Variable universe adaptive fuzzy sliding mode projective synchronization of hyperjerk system based on disturbance observer

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Abstract

In this paper, a sliding mode projective synchronization strategy based on disturbance observer and fuzzy system is presented to implement projective synchronization of hyperjerk system with low time-varying disturbance and white noise. Theoretical analysis and numerical calculation show that the disturbance observer can approach the low time-varying disturbance very well. The application of disturbance observer reduces the chattering of the controller. Variable universe adaptive fuzzy control (VUAFC) method is utilized to further reduce the chattering phenomenon. The simulation results demonstrate the effectiveness of the proposed controller.

Keywords Hyperjerk system · Sliding mode control · Disturbance observer · Low time-varying · Variable universe adaptive fuzzy control

1 Introduction

A hyperjerk system [1] is a dynamical system described by an n th ordinary differential equation where n is 4 or up to, i.e.

$$\frac{d^n x}{dt^n} = J\left(\frac{d^{n-1}x}{dt^{n-1}}, \dots, \frac{dx}{dt}, x\right) \quad (n \geq 4). \quad (1)$$

where x denotes displacement, $\frac{dx}{dt}$ denotes velocity, $\frac{d^2x}{dt^2}$ is acceleration, $\frac{d^3x}{dt^3}$ is jerk, and $\frac{d^4x}{dt^4}, \dots, \frac{d^nx}{dt^n}$ are hyperjerks. When $n = 3$, system (1) degenerates into a jerk system. The main concerns of the researchers are the chaotic or hyperchaotic performance, control and synchronization of jerk system or hyperjerk system with such simple structures [2–11].

As a synchronization strategy, the concept of projective synchronization of chaotic (or hyperchaotic) system

was first proposed by Mainieri and Rehacek [12], where the introduction of a scaling factor extended the range of synchronization. When projective synchronization is realized, the state outputs of the drive system and the response system are not only phase locked, but also the amplitude of the corresponding state evolves according to a fixed proportional relationship. Because projective synchronization can be applied to chaotic secure communication, binary number can be extended to base m number to achieve faster transmission, so many scholars have done in-depth research on projective synchronization [13–16].

For the inevitable external disturbance and uncertainty of control system, many control method is used, e.g., adaptive control [7, 11, 17], backstepping control [7, 17, 18], interval observer method [19, 20], sliding mode control [21–23]. A single control method cannot achieve the required precision of control, most of the control in the literature is the

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combination of several control methods. To address the problem of attitude adjustment for a quad-rotor unmanned aerial vehicle, Song et al. [17] apply the adaptive control and back-stepping control. One of the characteristics of sliding mode control is to overcome the uncertainty and disturbance of the system. Gao et al. [24] proposed the exponential reaching law method to design a sliding mode controller, which can not only accelerate the approach speed but also provide a measure for the reduction of chattering to some extent [25, 26]. For the purpose of reaching the stable state of the control system in limited time, terminal sliding mode control is proposed by Zak [27]. Many researchers have applied the terminal sliding control to various control systems. Fei et al. [21, 23] designed the fractional-order sliding mode controller based on neural network.

In [28], Kawamura et al. proposed disturbance observer to compensate the uncertainty and disturbance of the servo motor in the controlling system. Some literature is posted on this method [29–31]. Considering the observer error and some other disturbance which could not compensate with observer, such as white noise, fuzzy system is a great idea [32] to smooth the discontinuity of the sliding mode control. Usually the control precision in a fuzzy system can be enhanced by increasing the amount of fuzzy rules. This can lead to a problem of a great deal of computations and even an “rules explosion.” Variable universe adaptive fuzzy control method (VUAFC) introduced by Li [33, 34] can avoid the problem of “rules explosion” and improve the control precision by on-line contraction-expansion of variable universes.

Inspired by the above control methods, in this paper, a variable universe adaptive fuzzy sliding mode projective Synchronization (VUAFSMPS) based on disturbance observer is proposed for hyperjerk system. A sliding mode surface with nonlinear term is proposed. We quantitatively analyze the effect of the disturbance observer proposed in [28] and apply it to compensate the low time-varying disturbance of hyperjerk system. Variable universe adaptive fuzzy control method is used to smooth the switching term.

The organization of this work is as follows. In Sect. 2, problem description is given. In Sect. 3, variable universe adaptive fuzzy sliding mode projective synchronization is introduced in detail. In particular, we use linear system theory to analyze the observer error of disturbance observer. Simulation results given in Sect. 4. Finally, the conclusion is drawn.

2 Problem description

Hyperjerk system has a very simple structure. It is quite interesting because it presents a complex dynamic. In the literature, there are some hyperjerk systems presenting chaotic (hyperchaotic) phenomena. Here we consider the projective synchronization.

The drive system is depicted as

$$\begin{cases} \dot{x}_1 = y_1, \\ \dot{y}_1 = z_1, \\ \dot{z}_1 = w_1, \\ \dot{w}_1 = f_d(x_1, y_1, z_1, w_1) + d_1(t) + n_1(t) \end{cases} \quad (2)$$

where x_1, y_1, z_1, w_1 are state variables, f_d is continuous function, $d_1(t)$ is an unknown low time-varying external disturbance, $|d_1(t)| \leq D_1$, D_1 is a positive constant, $n_1(t)$ is white noise, $n_1(t) \leq N_1$.

The response system is described as

$$\begin{cases} \dot{x}_2 = y_2, \\ \dot{y}_2 = z_2, \\ \dot{z}_2 = w_2, \\ \dot{w}_2 = f_r(x_2, y_2, z_2, w_2) + d_2(t) + n_2(t) + u \end{cases} \quad (3)$$

where x_2, y_2, z_2, w_2 are state variables, f_r is continuous function, $d_2(t)$ is an unknown low time-varying external disturbance, $|d_2(t)| \leq D_2$, D_2 is a positive constant, $n_2(t) \leq N_2$, u is the controller to be designed.

Suppose the scaling factor is α . The errors are given by

$$\begin{cases} e_x = x_2 - \alpha x_1, \\ e_y = y_2 - \alpha y_1, \\ e_z = z_2 - \alpha z_1, \\ e_w = w_2 - \alpha w_1. \end{cases} \quad (4)$$

Differentiating (4), the error dynamic system is

$$\begin{cases} \dot{e}_x = e_y, \\ \dot{e}_y = e_z, \\ \dot{e}_z = e_w, \\ \dot{e}_w = f_r - \alpha f_d + (d_2(t) + n_2(t)) - \alpha(d_1(t) + n_1(t)) + u \end{cases} \quad (5)$$

Our goal is designing suitable input u such that system (3) synchronizes (2) up to the scaling factor α , i.e., system (5) converges to zero, especially, $\mathbf{e} = (e_x, e_y, e_z, e_w) \rightarrow 0$ as $t \rightarrow \infty$.

3 Design of fuzzy sliding mode projective synchronization based on disturbance observer

The sliding mode surface with nonlinear term is proposed in Sect. 3.1. The observer of slow time-varying disturbance is quantitatively analyzed in Sect. 3.2. Variable universe

adaptive fuzzy controller is restated in Sect. 3.3. The composite controller is introduced in Sect. 3.4.

3.1 Sliding mode control

Under the view of traditional sliding mode control, the control system (5) can be stabilized with the sliding mode surface as

$$s = e_w + c_3 e_z + c_2 e_y + c_1 e_x \quad (6)$$

where the coefficients c_1, c_2, c_3 satisfy the Hurwitz polynomial $p^3 + c_3 p^2 + c_2 p + c_1$, p is Laplace operator. By changing the first term of Eq. (6), we can get a sliding mode surface with nonlinear term, i.e.

$$s = e_w^{p/q} + c_3 e_z + c_2 e_y + c_1 e_x \quad (7)$$

where $q < p < 2q$, p, q are prime numbers, the coefficients c_1, c_2, c_3 is to be designed such that the sliding surface (7) is stable when $s = 0$.

Differentiating Eq. (7), we get

$$\begin{aligned} \dot{s} &= \frac{p}{q} e_w^{p/q-1} \dot{e}_w + c_3 \dot{e}_z + c_2 \dot{e}_y + c_1 \dot{e}_x \\ &= \frac{p}{q} e_w^{p/q-1} \left(\dot{e}_w + c_3 \frac{q}{p} e_w^{2-p/q} + c_2 \frac{q}{p} e_w^{1-p/q} e_z + c_1 \frac{q}{p} e_w^{1-p/q} e_y \right) \end{aligned} \quad (8)$$

Considering the error dynamic system (5), ignoring the external disturbances, the equivalent control term is solved by setting $\dot{s} = 0$

$$u_{eq} = -f_r + \alpha f_d - \frac{c_3 q}{p} e_w^{2-p/q} - \frac{c_2 q}{p} e_w^{1-p/q} e_z - \frac{c_1 q}{p} e_w^{1-p/q} e_y \quad (9)$$

The switching control term is

$$u_{erl} = -M \text{sign}(s) - \gamma s \quad (10)$$

Then the comprehensive controller is designed as

$$\begin{aligned} u &= u_{eq} + u_{erl} \\ &= -f_r + \alpha f_d - \frac{c_3 q}{p} e_w^{2-p/q} - \frac{c_2 q}{p} e_w^{1-p/q} e_z - \frac{c_1 q}{p} e_w^{1-p/q} e_y \\ &\quad - M \text{sign}(s) - \gamma s \end{aligned} \quad (11)$$

As is well known, the main cause of chattering is the existence of switching gain M . Switching term is used to overcome the disturbances and uncertainty. It is a good idea by using disturbance observer to compensate the disturbance.

3.2 Observer of slow time-varying disturbance

In this section, we give the disturbance observer based on Kawamura's work [28] and analyze the observation error quantitatively.

For $d_1(t)$ of system (2), we design the disturbance observer as

$$\begin{cases} \dot{\hat{d}}_1 = -k_1(\hat{w}_1 - w_1), \\ \dot{\hat{w}}_1 = f_d + \hat{d}_1 - k_2(\hat{w}_1 - w_1), \end{cases} \quad (12)$$

where $\hat{d}_1$ is the estimation of d_1 , $\hat{w}_1$ is the estimation of w_1 , $k_1 > 0, k_2 > 0$.

For $d_2(t)$ of system (3), we design the disturbance observer as follows

$$\begin{cases} \dot{\hat{d}}_2 = -k_3(\hat{w}_2 - w_2), \\ \dot{\hat{w}}_2 = f_r + \hat{d}_2 - k_4(\hat{w}_2 - w_2) + u, \end{cases} \quad (13)$$

where $\hat{d}_2$ is the estimation of d_2 , $\hat{w}_2$ is the estimation of w_2 , $k_3 > 0, k_4 > 0$.

We define observation errors $\tilde{d}_i = \hat{d}_i - d_i$, $\tilde{w}_i = \hat{w}_i - w_i$, ($i = 1, 2$), now we calculate them quantitatively. From system (2) and system (12), we have

$$\begin{cases} \dot{\tilde{d}}_1 = -k_1 \tilde{w}_1 - \dot{d}_1 \\ \dot{\tilde{w}}_1 = \tilde{d}_1 - k_2 \tilde{w}_1 \end{cases} \quad (14)$$

From system (3) and system (13), we have

$$\begin{cases} \dot{\tilde{d}}_2 = -k_3 \tilde{w}_2 - \dot{d}_2 \\ \dot{\tilde{w}}_2 = \tilde{d}_2 - k_4 \tilde{w}_2 \end{cases} \quad (15)$$

By using vector notation, we define

$$E_1 = \begin{pmatrix} \tilde{d}_1 \\ \tilde{w}_1 \end{pmatrix}, A = \begin{pmatrix} 0 & -k_1 \\ 1 & -k_2 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

and rewrite the system (14) as one 2-dimensional first-order vector differential equation

$$\dot{E}_1 = AE_1 + B\dot{d}_1 \quad (16)$$

Now we diagonalize the system matrix A . The eigenvalues of A are $\lambda_1 = \frac{-k_2 + \sqrt{k_2^2 - 4k_1}}{2}$ and $\lambda_2 = \frac{-k_2 - \sqrt{k_2^2 - 4k_1}}{2}$.

If we define

$$D = \begin{pmatrix} \frac{-k_2 + \sqrt{k_2^2 - 4k_1}}{2} & 0 \\ 0 & \frac{-k_2 - \sqrt{k_2^2 - 4k_1}}{2} \end{pmatrix},$$

$$P = \begin{pmatrix} \frac{k_2 + \sqrt{k_2^2 - 4k_1}}{2} & \frac{k_2 - \sqrt{k_2^2 - 4k_1}}{2} \\ 1 & 1 \end{pmatrix},$$

$$P^{-1} = \frac{1}{\sqrt{k_2^2 - 4k_1}} \begin{pmatrix} 1 & \frac{-k_2 + \sqrt{k_2^2 - 4k_1}}{2} \\ -1 & \frac{k_2 + \sqrt{k_2^2 - 4k_1}}{2} \end{pmatrix},$$

then

$$P^{-1}AP = D, \quad A = PDP^{-1}$$

Defining $E_1^* = P^{-1}E_1$, then system (16) change to

$$\dot{E}_1^* = DE_1^* + P^{-1}B\dot{d}_1. \quad (18)$$

For D is diagonalizable, from linear system theory, we know the solution of system (18) is

$$E_1^* = e^{Dt}E_1^*(t_0) + \int_0^t e^{D(t-\tau)}P^{-1}B\dot{d}_1(\tau)d\tau \quad (19)$$

Because $\lambda_1 = \frac{-k_2 + \sqrt{k_2^2 - 4k_1}}{2}$, $\lambda_2 = \frac{-k_2 - \sqrt{k_2^2 - 4k_1}}{2}$, then

$$E_1^* = \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix} E_1^*(t_0) + \frac{1}{\sqrt{k_2^2 - 4k_1}} \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix} \int_0^t \begin{pmatrix} -e^{-\lambda_1 \tau} \\ e^{-\lambda_2 \tau} \end{pmatrix} \dot{d}_1(\tau)d\tau$$

If $\dot{d}_1(t) = 0$, $E_1^* \rightarrow 0$ as $t \rightarrow \infty$.

Suppose $0 < |\dot{d}_1(t)| \leq N_1$, we have

$$|e^{\lambda_1 t} \int_0^t -e^{-\lambda_1 \tau} \dot{d}_1(\tau)d\tau| \leq N_1 e^{\lambda_1 t} \int_0^t e^{-\lambda_1 \tau} d\tau = \frac{N_1}{\lambda_1} (e^{\lambda_1 t} - 1)$$

$$|e^{\lambda_2 t} \int_0^t e^{-\lambda_2 \tau} \dot{d}_1(\tau)d\tau| \leq N_1 e^{\lambda_2 t} \int_0^t e^{-\lambda_2 \tau} d\tau = \frac{N_1}{\lambda_2} (e^{\lambda_2 t} - 1) \quad (20)$$

From linear system theory, we know $E_1^* \leq \frac{-N_1}{\sqrt{k_2^2 - 4k_1}} \begin{pmatrix} 1 \\ \lambda_1 \\ 1 \\ \lambda_2 \end{pmatrix}$

as $t \rightarrow \infty$, i.e.

$$E_1 \leq \frac{N_1}{2k_1 \sqrt{k_2^2 - 4k_1}} \begin{pmatrix} k_2^2 - 2k_1 \\ 2k_2 \end{pmatrix} \quad (21)$$

What we're interested in is $\tilde{d}_1(t)$. Now we know

$$|\tilde{d}_1(t)| \leq \frac{N_1(k_2^2 - 2k_1)}{2k_1 \sqrt{k_2^2 - 4k_1}} = \frac{N_1((\frac{k_2}{k_1})^2 - \frac{2}{k_1})}{2\sqrt{(\frac{k_2}{k_1})^2 - \frac{4}{k_1}}} \sim \frac{k_2 N_1}{2k_1} \quad (22)$$

$(k_1 \gg k_2 > 1)$

From (22), we know that if $k_1 \gg k_2$, observer error is very small, that is to say, the disturbance observer works very well.

Example 1 If $d_1 = \sin t$, then $\dot{d}_1(t) = \cos t$, we have

$$E_{11}^* = e^{\lambda_1 t} E_{11}^*(t_0) + \frac{1}{(1+\lambda_1^2)\sqrt{k_2^2 - 4k_1}} (-\sin t + \lambda_1 \cos t - \lambda_1 e^{\lambda_1 t})$$

$$= \left(E_{11}^*(t_0) - \frac{\lambda_1}{(1+\lambda_1^2)\sqrt{k_2^2 - 4k_1}} \right) e^{\lambda_1 t}$$

$$+ \frac{1}{\sqrt{1+\lambda_1^2}\sqrt{k_2^2 - 4k_1}} \left(\frac{-1}{\sqrt{1+\lambda_1^2}} \sin t + \frac{\lambda_1}{\sqrt{1+\lambda_1^2}} \cos t \right)$$

$$= \left(E_{11}^*(t_0) - \frac{\lambda_1}{(1+\lambda_1^2)\sqrt{k_2^2 - 4k_1}} \right) e^{\lambda_1 t}$$

$$+ \frac{1}{\sqrt{1+\lambda_1^2}\sqrt{k_2^2 - 4k_1}} \sin(t + \beta_1)$$

$$E_{12}^* = e^{\lambda_2 t} E_{12}^*(t_0) + \frac{1}{(1+\lambda_2^2)\sqrt{k_2^2 - 4k_1}} (\sin t - \lambda_2 \cos t - \lambda_2 e^{\lambda_2 t})$$

$$= \left(E_{12}^*(t_0) - \frac{\lambda_2}{(1+\lambda_2^2)\sqrt{k_2^2 - 4k_1}} \right) e^{\lambda_2 t}$$

$$+ \frac{1}{\sqrt{1+\lambda_2^2}\sqrt{k_2^2 - 4k_1}} \left(\frac{1}{\sqrt{1+\lambda_2^2}} \sin t - \frac{\lambda_2}{\sqrt{1+\lambda_2^2}} \cos t \right)$$

$$= \left(E_{12}^*(t_0) - \frac{\lambda_2}{(1+\lambda_2^2)\sqrt{k_2^2 - 4k_1}} \right) e^{\lambda_2 t}$$

$$+ \frac{1}{\sqrt{1+\lambda_2^2}\sqrt{k_2^2 - 4k_1}} \sin(t + \beta_2)$$

Based on linear system theory, we know

$$E_1^* \rightarrow \begin{pmatrix} \frac{1}{\sqrt{1+\lambda_1^2}\sqrt{k_2^2 - 4k_1}} \sin(t + \beta_1) \\ \frac{1}{\sqrt{1+\lambda_2^2}\sqrt{k_2^2 - 4k_1}} \sin(t + \beta_2) \end{pmatrix}$$

From $E_1 = PE_1^*$, we have

$$E_1 \rightarrow \begin{pmatrix} \frac{-\sqrt{(1+k_2^2-k_1)^2 + k_1^2 k_2^2}}{1+k_1^2+k_2^2-2k_1} \sin(t + \beta_1) \\ \frac{1}{\sqrt{(1-k_1)^2 + k_2^2}} \sin(t + \beta_2) \end{pmatrix} \quad (23)$$

Note 1: From (23), we know that the period of the observer error is the same as the period of the disturbance, the observer error boundary of $\tilde{d}_1$ is $\frac{\sqrt{(1+k_2^2-k_1)^2+k_1^2k_2^2}}{1+k_1^2+k_2^2-2k_1}$ and the observer error boundary of $\tilde{\omega}_1$ is $\frac{1}{\sqrt{(1-k_1)^2+k_2^2}}$.

The changes of observer error boundary with parameters evolving are shown as Table 1.

Table 1 further illustrates that as long as appropriate parameters k_1 and k_2 are selected, the observer can achieve better compensation. It is also demonstrated that the observation error is persistent as long as the derivative of the disturbance is not zero. Once again, it is proved that the observation error is related to the parameter proportion.

We also get the similar conclusions about the observer error system (15).

3.3 Variable universe adaptive fuzzy controller

From the discussion of the above subsection, we know the switching gain in sliding mode control can be significantly reduced by using the disturbance observer method. At the same time, as long as the derivative is not equal to zero, the observation error is persisting steadily existing. And if the disturbances don't have derivatives (inevitable, white noise, for instance), the observer cannot be employed to compensate them. To further reduce chattering is still a problem. Because the fuzzy system can estimate the switching gain effectively according to the reaching condition, this action can reduce chattering. Usually the control precision in a

fuzzy system can be improved by increasing the amount of fuzzy rules. Meanwhile, this can lead to a large number of computations and even an "rules explosion." VUAFC method improves the control precision by on-line contraction-expansion of variable universes without increasing the fuzzy rules. Variable universe adaptive fuzzy control method can establish high control precision with a few fuzzy rules.

Variable universe adaptive fuzzy controller was first proposed by Li [33, 34]. Because this is a high precision controller, many scholars have studied it [35–38]. Suppose $X_i = [-E_i, E_i]$ ($i = 1, 2, \dots, m$) be the universe of input variable x_i ($i = 1, 2, \dots, m$), $Y = [-U, U]$ be the universe of the output variable y . $\{A_{ij}\}$ ($1 \leq j \leq q_i$) stand for a fuzzy partition on X_i and B_l ($1 \leq l \leq h$) stand for a fuzzy partition on Y . Suppose the fuzzy inference rules is as:

$$\text{If } x_1 \text{ is } A_{1j_1}, x_2 \text{ is } A_{2j_2}, \dots, x_m \text{ is } A_{mj_m} \text{ then } y \text{ is } B_l. \quad (24)$$

Suppose x_{ij} is the peakpoint of A_{ij} , y_l is the peakpoint of B_l . The output of canonical fuzzy controller is

$$y = \sum_{j_1=1}^{q_1} \dots \sum_{j_m=1}^{q_m} \prod_{i=1}^m A_{ij_i}(x_i) y_l.$$

By introducing scale factors α_i and β , the universe discourse is changed with the changing variables x_i and y , i.e. $X_i(x_i) = [-\alpha_i(x_i)E_i, \alpha_i(x_i)E_i]$, $Y(y) = [-\beta(y)U, \beta(y)U]$. The fuzzy output of the variable universe fuzzy controller is as

$$y = \beta \sum_{j_1=1}^{q_1} \dots \sum_{j_m=1}^{q_m} \prod_{i=1}^m A_{ij_i}\left(\frac{x_i}{\alpha(x_i)}\right) y_l. \quad (25)$$

The parameter β can be adjusted by optimization.

3.4 The design of the controller

Based on (25), we firstly design the fuzzy system, the input of the fuzzy system is selected as sliding mode surface $s(t)$. Let the output of the fuzzy system is μ . The universes of discourse of $s(t)$ and μ are $|s_0| \times [-1, 1]$ and $[-1, 1]$, respectively.

Suppose the width of fuzzy layer is 2δ , the initial membership function of $s(t)$ is:

$$\begin{aligned} NB(s) &= \min\left(1, \max\left(-\frac{s}{\delta}, 0\right)\right), \\ ZO(s) &= \max\left(0, \min\left(\frac{s+\delta}{\delta}, -\frac{s-\delta}{\delta}\right)\right), \\ PB(s) &= \min\left(1, \max\left(\frac{s}{\delta}, 0\right)\right) \end{aligned} \quad (26)$$

The plot of initial membership function is depicted by Fig. 1.

The initial membership function of μ is selected as:

Table 1 Observer errors with parameters k_1 , k_2 changing

No.	k_1	k_2	Boundary of $\tilde{d}_1$	Boundary of $\tilde{\omega}_1$
1	1000	50	0.05	9.9975×10^{-4}
2	1000	100	0.0996	9.9602×10^{-4}
3	1000	150	0.1485	9.8990×10^{-4}
4	1000	200	0.1963	9.8152×10^{-4}
5	1000	250	0.2428	9.7106×10^{-4}
6	2000	100	0.05	4.9963×10^{-4}
7	3000	100	0.0333	3.3326×10^{-4}
8	4000	100	0.0250	2.4998×10^{-4}
9	5000	100	0.0200	2.000×10^{-4}
10	6000	100	0.0167	1.6667×10^{-4}
11	7000	100	0.0143	1.4286×10^{-4}
12	8000	100	0.0125	1.2501×10^{-4}
13	9000	100	0.0111	1.1112×10^{-4}
14	10,000	100	0.0100	1.0000×10^{-4}
15	3000	150	0.05	3.3303×10^{-4}

Fig. 1 The initial membership function of input $s(t)$

Fig. 2 The initial membership function of output μ

$$NB(\mu) = \min\left(1, \max\left(-\frac{\mu}{0.5}, 0\right)\right),$$

$$ZO(\mu) = \max\left(0, \min\left(\frac{\mu}{0.1}, -\frac{\mu - 0.3}{0.2}\right)\right),$$

$$PB(\mu) = \min\left(1, \max\left(\frac{\mu}{0.5}, 0\right)\right)$$

The plot of initial membership function is depicted by Fig. 2.

The natural initial fuzzy rules are:

$R^{(1)}$: If $s(t)$ is NB then μ is PB

$R^{(2)}$: If $s(t)$ is ZO then μ is ZO

$R^{(3)}$: If $s(t)$ is PB then μ is PB

The contraction-expansion factor is as follows

$$\alpha(s) = 1 - \lambda_1 \exp(-\lambda_2 s^2), \quad \lambda_1 \in (0, 1), \quad \lambda_2 > 0 \quad (27)$$

From (25), we know the structure of fuzzy system is

$$\mu = \left(p_1 \int_0^t s d\tau + \beta(0)\right) \left(0.5NB\left(\frac{s}{\alpha(s)}\right) + 0.1ZO\left(\frac{s}{\alpha(s)}\right) + 0.5PB\left(\frac{s}{\alpha(s)}\right)\right) \quad (28)$$

We add a coefficient μ to the switch term, the control law is designed as

Theorem 1 With the disturbance observers (12) and (13), projective synchronization of system (2) and system (3) up to scaling factor α can be realized under the following controller

$$u = -f_r + \alpha f_d - \hat{d}_2(t) + \alpha \hat{d}_1(t) - \frac{c_3 q}{p} e_w^{2-p/q} - \frac{c_2 q}{p} e_w^{1-p/q} e_z - \frac{c_1 q}{p} e_w^{1-p/q} e_y - \mu M_2 \text{sign}(s) - \gamma_2 s \quad (29)$$

where $M_2 \geq |D_1| + |\alpha D_2| + |N_1| + |\alpha N_2|$, μ is the output of variable universe adaptive fuzzy controller. The equivalent control part $u_{eq} = -f_r + \alpha f_d - \hat{d}_2(t) + \alpha \hat{d}_1(t) - \frac{c_3 q}{p} e_w^{2-p/q} - \frac{c_2 q}{p} e_w^{1-p/q} e_z - \frac{c_1 q}{p} e_w^{1-p/q} e_y$ and the reaching control part $u_{erl} = -\mu M_2 \text{sign}(s) - \gamma_2 s$.

Note 2: Obviously when $\mu = 1$, $u = u_{eq} + u_{erl}$. When $\mu \neq 1$, the control law is fuzzified. We select $|s_0| \in [-1, 1]$ as universe of discourse of $s(t)$ avoiding overflow of input.

4 Numerical simulation

We select the drive and response system given by [39]. The drive system is

$$\begin{cases} \dot{x}_1 = y_1, \\ \dot{y}_1 = z_1, \\ \dot{z}_1 = w_1, \\ \dot{w}_1 = -2x_1^3 - y_1^3 + (3x_1 - 4z_1)z_1^2 - 0.1w_1 + d_1(t) + n_1(t). \end{cases} \quad (30)$$

where $d_1(t) = 3 \sin(3t)$ is the low time-varying disturbance and $n_1(t) = 0.1 \text{rand}$ is uniform white noise.

The response system is

$$\begin{cases} \dot{x}_2 = y_2, \\ \dot{y}_2 = z_2, \\ \dot{z}_2 = w_2, \\ \dot{w}_2 = -2x_2^3 - y_2^3 + (3x_2 - 4z_2)z_2^2 - 0.1w_2 + d_2(t) + n_2(t). \end{cases} \quad (31)$$

where $d_2(t) = \sin(0.2t) + \cos(t)$ is the low time-varying disturbance and $n_2(t) = 0.2\text{rand}(t)$ is uniform white noise.

We select the scaling factor $\alpha = 1/2$, and the initial values of drive system and response system are $(3, 3, 3, -3)^T$ and $(-1, -1, -1, -1)^T$, respectively.

The sliding mode surface is designed as

$$s = e_w^{\frac{5}{3}} + 3e_z + 3e_y + e_x \quad (32)$$

Let the parameters $k_1 = 1000, k_2 = 200$ in Eq. (12). Let the parameters $k_3 = 1000, k_4 = 200$ in Eq. (13). Let the parameters $M_2 = 10, \gamma_2 = 50$.

4.1 Sliding mode control with disturbance observer

Regardless of fuzzy system, that is to say, let $\mu = 1$ in Eq. (29).

The synchronization error is depicted as Fig. 3. The evolution of sliding mode surface is depicted as Fig. 4.

The control u is depicted as Fig. 5. For the sake of clarity, the control u is enlarged as Fig. 6. The chattering phenomenon is very obvious.

Fig. 3 The synchronization errors

Fig. 4 The evolution of sliding mode surface s

The phase diagrams of projective synchronization are depicted as Figs. 7 and 8.

Figure 3 illustrates the projective synchronization be achieved with $\mu = 1$ of controller (28). Figures 5 and 6 illustrate the presence of chattering.

4.2 Sliding mode control with disturbance observer and variable universe adaptive fuzzy controller

Suppose the parameter $\delta = 1$ of Eq. (26), $\lambda_1 = 0.9, \lambda_2 = 10$ in Eq. (27), with the variable universe adaptive fuzzy

Fig. 5 The evolution of control u

Fig. 6 The enlargement of control u

Fig. 9 The evolution of the synchronization errors

Fig. 7 The phase diagram on the $x_1 - y_1 - z_1(x_2 - y_2 - z_2)$ space

Fig. 8 The phase diagram on the $y_1 - z_1 - w_1(y_2 - z_2 - w_2)$ space

Fig. 10 The evolution of sliding mode surface s

controller (28), the simulation results of composite controller (29) are as follows.

The synchronization error is depicted as Fig. 9. The evolution of sliding mode surface s is depicted as Fig. 10.

The control u is depicted as Fig. 11. For the sake of clarity, the control u is enlarged as Fig. 12.

The phase diagrams of projective synchronization are depicted as Figs. 13 and 14.

From Fig. 9, we know that the projective synchronization is achieved with controller (29). By comparing Figs. 5, 6, 11 and 12, we find, by using VUAF, chattering phenomenon is eliminated. Meanwhile, from Figs. (4) and (10), we find that s cannot stay at zero.

Fig. 11 The control u Fig. 12 The enlargement of the control u

5 Conclusion

In this paper, we have discussed fuzzy sliding mode projective synchronization of hyperjerk system with disturbances. Disturbance observer has been developed for the low time-varying disturbance. By quantitative analysis, given appropriate observer parameters, the disturbance can be compensated by the observer. For the chattering caused by the white noise and the persistent observer error, we have used VUAFC method to smooth it. VUAFC-SMPS method which has been designed is built on sliding mode control, disturbance observer and variable universe adaptive fuzzy control. Simulation results have shown that

Fig. 13 The phase diagram on the $x_1 - y_1 - z_1(x_2 - y_2 - z_2)$ space Fig. 14 The phase diagram on the $y_1 - z_1 - w_1(y_2 - z_2 - w_2)$ space

this control method can reduce chattering significantly without increasing the amount of control.

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Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of this paper.

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