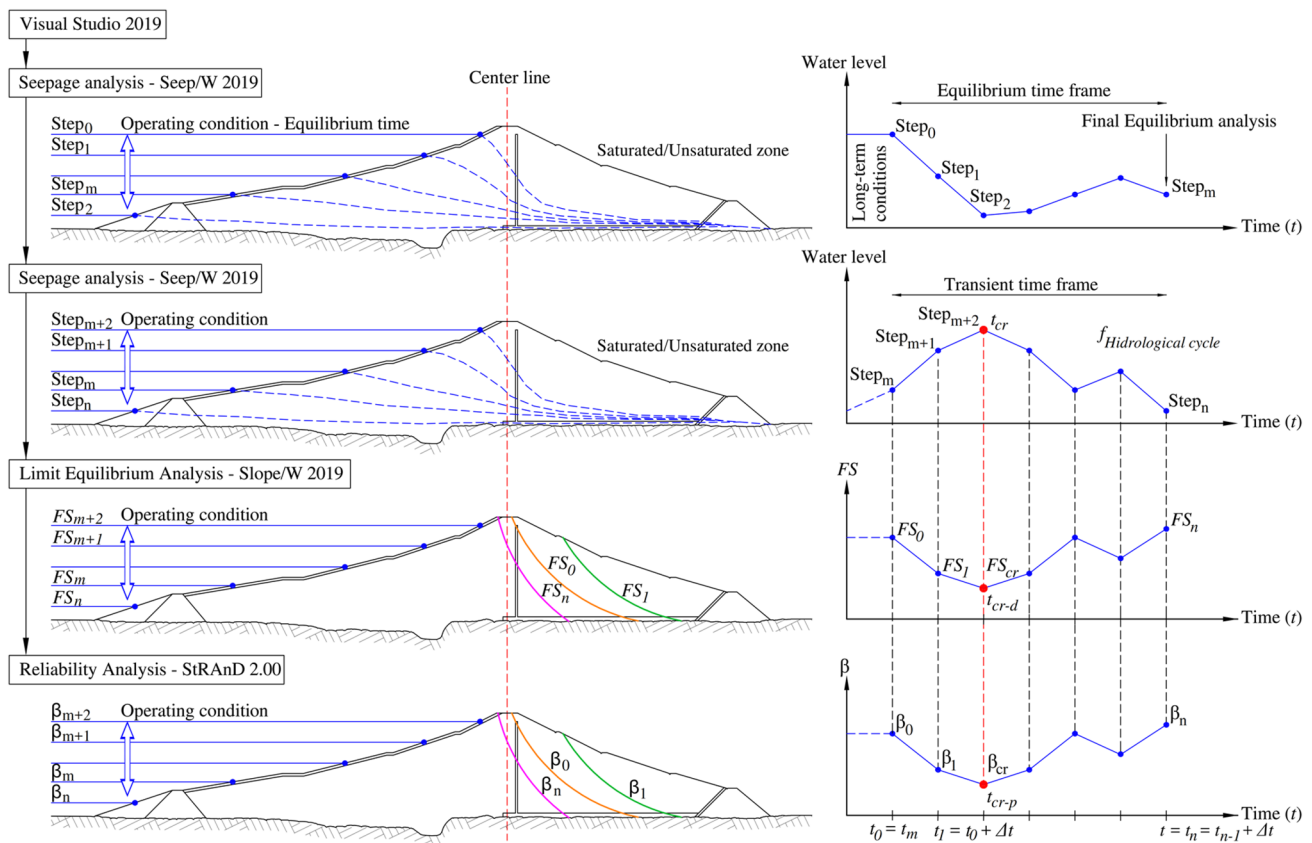


Fig. 2 Schematic procedure to perform deterministic and probabilistic analysis in long-term steady-state and transient conditions

- (7) Define deterministic (nominal) material properties for the seepage analysis and LEM.
- (8) Define the probabilistic conditions for geotechnical material properties.
- (9) Key program: all the analysis (deterministic and probabilistic) described in the flowchart are performed from the software Visual Studio 2019 compiler, which have the FORTRAN code as a language of programming.
- (10) Characteristics of the GeoStudio 2019 file: The file has a format ".gsz" which is a ZIP file that contains different input/output files. All input parameters are changed in the file extension ".xml", which is changed in every simulation or during a search for the design point (DP). The input parameters are organized/changed in the original position of the ".xml" file for every simulation. For saving output results, file ".gsz" is compressed/uncompressed from the 7-zip software saving the FS to find the DP. All the output results (e.g., FS, PWP, phreatic surface, ..., etc.) are found in file extension ".csv" for every step of time.
- (11) Probabilistic results of the model: the STRAND_OUTPUT.txt file of the StRAnD 2.00 software contains all the output results of the reliability analysis (number of evaluations and simulations, sensitivity coefficients at DP, reliability index, probability of failure, evaluation time, DP, ..., etc.). Visual results and additional information (e.g., pore water pressures in specific coordinates or critical slip surfaces) of the reliability results are found in the last ".gsz" file of the analysis. The probabilistic critical condition or minimum safety of the structure is represented by critical reliability index (β_{cr}) during the critical time (t_{cr-p}) (Fig. 2).

The FS is found from m to n steps of time or at a specific time of the transient analysis. From the FS, the StRAnD software performs the reliability analysis in both options of time (Fig. 2).

3.2 The application-problem and boundary conditions

3.2.1 Earth dam of the study

The application of the probabilistic dam slope and flow analysis methodologies described earlier in the paper are illustrated in the analysis of a well-documented Brazilian dam. The dam is not identified due to privacy concerns. The reservoir filling was started in July 1970, and the construction of the dam was finished in August 1971. The dam has about 50 years of normal operation conditions (NOC), and it is classified as an old earth dam. The following study is an academic one; therefore, all the conclusions are only suggestions to the company in charge of the dam.

The dam was subject of extensive studies quantifying the soil properties from instrumentation, field and laboratory tests. However, we were not given access to measured data; hence, we could not evaluate the actual variability of the soil properties. These data were taken from the literature, but adjusted to the actual mean values provided in previous studies of the dam.

The earth dam is a dam of homogeneous type with different elements. The dam has a horizontal (approximately 1.8 m in height) and vertical filter (approximately 1.0 m in thickness) to reduce the pore water pressure within the dam, conducting the water downstream.

As the foundations presented good quality and favorable conditions for implementing the project, the concrete cut-off and the injection curtain initially foreseen in the project were not executed. Nevertheless, the dam counts

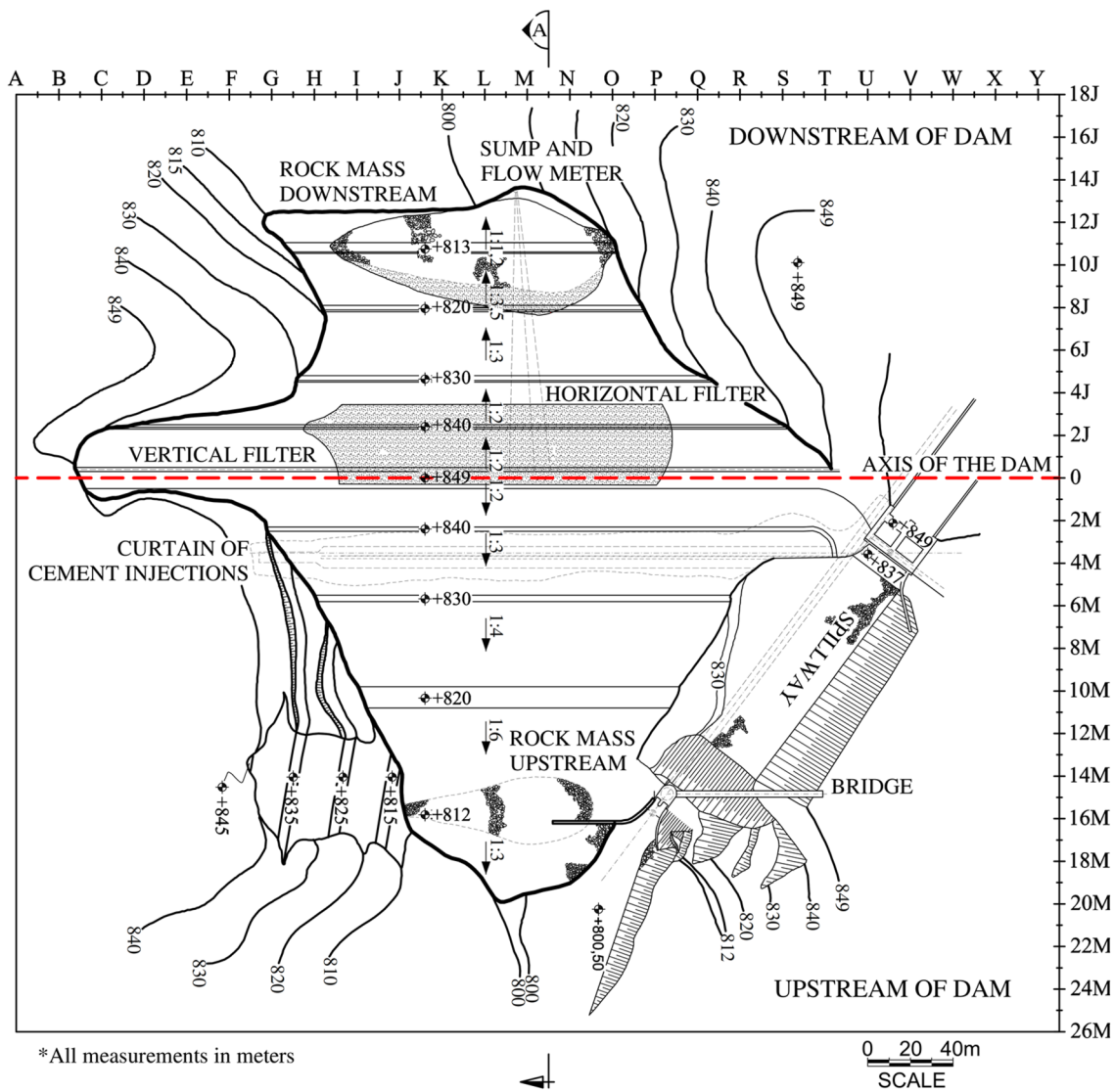


Fig. 3 The plan view and sections of the dam

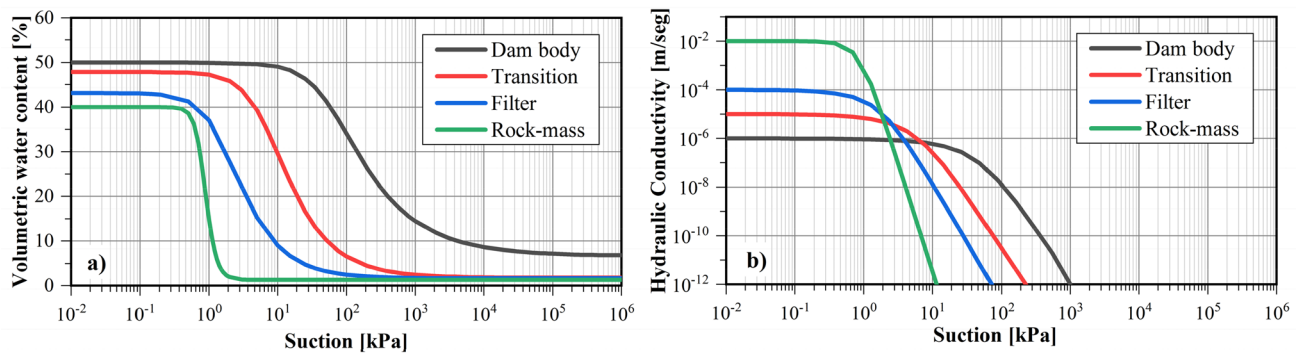


Fig. 5 Mean value of hydraulic parameters **a** SWRC, **b** hydraulic conductivity curve

The dam body consists of silty-clays, the filter of sands, and the rip-rap and rock mass for protecting the fractured rock. For avoiding computational issues by differences in hydraulic conductivity, a transitional material of 5 cm was used between the dam body and filter. The random seepage analysis is characterized by the uncertainty of two SWRC fitting parameters (a , n), the saturated and residual volumetric water contents (θ_s , θ_r), and the saturated hydraulic conductivity parameter (k_s). The random stability analysis is characterized by uncertain specific weight (γ), effective cohesion (c'), effective friction angle (ϕ') and the angle that increases shear strength (ϕ^b). The uncertain parameters of the stability and seepage analyses are assumed to have normal (N) or lognormal (LN) distributions. The mean (μ) and the coefficient of variation (COV) of all parameters are shown in Table 2. In terms of COV of soil parameters, Phoon [16] reported different range of values and Siacara et al. [31, 39] applied it in earth dams.

The SWRC and hydraulic conductivity curves corresponding to the mean values in Table 2, are shown in Fig. 5a, b, respectively.

The following remarks are made about the COV values and r_k ratio in Table 2:

1. The COV of k_s , θ_s , θ_r , a , n , γ , c' , ϕ' and ϕ^b for all the different soils variations was compilation of Siacara et al. [39].
2. The r_k ratio for existing dams varies from 1 to 15 [26, 46–48], Cruz [2] suggests a value of 1 for non-cohesive soils and from 4 to 10 for cohesive soils. In compacted earth fills, the r_k ratio may exceed 20 [49]. Fell et al. [50] and UCACE [51] give r_k ratios in the order of 1 to 100, covering the possible range of expected field conditions. A common value of 3 for r_k ratio is suggested by Verbrugge and Schroeder [52]. For compacted soils, the r_k ratio for non-cohesive soils varies from 1 to 40, and for cohesive soils from 0.4 to 4.1 [53]. From laboratory tests, the r_k ratio varies from 1 to 4.1 [53]. Leroueil

et al. [54] found r_k ratios from 1 to 1.4 in compacted soils in laboratory, and Smith and Konrad [55] found an r_k ratio of 5 using geostatistical analysis of construction control data from the core of an earth dam.

Concerning anisotropy, the data available did not allow using a random characterization of the anisotropy coefficients of the materials. The anisotropic coefficient (r_k) was considered a constant for the different materials. The r_k ratio was assumed as 5 for dam body and transition, and 1 for filter and rock-mass, according to the literature on cohesive and non-cohesive soils (Table 2).

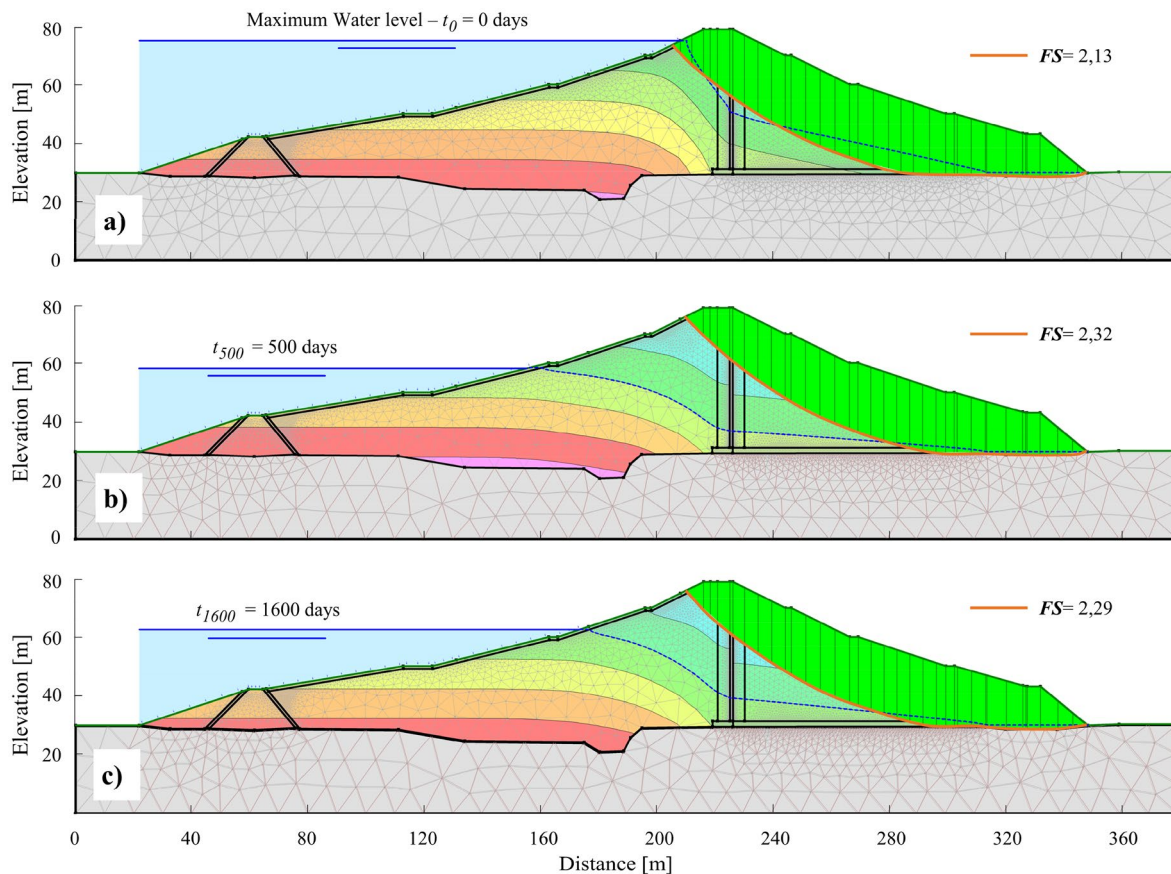
The USBR [56] indicates that typical dams will have r_k ratios ranging from 2 to 10, with higher values relating to higher water contents during placement. Older dams, such as those constructed in the early twentieth century or by hydraulic fill methods, may have anisotropy as high as 50 due to stratification during placement and earlier compaction methods that did not emphasize mixing and discing. However, coarse-grained materials, such as rockfill shells or filter and drain materials, are typically placed in thicker lifts without as much compactive effort, and they tend to have lower anisotropy. These types of soils are often assigned anisotropy values of 1.

3.3 Initial mean value analysis

Seepage and stability analyses were performed in the GeoStudio software (Seep/W and Slope/W), for normal operating conditions (NOC), to obtain deterministic (mean value) results of pore water pressure (PWP) and factor of safety (FS) for the most critical slip surface. The mean value (μ) of seepage properties (k_s , θ_s , θ_r , a and n) and properties involved in stability calculation (γ , c' , ϕ' and ϕ^b), presented in Table 2, were used in this analysis. A two-dimensional analysis is performed herein. Table 3 shows the duration in days of the different analyses performed.

Table 3 Duration of the different analyses performed herein

N	Step of analysis	Type of analysis	Initial time (days)	Final time (days)	Duration
1	Initial condition	Long term Steady-state			
2	Equilibrium time	Transient	0	1460	1460 (4 years)
3	Time of analysis	Transient	1460	1825	365 (1 year)
	Time period	A Transient	1466	1481	15 days
		B Transient	1600	1615	15 days
		C Transient	1645	1660	15 days
		D Transient	1810	1825	15 days

**Fig. 6** Critical slip surfaces for the dam: **a** $t=0$ days, **b** $t=500$ days and **c** $t=1600$ days

The initial PWP condition is the key input information for transient seepage analysis in an earth dam, and it can be difficult to determine. The following methodology used herein will help to define the correct initial conditions.

The first step (Table 3), an initial long term steady-state analysis was performed to determine the PWP for maximum water level conditions (845.0 m.a.s.l. or 75 m for numerical purpose), which is assumed at initial time ($t=0$ days). The events described in Table 3 correspond to the actual normal operational conditions for the dam,

following the official information available. The nomenclature of events states event duration in days (e.g., $t=570$ days represents 570 days of analysis). This nomenclature corresponds to, but is independent of dam age. The height of the dam will be taken from the height of the numerical model in meters (m) and not from the meters above sea level (m.a.s.l.).

The initial seepage analysis ($t=0$ days) yields the phreatic surface, PWP and total water head as shown in Fig. 6a. In this study, the dam stability is improved by negative

PWPs effects. The boundary between negative and positive PWPs (PWP = 0 kPa) is known as phreatic surface.

The dam was modeled using triangular elements with of approximate absolute size of 5 m. The discretization of the mesh was according to the size of the element and the importance of the element in the analysis. A total of 33,825 nodes and 67,232 nodes were automatically created by the mesh generator available in Seep/W.

Taking into account flow measurements during construction, the infiltration trough the dam foundation was estimated at around 1.6 l/s, which, when increased by external contribution, has led to values of the order of 3.6 l/s. However, the real value was about three times higher than the one initially estimated. These differences eventually impacted the performance of the horizontal filter. The water level was found to be above the horizontal filter, and the downstream PWP was very different. This was verified with historical field measurements [57, 58].

The numerical model employed herein was calibrated taking into account field measurements and instrumentation. The parameter used to calibrate the pore water pressures (PWP) is the saturated hydraulic conductivity (k_s) of the four materials of the dam. Real measurements of PWP are compared with the PWP of the numerical seepage analysis. The comparison was performed over time (150 continuous days) using different field instrumentations

(different locations) inside the dam. The calibration was realized to find the most similar result between the field measurements and numerical results. The calibrated k_s values were used in the numerical analysis, which yields representative results of real PWPs.

The second step (Table 3), the transient analysis (equilibrium time) is the variation of the reservoir (oscillation between 845.0 m.a.s.l. or 75 m and 827.0 m.a.s.l. or 57 m) over four years (from $t=0$ days to $t=1460$ days) to find the PWP equilibrium of the dam. The oscillation of the water reservoir is a variation from hydrological conditions, and by user demand request.

The behavior of the reservoir changes every day (there are always some water level variations), and the dissipation time of PWP is insufficient. In a transient analysis, the seepage analysis is performed to define the phreatic surface, PWP and total water head in every time step (in this case, one day). Figure 6b shows the seepage results for $t=500$ days.

The third step (Table 3), the transient analysis is the variation in the reservoir (oscillation between 844.0 m.a.s.l. or 74 m and 831.0 m.a.s.l. or 61 m) over one year (from $t=1460$ days to $t=1825$ days). During this step, the earth dam is considered to be in equilibrium, and PWP values are more realistic. Figure 6c shows the seepage results for $t=1600$ days.

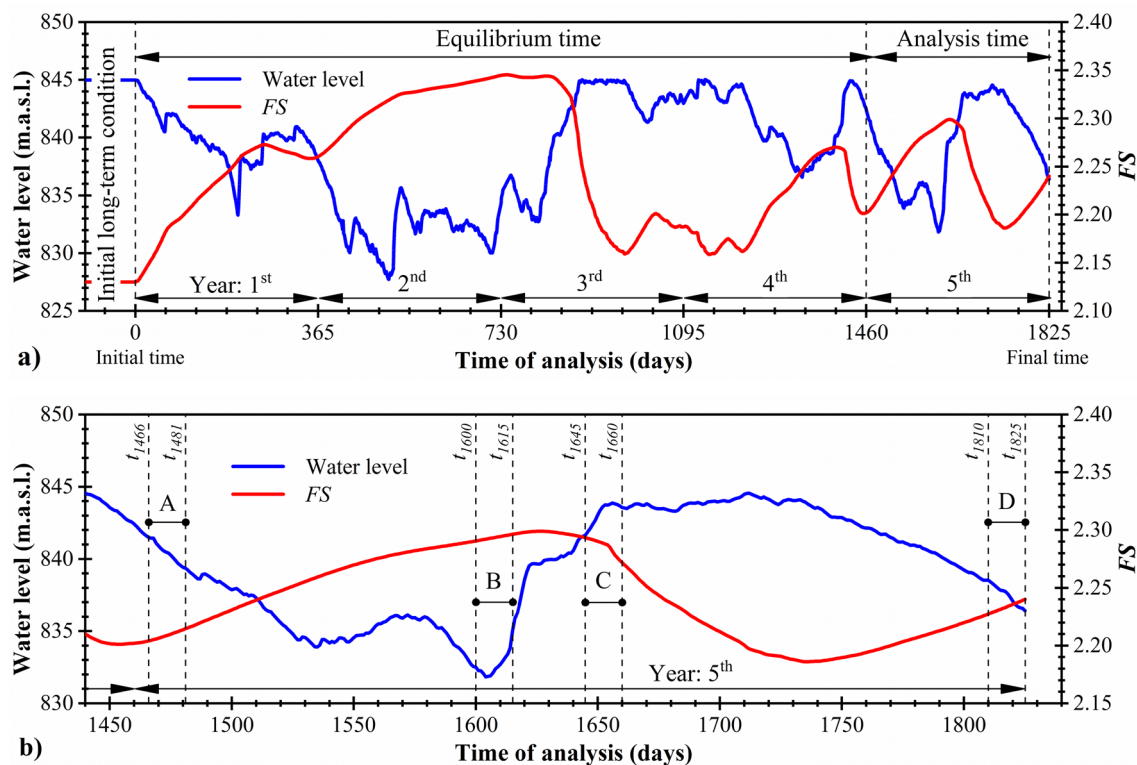


Fig. 7 Results of the limit equilibrium analysis for different times of analysis: **a** equilibrium + analysis time; **b** zoom at analysis time

The stability of the dam is calculated from the downstream slope using the results of the three seepage steps mentioned above. The LEM is used for all stability calculations following the Morgenstern and Price's procedure. In every step of time, the seepage results (phreatic surface, PWP and total water head) are used to find the most critical slip surface. This is automatically found by the entry and exit specification technique (an indepth explanation is found in GeoStudio [19, 20]). An initial configuration is defined by an 1.2 m discretization of the entry and exit slip surfaces, and radius tangent lines in the slip direction. The finite element mesh of seepage analysis, deterministic critical slip surface and the minimum factors of safety (FS) of the Slope/W software results are shown in Fig. 6a–c for times $t=0$, $t=500$ and $t=1600$ days, respectively.

The variation of the water reservoir level as function of time produces different FS and critical surfaces for every time step, as shown in Fig. 7a, b. In Seep/W, an interpolated PWP was used to define the phreatic surface. All the slip surfaces found in Slope/W must be contained in the domain of the numerical model. If a slip surface follows the boundary, the strength parameters are taken from the materials overlying the base of individual slices. The initial condition of stability ($t_0=0$ days) yielded the initial safety factor as $FS_0=2.13$. During the equilibrium analysis (from $t=0$ to $t=1460$ days), the oscillation of FS is between $FS_{max}=2.34$ and $FS_{cr}=2.13$. During transient analysis time (from $t=1460$ to $t=1825$ days), the oscillation of FS is between $FS_{max}=2.30$ and $FS_{cr}=2.18$. Although the maximum water level is reached several times, the critical FS is found for $t=0$ days, because the rise and fall velocity of the reservoir during NOC is not very large.

The mechanics behind the transient seepage analyses were developed more than 80 years ago. However, it is difficult to put the mechanics into immediate practical use owing to the large number of numerical calculations required. In recent years, transient seepage analysis programs, such as SEEP2D1, SEEP/W, and SLIDE, allowed these analyses to be conducted by personal computers in relatively modest execution times [59]. In this study, we consider unsaturated soils and employ long term steady-state analysis as an initial condition, followed by four years of equilibrium analysis, to find more realistic PWP before the actual transient analysis.

A minimum $FS=1.5$ for long-term steady-state analysis and $FS=1.1$ to 1.3 for rapid drawdown analysis in earth dams are recommend by different authors [2, 50, 60] and specialized organizations [61–65]. In this study, all the FSs of deterministic analyses meet these minimum stability criterions.

4 Reliability analyses and results

Reliability analyses for dam equilibrium were performed at the four time periods identified as A, B, C and D Fig. 7 and Table 3. (between $t=1460$ and $t=1825$ days). The direct coupling (DC) between GeoStudio/StRANd softwares was used to perform reliability analysis. The FORM method was performed using the HLRF algorithm to search for the design point (DP).

In the reliability analysis, the random seepage and stability properties (see Table 2) were considered for every material of the dam. The results are presented as follows:

- (1) Preliminary screening of random variable importance in NOC;
- (2) Normal operating conditions at time periods A, B, C and D;
- (3) Differences of pore water pressures;
- (4) Differences of critical surfaces;
- (5) Sensitivity of the random variables w.r.t. seepage and equilibrium analyses.

4.1 Preliminary screening of random variable importance in NOC

An advantage of FORM is the possibility of carrying out a sensitivity analysis through the direction cosines (α^2) at the Design Point (DP). The relative contribution of each random variable used in a reliability analysis is measured by α^2 . Large and low α^2 values represent the most important and irrelevant variables, respectively.

Initially, a reliability analysis was performed with 34 variables (Table 2) of the earth dam, for four different times. These results (not shown here) reveal that, out of 34 initial random variables, only four dam body parameters (k_{sat} , γ , c' and ϕ') and two filter parameters (k_{sat} and ϕ') are important, with individual contributions to reliability analysis greater than 0.1%. The other 28 random variables resulted in nearly zero sensitivity coefficients ($\alpha^2 \approx 0$); hence, these random variables have negligible contributions to the computed failure probabilities and were considered deterministic in the remaining analyses.

The numerical reliability analyses were performed by direct coupling, using a dual core workstation computer, with processor speed of 2.1 GHz, and RAM memory of 64 GB. A complete transient reliability analysis with 34 random variables took about 320 h to compute (with total analysis time of sixteen days and discretization of one day). Each day of transient analysis took about 20 h to compute. With the reduction to the six most important random variables, the same transient reliability analysis was reduced to 96 h, with each day of transient analysis reduced to

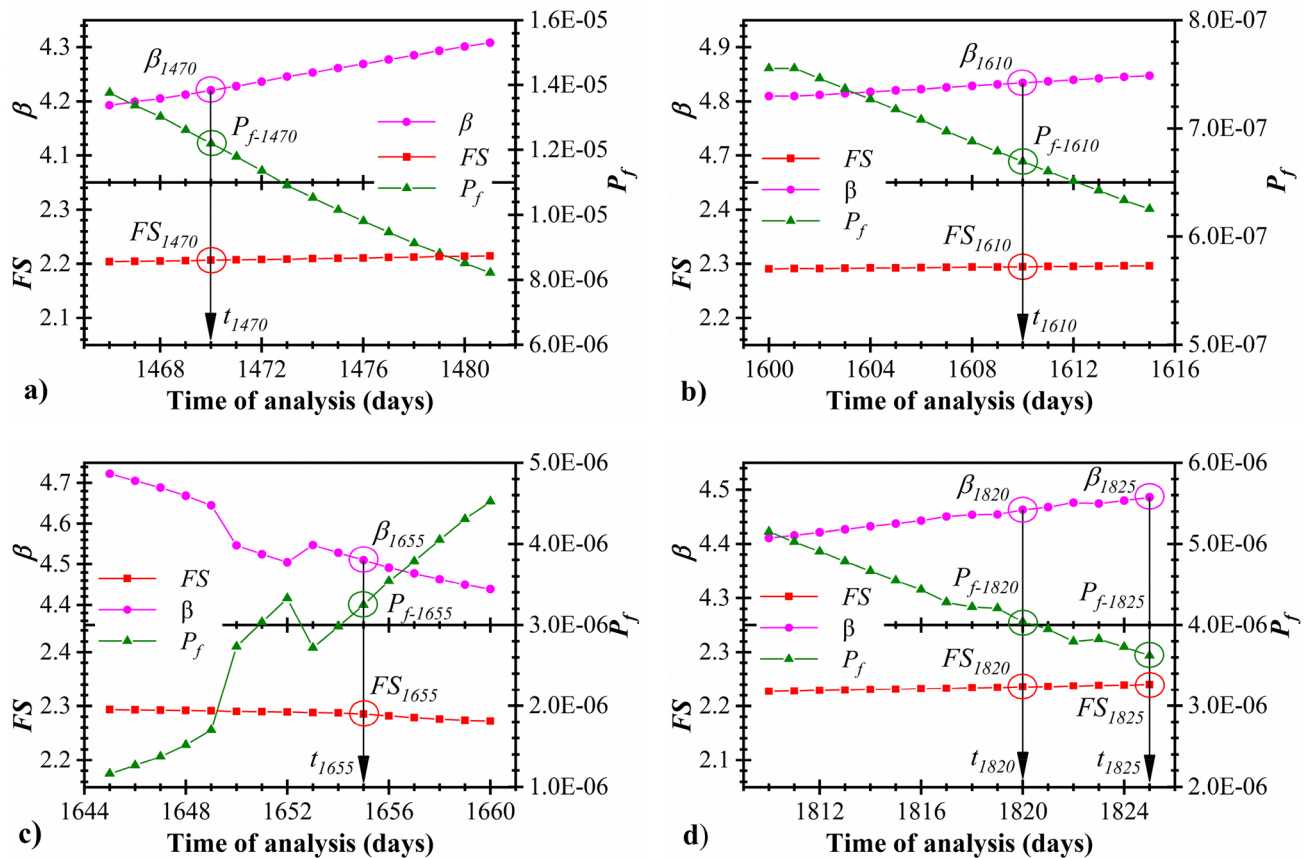


Fig. 8 Safety factors, reliability index and failure probabilities for time periods: a A, b B, c C and d D

about 6 h. This reveals the importance of pre-screening the importance of random variables, and eliminating those of irrelevant contribution. Further details on sensitivity coefficients of the remaining variables are presented in Sect. 4.5.

4.2 Analysis for normal operating conditions (NOC)

During normal operating condition (NOC), we can obtain both deterministic FS and probabilistic β of the unsaturated earth dam. Figure 7 shows the variation in water levels over the analysis time, as well as the safety factors found in the deterministic (mean value) analysis. The reliability analysis is performed assuming the reservoir water level known at the end of each day; the end condition of one day is the starting condition for the next day. In dam practice, water levels at the end of any day are well known, by way of manual measurements (ruler inserted into water reservoir), hydraulic installations (capacity of spillways) or water demand (electricity generation or water supply). The reliability analysis is performed using the FORM method in the periods A (from $t = 1466$ to $t = 1481$ days), B (from $t = 1600$ to $t = 1615$ days), C (from $t = 1645$ to $t = 1660$ days)

and D (from $t = 1810$ to $t = 1825$ days), as shown in Figs. 10 and 11.

The four time periods of analysis (A to D) have lengths of 15 days (Table 3). In NOC analysis, the FS and β follow similar trends for the same step of time (Fig. 8). Time periods A, B and D show a negative tendency of the P_f curve, but period C has a positive tendency. The FS and β curves have an opposite behavior to the P_f curves for the four periods of analysis, as expected. The determination of FS , β and P_f for every day of analysis reduces uncertainty regarding the behavior of the dam (e.g., $t = 1470$, 1610, 1655 and 1820 days), as shown in Fig. 8.

Figure 8d shows two results for specific times ($t = 1820$ and 1825 days) with a difference of 5 days and a 0.66 m drop of the reservoir. The difference between results are these two points in time, $\Delta FS = 4.3 \times 10^{-3}$, $\Delta \beta = 2.33 \times 10^{-2}$ and $\Delta P_f = 4.17 \times 10^{-7}$. These variations are important to prevent occasional unsafe behaviors.

In this manuscript, rapid drawdown is not considered part of normal operating conditions. Although rapid drawdown is typically considered the most critical equilibrium situation, the normal operating conditions could present other critical situations. An example is

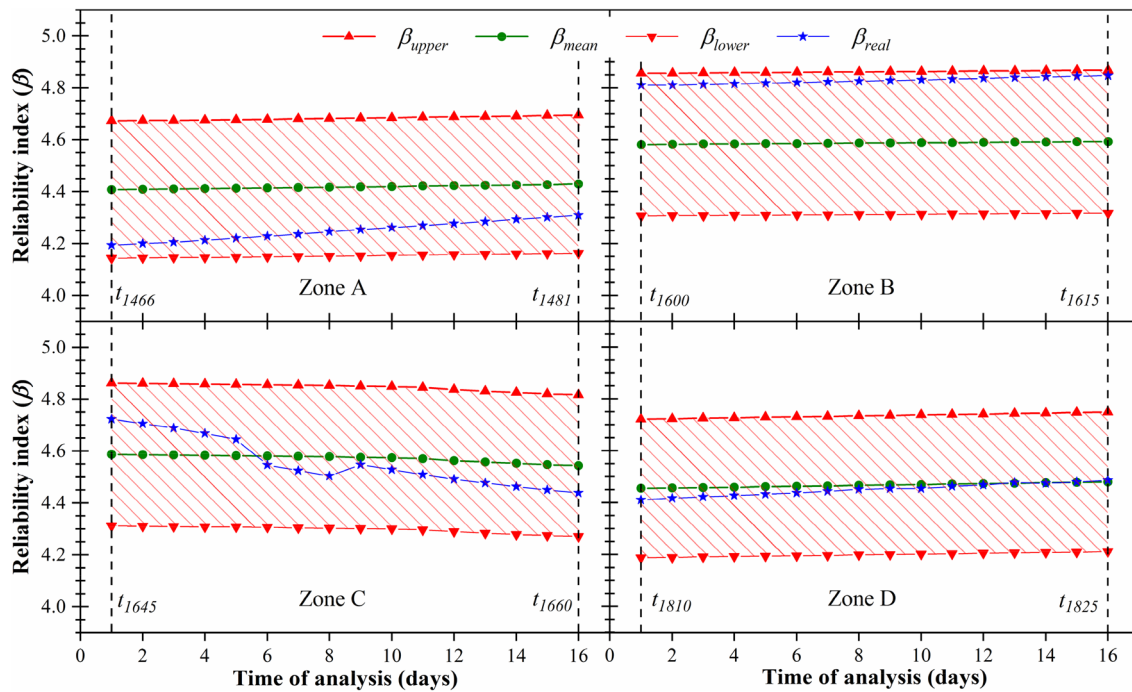


Fig. 9 Estimation of reliability index (β) from SF and bounds of ratio $R = \beta/FS$

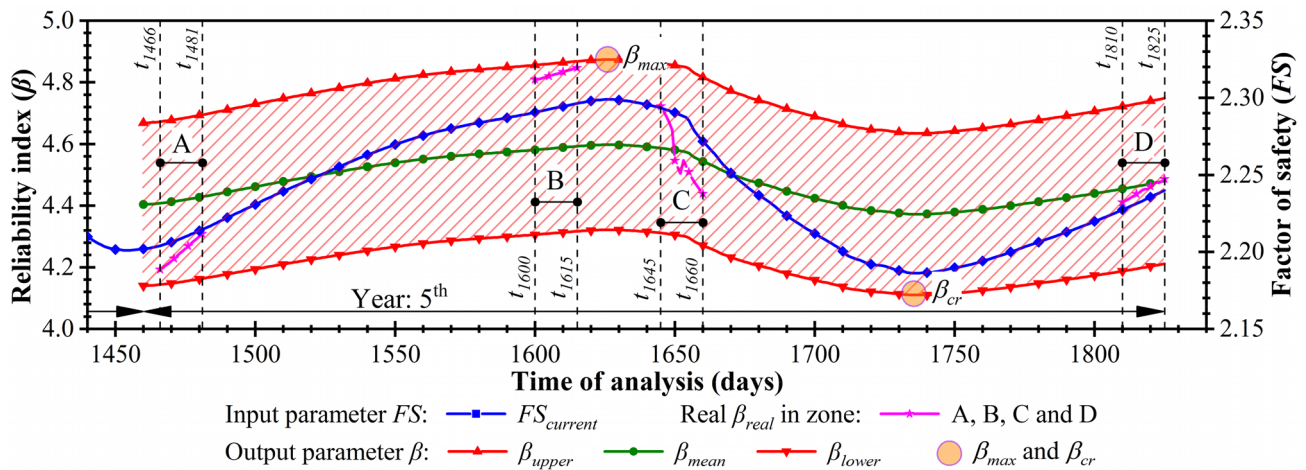


Fig. 10 Estimation of the reliability index (β) for the analysis time

pore water pressure increases in the old dam by continuous rainfall and/or decrease of the efficiency of the dam filter. These situations lead to critical conditions of the dam, which can be detected using constant monitoring, or can be predicted using reliability analysis, as demonstrated herein.

The methodology developed herein targets the long term ongoing safety analysis of dams, which is based on combining transient seepage analysis with equilibrium

analysis, and in relating safety factor changes with changes in failure probabilities.

Figure 8 shows how FS , β and P_f curves change during the normal operating conditions. As stated in Sect. 2.9, it may be possible to estimate reliability index β from the observed ratios $R = \beta/FS$ of for this dam and for the random parameters considered in Table 2. Based on the results in Fig. 8, and using Eqs. (7) and (8), the ratios were found as $R_{upper} = 2.12$, $E(R) = 2.00$ and $R_{lower} = 1.88$. Based on these ratios, and on the FS values computed for the

whole analysis period, the reliability index bounds were computed. The results are first compared for the four time periods (A, B, C and D) for which actual β values are known (Fig. 9). In Fig. 9, actual β values are observed to indeed be within the bounds given by Eqs. (7) and (8). This could be expected, since the ratios above were computed in the same time periods (A, B, C and D).

The usefulness of the proposed bounds can be appreciated when observing the whole (deterministic) analysis time, in Fig. 10. In this figure, the bounds are employed to estimate the maximum and minimum values of β for the whole interval, including the times between time periods A, B, C and D. From these bounds, it is estimated that the maximum and minimum (critical) values of β are $\beta_{max}=4.87$ at $t=1626$ days, and $\beta_{cr}=4.11$ at $t=1735$ days. Hence, this initial screening can be used to perform a reliability analysis at the most critical section, around time $t=1735$ days. This was done herein, after identifying $t=1735$ as the critical time. The actual reliability analysis at time $t=1735$ resulted in $\beta_{real}=4.07$; a difference of only $\Delta\beta=0.04$ to the estimated value. This confirms, by observation, that the bounds proposed in Eqs. (7) and (8) can be used for screening the most critical section in NOC. Note that $t=1735$ is critical because it is the end of a longer period for which the reservoir had been full or nearly full; hence, pore water pressures were high in a significant part of the dam.

Considering the dam performance levels listed in Table 1 [36] and the reliability index values computed herein, the analyzed dam is classified as between Good and High. This includes the estimated critical value β_{cr} .

Equations (7) and (8) are alternatives to estimate β from the FS as is shown in Fig. 10. Although this approach helps geotechnical engineers to estimate reliability values from deterministic results, these equations not avoid to perform reliability analysis. The observed ratios $R=\beta/FS$ of every dam can be improved in accuracy with more results from the reliability analysis, and a calibration with a high number of results gives more reliable estimation. These ratios R can be defined during the first years of NOC of the dam, and it can help during the safety monitoring of a dam. The principal advantage of using these ratios R are performing deterministic transient analysis to estimate the reliability results in function of time, avoiding more computational efforts. During NOC , the estimation of β is helpful (e.g. a dam during large increases or decreases of the reservoir; the state of the dam is known immediately). Factors of safety and reliability index complement each other and hence both are more useful than knowing either one alone.

4.3 Differences of pore water pressures

Differences between deterministic and probabilistic analyses in terms of seepage and stability are further investigated. In every step of time t_k , different PWP, phreatic and failure surfaces are illustrated over the dam. The results of the analyses (deterministic and probabilistic) are illustrated in the GeoStudio software. Deterministic results are represented by the mean values of the geotechnical variables (Table 2) and probabilistic results are represented by the DP values of the same geotechnical variables.

In Fig. 11, the deterministic and probabilistic PWP of the seepage analysis are compared, for times $t=1604$ and $t=1656$ days. The critical equilibrium situation occurs for higher PWP; therefore, the probabilistic PWP in the DP are higher than the deterministic PWP. The variation in PWP depends on the time period of analysis, and on the velocity of water level change. The differences between results in Fig. 14a, b are mainly explained by use of mean value of k_s (1.0×10^{-6} m/s) and the design point value of k_s (0.84×10^{-6} m/s for $t=1604$ and 1.24×10^{-6} m/s for $t=1656$ days). This difference is estimated to be larger in time period C.

Two cross-sections identified in Fig. 11 (A–A' and B–B') are presented in Fig. 12 to compare the PWP at times $t=1604$ and $t=1656$ days. The points T and U identified in Fig. 11 are represented in Fig. 13, where the PWP are shown in every time step of time periods B and C. Significant differences are observed between deterministic and probabilistic PWP in time periods B and C over time. Hence, the differences between deterministic and probabilistic analyses can be explained by the effects of seepage in the days preceding the equilibrium analysis, and dissipation of PWP in water level falls.

4.4 Differences of critical surfaces

A comparison of the corresponding critical deterministic and probabilistic slip surfaces, for times $t=1604$ and $t=1656$ days, is presented in Fig. 14a, b, respectively. The geometry of the deterministic critical slip surface does not change significantly as function of time, as seen in Fig. 6. The critical deterministic slip surface is located between the rock foundation and the rock mass downstream.

The reliability analysis using FORM involves a search for the design point (DP). When the random variables assume the values corresponding to the DP, the slip surface with the highest probability of occurrence is obtained at every time step. This is called the probabilistic slip surface.

As observed in Figs. 14 and 15, there are subtle differences between the PWP and phreatic surfaces observed in the deterministic and probabilistic analyses. As discussed in the next session, this is a consequence of the

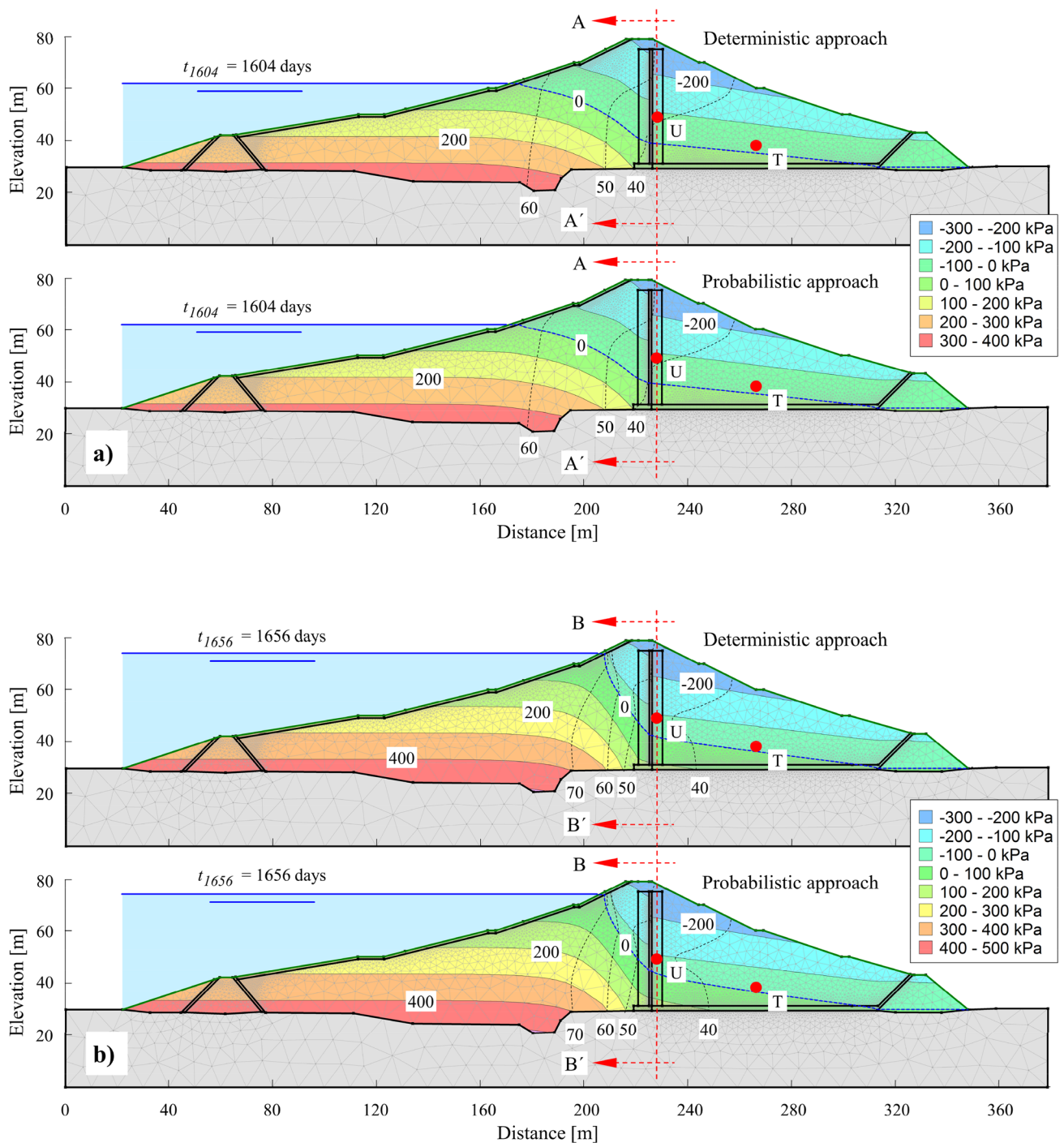


Fig. 11 Comparison between deterministic and probabilistic pore water pressures (PWP) in NOC: **a** $t = 1604$ days and **b** $t = 1656$ days

small contribution of random seepage parameters to the reliability problem. However, there are significant differences between deterministic and probabilistic slip surfaces, as observed in Fig. 14. The probabilistic slip surface is located above the downstream rock mass, in contrast to the deterministic slip surface. In transient NOC analyses, these results suggest that geometric differences between

deterministic and probabilistic slip surfaces in homogeneous earth dams are much larger than differences observed in long-term steady-state analyses [31, 39, 66, 67]. These differences need to be studied for zoned dams where the properties of the materials change in function of the geotechnical design.

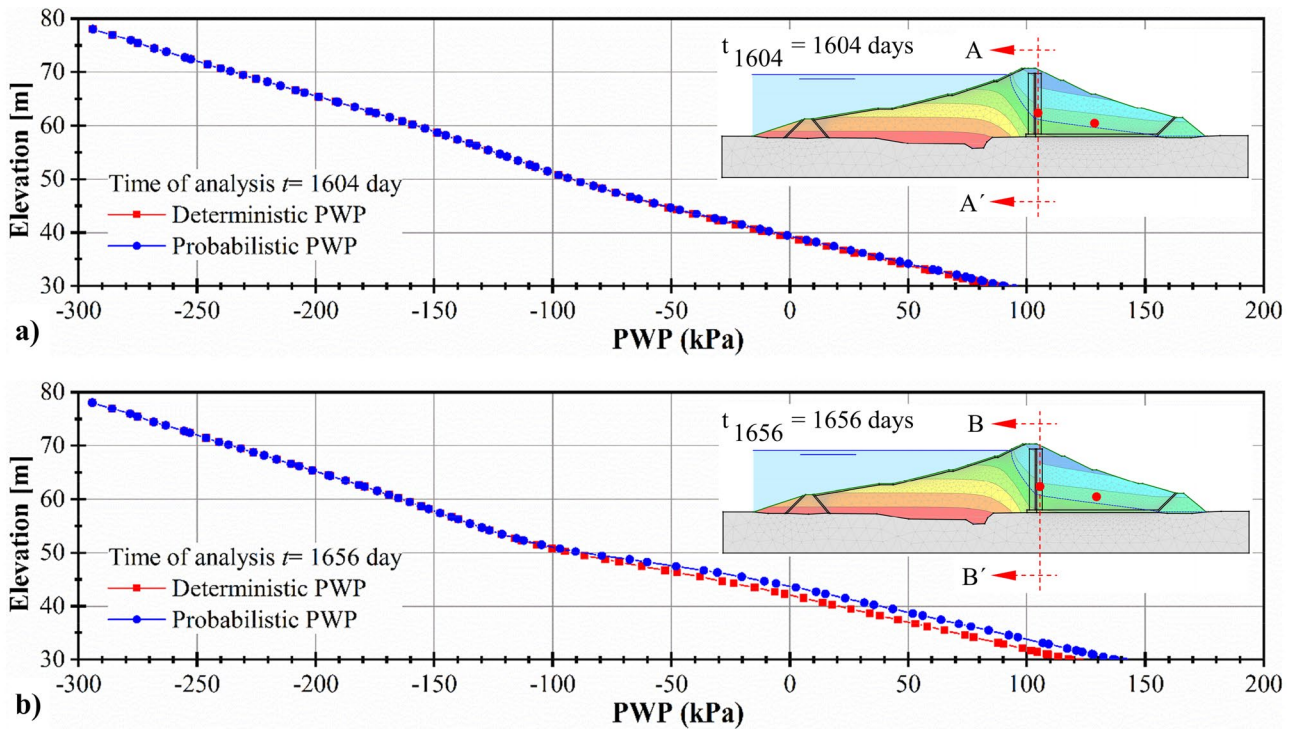


Fig. 12 Comparison between deterministic and probabilistic pore water pressures (PWP), for cross-sections **a** A-A' and **b** B-B', versus dam elevation

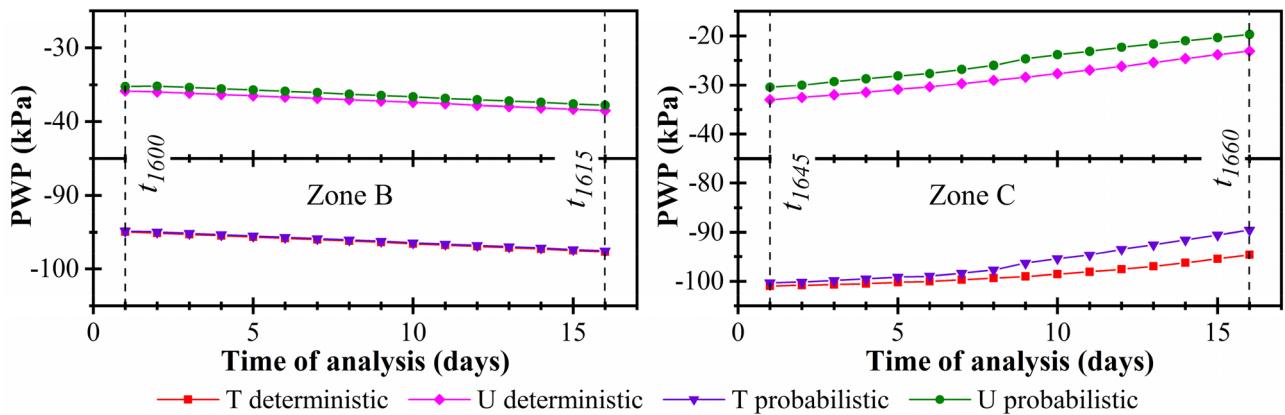


Fig. 13 Deterministic and probabilistic variation of pore water pressures (PWP) as function of the time, for time period B and C

In normal operating conditions (NOC), safety factors and reliability indexes at a given time depend on the history of the reservoir water level and PWP conditions. Finding the real condition of earth dams is hard work in a deterministic approach, and using of probabilistic approaches is a challenge for geotechnical engineers. This helps to see the importance of performing reliability analyses during the design and for assessing the safety of old dams.

4.5 Sensitivity of random variables

Figure 15 illustrates sensitivity coefficients for the six most relevant random variables, under NOC and for time periods A, B, C and D. Firstly, the sensitivities are observed not to change significantly in time, which is quite different from the case or rapid drawdown [31]. This also adds to the argument that five days of previous seepage analysis

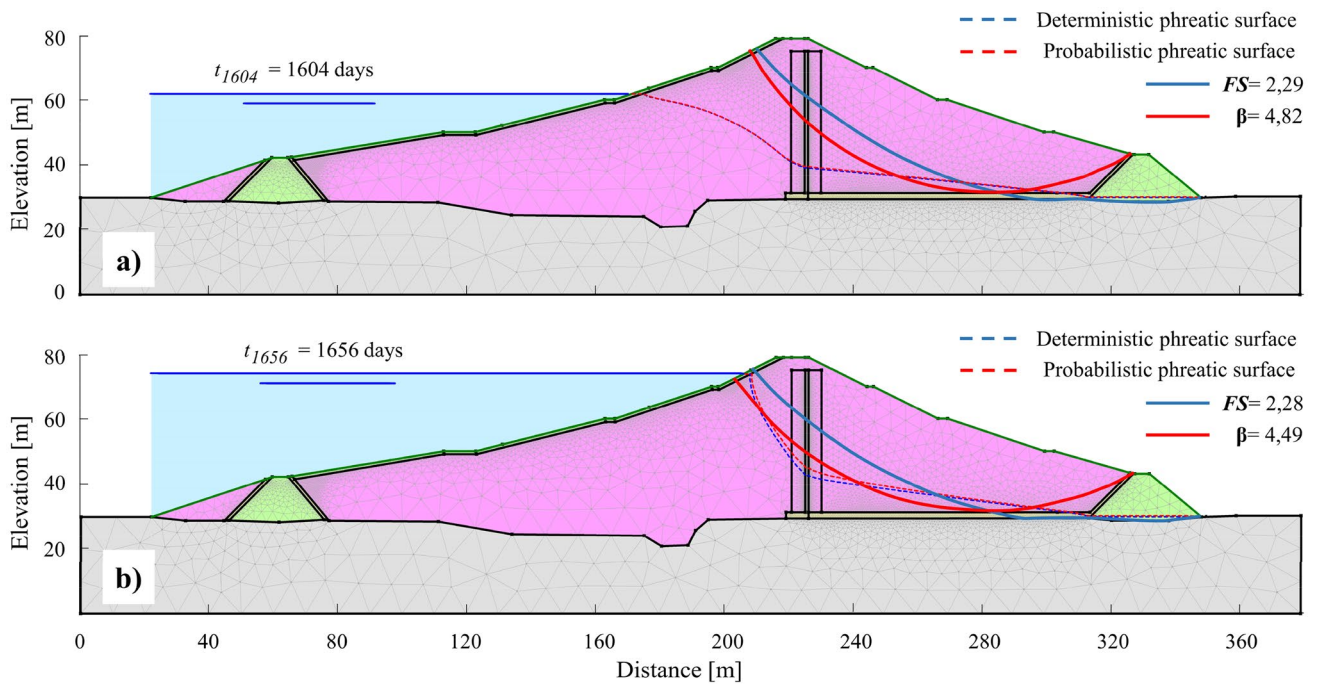


Fig. 14 Comparison of slip surfaces, deterministic and probabilistic approaches at times: **a** $t = 1604$ days and **b** $t = 1656$ days

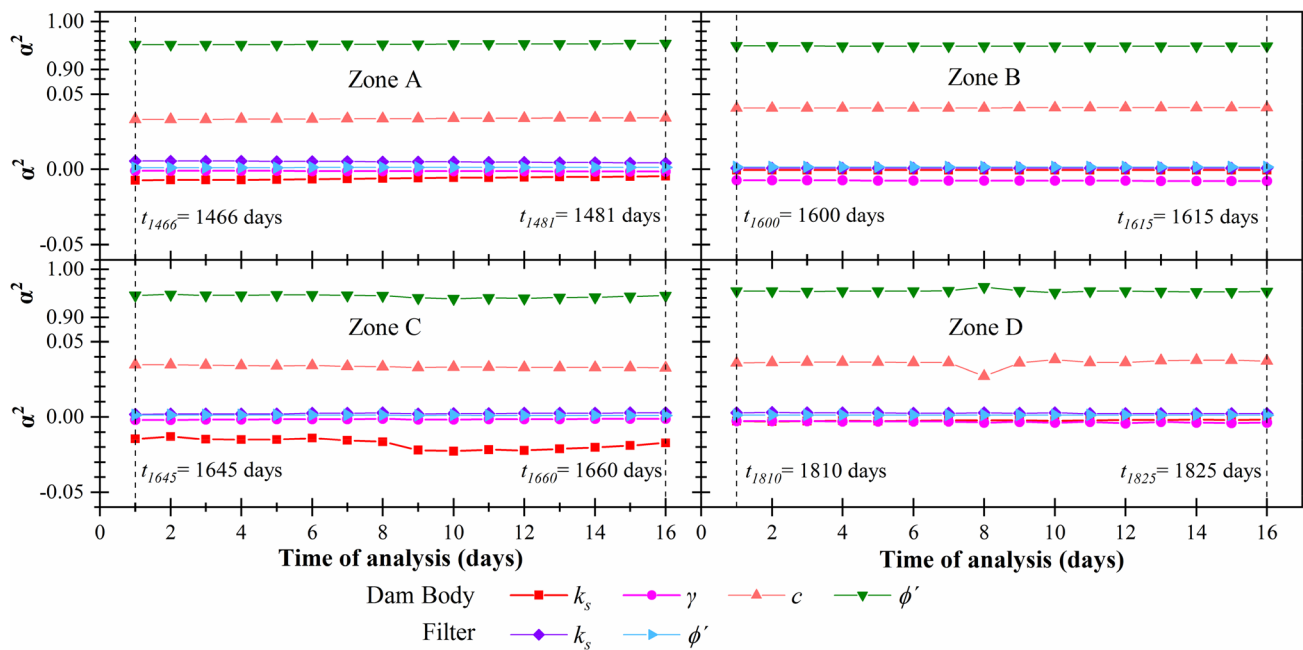


Fig. 15 Sensitivity coefficients of different random variables over time for time periods A, B, C and D

is sufficient for equilibrium analysis in NOC, whereas the whole time interval had to be considered in the rapid drawdown analysis [31].

The results in Fig. 15 also show that, of the six random variables with the greatest contribution to failure probabilities, the dam body friction angle ϕ' is the most important for all the NOC time periods considered herein. The second most important variable is dam body effective cohesion

c' . The other two parameters of the dam body (γ and k_s) and two parameters of the filter (k_s and ϕ') have smaller importance ($\alpha^2 \neq 0$).

Studies on stochastic pore pressure variability [68], transient analysis of rapid drawdown [31] and stochastic hydraulic conductivity [69], as well as other studies [70, 71], found seepage properties to have great importance to reliability analyses. The reliability index decreases as the COV of conductivity hydraulic increases. A comparison of our results with the references above reveals that the importance of the seepage parameters (K_s , θ_s , θ_r , a and n) of the materials in our study are smaller or nearly zero ($\alpha^2 \approx 0$).

In this case, the geometry of the dam, the geometry and position of the filter, the phreatic surface and critical slip surface produce seepage properties which have smaller or nearly zero ($\alpha^2 \approx 0$) incidence. The critical slip is located downstream of the dam, and the importance of phreatic surface is less relevant when the reservoir is at lower water level.

In deterministic analyses, it is well known that the increase of phreatic surfaces downstream of the dam reduce safety factors. In the NOC reliability analysis, this results in reduction of β 's and in a negative α^2 value for dam body k_s . Water flow through the lower part of the critical deterministic and probabilistic surfaces (Fig. 14a, b) helps to explain the smaller relevance of random seepage parameters in the reliability analysis.

5 Concluding remarks

In this paper, reliability analysis of an existing old earth dam was performed for normal operating conditions (NOC), considering transient seepage conditions. A direct coupling (DC) between the deterministic GeoStudio 2018 package (Seepage/W and Slope/W software) and the structural reliability StRAnD software was employed. Numerical analysis of seepage and stability were performed. The first-order reliability method (FORM) method was used to find the design point in reliability analyses.

For new dams, it is numerically possible to model all the history of the dam, in deterministic and probabilistic approaches, with admissible computational cost. However, an old dam is analyzed with a different approach. To find the most realistic pore water pressure (PWP), a long-term steady state analysis is performed to find the initial condition, followed by four years of equilibrium analysis before each transient seepage analysis. These PWPs are calibrated based on dam instrumentation readings. A computational analysis from the empty dam to the actual state of the dam is unfeasible, and reliability analysis would be very

hard using FORM. In this study, five days of previous seepage analysis were found to be sufficient, starting from the mean value condition, for performing reliability analysis of a dam under NOC.

Identifying the critical time corresponding to minimum reliability is very costly. A simple empirical equation was proposed comparing safety factors (FS) and reliability indexes (β) for a single dam in NOC. This equation yields the expected (minimum and maximum) reliability indexes, based on FS calculations for the whole life of the dam. Estimated minimum FS values can be used to find the (approximated) critical time, at which actual reliability analyses should be performed. Note also that the critical time corresponding to minimum FS can be slightly different from the critical time corresponding to minimal β [31]. The empirical relation is valid for a single dam, and for a stationary description of random variables.

The initial analysis listed 34 random geotechnical parameters: five seepage and four equilibrium parameters for each material. Each day of random seepage analysis with 34 parameters took about 20 h to compute. Sensitivity analyses revealed the six parameters with the greatest impact on evaluated failure probabilities; a reduction in random variable dimensionality reduced processing time for a daily transient analysis to about 6 h.

Using deterministic (mean values) data of laboratory and field test, and statistical data from the literature, the transient analysis reveals that four dam body parameters (k_{sat} , γ , c' and ϕ') and two filter parameters (k_{sat} and ϕ') presented the largest contribution to reliability analysis. The friction angle (ϕ') has the greatest impact on the reliability analysis for equilibrium. Different conditions of the dam (e.g., the geometry of the dam and position of the filter, the phreatic surface and critical slip surface) produce random seepage properties with smaller or nearly zero ($\alpha^2 \approx 0$) contribution. The critical slip is located downstream of the dam, and the importance of phreatic surface is less relevant when the reservoir is at lower water level.

The cumulative effect of random k_{sat} (mainly) in transient analysis produces worse critical seepage results (phreatic surfaces, PWP and total water head) and stability results (critical slip surface) in probabilistic analysis than in deterministic (mean value) analysis.

The most probable slip surface, found in the reliability analysis, is not the same as the critical slip surface found in a mean value analysis for the minimum factor of safety (FS). The difference is due to seepage parameters and the geometry of the dam.

Considering target reliability indexes suggested in ICOLD [72], the expected performance level of the studied dam is Good.

A limitation of the study was the determination of different conclusions using only four time periods. The

suggested relationship value between FS and β could be more accurate using a larger sample space. This paper suggests the methodology to find this value, and suggests use of a larger sample space for this purpose.

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Data availability Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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References

- de Vallejo LIG, Ferrer M, Ortuño L, Oteo C (2002) Geological engineering (in Spanish). Pearson Educación, Madrid
- Cruz PT (2004) 100 Brazilian dams (in Portuguese), 2 Ed. Oficina de textos, São Paulo
- Zhang L, Peng M, Chang D, Xu Y (2016) Dam failure mechanisms and risk assessment, 1st edn. Wiley, Singapore
- Menescal RA (2009) Dam safety management in Brazil—proposal for an integrated, decentralized, transparent and participatory system (in Portuguese). Doctoral thesis, Federal University of Ceara
- Fenton GA, Griffiths DV (2008) Risk assessment in geotechnical engineering. Wiley, Hoboken
- Phoon K-K (2008) Reliability-based design in geotechnical engineering: computations and applications. CRC Press, London
- Duncan JM (2000) Factors of safety and reliability in geotechnical engineering. *J Geotech Geoenviron Eng* 126:307–316. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2000\)126:4\(307\)](https://doi.org/10.1061/(ASCE)1090-0241(2000)126:4(307))
- Bhattacharya G, Jana D, Ojha S, Chakraborty S (2003) Direct search for minimum reliability index of earth slopes. *Comput Geotech* 30:455–462. [https://doi.org/10.1016/S0266-352X\(03\)00059-4](https://doi.org/10.1016/S0266-352X(03)00059-4)
- Mouyeaux A, Carvajal C, Bressolette P et al (2018) Probabilistic stability analysis of an earth dam by Stochastic Finite Element Method based on field data. *Comput Geotech* 101:34–47. <https://doi.org/10.1016/j.compgeo.2018.04.017>
- Li D-Q, Wang L, Cao Z-J, Qi X-H (2019) Reliability analysis of unsaturated slope stability considering SWCC model selection and parameter uncertainties. *Eng Geol* 260:105207. <https://doi.org/10.1016/j.enggeo.2019.105207>
- Chen Z, Du J, Yan J et al (2019) Point estimation method: validation, efficiency improvement, and application to embankment slope stability reliability analysis. *Eng Geol*. <https://doi.org/10.1016/j.enggeo.2019.105232>
- Kumar V, Samui P, Himanshu N, Burman A (2019) Reliability-based slope stability analysis of Durgawati Earthen Dam considering steady and transient state seepage conditions using MARS and RVM. *Indian Geotech J* 49:650–666. <https://doi.org/10.1007/s40098-019-00373-7>
- Sivakumar Babu GL, Srivastava A (2010) Reliability analysis of earth dams. *J Geotech Geoenviron Eng* 136:995–998. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000313](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000313)
- Guo X, Dias D, Carvajal C et al (2021) Modelling and comparison of different types of random fields: case of a real earth dam. *Eng Comput*. <https://doi.org/10.1007/s00366-021-01495-4>
- Wang L, Wu C, Tang L et al (2020) Efficient reliability analysis of earth dam slope stability using extreme gradient boosting method. *Acta Geotech* 15:3135–3150. <https://doi.org/10.1007/s11440-020-00962-4>
- Hekmatzadeh AA, Zarei F, Johari A, Torabi Haghghi A (2018) Reliability analysis of stability against piping and sliding in diversion dams, considering four cutoff wall configurations. *Comput Geotech* 98:217–231. <https://doi.org/10.1016/j.compgeo.2018.02.019>
- Johari A, Heydari A (2018) Reliability analysis of seepage using an applicable procedure based on stochastic scaled boundary finite element method. *Eng Anal Bound Elem* 94:44–59. <https://doi.org/10.1016/j.enganabound.2018.05.015>
- Johari A, Talebi A (2019) Stochastic analysis of rainfall-induced slope instability and steady-state seepage flow using random finite-element method. *Int J Geomech* 19:04019085. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001455](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001455)
- GeoStudio (2018) Heat and Mass Transfer Modeling with GeoStudio. <http://downloads.geo-slope.com/geostudioresources/books/10/0/HeatandMassTransferModeling.pdf>. Accessed 2 Feb 2022
- GeoStudio (2018) Stability Modeling with GeoStudio. <https://downloads.geo-slope.com/geostudioresources/8/0/6/books/slopemodeling.pdf?v=8.0.7.6129>. Accessed 2 Feb 2022
- Beck AT (2008) Structural risk analysis and design—StRANd, Release 1 (in Portuguese). Department of Structural Engineering, University of Sao Paulo, Sao Paulo, Brazil
- Napa-garcía GF (2014) Reliability analysis of rock slope with planar failure using the direct coupling approach. In: ISRM conference on rock mechanics for natural resources and infrastructure-SBMR 2014. International Society for Rock Mechanics, Goiania, Brazil
- Kroetz HM, Do N, Dias D, Beck AT (2018) Reliability of tunnel lining design using the Hyperstatic Reaction Method. *Tunn Undergr Sp Technol* 77:59–67. <https://doi.org/10.1016/j.tust.2018.03.028>
- Paniago LN (2018) Dam safety—Brazilian federal dam safety legislation commented (in Portuguese). https://drive.google.com/file/d/143Fe_jQr29xL1HNPc0x56ixOn7shQm1c/view. Accessed 2 Feb 2022
- Richards LA (1931) Capillary conduction of liquids through porous mediums. *Physics (College Park Md)* 1:318–333. <https://doi.org/10.1063/1.1745010>
- Mouyeaux A, Carvajal C, Bressolette P et al (2019) Probabilistic analysis of pore water pressures of an earth dam using a

- random finite element approach based on field data. *Eng Geol* 259:105190. <https://doi.org/10.1016/j.enggeo.2019.105190>
27. Rajesh S, Roy S, Madhav S (2017) Study of measured and fitted SWCC accounting the irregularity in the measured dataset. *Int J Geotech Eng* 11:321–331. <https://doi.org/10.1080/19386362.2016.1219541>
 28. Fredlund DG, Rahardjo H, Fredlund MD (2012) Unsaturated soil mechanics in engineering practice. Wiley, Hoboken
 29. Van Genuchten MT (1980) A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci Soc Am J* 44:892. <https://doi.org/10.2136/sssaj1980.03615995004400050002x>
 30. Fredlund DG, Morgenstern NR, Widger RA (1978) The shear strength of unsaturated soils. *Can Geotech J* 15:313–321. <https://doi.org/10.1139/t78-029>
 31. Siacara AT, Beck AT, Futai MM (2020) Reliability analysis of rapid drawdown of an earth dam using direct coupling. *Comput Geotech* 118:103336. <https://doi.org/10.1016/j.compgeo.2019.103336>
 32. Liang RY, Nusier OK, Malkawi a. H, (1999) A reliability based approach for evaluating the slope stability of embankment dams. *Eng Geol* 54:271–285. [https://doi.org/10.1016/S0013-7952\(99\)00017-4](https://doi.org/10.1016/S0013-7952(99)00017-4)
 33. Melchers EM, Beck AT (2018) Structural reliability analysis and prediction, 3rd edn. Wiley, New York
 34. Beck AT (2019) Reliability and safety of structures (in Portuguese). Elsevier Editora Ltda, Sao Paulo
 35. Beck AT (2017) Structural reliability (in Portuguese). Department of Structural Engineering, University of São Paulo, Sao Paulo
 36. USACE (1997) Introduction to probability and reliability methods for use in geotechnical engineering. <http://faculty.washington.edu/kramer/GeotechProbability.pdf>. Accessed 2 Feb 2022
 37. Westberg M (2010) Reliability-based assessment of concrete dam stability. Doctoral thesis, Lund University
 38. Ditlevsen O, Madsen HO (2007) Structural reliability methods. Technical University of Denmark
 39. Siacara AT, Napa-García GF, Beck AT, Futai MM (2020) Reliability analysis of earth dams using direct coupling. *J Rock Mech Geotech Eng* 12:366–380. <https://doi.org/10.1016/j.jrmge.2019.07.012>
 40. Seara RW (2010) Dam performance evaluation of Governador Parigot de Souza hydroelectric power plant (Capivari-Cachoeira) (in Portuguese). Dissertation, Federal University of Ouro Preto
 41. Heuko LE (2014) Definition of control parameters of an earth dam through statistical analysis and performance evaluation: case study of Capivari-Cachoeira hydroelectric power plant (in Portuguese). Dissertation, Federal University of Paraná
 42. Morgenstern NR, Price VE (1965) The analysis of the stability of general slip surfaces. *Géotechnique* 18:393–394. <https://doi.org/10.1680/geot.1968.18.3.393>
 43. Morgenstern NR, Price VE (1967) A numerical method for solving the equations of stability of general slip surfaces. *Comput J* 9:388–393. <https://doi.org/10.1093/comjnl/9.4.388>
 44. van Genuchten MT, Leij FJ, Yates SR (1991) The RETC code for quantifying the hydraulic functions of unsaturated soils. *U S Environ Res Lab*. <https://doi.org/10.1002/9781118616871>
 45. GeoStudio (2015) Seepage modeling with SEEP/W. <http://downloads.geo-slope.com/geostudioresources/8/0/6/books/seepmodeling.pdf?v=8.0.7.6129>. Accessed 2 Feb 2022
 46. Baligh MM, Levadoux JN (1980) Research report r80–11: pore pressure dissipation after cone penetration. Cambridge University Press, Cambridge
 47. Mayne PW (2007) NCHRP synthesis 368: cone penetration testing a synthesis of highway practice. Transportation Research Board, National Academic Press, Washington
 48. Kutzner C (1997) Earth and rockfill dams: principles for design and construction, 1st edn. CRC Press, New York
 49. Novak P, Moffat AIB, Nalluri C, Narayanan R (2014) Hydraulic Structures, 4th edn. CRC Press, Boca Raton
 50. Fell R, MacGregor P, Stapledon D et al (2015) Geotechnical engineering of dams, 2nd edn. CRC Press, London
 51. USACE (1994) Seepage analysis and control for dams, EM-1110-2-1901. In: U.S. Army Corps Eng. https://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=INPQyoyQDHXDtISpfJ9fXw%3D%3D. Accessed 2 Feb 2022
 52. Verbrugge J-C, Schroeder C (2018) Geotechnical correlations for soils and rocks. Wiley, Hoboken
 53. Chapuis RP, Gill DE, Baass K (1991) Laboratory permeability tests on sand: influence of the compaction method on anisotropy. *Can Geotech J* 28:172–173. <https://doi.org/10.1139/t91-022>
 54. Leroueil S, Le Bihan J-P, Sebaihi S, Alicescu V (2002) Hydraulic conductivity of compacted tills from northern Quebec. *Can Geotech J* 39:1039–1049. <https://doi.org/10.1139/t02-062>
 55. Smith M, Konrad J-M (2011) Assessing hydraulic conductivities of a compacted dam core using geostatistical analysis of construction control data. *Can Geotech J* 48:1314–1327. <https://doi.org/10.1139/t11-038>
 56. USBR (2011) Embankment Dams. Chapter 8 : Seepage. In: Des. Stand. No. 13 Embankment Dams Embankment Dams. <https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finals-pdfs/DS13-8.pdf>. Accessed 2 Feb 2022
 57. Sousa Pinto N (1971) Capivari-Cachoeira Dam - Project and construction (in portuguese). *Electr J* 71:162–172
 58. Sousa Pinto N (1972) Evolution of interstitial pressures at Capivari-Cachoeira Dam (in portuguese). In: VIII National Seminar of Large Dams. Sao Paulo, Brazil
 59. USACE (2016) Transient Seepage Analyses in Levee Engineering Practice. <https://apps.dtic.mil/sti/pdfs/AD1012946.pdf>. Accessed 2 Feb 2022
 60. Duncan JM, Wright SG, Brandon TL (2014) Soil Strength and Slope Stability, 2nd Editio. Wiley, New York
 61. USBR (1987) Bureau of Reclamation - Design of Small Dams. <https://www.usbr.gov/tsc/techreferences/mands/mands-pdfs/SmallDams.pdf>. Accessed 2 Feb 2022
 62. USBR (2011) Embankment Dams. Chapter 4 : Static Stability Analysis. In: Des. Stand. No. 13. Embankment Dams. <https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finals-pdfs/DS13-4.pdf>. Accessed 2 Feb 2022
 63. Bureau of Indian Standards (1975) Code of practice for stability analysis of earth dams. https://awrmis.assam.gov.in/sites/default/files/swf_utility_folder/departments/awrmis_webcomindia_org_oid_5/do_u_want_2_know/is.7894.1975.pdf. Accessed 2 Feb 2022
 64. USACE (2003) Slope Stability. https://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1902.pdf. Accessed 2 Feb 2022
 65. ELETROBRÁS (2003) Civil project criteria for hydroelectric plants (in portuguese). In: Brazilian Power Plants S.A. - ELETROBRÁS. <https://eletrobras.com/pt/AreasdeAtuacao/geracao/Manualis para Estudos e Projetos de Geração de Energia/Critérios de Projetos.pdf>. Accessed 2 Feb 2022
 66. Cheng YM, Li L, Liu LL (2015) Simplified approach for locating the critical probabilistic slip surface in limit equilibrium analysis. *Nat Hazards Earth Syst Sci* 15:2241–2256. <https://doi.org/10.5194/nhess-15-2241-2015>
 67. Low BK, Gilbert RB, Wright SG (1998) Slope reliability analysis using generalized method of slices. *J Geotech Geoenviron Eng*

- 124:350–362. [https://doi.org/10.1061/\(ASCE\)1090-0241\(1998\)124:4\(350\)](https://doi.org/10.1061/(ASCE)1090-0241(1998)124:4(350))
68. Bergado DT, Anderson LR (1985) Stochastic analysis of pore pressure uncertainty for the probabilistic assessment of the safety of earth slopes. *SOILS Found* 25:87–105. https://doi.org/10.3208/sandf1972.25.2_87
69. Gui S, Zhang R, Turner JP, Xue X (2000) Probabilistic Slope Stability Analysis with Stochastic Soil Hydraulic Conductivity. *J Geotech Geoenvironmental Eng* 126:1–9. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2000\)126:1\(1\)](https://doi.org/10.1061/(ASCE)1090-0241(2000)126:1(1))
70. Srivastava A, Babu GLS, Haldar S (2010) Influence of spatial variability of permeability property on steady state seepage flow and slope stability analysis. *Eng Geol* 110:93–101. <https://doi.org/10.1016/j.enggeo.2009.11.006>
71. Yi P, Liu J, Xu C (2015) Reliability analysis of high Rockfill Dam stability. *Math Probl Eng* 2015:1–8. <https://doi.org/10.1155/2015/512648>
72. ICOLD (2005) Bulletin 130: risk assessment in dam safety management. <https://www.icold-cigb.org/GB/publications/bulletins.asp>. Accessed 2 Feb 2022

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