Wave propagation analysis of magnetic nanotubes conveying nanoflow

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Abstract

According to the nonlocal strain gradient theory, wave propagation in magnetic nanotubes conveying magnetic nanoflow under longitudinal magnetic field is inspected. The nonlocal strain gradient Timoshenko beam model is coupled with magnetic nanoflow considering slip boundary condition to model fluid structure interaction. By applying Hamilton's principle, the size-dependent governing equations of motion have been obtained. Calculation of the wave frequency as well as phase velocity has been carried out based on the harmonic solution. The influences of strain gradient length scale, nonlocal parameter, Knudsen number, longitudinal magnetic field and magnetic nanoflow on nanotubes' wave propagation behavior have been examined. According to analytical results, the magnetic intensity related to the longitudinal magnetic field contributes significantly to increasing nanotubes' wave frequency as well as phase velocity. Besides, the magnetic nanotubes conveying magnetic nanoflow predict the highest phase velocity and wave frequency. Also, the wave frequency decrease when the nonlocal parameter increases or the strain gradient length scale decreases. Moreover, an increase in fluid velocity reduces the wave frequency and phase velocity.

Article highlights

1. The nonlocal strain gradient Timoshenko beam model is considered.
2. Wave propagation in magnetic nanotubes conveying magnetic nanoflow is studied.
3. Longitudinal magnetic field and magnetic nanoflow with considering slip boundary condition is inspected.
4. Wave frequency decrease when the nonlocal parameter increases or the strain gradient length scale decreases.
5. Increase in fluid velocity reduces the wave frequency and phase velocity.

Keywords Wave propagations · Nonlocal strain gradient theory · Magnetic nanoflow · Knudsen number · Timoshenko beam model

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1 Introduction

Since the discovery of carbon nanotubes (CNTs) by Iijima [1], an extraordinary perspective regarding the development of various aspects of the CNTs features has been established across different scientific areas. Given their great mechanical characteristics, the CNTs can be applied in potentially different areas, including storage and transport of fluids, exchanging heat, delivery of drugs, storage of hydrogen and nano-electromechanical systems (NEMS) [2–5]. Wave propagation analysis of nanotubes conveying nanoflow was prepared as a strong and suitable alternative field to improve features of nanofluidic dynamic system research in nanobiological devices and nanomechanical systems such as fluid filtration devices, fluid transport, and targeted drug delivery devices [6, 7]. Based on these aspects, the nanotubes can be considered as biosensors for detection and elimination of cancer cells and become emerged as pharmaceutical excipients in the construction of versatile drug delivery systems. More specifically, they can be also used in order to deliver anticancer drugs into target site to kill metastatic cancer cells and have a great deal of characteristics for drug delivery applications to inject small-molecule drugs and some proteins [8]. Besides, due to significant role of wave propagation analysis of nanotubes conveying nanoflow in magnetic field, a remarkable understanding of their wave velocity and wave frequency can be necessary to exhibit better performance of them. The wave propagation analysis of a single-walled carbon nanotube (SWCNT) according to the continuum mechanics as well as the molecular dynamics simulation was studied by Wang and Hu [9]. The wave propagation in carbon nanotubes using Euler–Bernoulli and Timoshenko beam models together with nonlocal continuum mechanics model was investigated by Wang [10]. Narendar and Gopalakrishnan [11] investigated the impact of nonlocal parameter on the wave propagation analysis of the SWCNT which conveyed fluid. Wang et al. [12] examined the wave propagation in double-walled CNTs which conveyed fluids, according to the nonlocal elasticity theory. The wave propagation of the SWCNT with following fluid according to the strain gradient elasticity theory with both Euler–Bernoulli and Timoshenko beam models was performed by Wang [13]. Kaviani and Mirdamadi [14] investigated the wave propagation in the CNTs which conveyed fluid using the slip boundary conditions as well as the theory of strain gradient. Ghorbanpour Arani et al. [15] examined the impact of longitudinal magnetic field on the wave propagation of a double-walled CNT which conveyed fluid. Filiz and Aydogdu [16] performed the wave propagation in the coupled functionally graded nanotube containing fluid flow. Sina et al. [17] reported the wave dispersion of the SWCNT

which conveyed fluid using Timoshenko beam model. Furthermore, the wave propagation in structures can be addressed by Refs. [18–21]. The propagation of axial stress waves in boron-nitride nanotubes using the molecular dynamics and harmonic method was studied by Patra and Batra [22]. Zhen [23] investigated the wave propagation in the viscoelastic SWCNTs containing fluid flow according to the nonlocal elasticity theory with surface effect. The longitudinal and transverse wave propagation in the CNTs with following fluid according to the theory of nonlocal elasticity and Knudsen number were analyzed by Oveissi and Ghassemi [24]. Arani et al. [25] studied the impact of visco-Pasternak foundation, magnetic field, fluid velocity, Knudsen number, nonlocal parameter and surface effects on the wave propagation of SWCNTs conveying viscous fluid. Furthermore, current research has mainly concentrated on the micro/nano structures mechanical behaviors [26–28]. Recently, Lim et al. [29] developed the nonlocal strain gradient theory (NSGT) for investigation of the wave propagation analysis in the nanobeams. According to their findings, the prediction of the wave dispersive by NSGT was considerably consistent with what was found in molecular dynamics simulations. Based on the NSGT, Li and Hu [30] examined the wave propagation in the viscoelastic CNTs with following fluid. Li et al. [31] performed the flexural wave propagation analysis in functionally graded beams by using the nonlocal strain gradient theory was performed. Also, Li et al. [32] studied the wave motion of viscoelastic SWCNTs with surface effects subjected to magnetic field according to the theory of nonlocal strain gradient. Bahaadini et al. [33] investigated the dynamics of nanotube which conveyed nanoflow with the use of NSGT along with slip boundary condition. Mohamadmehr et al. [34] studied the critical fluid velocity in micro pipes that contained fluid flow by using the size-dependent Timoshenko and Euler–Bernoulli beams. Based on the nonlocal Euler–Bernoulli beam theory, thermo-magnetic wave propagation in rotating graphene tubules was studied by Amuthalakshmi and Prabha [35]. Thermal wave propagation characteristics of functionally graded material nanotube conveying nanoflow was investigated by Dai et al. [36].

As far as the authors know, no papers have investigated the size-dependent wave propagation in SWCNTs which convey magnetic nanoflow by using the theory of nonlocal strain gradient Timoshenko beam. The theory of nonlocal strain gradient Timoshenko beam, magnetic nanoflow, Knudsen number as well as magnetic field have been used in this study with the aim of formulating the size-dependent SWCNT which convey magnetic nanoflow. Application of the extended Hamilton's principle aimed at obtaining the governing equations of motion. The harmonic solution has been used to evaluate the phase

velocity and wave frequency. The impacts of strain gradient length scale, nonlocal parameter, Knudsen number as well as magnetic field on the wave propagation behavior have been investigated.

This paper is organized as follows: The governing equations of motion are derived by using Hamilton's principle in Sect. 2. The harmonic solution is applied to evaluate the phase velocity and wave frequency in Sect. 3. The effects of strain gradient length scale, nonlocal parameter, Knudsen number and magnetic field on the wave propagation behavior are studied in Sect. 4. The conclusion of this paper is stated in Sect. 5.

2 Theoretical formulation

Figure 1 shows the model of the SWCNTs conveying magnetic nanoflow exposed to longitudinal magnetic field. A length of L , inner diameter of d , outer diameter of D , thickness of h , as well as mass/unit length of m_t have been defined for SWCNT. The nanotube conveyed incompressible fluid of mass/unit length of m_f , with axial flow at a velocity of U . The present work has employed nonlocal Timoshenko beam model to study wave propagation for the nanotube conveying nanoflow. Thus, the displacement fields can be written as

$$u_x(x,z,t) = u(x,t) - z\phi(x,t), \quad u_y(x,z,t) = 0, \quad u_z(x,z,t) = w(x,t) \quad (1)$$

where $u(x,t)$, $\phi(x,t)$ and $w(x,t)$ denote the axial, rotation as well as transverse deflections of the cross-section for time t , respectively. Besides, u_x , u_y and u_z indicate the components of displacement field in x , y and z directions, respectively. The following relation shows the nonzero strains of Timoshenko beam theory in which axial deflection is ignored:

$$\varepsilon_{xx} = -z \frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} - \phi \quad (2)$$

It is possible to write the virtual strain energy for the nonlocal strain gradient Timoshenko beam theory as [33]

$$\begin{aligned} \delta E = & \int_V [(\sigma_{xx} - \nabla \sigma_{xx}^{(1)}) \delta \varepsilon_{xx} + (\sigma_{xz} - \nabla \sigma_{xz}^{(1)}) \delta \gamma_{xz}] dV \\ & + \left[\int_A (\sigma_{xx}^{(1)} \delta \varepsilon_{xx} + \sigma_{xz}^{(1)} \delta \gamma_{xz}) dA \right]_0^L \\ = & \int_0^L \left[-M \delta \frac{\partial \phi}{\partial x} + Q \delta \left(\frac{\partial w}{\partial x} - \phi \right) \right] dx \\ & + \left[-M_{nc} \delta \frac{\partial \phi}{\partial x} + Q_{nc} \delta \left(\frac{\partial w}{\partial x} - \phi \right) \right] \Big|_0^L \end{aligned} \quad (3)$$

where $\nabla = \partial/\partial x$ represent the 1-D differential operator. Besides, σ_{xx} and σ_{xz} show the classical stresses while $\sigma_{xx}^{(1)}$ and $\sigma_{xz}^{(1)}$ represent the higher-order stresses. Also, M and Q show the resultant bending moment and shear force, respectively. Furthermore, M_{nc} and Q_{nc} represent the non-classical resultant bending moment and shear force, respectively defined in the following form:

$$M = \int_A z t_{xx} dA, \quad Q = \int_A t_{xz} dA. \quad (4)$$

$$M_{nc} = \int_A z \sigma_{xx}^{(1)} dA, \quad Q_{nc} = \int_A \sigma_{xz}^{(1)} dA$$

In which, A indicates the cross-section area while t_{xx} and t_{xz} show the total stresses of the nonlocal strain gradient theory, correspondingly, which are defined as [29]

$$t_{xx} = \sigma_{xx} - \nabla \sigma_{xx}^{(1)}, \quad t_{xz} = \sigma_{xz} - \nabla \sigma_{xz}^{(1)} \quad (5)$$

It is possible to give the classical nonlocal stress σ and the higher-order nonlocal stress $\sigma^{(1)}$ by Lim et al. [29]

$$(1 - (ea)^2 \nabla^2) \sigma = C : \varepsilon \quad (6)$$

$$(1 - (ea)^2 \nabla^2) \sigma^{(1)} = l_m^2 C : \nabla \varepsilon \quad (7)$$

In Eqs. (6) and (7), the classical strain, the strain gradient and the fourth-order elasticity tensors are denoted as ε , $\nabla \varepsilon$ and C , respectively. Also, ea and l_m represent the nonlocal parameter and strain gradient length scale, respectively.

It is possible to express the nanotube kinetic energy including the rotary inertia as:

$$T_t = \int_0^L \left(\frac{m_t}{2} \left(\frac{\partial w}{\partial t} \right)^2 + \frac{J_t}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 \right) dx \quad (8)$$

Fig. 1 The schematic of the SWCNT which convey magnetic nanoflow subjected to longitudinal magnetic field

In which m_t and J_t denote the mass/unit length and the nanotube mass moment of inertia, respectively.

It is possible to write the force applied to the CNT due to the magnetic fluid flow in the following form:

$$\delta W_f = \int_0^L \left(2m_f U \frac{\partial^2 w}{\partial t \partial x} + m_f U^2 \frac{\partial^2 w}{\partial x^2} + m_f \frac{\partial^2 w}{\partial t^2} + J_f \frac{\partial^2 \phi}{\partial t^2} + \sigma A_f B_0^2 \left(\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} \right) \right) \delta w dx \quad (9)$$

In which, the subscript f and t show the fluid and tube, correspondingly. To take the small-size effect on fluid flow into account, velocity correction factor (VCF) is applied as $v_{avg(slip)} = VCF \times U$ [37]. They used modified nanotube through application of VCF whose formula can be as follows [37]:

$$VCF = (1 + \alpha Kn) \left(4 \left(\frac{2 - \sigma_v}{\sigma_v} \right) \left(\frac{Kn}{1 + Kn} \right) + 1 \right) \quad (10)$$

where σ_v is equal to 0.7 and α is

$$\alpha = \frac{64}{3\pi \left(1 - \frac{4}{b} \right)} (2/\pi) (\tan^{-1} (\alpha_1 Kn^B)) \quad (11)$$

which $\alpha_1 = 4, B = 0.4$ and $b = -1$ [37].

The following equation shows the magnetic force across the y direction because of a longitudinal magnetic field according to the Maxwell relation:

$$M = (ea)^2 \left\{ (m_t + m_f) \frac{\partial^2 w}{\partial t^2} + m_f v_{avg(slip)}^2 \frac{\partial^2 w}{\partial x^2} + 2m_f v_{avg(slip)} \frac{\partial^2 w}{\partial x \partial t} + \sigma A_f B_0^2 \left(\frac{\partial w}{\partial t} + v_{avg(slip)} \frac{\partial w}{\partial x} \right) - \frac{1}{\eta} B_0^2 A_t \frac{\partial^2 w}{\partial x^2} \right\} - EI \left(1 - I_m^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial \phi}{\partial x} \quad (18)$$

$$Q = (ea)^2 \left\{ (m_t + m_f) \frac{\partial^3 w}{\partial x \partial t^2} + m_f v_{avg(slip)}^2 \frac{\partial^3 w}{\partial x^3} + 2m_f v_{avg(slip)} \frac{\partial^3 w}{\partial x^2 \partial t} + \sigma A_f B_0^2 \left(\frac{\partial^2 w}{\partial t \partial x} + v_{avg(slip)} \frac{\partial^2 w}{\partial x^2} \right) - \frac{1}{\eta} B_0^2 A_t \frac{\partial^3 w}{\partial x^3} \right\} + k_s GA \left(1 - I_m^2 \frac{\partial^2}{\partial x^2} \right) \left(\frac{\partial w}{\partial x} - \phi \right) \quad (19)$$

$$\delta W_m = \int_0^L \frac{1}{\eta} B_0^2 A_t \frac{\partial^2 w}{\partial x^2} \delta w dx \quad (12)$$

In which η indicate the permeability of the magnetic field and B_0 shows the magnetic intensity related to the longitudinal magnetic field.

It is also possible to use extended Hamilton's principle to derive the governing motion equations:

$$\delta \int_{t_1}^{t_2} (T_t - E + W_f + W_m) dt = 0 \quad (13)$$

Substitution of Eqs. (3), (8), (9) as well as (12) into Eq. (13) along with the application of typical variational procedures will lead to the governing equations of motion for nanotubes which convey magnetic nanoflow subject to magnetic field:

$$\delta w : \frac{\partial Q}{\partial x} - \left[(m_t + m_f) \frac{\partial^2 w}{\partial t^2} + m_f v_{avg(slip)}^2 \frac{\partial^2 w}{\partial x^2} + 2m_f v_{avg(slip)} \frac{\partial^2 w}{\partial x \partial t} + \sigma A_f B_0^2 \left(\frac{\partial w}{\partial t} + v_{avg(slip)} \frac{\partial w}{\partial x} \right) - \frac{1}{\eta} B_0^2 A_t \frac{\partial^2 w}{\partial x^2} \right] = 0 \quad (14)$$

$$\delta \phi : \frac{\partial M}{\partial x} - Q - (J_t + J_f) \frac{\partial^2 \phi}{\partial t^2} = 0 \quad (15)$$

It is possible to write the nonlocal strain gradient constitutive relationship for Timoshenko beam theory as:

$$\left[1 - (ea)^2 \frac{\partial^2}{\partial x^2} \right] M = -EI \left(1 - I_m^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial \phi}{\partial x} \quad (16)$$

$$\left[1 - (ea)^2 \frac{\partial^2}{\partial x^2} \right] Q = k_s GA \left(1 - I_m^2 \frac{\partial^2}{\partial x^2} \right) \left(\frac{\partial w}{\partial x} - \phi \right) \quad (17)$$

In Eq. (17), the shear correction factor as well as inertia cross-sectional moment are denoted by $k_s = \pi^2/12$ and I , respectively. It is also possible to obtain M and Q through substitution of Eqs. (14) and (15) into Eqs. (16) and (17):

If Eqs. (18) and (19) are substituted into Eqs. (14) and (15) according to the nonlocal strain gradient Timoshenko beam model, the governing equation of motion is obtained:

$$\delta w : k_s GA \left(1 - l_m^2 \frac{\partial^2}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \phi}{\partial x} \right) - \left(1 - (ea)^2 \frac{\partial^2}{\partial x^2} \right) \left[(m_t + m_f) \frac{\partial^2 w}{\partial t^2} + m_f v_{avg(slip)}^2 \frac{\partial^2 w}{\partial x^2} + 2m_f v_{avg(slip)} \frac{\partial^2 w}{\partial x \partial t} + \sigma A_f B_0^2 \left(\frac{\partial w}{\partial t} + v_{avg(slip)} \frac{\partial w}{\partial x} \right) - \frac{1}{\eta} B_0^2 A_t \frac{\partial^2 w}{\partial x^2} \right] = 0 \quad (20)$$

$$\delta \phi : \left(1 - l_m^2 \frac{\partial^2}{\partial x^2} \right) \left[k_s GA \left(\frac{\partial w}{\partial x} - \phi \right) + EI \frac{\partial^2 \phi}{\partial x^2} \right] - \left(1 - (ea)^2 \frac{\partial^2}{\partial x^2} \right) \left((J_t + J_f) \frac{\partial^2 \phi}{\partial t^2} \right) = 0 \quad (21)$$

3 Solution procedure

The harmonic solution for CNT conveying magnetic nanoflow can be defined as follows

$$w = W e^{i(kx - \omega t)} \quad (22)$$

$$\phi = \Phi e^{i(kx - \omega t)} \quad (23)$$

In which k and ω indicate wave number as well as frequency, correspondingly. Also, W and Φ represent the wave amplitudes. It is possible to determine the dispersion relation by inserting Eqs. (22) and (23) into Eq. (20) and (21):

$$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{Bmatrix} W \\ \Phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (24)$$

where:

$$\begin{aligned} Q_{11} &= ((m_t + m_f)(1 + (ea)^2 k^2))\omega^2 + (2m_f v_{avg(slip)}(-k + (ea)^2 k^3) + \sigma A_f B_0^2 i(1 + (ea)^2 k^2))\omega - k_s GA(k^2 + l_m^2 k^4) \\ &\quad + m_f v_{avg(slip)}^2(k^2 + (ea)^2 k^4) - \sigma A_f B_0^2 v_{avg(slip)} i(k + (ea)^2 k^3) - \frac{1}{\eta} B_0^2 A_t(k^2 + (ea)^2 k^4), \\ Q_{12} &= -k_s GA i(k + l_m^2 k^3), \\ Q_{21} &= k_s GA i(k + l_m^2 k^3), \\ Q_{22} &= (J_t + J_f)\omega^2 + k_s GA(-1 + l_m^2 k^2) - EI(k^2 + l_m^2 k^4) \end{aligned} \quad (25)$$

To determine a nontrivial solution, the determinant of the coefficient matrix in Eq. (24) has to be set equal to zero. Besides, the relation below gives the phase velocity of waves:

$$c_p = \frac{\omega}{k} \quad (26)$$

Fig. 2 Comparisons between the phase velocity for wave propagation related to SWCNT obtained in the present study with those presented by Wang and Hu [6] for $v_{avg(slip)} = 0$, $Kn = 0$ and $B_0 = 0$

4 Results and discussion

The present research investigated the wave propagation behaviors of nanotubes which convey magnetic nanoflow under longitudinal magnetic field using the theory of nonlocal strain gradient Timoshenko beam. The impacts of strain gradient length scale, nonlocal parameter, velocity of the fluid, Knudsen number as well as magnetic field on the wave frequency and the system phase velocity have been examined. A comparison of the phase velocity of different nonclassical theories with the molecular dynamics (MD) simulations has been made to investigate the accuracy of the current research. Furthermore, it is assumed that the nanotube of armchair (5, 5) in absence

of magnetic nanoflow and longitudinal magnetic field. It is noteworthy that the armchair nanotube diameters (n, m) can be expressed as [6]

$$d = \frac{a}{\pi} \sqrt{3(n^2 + nm + m^2)} \quad (27)$$

Fig. 3 The phase velocity against the wave number regarding magnetic fluid effect with $v_{avg(slip)} = 1000$ m/s, $Kn = 0.001$ and $B_0 = 50$ T

Fig. 5 The phase velocity in terms of the Knudsen number for $k = 10$ (1/nm), $Kn = 0.001$, $l_m = 0.19$ nm, $ea = 0.3$ nm and various fluid velocity

Fig. 4 The phase velocity against the wave number with $v_{avg(slip)} = 1000$ m/s, $Kn = 0.001$, $l_m = 0.19$ nm, $ea = 0.3$ nm and various intensity of magnetic field

In Eq. (23), a denotes the length of the carbon–carbon bond, which is considered to be 0.142 nm. The physical parameters of (5, 5) SWCNT are defined in Table 1. Figure 2 indicates the phase velocity c_p against the wave number k based on molecular dynamics simulation and various continuum elasticity theories. Based on the results of MD simulation [6], values of 0.3 nm and 0.19 nm have been considered for the nonlocal parameter and strain gradient length scale, respectively. According to the results, good consistency was shown with the MD simulations [6]. Furthermore, it is possible to convert the NSGT to a

Table 1 The values for physical parameters of CNT [6]

Physical properties and parameters	Values
Elastic modulus, E (TPa)	0.39
Nanotube density, ρ_t (kg/m ³)	2237
Thickness, h (nm)	0.34
Poisson ratio, ν	0.22
Fluid density, ρ_f (kg/m ³)	997
Magnetic density, ρ_s (kg/m ³)	5150
Electrical conductivity, σ (S/m)	0.55
Magnetic permeability, η	$4\pi \times 10^{-7}$
Solid volume fraction, φ	1%
Magnetic fluid density, $\rho_{mf} = \rho_f(1 - \varphi) + \rho_s\varphi$ (kg/m ³)	1038.5

strain gradient theory (SGT) in the case of $l_m = 0.19$ nm and $ea = 0$, a nonlocal theory (NT) in the case of $l_m = 0$ and $ea = 0.3$ nm and a classical theory (CT) if $l_m = 0$ and $ea = 0$. As can be observed, in lower wave number (corresponding to large wavelength), the phase velocity is not dependent on nanostructural characteristics and consequently various elasticity theories lead to similar predictions. In other words, the phase velocity is dependent nanostructural properties at higher wave numbers (corresponding to small wavelength) and the difference between these results is prominent. Besides, stiffness enhancement as well as stiffness softening effects for the nanotube can be predicted by the strain gradient and the nonlocal elasticity models, respectively. Based on the findings, the SWCNT wave propagating analysis according to the nonlocal strain gradient model can possibly show higher reliability.

Fig. 6 The phase velocity regarding the fluid velocity in case of $k = 10(1/\text{nm})$, $B_0 = 100\text{ T}$, $l_m = 0.19\text{ nm}$, $ea = 0.3\text{ nm}$ and various magnetic intensity

Fig. 8 The wave frequency versus the magnetic intensity for $Kn = 0.001$, $v_{\text{avg(slip)}} = 1000\text{ m/s}$ and $k = 5(1/\text{nm})$

Fig. 7 The wave frequency as the wave number function in case of $Kn = 0.001$, $v_{\text{avg(slip)}} = 1000\text{ m/s}$ and $B_0 = 100\text{ T}$

Figure 3 investigates the impacts of magnetic nanoflow on the wave propagation behavior of SWCNT which conveys magnetic nanoflow subject to longitudinal magnetic field for various elasticity theories, indicating the results for $v_{\text{avg(slip)}} = 1000\text{ m/s}$, $Kn = 0.001$ and $B_0 = 50\text{ T}$. Based on the results, the magnetic fluid effect increases SWCNT bending stiffness, and the phase velocity also increases. A lower phase velocity was predicted through the NT compared to the NSGT, while higher phase velocities were predicted through the SGT and CT compared to the NSGT.

Fig. 9 The wave frequency variations with the strain gradient length scale and nonlocal parameters considering $v_{\text{avg(slip)}} = 1000\text{ m/s}$, $k = 5(1/\text{nm})$, $Kn = 0.001$ and $B_0 = 100\text{ T}$

Figure 4 shows similar calculations to illustrate the impact of intensity of longitudinal magnetic field on the phase velocity, indicating the results for $v_{\text{avg(slip)}} = 1000\text{ m/s}$, $Kn = 0.001$, $l_m = 0.19\text{ nm}$ and $ea = 0.3\text{ nm}$. According to the results, the phase velocity increases when the intensity of longitudinal magnetic field increases. Notably, the phase velocity of the wave propagation changes according to magnetic intensity at the whole range of wave numbers.

Figure 5 indicates the effect of Knudsen number on the phase velocity of magnetic nanotube which

Fig. 10 The wave frequency variations with the nanoflow velocity and magnetic intensity for $k = 10(1/\text{nm})$, $Kn = 0.001$, $l_m = 0.19\text{nm}$ and $ea = 0.3\text{nm}$

conveys nanoflow according to the NSGT considering $k = 10(1/\text{nm})$, $B_0 = 100\text{ T}$. Various flow regimes including those for the continuum flow ($0 < Kn < 10^{-2}$), the slip flow ($10^{-2} < Kn < 10^{-1}$), the transition flow ($10^{-1} < Kn < 10$), and the free molecular flow ($Kn > 10$) are identified using the Knudsen number [38, 39]. Considering the continuum flow regime, it has been shown that the phase velocity is not changed with the increase in the Knudsen number. Also, the phase velocity shows a slight reduction with the increase in the Knudsen number in the slip flow regime. Moreover, the phase velocity sharply decreases with the increase in the Knudsen number in the transient flow regime. Considering the $Kn > 10$, phase velocity almost vanishes and the flow velocity effect on the phase velocity fades. In addition, flow velocity has significant effects on phase velocity in the transient flow regime.

Figure 6 plots for $Kn = 0.001$, $l_m = 0.19\text{ nm}$ and $ea = 0.3\text{ nm}$ to examine the influences of the magnetic nanoflow velocity on SWCNT phase velocity. According to the findings, when the magnetic nanoflow velocity increases, the phase velocity decreases. Also, as magnetic intensity increases, there is an increase in phase velocity. It is noteworthy that the nonclassical continuum theories provide higher precision of the mechanical models on the nanoscale materials. Thus, the impacts related to the theories of nonclassical continuum on the wave frequency are illustrated in Figs. 7, 8, 9. As it can be seen, there is potentially a smaller SWCNT wave frequency according to the NSGT compared to the classical model, which is justifiable through the stiffness-softening effects because

of the scaling parameters values. The stiffness-softening effect can be observed in the case of $l_m < ea$. It was shown that increasing the wave number has significant effects on the wave frequency in the case of $k > 1(1/\text{nm})$. According to Fig. 8, the wave frequency increases as the magnetic intensity increases; in other words, taking magnetic field into account increases the SWCNT bending rigidity. Besides, Fig. 9 indicates the impacts of both strain gradient length scale and nonlocal parameters on the SWCNT wave frequency, which contains magnetic nanoflow considering $v_{avg(slip)} = 1000\text{ m/s}$, $k = 5(1/\text{nm})$, $Kn = 0.001$ and $B_0 = 100\text{ T}$. The results show that the wave frequency decreases with the increase in the nonlocal parameters; in other words, taking the nonlocal effects into account decreases the SWCNT bending rigidity. Besides, there is an increase in the wave frequency when the strain gradient length scale increases. In other words, there is an increase in the SWCNT stiffness with the increase in the strain gradient length scale. In addition, the variation of wave frequency versus the fluid velocity and magnetic intensity is plotted in Fig. 10 for $k = 10(1/\text{nm})$, $Kn = 0.001$, $l_m = 0.19\text{ nm}$ and $ea = 0.3\text{ nm}$. According to the results, there is a decrease in the wave frequency when the fluid velocity increases. Besides, as the nanoflow velocity increases, the wave frequency of SWCNT decreases.

5 Conclusion

The present study analyzed the wave propagation of the magnetic SWCNT which conveyed magnetic nanoflow under longitudinal magnetic field. The governing equations of motion using the nonlocal strain gradient Timoshenko beam theory and magnetic nanoflow considering slip boundary condition according to the Hamilton's principle were derived, after which they were solved applying the harmonic method. The study investigated the effects of strain gradient length scale, nonlocal parameter, velocity of the fluid, Knudsen number as well as magnetic field on the SWCNTs wave propagation behaviors. Based on the research findings, the magnetic fluid affected the wave frequency significantly. Further, with the increase in the Knudsen number, the phase velocity sharply decreases considering the transient flow regime. Besides, it was observed that considering nonlocality and strain gradient parameters exert inverse impacts on the wave frequency. The results of this study illustrate that nonlocal strain gradient theory can be a suitable substitute for the classical theory. The analytical examination is essential to estimate the wave frequency of the SWCNT conveying magnetic nanoflow. Thus, the future work will focus on the examination the nonlinear hygro-thermal wave propagation in nanotube conveying magnetic fluid.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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