Analysis of biaxial proportional low-cycle fatigue crack propagation for hull inclined-crack plate based on accumulative plasticity

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Abstract

Fracture failures of ship plates subjected to in-plane biaxial low-cycle fatigue loading are generally the coupling result of accumulative plasticity and biaxial low-cycle fatigue damage. A biaxial low-cycle fatigue crack growth analysis of hull structure that accounts for the accumulative plasticity effect can be more suitable for the actual evaluation of the overall fracture performance of the hull structure in severe sea conditions. An analytical model of biaxial low-cycle fatigue crack propagation with a control parameter for $\Delta CTOD$ is presented for hull inclined-crack plate. A test was conducted for cruciform specimens made of Q235 steel with an inclined crack to validate the presented analysis. The biaxial accumulative plasticity behavior and the effects of biaxiality and stress ratios were investigated. The results of this study reveal a strong dependence of biaxial low-cycle fatigue crack propagation on biaxial accumulated plasticity.

Keywords Hull cracked plate · Biaxial low-cycle fatigue · Accumulative plasticity · $\Delta CTOD$

1 Introduction

Multiaxial irregular wave loads are typical in severe sea conditions. These cause multi-axial low-cycle fatigue and accumulative plastic damage in ship structures. Although a few scholars have a profound comprehension of this multi-axial low-cycle fatigue crack propagation, no substantive results have been obtained from the specific research. This scenario does not satisfy the requirements of ship engineering development. It is urgent to initiate research to solve the problems in engineering application.

In the research on the linear elastic range of multi-axial fatigue, scholars prefer to extend the stress intensity factor K . It introduced the influence coefficient of multi-axial loading to establish the multi-axial crack propagation model on the basis of uniaxial fatigue. Mall et al. [1, 2] observed that the relationship between crack growth rate and strain energy release rate of Model I under biaxial

tensile loading with different stress ratios can be divided into two regions: crack initiation and crack propagation. Gotoh [3] explored and analyzed the variation in the stress intensity factor of a cross specimen under various load combinations (including proportional load and non-proportional load). However, significant plastic deformation occurs in material structures in most engineering environments. K would be limited by plastic deformation. However, the elastic–plastic evaluation method based on crack tip opening displacement (CTOD) or J-integral can compensate for this shortcoming. In 1999, Itho [4] proposed a simple J-integral equation for Type I crack under biaxial loading. It is based on crack opening displacement (COD) and uses the finite element method. However, the accuracy of this method is low. This is because of the insufficient precision of finite element mesh models as a result of the limitation of software technology. Subsequently, Tchankov [5] explored the strain intensity in front of Type

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I crack under a multiaxial stress state, on the basis of Itho's work. Then, he proposed a general J-integral method without considering the material coefficient and accurately predicted the multi-axial low-cycle fatigue life of various metal materials.

The conservation of J-integral facilitates its calculation. This can prevent the complex stress–strain field at crack tips. However, when plastic damage is considered, the crack tip would be unloaded, and the J-integral would no longer be conserved for its calculation. Unlike J-integral, CTOD can reflect the capability of the crack tip material to resist fracture and effectively measure the plastic deformation of crack tip during loading or unloading. Dong [6] studied the fatigue crack propagation behavior of AH32 steel under overload condition by considering the crack closure effect of the CTOD experiment. It was observed that the plastic-induced crack closure has a substantial influence on the delay effect of fatigue load. Antunes [7] used the finite element method to determine the plastic CTOD values under the action of different crack increments, including different crack tip mesh sizes and different loading cycles in the numerical method. In recent years, the research based on 3D-DIC of CTOD has advanced rapidly with the development of digital image correlation technology. Vasco-Olmo et al. [8–10] used DIC to measure the range of CTOD. They decomposed the data into elastic and plastic parts by offset flexibility technology. The research expounded the mechanism involved in plasticity-induced crack tip deformation. It also demonstrated the feasibility and accuracy of using $\Delta CTOD_p$ (rather than ΔK) to characterize the crack growth rate. Recently, Meng et al. [11] conducted biaxial tensile–tensile fatigue tests on magnesium alloys using cruciform specimens. They analyzed the fatigue crack propagation mechanism under different stress ratios and different phases. Baptista [12] also used this optimized cross-shaped fatigue specimen with elliptical fillet while exploring the variation law of K and J-integral under different phase biaxial fatigue cyclic loading.

Although many studies have been conducted on CTOD in the plastic range, few have considered the effect of accumulative plasticity. Deng et al. [13] considered the accumulative plastic damage of hull plate with a central through-crack under high stress load. They proposed a low-cycle fatigue analysis method to evaluate the accumulative plastic strain based on $\Delta CTOD$. In addition, they extended this method to study the low-cycle fatigue crack initiation life of notch plates. Jiang [14] carried out low-cycle fatigue crack propagation experiments on stiffened plates with different heights and cracks by using the MTS322 fatigue test machine. They analyzed the accumulative plastic strain during the propagation process. The research on CTOD and

accumulative plasticity under low-cycle fatigue load has been limited mainly to the uniaxial range. However, the stress state of uniaxial loading is excessively idealized, whereby it cannot reflect the multiaxial high stress characteristics of actual sea state loads. In addition, the internal mechanism of this coupling relationship and its influence on crack propagation in the hull structure remain elusive. Therefore, it is more effective to analyze biaxial low-cycle fatigue crack growth by considering the accumulative plasticity at the crack tip, to evaluate the overall fracture load-bearing capacity of ship structural components.

In the author's previous study [15], the biaxial accumulative plasticity and the biaxial non-proportional low-cycle fatigue crack growth behaviour of hull inclined-crack plate subjected to biaxial nonproportional low-cycle fatigue loading with different phase differences of approximately 45°, 90°, and 180° were investigated. The biaxial low-cycle fatigue crack growth rate of hull inclined-crack plate is represented using a power law expression with the controlled parameter ΔJ -integral in the study.

However, in this study, a series of tests were conducted to study the biaxial proportional low-cycle fatigue fracture behavior of hull inclined-crack plate considered the biaxial accumulative plasticity effect subjected to biaxial proportional low-cycle fatigue loading. A simple predictive model for biaxial proportional low-cycle fatigue crack propagation of hull cracked plate with $\Delta CTOD$ as the control parameter is proposed based on biaxial accumulative plasticity. The effects of biaxial accumulative plasticity, biaxiality ratio, and stress ratio on biaxial proportional low-cycle fatigue crack propagation are discussed. The tests were conducted to fit the material parameters of the predicted model. A detailed investigation of the fracture morphology is presented to comprehensively analyze the biaxial proportional low-cycle fatigue damage mechanism of the inclined-crack plate using the experiment results. The obtained results provide a basis for the accurate evaluation of the biaxial proportional low-cycle fatigue crack growth fracture behaviour of hull cracked plate.

2 Theoretical analysis

In this study, the stress–strain hysteresis loop of hull cracked plate under biaxial low-cycle fatigue load is studied, and then the $\Delta CTOD$ based on accumulative plasticity is established. Finally, a simple predictive model for biaxial proportional low-cycle fatigue crack propagation of hull cracked plate with $\Delta CTOD$ as the control parameter is proposed based on biaxial accumulative plasticity.

2.1 Determination of $\Delta CTOD$

According to the author's previous study [15], the biaxial accumulative plasticity at crack tip of an inclined-crack specimen had been calculated under biaxial low-cycle fatigue loading, and the detailed expression for the ΔJ integral based the biaxial accumulative plasticity at crack tip had also been obtained in the study.

The accumulative plastic strain of the hull structure at the hazardous position after N variable amplitude multi-axial low-cycle fatigue load is calculated according to the elastic–plastic theory and the accumulative incremental plastic process. In this study, the equivalent $\Delta CTOD$ is used as the control parameter to characterize the low cycle fatigue crack growth rate model of a hull plate under biaxial low cycle fatigue load. Combined with the accumulative plastic strain at the crack tip, the theoretical solution process of the hull plate $\Delta CTOD$ formula under biaxial low cyclic loading is as follows:

For an ideal elastic–plastic material, the following calculation formula relating ΔJ integral and $\Delta CTOD$ is adopted by referring to Shih's research on infinite plate with central penetrating crack under unidirectional load:

$$\Delta CTOD = d_n \frac{\Delta J}{\sigma_y} \quad (1)$$

The cyclic J-integral increment ΔJ has a reliable theoretical basis for cyclic loading. All the parameters in Eq. (1) can be expressed by the corresponding cyclic parameters and can be approximately superposed by the elastic and plastic parts. Thus, it can be obtained as follows:

$$\Delta CTOD_e = \frac{d_{n'}}{\sigma_{cy}} \left(1.45 \frac{\Delta \sigma_{eff}^2}{E} + 2.4 \frac{\Delta \sigma \Delta \epsilon_{pl}}{\sqrt{1 + 3n'}} \right) a \quad (2)$$

where $\Delta \sigma_{eff}$ and $\Delta \epsilon_{pl}$ are respectively the effective stress amplitude accumulative plastic strain under biaxial low-cycle fatigue loading. The effective stress amplitude $\Delta \sigma_{eff}$ is conducted to consider the effect of crack closure $\Delta \sigma_{eff} = \Delta \sigma_{max} - \Delta \sigma_{op}$. σ_{max} and σ_{op} are the maximum and crack opening stress respectively [6].

The Eq. (2) for $\Delta CTOD$ to a fatigue crack under biaxial low-cycle fatigue load can be used in the following analysis.

2.2 Biaxial fatigue crack growth rate

The biaxial low-cycle fatigue crack growth rate of hull inclined-crack plate was represented by the following power-law expression with the $\Delta CTOD$ as the control parameter:

$$\frac{da}{dN} = f(\delta, a) = C(\Delta CTOD)^m \quad (3)$$

where C and m are the material parameters, which were determined by biaxial low-cycle fatigue tests under different experimental conditions.

3 Experiment investigation

3.1 Experimental setup

A series of tests were conducted using a servo-hydraulic system to study biaxial low-cycle fatigue crack propagation of cruciform specimens with inclined crack subjected to biaxial tension–tension low-cycle fatigue loading with different biaxiality and stress ratios and without phase differences. The biaxial accumulative plasticity was also studied. The cruciform specimen had a length, width, and plate thickness of 190 mm, 76 mm, and 8 mm, respectively. A 45° prefabricated crack with a length of 8 mm was located at the center of the specimen. Two pin shaft holes with a diameter of 20 mm were made in the specimen for tooling connection between the loading end and fixed end. The thickness of the circular region (with a radius of 30 mm) in the central area of the specimen was reduced to 4 mm to ensure a high uniform stress level in the central region of specimen. Meanwhile, an elliptical fillet was set at the junction to reduce the stress concentration and thereby ensure that the central prefabricated crack tip could initiate new cracks smoothly. The cruciform specimens with inclined crack and the other geometrical dimensions are shown in Fig. 1. The mild carbon Q235 steel was used to machine the cruciform specimens with an inclined crack. The chemical composition (in wt%) of the experimental material was as follows: Mn_{0.48} C_{0.18} P_{0.027} Si_{0.22} S_{0.023}. The cruciform specimens were fabricated using an electro-discharge method. A circular sheet with a diameter of 60 mm was set up at the center of the cruciform specimen. Here, the thickness of the web area was 4 mm, and a 2.0 mm-long notch was produced to accelerate biaxial crack initiation. Before the test, an inclined pre-crack with a length of 8 mm and at an initial angle of 45° to the horizontal axis was prepared in the central area of the cruciform specimen.

The experimental setup shown in Fig. 2 was used to collect experimental data to analyze the mechanical properties of the experimental material. The MTS 505.60 four-channel electro-hydraulic servo loading experiment and analysis system was adopted to perform the biaxial low-cycle fatigue experiment. Two actuators MTS 244.31 and MTS 244.41 cooperate with the I-beam to realize biaxial fatigue loading. The MTS 505.60 four-channel electro-hydraulic servo loading system has four separate loading

channels with load ranges of 250T, 250T, 500T, and 1000T. Furthermore, the loading frequency of the biaxial low cycle fatigue test in this study was set as 1 Hz. In terms of load amplitude, the system can realize the synchronous sinusoidal periodic loading by setting the intermediate value. It can be suspended at any time during the course of the experiment and transformed into static control. This is convenient for measuring crack length and crack tip opening displacement. The waveforms applied for the cruciform specimens were sinusoidal and are shown in Fig. 3.

The horizontal instrument and plumbum auxiliary installation are used to ensure the horizontality and verticality, respectively, of the two actuators during their installation. Thereby, the in-plane orthogonality of the loading force is realized. The fatigue specimen and experimental tooling are closely connected through the pin shaft. Toolings ① and ② are fixed in the orthogonal I-beam frame by bolts. Toolings ③ and ④ are installed on the two actuators in the horizontal and vertical directions, respectively, as shown in Fig. 2.

The cruciform specimens were loaded in both horizontal axial and vertical axial directions simultaneously. Furthermore, different biaxiality ratios ($\lambda=0, 0.5, 1$, and 1.5) of the biaxial low-cycle fatigue loads were applied to the test specimen. Figure 5 shows the tensile stress–strain curve of Q235 steel determined using an MTS fatigue test machine. Table 2 lists the mechanical parameters of Q235 steel. The strain gauges were used to measure the stress–strain field in the vicinity of the crack tip. Fractography was conducted to study the microscopic fracture characteristics of the test specimens.

A high-resolution optical camera (CCD) was used to capture the biaxial crack growth morphology of the test specimens. The CCD camera supports a magnification by 270 times. Its supporting software Pro2 has many practical functions such as photographing video, marking line, measurement assessment, measurement, and scale. Pro2 has a measurement accuracy of 0.001 mm. To ensure measurement accuracy, it is necessary to examine the measurement by a 1×1 mm grid line pasted on the surface of the specimen before each measurement. The relative parallel position between the CCD camera lens and the specimen surface was ensured to prevent measurement error caused by lens tilt. The experimental layout of the CCD camera and its measurement image are shown in Fig. 4. A LabVIEW-operated camera was used to obtain images of the extended fractures at different time intervals. A BE120-3CA high-precision strain gauge with a resistance of 120Ω was used to collect the surface strain of the specimen in this study.

3.2 Test conditions

In the loading design of the biaxial low-cycle fatigue experiment, the x-axis (horizontal direction) and y-axis (vertical direction) were considered as the benchmarks. The influences of the biaxial stress ratio λ , stress amplitude $\Delta\sigma$, and stress ratio R were considered separately through the control variable method. Thereby, the influence of various load forms on the crack propagation path and the variation law in the process of fatigue crack

Fig. 3 Applied biaxial loading waveforms

Fig. 4 CCD Camera and its measurement image

Table 1 Loading conditions of biaxial low-cycle fatigue test

Num	F_x (kN)	F_y (kN)	R	λ	Actual plate thickness in central area (mm)
1	58	58	0.1	1	3.3
2	58	58	0.2	1	3.3
3	58	58	0.3	1	3.3
4	0	74.57	0.2	0	3.0
5	33.35	66.70	0.2	0.5	3.0
6	52.73	52.73	0.2	1	3.0
7	62.00	41.34	0.2	1.5	3.0
8	66.42	66.42	0.1	1	3.7

Table 2 Material properties of Q235 steel

Yield stress (MPa)	Ultimate tensile strength (MPa)	Elastic modulus (GPa)	Poisson's ratio	(MPa)	Hardening coefficient
σ_y	σ_b	E	ν	k'	n'
430	680	206	0.3	1193	0.412

propagation could be explored. The specific experimental loading conditions are shown in Table 1.

In particular, while discussing the influence of different biaxial stress ratios λ , the magnitude of the loading force in Working conditions 5–7 are fixed at 74.57 kN to control the single variable.

Fig. 5 Tensile stress–strain curve of the Q235 steel specimen

4 Experimental results and discussion

4.1 Near-tip stress–strain fields

The tension stress–strain relationship curve for the Q235 steel test specimen is presented in Fig. 5. The mechanical parameters of Q235 steel were determined by the test, as given in Table 2.

Figure 6 shows the hysteresis loop at the crack tip under the same biaxial ratio but with different stress ratios. Figure 7 indicates that the stress and strain fields in the X- and Y-axis directions exhibit similar accumulative incremental plastic deformation phenomena as in the uniaxial state. Moreover, the accumulative plastic strain in the X- and Y-axis directions increases gradually

Fig. 6 Hysteresis loops for inclined-crack specimens with different biaxial stress ratios

with the increase of the number of cycles, and the greater the stress ratio, the higher the accumulative plastic strain rate is. These above results show that, when the stress amplitude remains changes correspondingly, the biaxial accumulative plastic strain at the crack tip increases gradually and associated with the stress ratio and the maximum external load. This accumulative plastic strain at the crack tip may eventually cause separation of structural materials in the area near the crack tip and lead to biaxial low-cycle fatigue crack propagation

forward. This phenomenon was explained by Kapoor et al. [16].

In summary, obvious accumulative plastic strain was discovered at the crack tip under biaxial ($\lambda \neq 0$) low-cycle fatigue loading. This induced the final separation of the material along the crack extension path and caused the crack tip to move forward. Zheng et al. [17] obtained similar conclusions through analysis of crack growth based on the accumulative plastic dissipation energy density. Compared with the stress ratio of the external load, the accumulative plastic strain at the crack tip was prominently affected by the local stress ratio at the crack tip, which should be considered in the analysis of biaxial crack extension.

4.2 Accumulative plasticity effect

Figure 8 presents the relationship curves of the horizontal, vertical, and resultant strains at the crack tip with the cycle number under biaxial proportional low-cycle fatigue loading. According to the results, the plastic accumulation phenomenon appears in the horizontal and vertical strains. In addition, the maximum strain value in each cycle gradually accumulates and increases. The accumulative plastic strain rate subjected to different biaxial low-cycle fatigue loadings is shown in Fig. 9.

Figure 9 shows that the biaxial accumulative plastic strain rate at the crack tip decreases rapidly during the initial unstable stage and tends to stabilize gradually with

Fig. 7 Hysteresis loops of X- and Y-axis stress and strain with different stress ratios

Fig. 8 Relationship curves of strain and cycle numbers for test specimen under different biaxial proportional low-cycle fatigue loads

the increase in the biaxiality and stress ratios. The results show that the biaxial accumulative plastic strain of the plates with inclined crack is significantly affected by the biaxiality and stress ratios. With continuous loading, the hysteresis loop at the crack tip gradually tends to be stable. However, the strain value of the hysteresis loop continues to accumulate after the stabilization. The strain level of the stable hysteresis loop increases with the increase in R . This is mainly because the larger the value of R , the higher is the corresponding average load stress and thereby, higher the strain level.

The experimental results show that the accumulative rate of plastic strain decreases gradually to a stable value during the loading progress. This causes the plastic strain to grow continuously. Tchankov et al. [5] arrived at similar conclusions when they studied the corresponding relationship between the COD strain range and number of cycles.

However, the accumulative plastic strain rate transforms from a rapid decrease in the initial stage to gradual stabilization with the increase in the number of cycles. The results show that the stress ratio has a more significant influence on the accumulative incremental plastic strain of specimens with inclined crack.

4.3 $\Delta CTOD$ during biaxial low-cycle fatigue crack propagation

In the biaxial low-cycle fatigue crack propagation experiment, the crack length was measured using a high-definition optical electronic digital CCD industrial microscope.

Meanwhile, the $\Delta CTOD$ during the crack propagation was measured synchronously. The $\Delta CTOD$ values at different times under various operating conditions are shown in Fig. 10, and the relationship between crack length and loading cycles is shown in Fig. 11.

Figure 10 shows that the $\Delta CTOD$ curves under different biaxial low-cycle fatigue loads have an increasing trend. The biaxial low-cycle fatigue crack propagation rate is inversely proportional to the stress ratio R under biaxial tensile–tensile loading condition (see Fig. 11a). Therefore, the crack propagation becomes slower with the increase in R . This is also reflected in the variation in $\Delta CTOD$. The increase in R improves the average stress of the load in the case of a certain maximum load value. However, the amplitude range of the load is identical. It is established that the influence of stress amplitude on biaxial low-cycle fatigue crack growth rate and $\Delta CTOD$ value is significantly higher than that of mean stress. Figures 10b and 11b show the results of different stress amplitudes. The biaxial low-cycle fatigue crack growth rate and $\Delta CTOD$ increase with the stress amplitude. This appears to be reasonable. The stress–strain field at the crack tip becomes stronger with the increase in stress amplitude. This accelerates the biaxial low-cycle fatigue crack propagation. Meanwhile, the increase in tensile stress also results in a larger $\Delta CTOD$.

In the study all the measured values for accumulation plastic strain rates are essentially similar with different biaxial stress ratios λ , because the combined stress at the crack tip is identical. However, the experimental results in Figs. 10c and 11c show that the unidirectional stress field can promote the crack propagation more significantly

Fig. 9 Accumulation plastic strain rate for different biaxial loadings

than the perpendicular composite stress field. Therefore, the crack propagation rate is highest in the uniaxial loading experiment ($\lambda=0$). Meanwhile, when $\lambda \neq 1$, the crack propagation rate behaves differently because of the turning direction of crack propagation. In general, $\lambda=1$ appears to be the most stable state of crack propagation. In this condition, the horizontal and vertical stress levels are equal, and the variations in crack growth rate and $\Delta CTOD$ become gradual.

Due to the limitation of stress method in evaluating the mechanical mechanism of crack propagation and fracture interruption, the accumulative plastic phenomenon at the crack tip of biaxial low-cycle fatigue is considered. The evaluation equation for $\Delta CTOD$ based on the accumulative plastic strain is established to evaluate the biaxial fatigue crack propagation behavior of hull cracked plate under biaxial low-cycle fatigue load.

Figure 12 systematically analyzes the variation curves of $\Delta CTOD$ based on accumulative plastic strain

Fig. 10 Variation curves of $\Delta CTOD$ for different biaxial low-cycle fatigue loads

increment $\Delta\epsilon_p$ for different biaxial low-cycle fatigue loads with different biaxial stress ratios, stress ratios and stress amplitudes. Figure 12a presents that $\Delta CTOD$ and accumulative plastic strain increment $\Delta\epsilon_p$ basically presents an increasing relationship, and the increasing trend is basically the same. The accumulative plastic strain increment $\Delta\epsilon_p$ increases with the decrease of stress ratio R . The smaller stress ratio R is, the higher the accumulative plastic strain increment $\Delta\epsilon_p$, so the faster the biaxial low-cycle fatigue crack propagation. It also has a similar linear increasing relationship under the action of

different stress amplitudes and different biaxiality ratios, as shown in Fig. 12b, d.

The overall performance is that the greater the stress amplitude, the higher the accumulative plastic strain increment $\Delta\epsilon_p$, and the growth trend is also more significant. When the biaxiality ratio is equal to 1 ($\lambda = 1$), the corresponding accumulative plastic strain increment $\Delta\epsilon_p$ and $\Delta CTOD$ are the smallest. It indicated that the biaxial low-cycle fatigue crack propagation is the most stable when the horizontal load is equal to the vertical load.

Fig. 11 a–N curves of biaxial low-cycle fatigue test for different biaxiality ratios

4.4 Relationship between crack growth rate and $\Delta CTOD$

Figure 13 summarizes the crack propagation results of da/dN – $\Delta CTOD$ under biaxial low cycle fatigue loading conditions of maximum stress $\Delta\sigma = 100$ MPa, $R = 0.2$, and $\lambda = 0, 0.5, 1.0$, and 1.5 . Different biaxial stress ratios have a significant effect on the propagation rate of biaxial low-cycle fatigue cracks. The results show that $\Delta CTOD$ is an

appropriate parameter for characterizing fatigue crack tip state. The propagation of biaxial low-cycle fatigue crack is caused mainly by the continuous passivation and remodeling of the crack tip. In this process, $\Delta CTOD$ also varies continuously. To further clarify the relationship between da/dN and $\Delta CTOD$, Eq. (3) was fitted to the experimental data. In the results for $\lambda = 0, 0.5, 1.0$, and 1.5 , $C = 0.0314, 0.0138, 0.0471$, and 0.0228 , respectively, and $m = 1.7467, 1.2016, 2.5280$, and 1.9621 , respectively. Figure 14 shows that the

(a) With different stress ratios R (b) With different stress amplitudes $\Delta\sigma$ (b) With different biaxial stress ratios λ **Fig. 12** Relationships curves between ΔCTOD and accumulative plastic strain rate under different biaxial loadings

propagation prediction of biaxial low-cycle fatigue crack is in agreement with the experimental results.

Compared with the experimental results of biaxial low-cycle fatigue crack propagation, the fitting curve shows good consistency with the experimental results at the stable propagation stage. Therefore, all the values of the fitting parameters C and m are reasonable, and these can be used as the material constants for the crack propagation rate prediction model. The proposed propagation rate prediction model considering accumulative plasticity can accurately reflect biaxial low-cycle fatigue crack

propagation behavior of hull plates. The overall fitting results are shown in Table 3.

4.5 Fracture morphology analysis

Figure 14 shows the observed experimental crack growth paths for the four biaxiality ratio cases ($\lambda = 0, 0.5, 1.0$, and 1.5) with the $100\times$ magnification of CCD camera. These paths are highlighted as white lines along the actual crack extension profile. The resultant force under a uniaxial fatigue load with a biaxiality ratio of $\lambda = 0$ is in the y -direction. Therefore, biaxial crack growth, which initially propagated at an angle

Fig. 13 Biaxial low-cycle fatigue crack growth curve for different biaxiality ratios

of 45° with the horizontal, is likely to propagate perpendicular to the applied load. The directions of the resultant force for cruciform specimens relative to the initial inclined crack in the tests with different biaxiality ratios $\lambda = 0, 0.5, 1.0$, and 1.5 were $\pm 45^\circ, \pm 18.4^\circ, 0^\circ$, and $\pm 11.3^\circ$, respectively. As the biaxiality ratio was increased, the biaxial low-cycle fatigue crack propagation path tended to deviate from the original center line. Thereby, it exhibited apparent path instability

with a marginal “zigzag” pattern. Figure 15d shows that in the case with $\lambda = 1.5$, the biaxial low-cycle fatigue crack propagated from the initial Point line 6 to Point line 7 and then passed Point line 8 to Point line 10, which rapidly moved up the vertical axis in the end. This behavior occurred mainly because the direction of the resultant force of the biaxial fatigue loading was perpendicular to the vertical axis. The biaxial crack growth path in the case with $\lambda = 1.5$ exhibited an

Fig. 14 Images of crack propagation paths for **a** $\lambda=0$, **b** $\lambda=0.5$, **c** $\lambda=1$, and **d** $\lambda=1.5$. ($\times 100$ magnification)

Table 3 Fitting results of relationship between da/dN and $\Delta CTOD$

λ	$\Delta\sigma$ (MPa)	R	C	m
1.0	110	0.1	0.0172	1.1217
1.0	110	0.2	0.0008	0.4906
1.0	110	0.3	0.0008	0.5327
1.0	100	0.1	0.0049	1.0094
1.0	120	0.1	0.0011	0.3444
1.0	140	0.1	0.0157	1.0155
0	100	0.2	0.0314	1.7476
.5	100	0.2	0.0138	1.2016
1.0	100	0.2	0.0471	2.5280
1.5	100	0.2	0.0228	1.9621

overall non-coplanar deviation from the initial 45° crack line direction. Lee et al. [18] observed biaxial crack propagation

paths similar to those observed in this study by performing biaxial fracture tests of center-cracked plates.

5 Numerical analysis and discussion

5.1 Numerical model

In this study, ABAQUS software was used to establish the FE model for the cruciform specimen with an inclined crack, as shown in Fig. 15. The cross-tension whole cruciform specimen with a 45° pre-crack was modelled under equal proportional loading. The specific geometric dimensions are shown in Fig. 1. The four-node reduced integral element (CPS4R) was used to divide the mesh. The reduced integral element is helpful to improve the calculation efficiency, and the analysis accuracy will not

Fig. 15 Finite element model for cruciform specimen with an inclined crack and the refined mesh

Fig. 16 Three-layer transition grid for the crack growth region

Table 4 Material parameters for the Chaboche combined model

E (GPa)	ν	Q	b	k		
206	0.3	162	8.5	145		
C_1	α_1	C_2	α_2	C_3	α_3	
140,000	8750	75,000	238	1950	0	

be significantly affected when the distortion is merged in the vicinity of the crack tip.

In order to improve the convergence speed and convergences, reasonable mesh division is carried out to ensure that local elements do not have distortion and torsion. At the same time, to improve the calculation efficiency, the element grid in the crack tip region was refined. The element size in the crack tip region along the crack growth path was unified as $0.05 \text{ mm} \times 0.05 \text{ mm}$. Figure 16 shows the transition grid with a total of three layers that was used between the refined area and the periphery of the crack. The accuracy of the FE analysis of the stress field at the crack tip was directly affected by the refinement degree

Fig. 17 Convergence judgment of accumulative plastic strain at crack tip

of the crack extension region, which is more conducive to the use of node release technology to simulate the crack extension. In addition, because the cracked plate is usually relatively thin, the plane-stress assumption was adopted.

In the present FE study, Chaboche's kinematic hardening model was used to accurately simulate the Masing-type behaviour of materials. The elasto-plastic characteristic of the material constitutive model was described using the von Mises yield criterion associated with the plastic flow rule. The material constitutive function in Chaboche's model contains parameters E , ν , k , Q , C_1 , α_1 , C_2 , α_2 , C_3 , and α_3 . All the parameter values in Chaboche's kinematic hardening model were optimised from the fatigue test data at room temperature, which are listed in Table 4. Chaboche's

kinematic hardening model was adopted in the FE analysis to analyse the material's behaviour under biaxial fatigue loads.

The extension of a biaxial crack in the inclined-crack specimen was simulated by releasing sequentially the nodes on the extended path ahead of the crack tip. The nodes to be released were initially connected with the rigid surface along the original crack centre line. The contact elements were adopted to prevent the crack surface from overlapping under unloading.

5.2 Adopted element type and convergence study

In order to ensure the reliability of the finite element model calculation results for the stress–strain field at the crack tip, the accumulative plastic convergence at the crack tip of cruciform specimens with inclined crack obtained by the finite element method had been verified under the biaxial low-cycle fatigue loadings in this study. The relationship curve between the accumulative plastic deformation at crack tip and the elements numbers of finite element model obtained by calculation is shown in Fig. 17 as follows.

Figure 17 displayed that the accumulative plastic deformation at crack tip of cruciform specimens with inclined crack varied with the continuous refinemesh at crack tip region under biaxial low-cycle fatigue load, where i is the number of mesh element in the calculation model. The accumulative plastic deformation at crack tip tends to be stable with the increase of the nodes number in the numerical model, which shown that the accumulative plastic deformation at crack tip calculated by the finite

element model of cruciform specimens with inclined crack is convergent.

Compared with the test results under the same loading conditions, it can be seen that the numerical simulation results are basically consistent with the test results, which can ensure the reliability of the calculation results. Combined with the calculation results in Fig. 17 and considered the calculation accuracy and efficiency, the finite element model with a total mesh element numbers of 23,296 were selected to calculate the accumulative plastic deformation at crack tip of cruciform specimens with inclined crack under biaxial low-cycle fatigue load.

5.3 Residual stress distribution

Figure 18 presents the normal residual stress distribution of nodes at different positions from the crack tip under the minimum value of biaxial low-cycle fatigue loading with different biaxial and stress ratios. Figure 18 demonstrates that obvious stress concentration appears at the crack tip under biaxial fatigue loading. The residual stress reaches its peak value at the crack tip, then gradually decreases with increasing distance from the crack tip. Excessive stress will produce plastic yield and deformation at the crack tip, inducing the crack tip to gradually separate and move forward and expand. The greater the biaxial stress ratio is, the relatively higher the corresponding residual stress also is. Therefore, a parallel crack load enhances the stress field in front of the crack tip and accelerates the biaxial low-cycle fatigue crack growth rate for an inclined-crack specimen, which had been proposed by Tchankov [5].

(a) With different biaxial stress ratios

(b) With different stress ratios

Fig. 18 Distribution of residual stress at the crack tip along the crack propagation direction under different conditions

(a) Effect of biaxial stress ratios

(b) Effect of stress ratios

Fig. 19 Dimensionless stress histories at the crack tip with different biaxial fatigue loads

Figure 19 presents the dimensionless equivalent stress distribution history over an inclined-crack central line of the inclined-crack specimen under different biaxial fatigue loads. Each equivalent uniaxial stress was calculated according to Eq. (2) in the literature [19], which enables analysis of the biaxial load problem by transforming the biaxial load problem into an equivalent uniaxial stress problem. Figure 19a and b show that the equivalent stress along the crack line direction follows a sinusoidal function with time under different biaxial stress ratios and stress ratios. The residual stress in the

front of the crack tip gradually decreases along the crack line direction as the biaxial stress ratios and stress ratios change. In addition, it is also found that the equivalent x-axis stress under each average stress condition is basically consistent with the trend of the external load. Therefore, the equivalent uniaxial stress is higher than the original biaxial loading condition with different average stresses, and therefore the biaxial low-cycle fatigue crack propagation rate is accelerated.

Fig. 20 Stress and strain nephograms of the inclined-crack specimen

(a) With different stress ratios

(b) With different biaxial stress ratios

(c) With different stress amplitudes

Fig. 21 Plastic zone radius sizes of the inclined-crack specimen for different biaxial loads

5.4 Size of the plastic zone

The fatigue crack surface will be open during the tensile loaded stage, which causes forward biaxial low-cycle fatigue crack propagation. During the unloading phase, the crack closes, and fatigue crack propagation during this stage is significantly affected by the accumulative plasticity at the crack tip. The local plastic zone radius at the crack tip is the basis for a reasonable evaluation for biaxial

crack growth behaviours of the cruciform specimens with an inclined crack. Figures 20 and 21 present, respectively, the stress and strain diagrams and the plastic zone radius size formed at the crack tip with different biaxial loadings obtained from the elastic–plastic FE analyses.

Figure 21 demonstrates that the crack plastic zone radius increases with increasing biaxiality ratio, stress ratio, and stress amplitude. Because the driving force for the extension of the biaxial crack in the inclined-crack specimen originated from the plastic hysteresis of the crack tip, the crack plastic zone radius is considered to

be associated with the biaxial crack growth rate for the cruciform specimens with an inclined crack. Figure 21a and b show that the crack plastic zone radius increases with increasing biaxiality ratio and increasing stress ratio R with the same other load factors, compared with the uniaxial load condition. Figure 21c shows that the maximum and minimum plastic zone radii gradually increase as the average stress increases. The maximum and minimum plastic zone size varies with the effect of the stress ratio. In general, the greater the biaxial stress ratio, stress ratio, and stress amplitude are, the larger the crack plastic zone radius of the inclined-crack specimen is. Loads parallel to the crack promote the increase of the plastic zone radius size at the crack tip of the cruciform specimens with an inclined crack and accelerates biaxial low-cycle fatigue crack propagation. By measuring the crack opening displacement in push–pull tests, Tchankov [5] also indicated that the parallel stress increases the stress intensity factor at the crack tip and accelerates the biaxial fatigue crack propagation rate.

6 Conclusion

In this study, the biaxial low-cycle fatigue crack propagation rate of hull inclined-crack plate and $\Delta CTOD$ considered accumulative plasticity were obtained through theoretical analysis and experiment investigations under biaxial low-cycle fatigue loads. The main conclusions are as follows:

(1) The presented prediction model for the biaxial low-cycle fatigue crack growth of hull inclined-crack plate with $\Delta CTOD$ as a control parameter can essentially represent the evolution of biaxial low-cycle fatigue cracks. The results show that the $\Delta CTOD$ based on biaxial accumulative plasticity can characterize the biaxial low-cycle fatigue crack growth.

(2) Prominent accumulative plasticity was observed at crack tip under biaxial proportional low-cycle fatigue loading. It resulted in the final separation of the material along the crack tip propagation path. Thus, the method for predicting the biaxial low-cycle fatigue crack growth rate of hull inclined-crack plate considering the biaxial accumulative plastic effect is feasible.

(3) The results of the da/dN – $\Delta CTOD$ curve were obtained through the crack growth experiment. The values of the material parameters C and m under different biaxial low-cycle fatigue loads were fitted based on the experimental results. The result shows that C and m are suitable as material constants for the biaxial low-cycle fatigue crack propagation prediction model.

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Declarations

Conflict of interests The authors declare that they have no competing interests.

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