Uncertainty and heuristics for underactuated hands: grasp pose selection based on the potential grasp robustness metric

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Received: 4 August 2020 / Accepted: 20 April 2021

Published online: 31 May 2021

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Abstract

The Potential Grasp Robustness (PGR) metric considers different states of the contact points, relaxing the requirement of all points being far from the friction cone boundary. The addition of new states for each contact point increases the computational complexity, which is combinatorial on the number of states and takes a long time for grasping configurations with large number of hand-object contacts. In this work we analyse the computational complexity of two recently proposed heuristics, which consider that: (i) the minimum number of contact points needed could be in two states and (ii) an analysis of grasp contact data provides the most common combinations of contact points that lead to an accurate estimation of PGR. For selecting grasp configurations, the PGR computation approach is not robust because assumes that measured forces at the contact points do not have uncertainty. In addition to the heuristics, we propose a new uncertainty based metric, the coefficient of variation of PGR. The grasp selection experiments show that the coefficient of variation provides similar results to the pose variation metric. The grasp selection that uses the uncertainty based computation of PGR find more stable contact points than the maximization of the conventional PGR.

Article highlights

- Development of new heuristics for computation of grasp metrics of underactuated hands.
- Definition of uncertainty-based metrics for grasp selection
- Reduction of reality gap for physics-based grasping-metrics of underactuated hands

Keywords Grasp metric · Underactuated hands · Robustness to pose uncertainty · PGR heuristics

1 Introduction

Grasping metrics belong to the approaches based on physics models for grasping, which assume that all perceptual information (kinematics and dynamics) is fully observable, and usually does not consider uncertainty due to computational complexity issues. Although limited in application

scope due to the reality gap of model assumptions, the grasping metrics allow to (i) evaluate the mechanical design of robotic hands in simulation [18] and (ii) predict results of actual grasping experiments [17]. Thus, metrics of the physical models are a useful tool for designing mechanics and control without prototyping. On the more recent tactile approaches based on learning algorithms,

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Fig. 1 Vizzy's hand grasping a ball in simulated environment (left) and the actual robotic hand grasping a ball (right)

the focus is on the estimation of grasp adjustments [8, 15], slippage and manipulation of non-rigid objects [24] amongst others, which are actual applications of tactile-sensing-based grasping.

This work focuses on physics-based models for grasping. While most of these approaches consider fully actuated hands [22], more recent approaches have addressed underactuated hands [4] like the one showed in Fig. 1. These type of hands are a viable option due to low-cost, compliance and high grasping performance with unstructured objects [12].

The Potential Grasp Robustness (PGR) metric considers underactuated hands [18]. This metric assumes that each contact point is in one of the three states as follows: (i) Fully attached (positive constraint and Coulomb constraint), (ii) only positive constraint and (iii) detach. By considering these states, PGR is computationally heavy because it grows exponentially with the number of contact points (n_c), which yields 3^{n_c} possible combinations for the contact point states. To reduce the computational time, [17] developed two heuristics that choose a subset of these combinations. In this work we present two additional heuristics, which will be tested in simulations, where the pose of the object relative to the hand is known without any uncertainty. Since the physics based grasping metrics rely on fully observable data without uncertainty, the grasp metrics suffer of the reality gap. In order to reduce the reality gap and obtain values closer to realistic conditions, we consider uncertainty in the object pose while computing the PGR metric. For instance, the pose detection with respect to the hand by applying vision-based techniques suffers from intrinsic uncertainty due to noise in the sensor and also the arm and hand joints may add more uncertainty to the grasp execution. Thus, slight changes in the object pose will change the PGR metric values, which might change the grasping likelihood significantly for some scenarios. From an application point

of view, grasp selection in simulation will provide different results from real ones if uncertainty is not considered in PGR computation. We propose two uncertainty-based metrics for PGR: (i) Analysing the maximum variation on object pose while maintaining a similar PGR value and (ii) considering random sampling of PGR around a point to compute PGR coefficient of variation. The first metric corresponds to our previous work [1], the $PGR_{PoseVar}$, which considers the variations of PGR in the index computation. The second one corresponds to main difference with our previous work, the c_v^{PGR} , which considers random sampling of PGR in the neighborhood of the object poses in order to model uncertainty. We evaluate the grasp selection of a box and an sphere using: PGR, $PGR_{PoseVar}$ and c_v^{PGR} , showing that the uncertainty-based metrics select grasps that are more robust. The article is organized as follows: Section 2 explains the background knowledge and notation for the computation of PGR, Section 3 explains the heuristics for computing PGR, Section 4 the uncertainty-based metric computation, Section 5 the experiments and Sect. 6 conclusions and future work.

2 Potential grasp robustness

The contact forces are the forces applied by the hand on the contact points to prevent object movement through external wrenches. Bicchi in [2] has defined a subspace of controllable internal forces, which can be modified by the actuators, and he also developed a method for evaluating force closure properties based on these controllable internal forces. Since the contact force distribution is indeterminate and the static equilibrium equations does not define a unique solution, Bicchi replaced the rigid body kinematic assumption for the contact points with a lumped linear elastic stiffness model, assuming a quasi-static model.

The Potential Contact Robustness (PCR) metric relies on the elastic stiffness model and computes, for each contact point, the distance which the contact force is from violating the frictions constraints. The PCR index is the shortest distance obtained from all contact points. Even though PCR considers elastic stiffness for the contact points, in [20], Prattichizzo et al. claim that the PCR is too conservative because it considers that all contact points needs to apply a contact force to the object for a grasp to be stable. This may not be true because not every contact point needs to apply a force to the object to guarantee the grasp stability, that's why Prattichizzo et al. introduced the PGR index.

In Fig. 2 can be seen an object grasped by a finger that is actuated by an actuator. It can also be seen that it is in contact with the object at four contact points, where the red arrows represent the contact forces ($\lambda_{0,i}$ for $i = 1, \dots, 4$).

Fig. 2 Example of a single compliant underactuated robotic arm grasping an object through four contact points. Reprinted from [17]

Three of these forces are within the friction cone, while $\lambda_{0,3}$ is near the friction cone boundary. Thus, according to the PCR metric, a value close to zero is obtained. However, this grasp is stable once the remaining three contact points can guarantee its stability. The PGR index can identify which contact points are needed to have stable grasps, like this one, making it more suitable than PCR.

2.1 Algorithm definition

Considering that the grasp is in static equilibrium, one can define reference fixed frames on the workspace ($\{\mathbf{N}\}$) and on the object ($\{\mathbf{B}\}$), where $\mathbf{u}$ is a vector that represents the position and orientation of $\{\mathbf{B}\}$ relative to $\{\mathbf{N}\}$. The hand's configuration is represented by the vector $\mathbf{q} \in \mathbb{R}^{n_q}$, where n_q is the number of hand joints. The object can be subject to external wrenches that can be described by $\mathbf{g} = [\mathbf{F}^T \mathbf{m}^T]^T \in \mathbb{R}^6$, where $\mathbf{F} \in \mathbb{R}^3$ and $\mathbf{m} \in \mathbb{R}^3$ are the force and torque applied to the object, respectively. The torque is computed with respect to the origin of $\{\mathbf{B}\}$.

Between the hand and the object there are contact points that transmit the force to the object, which are modeled by Single Point with Friction (SPwF) model. This model allows to decompose the force applied in a normal and tangential components to the contact boundary [9, 16]. At the i -th contact point, the applied force $\lambda_i = [\lambda_{i,n}, \lambda_{i,t}, \lambda_{i,o}] \in \mathbb{R}^3$ is expressed in the contact reference frame, where $\lambda_{i,n}$ is the normal force component $\lambda_{i,t}$ and $\lambda_{i,o}$ are its tangential components. In order to avoid detachment and slippage of the i -th contact point, the force must satisfy the positive constraint (1) and Coulomb's friction constraint (2).

$$\lambda_{i,n} \geq 0 \quad (1)$$

$$\sqrt{\lambda_{i,t}^2 + \lambda_{i,o}^2} \leq \mu_i \lambda_{i,n} \quad (2)$$

The μ_i is the static friction coefficient, which depends on the surface material. If the grasp has n_c contact points, then it can be defined a collection of all contact forces in a single vector $\lambda \in \mathbb{R}^{n_\lambda}$. If the SPwF model is assumed, then $n_\lambda = 3n_c$.

For a grasp to be in static equilibrium, the following conditions must be met:

$$\mathbf{g} = -\mathbf{G}\lambda, \quad \boldsymbol{\tau} = \mathbf{J}^T \lambda \quad (3)$$

where the Grasp matrix and the hand Jacobian matrix is represented by $\mathbf{G} \in \mathbb{R}^{6 \times n_\lambda}$ and $\mathbf{J} \in \mathbb{R}^{n_\lambda \times n_q}$ [16]. The two equations in (3) do not admit a unique solution for the contact forces λ when the $\mathcal{N}(\mathbf{G}) \cap \mathcal{N}(\mathbf{J}^T) \neq 0$, where $\mathcal{N}(\mathbf{A})$ denotes the null space of matrix $\mathbf{A}$. However, in [2], the problem has been solved assuming a linearized quasi-static model, where the violation of the kinematics contact constraint is associated with the contact force variation, through a contact stiffness model. If it is assumed that the contacts and joints are not completely rigid, then their rigidity can be modeled by $\mathbf{K}_c \in \mathbb{R}^{3n_c \times 3n_c}$ (contact stiffness matrix) and $\mathbf{K}_q \in \mathbb{R}^{n_q \times n_q}$ (joint stiffness matrix) [6]. In addition, if a grasp is in equilibrium and if there is a small disturbance, then the variations of the variables in relation to the equilibrium can be described by (4) and (5) [21],

$$\Delta \lambda = \mathbf{K}_c (\mathbf{J} \Delta \mathbf{q} - \mathbf{G}^T \Delta \mathbf{u}) \quad (4)$$

$$\Delta \boldsymbol{\tau} = \mathbf{K}_q (\Delta \mathbf{q}_r - \Delta \mathbf{q}) \quad (5)$$

where $\Delta \boldsymbol{\tau}$ is the joint action variation, $\Delta \mathbf{q}_r$ is the reference joints value and $\Delta \mathbf{q}$ their actual values. Moreover, if a set of postural synergies can coordinate the hand joints, then the joints reference values are given by $\Delta \mathbf{q}_r = \mathbf{S} \Delta \mathbf{z}$, where $\mathbf{S} \in \mathbb{R}^{n_q \times n_z}$ is the synergy matrix and $\mathbf{z} \in \mathbb{R}^{n_z}$ the input synergies vector (n_z is the number of synergies considered) [19].

The general solution of the systems of equations in (3), considering a slight variation from the initial grasp configuration, is given by

$$\Delta \lambda = \mathbf{G}_K^R \Delta \mathbf{g} + \Delta \lambda_h + \Delta \lambda_s \quad (6)$$

where the $\mathbf{K}$ -weighted pseudo-inverse of $\mathbf{G}$ is represented by $\mathbf{G}_K^R = \mathbf{K} \mathbf{G}^T (\mathbf{G} \mathbf{K} \mathbf{G}^T)^{-1}$, the internal controllable forces (the forces that can be modified by the actuators) by $\Delta \lambda_h$ and the internal structural forces by $\Delta \lambda_s$. The stiffness matrix $\mathbf{K}$ combines the contact and joint matrices into one and can be calculated as: $\mathbf{K} = (\mathbf{K}_c^{-1} + \mathbf{J} \mathbf{K}_q^{-1} \mathbf{J}^T)^{-1}$.

A more detailed explanation of how to get a solution for controllable and structural internal forces is given in [2] and extended to postural synergies in [19].

The internal controllable forces belong to a subspace $\mathcal{F}_h$ that can be expressed like

$$\mathcal{F}_h = \mathcal{R}(\mathbf{E}) = \mathcal{N}(\mathbf{G}) \cap (\mathcal{R}(\mathbf{KJS}) + \mathcal{R}(\mathbf{KG}^T)) \quad (7)$$

where the matrix $\mathbf{E}^1 \in \mathbb{R}^{n_i \times h}$ is a basis for that subspace ($\mathcal{F}_h$), so the controllable internal forces can be defined as $\Delta \lambda_h = \mathbf{E} \mathbf{y}$, where $\mathbf{y} \in \mathbb{R}^h$ is a vector that parameterizes $\mathcal{F}_h$. The new distribution of contact forces, considering a small disturbance of the initial equilibrium conditions are given by $\lambda = \lambda_0 + \Delta \lambda$.

A $d(\lambda)$ vector can be defined as a vector that stores the values of the distances from which a grasp is violating the frictions constraints and their maximum force limits. This vector is defined in (8), where $d_{i,c}$ is the normal component of the i th contact force ($d_{i,c} = \lambda_{i,n}$), $d_{i,f}$ is the distance of λ_i from the friction cone boundary and $d_{i,max} = \lambda_{i,max} - \|\lambda_i\|$ is the distance at which the force exerted is from the maximum force limit ($\lambda_{i,max}$).

$$d(\lambda) = [d_{1,c}, d_{1,f}, d_{1,max}, \dots, d_{n_c,c}, d_{n_c,f}, d_{n_c,max}] \quad (8)$$

Assuming that $d_{min}^{\mathcal{F}_h}$ is the lowest value of vector $d(\lambda)$, calculated with only the controllable internal forces $\lambda = \mathbf{G}_K^R \mathbf{g} + \mathbf{E} \mathbf{y}$, then it can exist a disturbance in the contact forces such that the friction constraints and the maximum force constraint are satisfied: $\|\Delta \lambda\| \leq d_{min}^{\mathcal{F}_h}$. The maximum external wrench disturbance that satisfies the previous constraints are expressed in equation (9), where $\sigma_{max}(\mathbf{G}_K^R)$ is the maximum singular value of $\mathbf{G}_K^R$. The deduction of this equation is given in [20].

$$\|\Delta \mathbf{g}\| \leq \frac{d_{min}^{\mathcal{F}_h}}{\sigma_{max}(\mathbf{G}_K^R)} \quad (9)$$

Since $d_{min}^{\mathcal{F}_h}$ depends on the choice of $\mathbf{y}$, then a optimal contact distribution $\hat{\lambda} = \mathbf{G}_K^R \mathbf{g} + \mathbf{E} \hat{\mathbf{y}}$ can be obtained in order to maximize the $d_{min}^{\mathcal{F}_h} / \sigma_{max}(\mathbf{G}_K^R)$.

The Potential Contact Robustness (PCR) can be defined as follows:

$$PCR = \max_y \frac{d_{min}^{\mathcal{F}_h}}{\sigma_{max}(\mathbf{G}_K^R)} \quad (10)$$

Let $\hat{\mathbf{y}} = \text{argmax}(d_{min}^{\mathcal{F}_h} / \sigma_{max}(\mathbf{G}_K^R))$, then any external wrench disturbance from the initial equilibrium configuration such

that $\|\Delta \mathbf{g}\| \leq PCR$ can be resisted as long as the internal forces $\mathbf{E} \hat{\mathbf{y}}$ are actuated.

The PCR considers that all points of contact must exert force on the object, but in reality the grasp can be stable even if some points of contact are detached or slipping, what is why PCR is considered too over conservative. The PGR is a generalization of PCR because it considers that a grasp does not need that all friction constraints to be met to verify the grasp stability. Therefore, the PGR considers that each contact point can be in one of the three following states:

- State 1: The constraints (1) and (2) are both satisfied. It is considered that the contact force is transmitted in any direction through the i th contact. The contact stiffness matrix can be defined as: $\mathbf{K}_{i,c} = \text{diag}(K_{itx}, K_{ity}, K_{in})$, where K_{itx} and K_{ity} are the tangential stiffness and K_{in} is the normal stiffness.
- State 2: Only the constraint (1) is verified. It is consider that a contact force can only be transmitted by the normal direction to the i th contact. The contact stiffness matrix can be defined as: $\mathbf{K}_{i,c} = K_{in}$.
- State 3: Both constrains are violated and thus it's consider that the i th contact is detached. The contact stiffness matrix is an empty matrix: $\mathbf{K}_{i,c} = []$.

If it is assumed that n_{st1} contact points are in state 1, n_{st2} in state 2 and consequently $n_{st3} = n_c - n_{st1} - n_{st2}$ in state 3, then a new contact forces can be defined as $\tilde{\lambda} \in \mathbb{R}^{3n_{st1} + n_{st2}}$ and the new static equilibrium is given by $\mathbf{g} + \tilde{\mathbf{G}} \tilde{\lambda} = 0$, where $\tilde{\mathbf{G}} \in \mathbb{R}^{6 \times (3n_{st1} + n_{st2})}$.

Considering that each of the n_c contact points can be in one of the three states mentioned above, and since it is not possible to know *a priori* which states each contact point is in, it is necessary to consider all states' combinations of the contact points. Thus, there are 3^{n_c} possible states' combinations, and each combination will have a specific global stiffness matrix. This global stiffness matrix of a given combination C_j can be expressed as $\mathbf{K}(C_j) = (\mathbf{K}_c^{-1} + \mathbf{J} \mathbf{K}_q^{-1} \mathbf{J}^T)^{-1}$, where $\mathbf{K}_c = \text{diag}(\mathbf{K}_{1c}, \dots, \mathbf{K}_{n_c c})$. It's worth mention that the PCR only considers one grasp configuration where all contact points are in state 1.

The Potential Grasp Robustness (PGR) can be computed as

$$PGR = \max_{C_j} \max_y \frac{d_{min}^{\mathcal{F}_a}(C_j)}{\sigma_{max}(\mathbf{G}_K^R(C_j))} \quad (11)$$

$$\text{subject to } \mathcal{N}(\mathbf{K}(C_j) \mathbf{G}^T) = \mathbf{0} \quad (12)$$

This algorithm maximizes the distance at which contacts are from violating the friction constraints through the

¹ $\mathbf{E} = \mathcal{R}(P)$, where P is a matrix that relates the contact forces variations with the reference synergies variation [19].

appropriate choice of vector $\mathbf{y}$ and the combination of contact point states C_j . Also, the grasp has to be able to have the object immobilized, being necessary to verify the condition (12) [21]. The $G_k^R(C_j)$ and $E(C_j)$ will depend on the contact points states for each combination C_j .

The PCR was first introduced in [20] and was adapted to hands actuated by postural synergies in [18], where in [19] the authors deducted a way of computing the subspace of internal controllable forces taking into account hands actuated by synergies.

Moreover, in order to improve the efficiency of the PGR computation and to avoid convergence problems, [3] developed an efficient method that aims to minimize a cost function that takes into account the friction constraints and the maximum forces magnitudes.

2.2 Heuristic H2

H2 is based on the theoretical result on the minimum number of contact points that satisfy constraints (1) and (2) (State 1 - Single Point with Friction (SPwF) [16]). Since it was shown in [21] that three SPwF points achieve a force closure grasp, the heuristic H2 assume that the hand can only transmit force through three contact points to the object. Thus, three contact points are considered as attached (set in State 1), and the rest of them are considered as detached (set in state 3). This assumption disregard all the possible combinations with State 2, reducing largely the computational complexity.

The number of combinations performed in this heuristic becomes $NOC_{H2} = \binom{n_c}{3}$, which is the number of three element combinations in the set of n_c contact points. NOC_{H2} is significantly less than $NOC_{BF} = 3^{n_c}$.

3 New heuristics for the potential grasp robustness

Previous heuristics for the computation of PGR have focused on the computational complexity, which correspond to lower accuracy of these heuristics. In this section we propose new heuristics that focus on the accuracy rather than computational time.

3.1 Heuristic H3—H2 based heuristic

The heuristic 3 was developed based on the heuristic H2, but considering the state 2 as well on the set of combinations. As in H2, H3 has a fixed number of points on state 1. The remaining points are considered to be either in state 2 or 3 (as opposed to H2, that only consider the state 3). By considering states 2 and 3 in the remaining points, it is

(a) Combinations with three states. The numbers [0, 1, ..., 7] correspond to the contact point label

(b) Combinations with states 1 and 2. The numbers [0, 1, 2, 3] correspond to the contact point label

Fig. 3 Histogram of the combination states for two and three states

expect to obtain more accurate values, practically equal to the Brute Force (BF) ones.

The number of combinations evaluated by this heuristic is $NOC_{H3} = \binom{n_c}{3} \cdot 2^{n_c-3}$, which is bigger than the combinations evaluated by H2.

3.2 Heuristic H4—data driven selection of contact point combinations

The idea behind heuristic H4 was to analyze a large set of grasps executed in simulation, and counting the total sum of contact points at each state, using the underactuated hand of the robot Vizzy [14]. In Fig. 3a, it can be seen the most common combinations occur when the number of

Fig. 4 Number of combinations evaluated in each method given the number of contact points, computed from NOC_{BF} , NOC_{H2} , NOC_{H3} and NOC_{H4}

contact points which are in states (1,2) is [2,2] and [2,3], respectively, regardless of the number of contact points which are in the state 3. In Fig. 3b, it can be seen in more detail that the combination [2,2] is the one that happens in most grasps (about 71.7% of all grasps), whereas combinations [2,3] and [3,0] represent about 14.5% and 6.5%, respectively.

Thus, in order to cover most combinations of the number of contact points in each state most likely to happen and yet the heuristic doesn't take too long to compute, only the combinations [2,2] and [2,3] in states (1,2) are chosen, the rest of the contact points are fixed at state 3. The heuristic H4 hit 86.2% of the combinations obtained by brute force method.

The number of combinations evaluated by this heuristic is $NOC_{H4} = \binom{n_c}{2} \left(\binom{n_c-2}{2} + \binom{n_c-2}{3} \right)$, where the first term corresponds to the contact points being in states (1,2), the second term to the [2,2] combinations and the third term to the [2,3] combinations.

Moreover, it is expected that this heuristic will have similar accuracy for anthropomorphic hands that have the same ways of closing the hand that Vizzy's hand has.

In Fig. 4, it is possible to see the number of combinations evaluated in each method for different numbers of contact points. We compute NOC_{BF} , NOC_{H2} , NOC_{H3} and NOC_{H4} for a given number of contacts n_c . It is also possible to see that heuristics H2 and H4 are the mildest growing due to its purely combinatorial nature. The heuristic H3 has an exponential and a combinatorial part. As can be seen in the figure, H3 follows the same trend as BF (purely

exponential), but it slows down a bit with the growth of the number of contact points.

4 PGR indexes that consider pose uncertainty

Grasp quality metrics are computed by taking into account the characteristics of well-defined contact points between the object and the hand. When the best simulated grasp is chosen and is applied to an object in the real-world, the experimental grasp is rarely the same as the one simulated. This is due to the object pose uncertainty obtained by vision-based systems and therefore the object is always grasped in the vicinity of the object pose chosen in simulation, thus the utility of metric evaluation is reduced because the grasp may be inadequate in the surrounding of the simulated object's pose. Approaches that deal with uncertainty include grasp control methods, grasp quality evaluation, pose error estimation and grasp task space [7].

There are some methods to more accurately estimate the pose of the object. Chalon et al. [5] used a particle filter to estimate the object's pose. Zhang and Trinkle [26] also used a particle filter to estimate the pose even when the object was hidden, not only with the help of vision-based system but also integrating tactile sensor data. Moreover this filter also estimates physical parameters such as the friction's coefficient.

Weisz and Allen [25] have developed an indicator that assesses if the grasp likelihood respects the force closure property, considering a certain uncertainty in the pose. The estimation of the likelihood is done in an empirical way, counting the number of grasp that succeeded over the number of trials. Kim et al. [10] [11] considers the effects of objects dynamic and pose uncertainty on the performance of estimating grasps success through real experiments on a robotic hand.

This work proposes two new metrics that consider the variation of PGR: (i) $PGR_{PoseVar}$, a larger scale variation and c_v^{PGR} , a smaller scale variation. On one hand, at an initial object pose, $PGR_{PoseVar}$ searches for the farthest pose that keeps the PGR index within a close range to the one at the initial pose. On the other hand, at a given pose, c_v^{PGR} computes the coefficient of variation² [23] at a small neighborhood around the given pose. While $PGR_{PoseVar}$ considers PGR robustness at larger scales (i.e. pose space), c_v^{PGR} models PGR robustness at smaller scales (i.e. pose space). These metrics are design for the problem of grasp pose selection for realistic scenarios, where the computation/estimation of variables such as contact point location,

² $c_v = \mu/\sigma$ for exponential distributions.

normal vector computation and contact forces must consider uncertainties.

4.1 Pose variation of PGR

To compute the maximum robustness of PGR to position uncertainty for an object that can move on a three dimensional space (e.g. precision grasps), first, one should find the maximum variation on each axis (X, Y, Z-axis). Then, with the maximum variation in each axis computed, the maximum robustness of PGR to position uncertainty can be given by:

$$PosVar = \sqrt{PosVarX^2 + PosVarY^2 + PosVarZ^2} \quad (13)$$

where $PosVarX$, $PosVarY$ and $PosVarZ$ are the maximum variation that the grasp can resist on the X, Y and Z position axis, respectively, without compromising the grasp quality.

For power grasps, the hand wraps around the object against the palm, and therefore the position uncertainty on the Z-axis is not very relevant. Thus, to compute the maximum robustness of PGR to position uncertainty, it is only worth considering the maximum variation on the X and Y-axis. The formula for this computation can be expressed as

$$PosVar = \sqrt{PosVarX^2 + PosVarY^2} \quad (14)$$

The computation of the maximum variation on an axis can be done as follows: First, it is defined a $VarStep$ (in this work, it is set to 3mm). Then two new positions are computed, which are the initial position displaced by $VarStep$ in the positive and negative axis direction. After that, the PGR metric for these new positions is calculated, and it is checked if any of them is less than a minimum value that depends on the initial PGR (stopping condition): if so, it is found the variation the axis can tolerate and the algorithm ends; otherwise the algorithm computes new positions that are shifted by $\pm k \cdot VarStep$ of the initial position (k is the number of the current iteration) and continues until the stopping condition is verified. The stopping condition becomes true when either Eq. (15) or Eq. (16) holds.

$$PGR_{curr+} < PGR_{init} - threshold \quad (15)$$

$$PGR_{curr-} < PGR_{init} - threshold \quad (16)$$

PGR_{curr+} and PGR_{curr-} represent the PGR index computed when the object position on the axis is displaced by $k \cdot VarStep$ along the positive and negative direction, respectively. The PGR_{init} is the PGR index for the initial object position, which is the actual value of the pose (i.e. no pose uncertainty in any axes).

In the experiments of this work, the threshold was set to 3mm. This value was chosen in order to consider only the most stable grasps, therefore a stable grasp is the one that had at least a PGR close to the initial PGR (PGR_{init}), grasps with significantly lower PGR values are not considered. Values close to this threshold could eventually be chosen.

The pseudo-code to compute the maximum variation in a generalized A-axis can be seen in algorithm 1.

Algorithm 1 Algorithm to compute the maximum position variation in a generalized axis A

```

1: procedure MAXPOSVARA(InitPos)
2:   Compute:  $PGR_{init}$ 
3:   Define a variation step:  $VarStep$ 
4:   Define a threshold:  $\psi$ 
5:   Define the iteration number:  $k = 1$ 
6:    $CurrPosA_+ \leftarrow InitPosA + VarStep$ 
7:    $CurrPosA_- \leftarrow InitPosA - VarStep$ 
8:   Compute:  $PGR_{curr+} \triangleright$  in  $CurrPos_+$  object position
9:   Compute:  $PGR_{curr-} \triangleright$  in  $CurrPos_-$  object position
10:  while  $PGR_{curr+} > PGR_{init} - \psi$  ||  $PGR_{curr-} >$ 
     $PGR_{init} - \psi$  do
11:     $k \leftarrow k + 1$ 
12:     $CurrPosA_+ \leftarrow InitPosA + k \cdot VarStep$ 
13:     $CurrPosA_- \leftarrow InitPosA - k \cdot VarStep$ 
14:    Compute:  $PGR_{curr+}$ 
15:    Compute:  $PGR_{curr-}$ 
16:   $PosVarA \leftarrow 2 \cdot (k - 1) \cdot VarStep$ 

```

Where $InitPos$ is the initial 3D position of the object and $InitPosA$ is the initial position of the object in A-axis, $VarStep$ is the sampling interval at which the object position is shifted in A-axis, thus obtaining $CurrPosA_+$ and $CurrPosA_-$ position if this shift is done along the positive or negative axis direction, respectively. The current object position in 3D is given by $CurrPos_+$ and $CurrPos_-$ when it is shifted along the A-axis to the $CurrPosA_+$ and $CurrPosA_-$ position, respectively. The maximum variation in the A-axis is then given by $PosVarA$.

The computation of the maximum variation in the object's rotation is identical to the one of the position although no difference exists for power grasps, i.e., for all grasps the maximum variation of rotation is represented by:

$$RotVar = \sqrt{RollVar^2 + PitchVar^2 + YawVar^2} \quad (17)$$

where $RollVar$, $PitchVar$ and $YawVar$ are the maximum variation that the grasp can resist on the Roll, Pitch and Yaw angles, respectively, without compromising the grasp quality.

Similar to the computation of the maximum position variation of each axis, the same idea was applied to the maximum rotation variation for each angle. This means that algorithm 1 was applied, but instead of varying the

position, the rotation angles were varied (in this case, a *VarStep* is defined to 1.6 degrees).

Thus, to choose the best grasp for a given object, it is proposed an index that takes into account the PGR index and the maximum variation of position and rotation. This index is given by:

$$PGR_{PoseVar} = PGR \cdot PosVar \cdot RotVar \quad (18)$$

4.2 Coefficient of Variation of PGR

In this section, the PGR index is considered as a function of the object pose $\mathbf{p} \in \mathbb{R}^6$:

$$\mathbf{p} = [t_x, t_y, t_z, R_x, R_y, R_z] \quad (19)$$

where t_x, t_y, t_z correspond to object translations and R_x, R_y, R_z to object rotations in Euler representation. We model pose uncertainty in the neighborhood of a given pose $\mathbf{p}_i$, by sampling object poses randomly³ in the neighborhood of $\mathbf{p}_i$. We denote $\mathbf{p}_{ij}$, as the j -th sample in the neighborhood of $\mathbf{p}_i$, so the empirical mean and standard deviation at $\mathbf{p}_i$ is as follows:

$$\overline{PGR}(\mathbf{p}_i) = \frac{1}{N} \sum_{j=1}^N PGR(\mathbf{p}_{ij}) \quad (20)$$

$$PGR^s(\mathbf{p}_i) = \sqrt{\frac{\sum_{j=1}^N (PGR(\mathbf{p}_{ij}) - \overline{PGR}(\mathbf{p}_i))^2}{N-1}}, \quad (21)$$

where $\overline{PGR}(\mathbf{p}_i)$ corresponds to the empirical mean and $PGR^s(\mathbf{p}_i)$ to the standard deviation. The coefficient of variation

$$c_v^{PGR} = \frac{\sigma}{\mu} = \frac{PGR^s(\mathbf{p}_i)}{\overline{PGR}(\mathbf{p}_i)}, \quad (22)$$

computes a dimensionless ratio that is close to zero when $PGR(\mathbf{p}_{ij})$ is a constant value, and a larger value when $PGR(\mathbf{p}_{ij})$ have very large variations. Thus, an stable grasp will have a close to zero c_v^{PGR} , and the most stable grasp in the object pose space is selected by

$$\hat{\mathbf{p}} = \min_{\mathbf{p}_i} c_v^{PGR}. \quad (23)$$

³ pseudo-random sampling, assuming a normal distribution.

5 Experiments

All experiments were developed in the simulation environment for Underactuated and Compliant Hands called Syngrasp [13]. The software developed to compute the PGR by brute force, the heuristics proposed in [17], and the new ones proposed in Section 3, as well as the $PGR_{PoseVar}$ index and other auxiliary functions, are available online⁴

5.1 New heuristics for PGR

In this section, the accuracy of the indices and the computational time taken by each heuristic are computed and analyzed for two types of hands (Vizzy's and human hand).

5.1.1 Vizzy Hand

To test whether these new heuristics are capable of reproducing identical PGR indices to those obtained with brute force or at least can reproduce identical PGR indices for the majority of grasps that can be analysed, these new heuristics were tested on a set of 138 grasps for Vizzy's hand [14] and compared to brute force and Pozzi et.al. heuristic H2 [17]. The executed grasps include a box and a cylinder at several poses with respect to Vizzy's hand.

To evaluate the most adequate heuristic, it should be considered a trade-off between accuracy and computation time, when compared to ground-truth values obtained by brute force computation. On one hand, in the case of brute force, the PGR value is very accurate but requires a long time to be computed. On the other hand, the PGR values computed by the heuristic H2 are not accurate but the computation time is low.

To evaluate whether the PGR values of the new heuristics are equal/similar to those of the brute force, we compute the accuracy of the PGR computation as follows:

$$accuracy = \frac{1}{n_g} \sum_{g=1}^{n_g} \mathbf{1}_{PGR_{Hi}=PGR_{BF}}, \quad (24)$$

where $\mathbf{1}_{PGR_{Hi}=PGR_{BF}}$ is the indicator function of the condition: If the brute force value is equal to the one of the heuristic, the value is 1, otherwise 0. n_g is the number of grasps ($n_g = 138$). Accuracies of the heuristics H2, H3 and H4 are 6.06%, 9.09% and 89.39%, correspondingly.

The average percentage errors are as follows:

$$Error = \frac{1}{n_g} \sum_{g=1}^{n_g} |PGR_{BF} - PGR_{Hi}| / PGR_{BF}, \quad (25)$$

⁴ <https://github.com/LuisMiguelAlmeida/Syngrasp>

PGR indices of brute force and the heuristics for a set of grasps

Fig. 5 PGR values for a set of 60 grasps executed on two objects. Brute Force (BF) corresponds to the PGR index for all possible combinations and Heuristics H2, H3 and H4 to the reduced set of contact point combinations

where H_i corresponds to heuristic $i = [2, 3, 4]$. The average errors for heuristic H2, H3 and H4 are, respectively, 78.86%, 52.94% and 6.68%, and the standard deviation are 26.76%, 28.91% and 22.23%.

Thus, by analyzing these results it can be concluded that the best heuristic to predict the quality of the grasps is H4, being able to successfully predict approximately 90% of the brute force indices of the set. For heuristic H3, although there are few indices equal to those obtained by brute force, it can still predict with some certainty the quality of the grasp. Regarding H2 heuristic, it is possible to conclude that this heuristic can rarely obtain indices equal or similar to those obtained by the brute force.

Fig. 5 shows the values of PGR computed from the brute force procedure and the heuristics, for a subset of 60 out of the 138 grasps.

As explained in section 3, only a few combinations of the contact point's states are considered in heuristics H3 and H4, thus taking less time to be computed than brute force.

In Fig. 6a is visible the time that each of the indices took to be computed. Where it can be observed that for times when BF takes about 1 second, H3 and H4 take about half that time. Also, it can be seen that for a 13 contact points grasp, BF takes 6351 seconds while H3 takes 1262 seconds (19.87% of BF time) and H4 takes 57.08 seconds (0.90% of BF time).

In Fig. 6b it is shown the ratio between each heuristic computational time and BF time. It is observed that the heuristic that takes on average less time is H2. For few contact points, heuristics H3 and H4 take almost the same

(a) Computational time between BF and heuristics

(b) Ratio between heuristic time and BF time

Fig. 6 Computational time of the PGR index for a set of 60 grasps executed on two objects in simulation. Brute Force (BF) corresponds to the PGR index for all possible combinations and Heuristics H2, H3 and H4 to the reduced set of contact point combinations

time, but as the number of contact points increases, H4 becomes faster than H3.

5.1.2 Human Hand

In order to see if these new heuristics are able to predict with success the indices obtained through brute force on the human hand, the average percentage errors and standard deviation were computed. For heuristics H2, H3 and H4, the average percentage errors are, respectively, 16.3%, 9.4% and 14.0%, and the standard deviation are 18.7%, 17.7% and 10.0%. The heuristic H3 was the best one to identify the grasp quality, but it still takes some

Fig. 7 Top view of Vizzy's hand on the respective X and Y axes. Object positions in the subsequent figures are with respect to Vizzy's hand reference frame

considerable time to be computed. Although the heuristic H4 is less accurate than H3, it still takes less time to compute.

5.2 Robustness to the uncertainty object's pose

In this section, we select the object position with respect to the hand that provides the best power grasp for a 30 mm radius sphere and a 60 mm edge cube. The selection is done by maximizing $PGR_{PoseVar}$ of Eq. (18) and minimizing c_v^{PGR} as in Eq. (23). The power grasps are executed by Vizzy's robotic hand.

To find out the position of the object that leads to the best grasp, it was made a grid with values from the object's center where its position on the X and Y axis were varied. We begin with the sphere, a smoother shape for normal vector estimation and follow with the cube, a yet simple but less smoother surface than the sphere from the point of view of normal estimation. To further illustrate where the various locations of the cube centers relative to the hand (defined by the grid previously mentioned) are, is shown in Fig. 7, the hand on the respective X and Y axes. For each of these positions, the grasp was executed and then evaluated with the $PGR_{PoseVar}$ index.

5.2.1 Choosing an optimal grasp for a sphere

A grid with several sphere center positions in the X-Y plane were made while keeping the Z value constant, because it is a power grasp. For this grid, the values on the X-axis were varied between 55 and 80mm, with an interval of 3mm between each value. For the Y-axis, the values were varied between -25 and 4mm with an interval of 4mm. Since the sphere is symmetrical with respect to the any axis selection, rotations with respect to the center do not

Fig. 8 PGR indices for each (X,Y) positions of the sphere

Fig. 9 Robustness to position uncertainty for each (X,Y) positions of the sphere

Fig. 10 $PGR_{PoseVar}$ index for each (X,Y) position of the sphere

Fig. 11 $\overline{PGR}(\mathbf{p}_i)$ and $PGR^s(\mathbf{p}_i)$, where poses consider only variations in the X-Y plane, so $\mathbf{p}_i \in \mathbb{R}^2$

Fig. 12 c_v^{PGR} index for each (X,Y) sphere's position

change the contact points at a given position (i.e., the PGR index is equal regardless of the rotation angle). Therefore the choice of the optimal position of the sphere can be done in a lower dimensional space (2D). This analysis is valid for both $PGR_{poseVar}$ and c_v^{PGR} .

Regarding $PGR_{poseVar}$ in Figs. 8 and 9 we see that object center location at (76, -9)mm corresponds to the highest PGR index (54.28), it has low robustness in position uncertainty (about 6.7mm). The position (67, -5)mm has the largest robustness to position uncertainty, with a value of 34.21mm, and the PGR value at this position is 28.34. This position leads to the optimal grasp, once it has the highest value of the $PGR_{poseVar}$ index (969.4), as can be seen in Fig. 10. Moreover, it is also possible to see that the position (67, -13)mm has an index of 964.3, which is approximately equal to the index of the optimal pose of the sphere and eventually may also be chosen.

Figure 11 shows the mean and variance of PGR computed in the neighborhood of each $\mathbf{p}_i \in \mathbb{R}^2$. We see that the mean value is close to the PGR values in Fig. 8, meaning that uncertainty in the pose does not bring large uncertainties to PGR computation for the majority of the poses. However, there is a section $Y > 21$ mm and $55 < X < 75$ mm where there is a large uncertainty and should be discarded. This observation is very clear on the c_v^{PGR} index in Fig. 12, which has very large values in the same section $Y > 21$ mm and $55 < X < 75$ mm. Considering the insights from PGR , $PGR_{poseVar}$ and c_v^{PGR} , the areas with large uncertainty are common to all indexes. However, the selection of the best pose by each method provide different results, according to the task in mind. PGR selects configurations that work well in deterministic scenarios, while $PGR_{poseVar}$ and c_v^{PGR} select poses where PGR maintains a close to contact value in larger and smaller neighborhoods correspondingly. The best 2D pose for grasping an sphere is located at $X = 64$ mm, $Y = -17$ mm, which is displayed in Fig. 13.

(a) According to the PGR index

(b) According to the $PGR_{poseVar}$ index

(c) According to the c_v^{PGR} index

Fig. 13 The best power grasp for an 30mm radius sphere

Fig. 14 $PGR_{PoseVar}$ index for each (X,Y) position of the cube

Figures 13a, c and b display the hand closing the fingers at the selected poses. Although the grasp in Fig. 13a is more stable at that position, it is less robust to pose uncertainty and more likely will fail in the real world, so it is preferable to choose a grasp that is less stable but is quite robust to the pose uncertainty (like the grasp of Fig. 13b, or the one of Fig. 13c).

5.2.2 Choosing an optimal grasp for a cube

We follow the same 2D grid approach used in the sphere results for the selection of the best power grasp of a cube. The values on the X-axis were varied between 65 and 85mm with an interval of 3mm between each value, and the values of Y-axis were varied between -25 and 4mm with an interval of 4mm between each value. Other cube center positions are not worth considering as they were not within the range of the fingers (when they close). The best position of the cube's center is one that maximizes the PGR index and the variance in the position and rotation in the case of $PGR_{PoseVar}$ index. In the c_v^{PGR} case the best pose is the one that minimizes the index. It is important to note that the grasp is a power grasp, so the cube's position on the Z-axis remains constant to keep one face of the cube in contact with the palm.

In Fig. 14 is shown the $PGR_{PoseVar}$ index as a function of the position (X and Y) of the cube's center. We observe that position (71, -5)mm maximizes the PGR index and the maximum variance the pose can have ($PGR_{PoseVar}$) and therefore it is the best choice for the cube's position to be grasped. Adjacent positions on the X-axis position ((68, -5)mm and (74, -5)mm) also show good PGR index and good variance robustness, where, eventually could also be chosen.

In Figs. 15, 16 and 17, it can be seen that the position (83, -5)mm holds the highest PGR index (49.56) of all

Fig. 15 PGR index for each (X,Y) position of the cube

Fig. 16 Maximum position variance for each (X,Y) cube position

Fig. 17 Maximum rotation variance for each (X,Y) cube position

Fig. 18 $\overline{PGR}(\mathbf{p}_i)$ and $PGR^s(\mathbf{p}_i)$, where poses consider only variations in the X-Y-Z plane, so $\mathbf{p}_i \in \mathbb{R}^3$

Fig. 20 $\overline{PGR}(\mathbf{p}_i)$ and $PGR^s(\mathbf{p}_i)$, where poses consider variations in the full pose space, $\mathbf{p}_i \in \mathbb{R}^2$

Fig. 19 c_v^{PGR} index for each (X,Y) cube's position, considering 3-dimensional translations only

Fig. 21 c_v^{PGR} index for each (X,Y) sphere's position, considering full pose variations, $\mathbf{p}_i \in \mathbb{R}^6$

positions tested, but the robustness concerning pose's uncertainty is not very large (6.7 mm for position and 0.19 radians for rotation). At position (83, -9)mm the robustness to pose's uncertainty is sufficient (18.25 mm in position and 0.24 radians) but the PGR index is a little low (around 16.16).

Thus, the choice of the best cube's position to be grasped passes through a tradeoff between the best PGR index with the largest variance that the pose can have. The position (71, -5)mm turns out to be the one with the best PGR (40.22) and the largest variation in pose (18.97 mm in position and 0.30 radians in rotation), therefore is the cube center pose that corresponds to the best power grasp to be applied to the cube.

Regarding the locally computed mean and variance of PGR , we begin with low dimensional pose variation $\mathbf{p}_i \in \mathbb{R}^3$, considering small object translations in X,Y and

Z, followed by a larger pose variation $\mathbf{p}_i \in \mathbb{R}^6$ that considers all the object translations and rotations. Fig. 18 shows $\overline{PGR}(\mathbf{p}_i)$ and $PGR^s(\mathbf{p}_i)$, where we can see already larger variances than the ones obtained for the sphere. The normal vector discontinuities at the intersections between the planes of the cube, and the abrupt changes in the contact points at certain neighborhoods explain the larger variances in the cube case. Nevertheless, there are some object poses with small variation in PGR that are present in Figs. 15, 19 and 14, namely the region $5\text{mm} < Y < 15\text{mm}$ and $65\text{mm} < X < 85$. The optimal pose selected by Eq. (23) is $X = 83\text{mm}$ and $Y = -5\text{mm}$. In the case of PGR is a plateau, of $PGR_{posevar}$ is the highest part of the hill and of c_v^{PGR} is the valley.

Figure 20 shows $\overline{PGR}(\mathbf{p}_i)$ and $PGR^s(\mathbf{p}_i)$, for a larger pose variation space, $\mathbf{p}_i \in \mathbb{R}^6$. The standard deviation now is

Fig. 22 The best power grasp for an 60mm edge cube

larger across most of the regions of the 2D space, but with the exception on the points next to $X = 68$ and $Y = -9$, where the minimum of Eq. (23) is located. In addition, some points of the valley in Fig. 19 are no longer that robust to pose variations. This change can be observed in Fig. 21.

Figure 22 display the hand closing the fingers at the best poses selected by each index. Although the grasp in Fig. 22a is more stable at that position, the thumb is touching the cube at an edge, which is less robust to pose uncertainty and more likely will fail in the real world. Thus, it is preferable to choose a grasp that is less stable but is quite robust to the pose uncertainty (like the grasp of Fig. 22b, or the one of Fig. 22c).

5.3 Discussion of results

On one hand, the heuristics proposed by Pozzi et al. are computationally faster than our proposals, their estimation performance is significantly lower than our heuristics (H3 and H4). On the other hand, heuristics H3 and H4 take longer time to compute than previous heuristics, but still faster than the brute force computational time. In the case of Vizzy's hand, H4 obtained the best accuracy prediction score (90%) when compared to brute force computation. Regarding computational time, heuristic H4 saves significant time on larger amount of contact points (0.9 ratio between H4 time and BF time). Regarding heuristic H3, its prediction score is lower than H4, but it is able to rank grasps similarly to BF. In the case of human's hand, H3 shows better prediction accuracy (error of 9.4%), followed by H2 (error of 18,7%) and H4 (error of 14%).

The indexes $PGR_{PoseVar}$ and c_v^{PGR} are able to choose the object pose such that is robust and stable in its surroundings poses, meaning that the object pose chosen is robust to the pose uncertainty. These two indexes are useful because, in the real world, the robot has some uncertainty about the object pose, the normal estimation of the contact points suffers from uncertainty for complex shapes,

and the object most likely will be grasped in a neighborhood of the pose chosen by that index. With the optimal object poses chosen through $PGR_{PoseVar}$ and c_v^{PGR} indexes, the real-world grasp will be more likely to succeed than the pose chosen by the PGR index.

As can be seen from Fig. 22b, the $PGR_{PoseVar}$ index chooses an object pose that maximizes both the PGR index and the maximum variation of the pose relative to the initial pose. In Fig. 22b, the object pose of the PGR index only maximizes the PGR index, where for the chosen grasp, the thumb only touches the edge of the cube, and therefore, any variation from the initial cube pose most likely makes the real-world grasp unviable. Therefore, the $PGR_{PoseVar}$ and c_v^{PGR} indexes are useful for choosing more stable grasps to be performed in the real world.

6 Conclusions

We propose modifications to the Potential Grasp Robustness (PGR) index: (i) New heuristics for PGR computation and (ii) considering robustness to uncertainty in the metric. Our contributions fit very well the offline grasp selection, where the goal is to evaluate a large set of object poses, compute the PGR with robustness to uncertainty and finally select the object pose with respect to the hand that has the largest PGR. The proposed heuristics (H3 and H4) provide more accurate results than previous heuristics for PGR (H2). More accurate results come at a price, the increased computation time with respect to H2. Nevertheless, the accuracy vs. computation time trade-off of our heuristics, fit better our grasp selection problem. The uncertainty-based modifications of PGR, namely the $PGR_{PoseVar}$ and c_v^{PGR} indexes, provide a more robust grasp selection by analysing a larger and smaller neighborhood respectively. $PGR_{PoseVar}$ includes the maximum allowed pose variation around a certain object pose, multiplying the PGR time the pose variation. The point where $PGR_{PoseVar}$

reaches its maximum it is the most stable point to execute the grasp. c_v^{PGR} is based on random sampling around a pose, computing the empirical mean and standard deviation of PGR. Then, the coefficient of variation is computed at each pose. The pose that reaches the global minimum of c_v^{PGR} provides a robust point by computing statistics on a local neighborhood. The results presented on grasping two objects show that $PGR_{poseVar}$ and c_v^{PGR} provide poses that are more robust to sensor uncertainties. In future work, we are evaluating the advantages in actual underactuated robotic hands.

Acknowledgements This work corresponds to an extended version of the published article in [1]. This work is partially funded by LARSyS - FCT Project UIDB/50009/2020 and Infrastructure project Robotics, Brain and Cognition Laboratory (RBCog-Lab) N° 022084

Declarations

Conflicts of interest The authors declare that they have no conflict of interest.

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