Homogeneous and re-circulatory gas–liquid flow in a bubble column: A numerical investigation using OpenFOAM

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Abstract

This work presents the influence of the sparger opening area, gas velocity, and bubble size on hydrodynamics and transition of the flow regime from uniform to re-circulatory in a rectangular bubble column using OpenFOAM. In the course of development of the model, the effect of several drag closures and lift on the predictability of the CFD model was studied by comparing the predictions with published experimental results. *Reynolds number*-based drag closure was found to be suitable for uniform sparger whereas *Tsuchiya drag* (Tsuchiya et al. in Chem Eng Sci 52:3053–3066, 1997. [https://doi.org/10.1016/S0009-2509\(97\)00127-9](https://doi.org/10.1016/S0009-2509(97)00127-9)) was used to simulate gas–liquid flow for other spargers. Simulations were performed for seven different spargers with opening area 18–100% (superficial gas velocity of 2.9–5.8 cm/s) and bubble size of 2–8 mm. The smaller opening area and higher gas velocity promote the re-circulatory flow in the bubble column. Change in bubble size affects the hydrodynamics due to change in lift and drag forces.

Highlights

- Predictions of gas volume fraction distributions and vertical liquid velocity.
- Effect of bubble size on dominant frequency and hydrodynamics inside the column.
- Combined effect of sparger opening-gas velocity and bubble size on the hydrodynamics using flow structure map.

Keywords Euler–Euler simulations · Bubble column · Homogeneous · CFD · OpenFOAM · Gas–liquid flow

List of symbols

C_1, C_2	Constant parameters, dimensionless
C_D	Drag coefficient, dimensionless
C_L	Lift coefficient, dimensionless
C_μ	Constant parameter, dimensionless
d_d	Diameter of bubble, mm
$Eö$	Eotvos number, dimensionless
g	Gravitational constant, m/s ²
k	Turbulent kinetic energy, m ² /s ²

$\vec{M}$	Interphase momentum transfer, N/m ²
P	Pressure, N/m ²
Re	Reynolds number, dimensionless
t	Time, s
$\vec{U}_d$	Superficial gas velocity, cm/s
$\vec{U}_c$	Liquid velocity, cm/s

Greek letters

α_c	Liquid volume fraction, dimensionless
α_d	Gas volume fraction, dimensionless

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ρ_c	Liquid density, kg/m ³
ρ_d	Gas density, kg/m ³
$\bar{\tau}$	Stress tensor, Pa
σ	Interfacial tension, mN/m
μ	Dynamic viscosity, Ns/m ²
ε	Energy dissipation rate, m ² /s ³

Subscript/superscript

d	Dispersed phase
c	Continuous phase

1 Introduction

In chemical, petrochemical, biochemical and food processing industries, innumerable processes involve dispersed gas–liquid flows. Due to this wide application, understanding of hydrodynamics of gas–liquid flow has drawn the attention of researchers, developers, and designers from both academia and industries. Increasing demand for emerging technology that improves process efficiency has renewed the interest in this subject time and again.

To understand the hydrodynamics of the gas–liquid flow, 2D-rectangular bubble column is frequently used in laboratory-scale experiments as this low-cost equipment facilitates flow visualization at different regimes. In addition, the simplicity and compactness of its construction permit to maintain the isothermal condition of the experiments on demand. Some notable studies carried out in the near past using 2D-rectangular bubble columns [e.g., 1–3]. The complexity of gas–liquid flow can be clearly acknowledged from these studies. However, the complex flow in a bubble column can be broadly divided into two regimes based on bubble size and gas velocity; homogeneous and heterogeneous regime.

The homogeneous regime is characterized by uniform size of nearly spherical bubbles and the heterogeneous regime is distinguished by a wider bubble size distribution in presence of large-scale vertical liquid circulation. For a given bubble column, the homogeneous flow occurs at relatively lower gas velocity and when the velocity is increased, the flow gradually becomes heterogeneous. Since both heat and mass transport processes substantially dependent on the flow regimes, the velocity at which the transition occurs becomes a matter of concern for the users [2, 4]. For example, the influence of velocity on hydrodynamics is discussed by Buwa and Ranade [1], Selma et al. [5], Julia et al. [6] and Zahradnik et al. [7] in detail.

In addition to the influence of velocity, some of the studies have also informed the effect of spargers (bubble formation strategies) on the flow pattern [e.g., 6,8,9]. The spargers which have been used in the past to study

hydrodynamics in bubble column are Orifice [10], sieve plate sparger [11], porous plate [12, 13], needles [6, 14], perforated plates [15], sintered plate distributor [16]. All these spargers produce different aeration patterns and bubble size distributions. To understand the effect of aeration patterns on hydrodynamics, experiments were carried out by Hartevelde [17] and then by Julia et al. [6]. The data acquired from these experiments have great archival value for the understanding of gas–liquid flow in detail.

All these information available in the literature connotes the complexity that can arise from a slight change in design features such as size, type, and arrangement of the spargers. The complexity multiplies many folds when more than two parameters are altered simultaneously. Therefore, the objective of the present work is to study the combined effect of multiple parameters such as superficial gas velocity, sparger type and bubble size on the flow pattern.

Though interesting, studying the flow pattern by simultaneously changing many parameters through experiments is often cumbersome and profligate. In such cases, an experimentally verified computational fluid dynamics (CFD) model becomes extremely helpful. In this context, the experiments by Hartevelde [17] were later simulated by Monahan and Fox [18] using an Eulerian two-fluid model. The simulations were carried out using the superficial gas velocity of 2 cm/s and input bubble diameter of 4 mm to study the flow pattern with different spargers. The results obtained from the simulation show good agreement with the experimental data obtained by Hartevelde [17] under some conditions and reasonable agreement in other conditions. In particular, the deviation of the predicted velocity vector field and time-averaged axial liquid velocity from the experimental results were significant for patterns 2 and 4. The deviation was correlated with the quick movement of the bubble towards the center and then to one side of the column in the simulation. The phenomenon was also partly attributed to the choice of drag coefficient in the two-fluid model. The drag model proposed by Tsuchiya et al. [19] with other drag models were considered in the two-fluid CFD model. From the study by Tomiyama et al. [20], it has been identified that the bigger bubbles have a propensity to stay near the center of the column while smaller bubbles migrate towards the column wall. This segregation is the consequences of the radial force acting on the bubbles inside the column. The larger bubble experiences a positive lift coefficient whereas the smaller bubble experiences negative lift coefficient [20]. This indicates that the lift force must be calculated on the basis of bubble size to predict the hydrodynamics accurately. The use of a constant lift force for all sizes of bubbles in the simulation as done by Monahan and Fox

[18] may be a contributing factor to the deviation. This is a clear indication that the model may not be suitable for the cases where the effect of bubble size on flow patterns has to be studied. From the above discussions, it is clear that to study the combined effect of different parameters on flow pattern, it is a prerequisite to develop a suitable CFD model.

The first part of the paper focuses on the systematic development of the existing two-fluid model in the open source CFD code OpenFOAM. In the course of development, the model has been validated against available experimental and numerical results to test its predictive capability. Subsequently, the suitability of various drag closures and lift closure has been rigorously studied and compared with the available experimental data. The second part reports the main results that include the effect of sparger configurations, superficial gas velocity and bubble size on hydrodynamics inside the bubble column. For this purpose, the time-averaged liquid velocity profiles, gas volume fraction distributions and liquid velocity fluctuations power spectra are reported. In addition, the influence of all the aforementioned parameters is presented through flow regime maps.

2 CFD model

For this work Eulerian–Eulerian (EE) two-fluid model was chosen which is based on the interpenetrating continuum assumption that allows co-existence of both phases at a given location. The presence of each phase at a location is characterized by its volume fraction. The existing *twoPhaseEulerFoam* module of OpenFOAM was developed further by (i) implementation of $k - \epsilon$ turbulence model with additional source term [5, 16], (ii) time-average user-defined terms and (iii) implementation of various user-defined drag and lift closures. Appropriate turbulence model was implemented to study both steady and unsteady flows. The customized code was subsequently implemented in a rectangular bubble column to simulate two-phase gas–liquid flow. The inter-phase momentum exchange was accounted through drag and lift forces acting on the dispersed phase (bubbles).

2.1 Governing equations

The equations of mass conservation for liquid and gas phases are

$$\frac{\partial}{\partial t}(\alpha_d \rho_d) + \nabla \cdot (\alpha_d \rho_d \vec{U}_d) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(\alpha_c \rho_c) + \nabla \cdot (\alpha_c \rho_c \vec{U}_c) = 0 \quad (2)$$

where, d and c denote dispersed (gas) and continuous (liquid) phases respectively. ρ and α are the density and volume fraction of the respective phases.

The equations of momentum conservation are

$$\frac{\partial}{\partial t}(\alpha_d \rho_d \vec{U}_d) + \nabla \cdot (\alpha_d \rho_d \vec{U}_d \vec{U}_d) = -\alpha_d \nabla P - \nabla \cdot \alpha_d \vec{\tau}_d + \alpha_d \rho_d \vec{g} + \vec{M}_{d-c} \quad (3)$$

$$\frac{\partial}{\partial t}(\alpha_c \rho_c \vec{U}_c) + \nabla \cdot (\alpha_c \rho_c \vec{U}_c \vec{U}_c) = -\alpha_c \nabla P - \nabla \cdot \alpha_c \vec{\tau}_c + \alpha_c \rho_c \vec{g} - \vec{M}_{c-d} \quad (4)$$

where, $\vec{U}_d$ and $\vec{U}_c$ denote velocities of gas and liquid phases, respectively and P is the pressure shared by both the phases. The stress tensor ($\vec{\tau}$) was calculated as $\vec{\tau} = -\mu(\nabla \vec{U} + \nabla \vec{U}^T)$. $\vec{M}_{d-c}$ and $\vec{M}_{c-d}$ represent interphase momentum exchange terms between the phases and modeled as

$$\vec{M}_D = \frac{3}{4} \rho_c \alpha_c \alpha_d \frac{C_D}{d_d} (\vec{U}_d - \vec{U}_c) |\vec{U}_d - \vec{U}_c| \quad (5)$$

where, d_d is the average diameter of the bubbles and C_D is the drag coefficient proposed by Tsuchiya et al. [19] as

$$C_D = \max \left\{ \frac{24}{\text{Re}} (1.0 + 0.15 \text{Re}^{0.687}), \frac{8}{3} \frac{\text{Eö}}{\text{Eö} + 4} \right\} \quad (6)$$

where, Re is the drop Reynolds number and is defined as $\text{Re} = d_d |\vec{U}_d - \vec{U}_c| \rho_c / \mu_c$ and Eö is the Eotvos number and is defined as $\text{Eö} = \vec{g} \Delta \rho d_d^2 / \sigma$. It is to be noted that the above drag closer has been chosen to study the effect of different parameters on flow patterns after evaluating other drag closures proposed/used by Harmathy, Ishi and Zuber, Mendelson and Monahan and Fox. The reason for selecting the drag closer proposed by Tsuchiya et al. [19] is discussed in Sect. 3.1.2.

$$\vec{M}_L = -C_L \rho_c \alpha_c \alpha_d |\vec{U}_d - \vec{U}_c| \times (\nabla \times \vec{U}_c) \quad (7)$$

where, C_L is the lift coefficient proposed by Tomiyama et al. [20] as

$$C_L = \begin{cases} \min [0.288 \tanh(0.121 \text{Re}), f(Eo_d)] & \text{if } Eo_d < 4 \\ f(Eo_d) & \text{if } 4 \leq Eo_d \leq 10.7 \\ -0.27 & \text{if } Eo_d > 10.7 \end{cases} \quad (8)$$

The liquid phase turbulence was simulated using the standard $k - \varepsilon$ model by solving the following equations:

$$\frac{\partial}{\partial t}(\alpha_c k_c) + \nabla \cdot (\alpha_c k_c \vec{U}_c) = \nabla \cdot \left(\frac{\nu_{c,eff}}{\sigma_k} \nabla k_c \right) + \alpha_c (G_{k,c} - \varepsilon_c) \quad (9)$$

$$\frac{\partial}{\partial t}(\alpha_c \varepsilon_c) + \nabla \cdot (\alpha_c \varepsilon_c \vec{U}_c) = \nabla \cdot \left(\frac{\nu_{c,eff}}{\sigma_\varepsilon} \nabla \varepsilon_c \right) + \alpha_c \frac{\varepsilon_c}{k_c} (C_1 G_{k,c} - C_2 \rho_c \varepsilon_c) \quad (10)$$

where, k and ε are the kinetic energy and the rate of energy dissipation, respectively. The effective viscosities of the liquid and gas phases $\mu_{c,eff}$ and $\mu_{d,eff}$, respectively and are calculated as:

$$\mu_{c,eff} = \mu_{c,lam} + \mu_{c,tur}, \quad \mu_{c,tur} = \rho_c C_\mu \left(\frac{k_c^2}{\varepsilon_c} \right) \text{ and } \mu_{d,eff} = \mu_{c,eff} \left(\frac{\rho_d}{\rho_c} \right) \quad (11)$$

2.2 Solution domain

The solution domain consists of a rectangular bubble column and is shown in Fig. 1. It was meshed (uniform structured) using a third-party software and was exported to OpenFOAM. The dimensions of the bubble column and different flow velocities used for the simulations are as follows

- (i) Width, $W = 0.2$ m; height, $Y = 0.6$ m; depth, $D = 0.05$ m with local sparger (0.024×0.012 m²) as used by Buwa and Ranade [1] and Pflieger et al. [21]. Simulations for this dimension of the column were performed with superficial gas velocity (U_G) of 0.14 cm/s. The detailed dimensions and other conditions are given in Table 1.
- (ii) Width, $W = 0.264$ m; height, $Y = 1.5$ m; depth, $D = 0.031$ m) with seven different types of sparger (that includes one uniform sparger and rest are non-uniform spargers) (see Fig. 2) as used by Julia et al. [6]. This covers a wide range of sparging area (from fully aerated to locally aerated) that is characterized by X/W (aerated width-to-column width) for superficial gas velocities (U_G) of 2.9, 4.3 and 5.8 cm/s. The detailed dimensions of the bubble column and spargers used for the main study are given in Table 2.

It is to be noted that the dimensions and velocity used given in case (i) were used to validate the numerical model and the data given in case (ii) were used for the present analysis.

2.3 Boundary conditions and numerical methodology

The simulations were performed using pure water as continuous phase ($\mu_c = 0.001$ kg m⁻¹ s⁻¹, $\rho_c = 999$ kg m⁻³) and gas as dispersed phase ($\mu_c = 1.7894 \times 10^{-5}$ kg m⁻¹ s⁻¹, $\rho_c = 1.187$ kg m⁻³). At the inlet, *velocity inlet* boundary condition was specified and *pressure outlet* was specified at the top surface. All the side faces (except sparger and top outlet) were specified as *wall* and a *no-slip* boundary condition was implemented. A uniform value of $1e-8$ was used for both k and ε in the internal field. A summary of all the boundary conditions used is given in Table 3. For all the simulations, *adjustable time step* and *convergence criteria* of 10^{-6} was used.

In order to optimize the grid, initial unsteady simulations were performed in OpenFOAM with uniform sparger using coarse (7100 cells), medium (14,330 cells), fine (27,150 cells) and very fine (55,280 cells) grids at 5.8 cm/s superficial gas velocity. The time-averaged gas volume fraction profiles obtained from these preliminary simulations were analyzed to examine the grid dependency of the results (see Fig. 3). A substantial change in time-averaged gas fraction profiles was observed with the refinement of the grid from coarse to medium and even

Fig. 1 **a** Schematic diagram and **b** computational mesh of the bubble column

Table 1 Dimensions and other conditions of the bubble column used for the validation study

Parameters	Specifications
Column height, Y	60 cm
Column width, W	20 cm
Column depth, D	5 cm
Sparger width	2.4 cm
Sparger depth	1.2 cm
Bubble diameter at inlet	5 mm
Superficial gas velocity at inlet	0.14 cm/s

medium to fine levels. However, with the refinement of the grid from fine to very fine levels, the time-averaged profiles remained similar. This refinement only increased the computational time. Therefore, keeping the computational time in mind, the grid with 27,150 cells was chosen for all further simulations. The number of points chosen to generate the fine grid is 50, 85 and 7 in x, y and z-direction, respectively. The y-axis corresponds to the vertical height of the cylinder whereas z is the depth.

3 Result and discussion

3.1 Model refinement

3.1.1 Validation of the computational model against measurements and predictions of Buwa and Ranade [1], Pfleger et al. [21]

The comparison of predicted and measured [1, 21] time-averaged gas volume fraction and vertical liquid velocity at superficial gas velocity (U_G) of 0.14 cm/s is shown in Fig. 4. It can be seen that the predictions by the CFD model are in good agreement with the experimental results. Buwa and Ranade [1] have also reported the simulations performed using a commercial flow solver and the predictions were in excellent agreement with their measurements. Therefore, it can be reasoned that the current model is on par with the commercial flow solver used by Buwa and Ranade [1].

Fig. 2 Schematic diagram of the sparger configuration used in numerical simulations **a** A1, **b** A2, **c** A3, **d** A4, **e** A5, **f** A6 and **g** A7**Table 2** Dimensions of the bubble column and sparger used for the main study

Column details	Dimensions (cm)
Column height, Y	150
Column width, W	26.4
Column depth, D	3.1
Sparger type	Width × Depth
Sparger A1	26.4 × 3.1 (100%)
Sparger A2	25.08 × 3.1 (95%)
Sparger A3	22.44 × 3.1 (85%)
Sparger A4	18.21 × 3.1 (69%)
Sparger A5	15.57 × 3.1 (59%)
Sparger A6	11.61 × 3.1 (44%)
Sparger A7	4.75 × 3.1 (18%)

Table 3 Boundary conditions for the simulations of gas–liquid flow in bubble column

	Inlet (at sparger)	Outlet (top surface)	Side walls	Bottom wall (except sparger)
P	Zero gradient	Fixed value 0	Zero gradient	Zero gradient
U_a	Fixed value (inlet velocity)	Zero gradient	Fixed value 0	Fixed value 0
U_b	Fixed value 0	Zero gradient	Fixed value 0	Fixed value 0
k	Fixed value $1.44\text{e}-6$	Fixed value $1.44\text{e}-6$	Zero gradient	Zero gradient
ϵ	Fixed value $5.09\text{e}-8$	Fixed value $5.09\text{e}-8$	Zero gradient	Zero gradient
α	Fixed value 1	Zero gradient	Zero gradient	Zero gradient
β	Fixed value 0	Fixed value 0	Zero gradient	Zero gradient

Fig. 3 Effects of grid resolution on time-averaged gas phase fractions using A1 (100%) sparger and gas velocity of 5.8 cm/s

3.1.2 Validation of the computational model against measurements of Julia et al. [6]

To simulate the gas–liquid flow using different aeration patterns (sparger), the CFD model was employed using a bubble column with dimensions as mentioned in Julia et al. [6] and specified in case (ii) of Sect. 2.2. Upon comparison of the experimental results [6] with the results obtained from simulation, it was found out that the predictions by the CFD model with the drag proposed by Tsuchiya et al. [19] show better agreement with the experimental results for spargers with opening area < 100%. Numerical results obtained using the drag proposed by Tsuchiya et al. [19] for uniform sparger, significantly deviates from the measurements carried out by Julia et al. [6]. This signifies that drag proposed by Tsuchiya et al. [19] is not suitable for all types of spargers. Hence, the suitability of additional drag closures needs to be investigated. Different drag closures proposed by different researchers are described in the following section.

3.1.2.1 Effect of drag force Methodical analysis of the results reported by Julia et al. [6] indicates that the bubbles of uniform size are distributed uniformly throughout the column. The velocity vector shown for the experiment was uniform with a constant magnitude. These are the features of the homogeneous flow, where the interaction between the liquid and dispersed gas phase is insignificant. The relative strength of inertial forces to viscous forces becomes the characteristic of interacting force between the dispersed and continuous phase. Since the drag correlation proposed by Tsuchiya et al. [19] contains both Reynolds (Re) and Eotvos number (Eo), the use of this drag closure for sparger A1 (uniform sparger) was supposed to be the cause of the difference between the experimental and numerical results. This is due to the homogeneous flow generated by the uniform sparger (A1) and the bubbles generated are expected to be spherical.

To verify the effect of the drag closure on the hydrodynamics, different drag closures were proposed by different researchers such as Harmathy, Mendelson, Ishii and Zuber, Tsuchiya [19] and the drag closure used by Monahan and Fox [18], etc. are implemented in OpenFOAM. The results obtained from the simulations after the implementation of these drag closures are shown in Fig. 5. It can be seen that Re based drag closure predicts the experimental value more accurately compared to all other drag closures for the uniform sparger (A1). Therefore, the CFD model was modified by replacing the Tsuchiya drag with drag based only on Re. However, drag proposed by Tsuchiya et al. [19] predicts the time-averaged liquid velocity profiles better compared to other drag closures for A4 and A7 spargers (other than uniform sparger). The re-circulatory motion which is the next transition phase of the homogeneous regime, causes the Tsuchiya drag more appropriate for these spargers (A4 and A7). However, Tsuchiya drag along with the lift force predicts the time-averaged profile and a good agreement between experimental and numerical results can be noticed. The importance of the lift force in

Fig. 4 Comparison of two-fluid time-averaged **a** gas volume fraction distribution, **b** vertical liquid velocity and **c** sparger configuration at superficial gas velocity 0.14 cm/s ($H/W=2.25$, $Y=37$ cm)

addition to the drag force is discussed in the following section.

3.1.2.2 Effect of lift force It is also well known that the lift force is as important as the drag force. Lift force acts in radial direction, i.e. the direction of decreasing liquid velocity and depends on the size of the bubble [20]. The lift model proposed by Tomiyama et al. [20] was used with the Tsuchiya drag model [19] for spargers except A1. While the velocity profiles were highly over predicted with no lift force in the model, implementation of the lift model improved the time-averaged liquid velocity profile quite well compared with the measurements (see Fig. 5b, c). Therefore, the Tsuchiya drag with the lift force (for selective cases) in the present work predicts all the behavior quite well compared to the other drag closures. It is therefore concluded that a proper drag and lift closure/model needs to be implemented in the CFD model for accurate prediction of dynamics inside a bubble column. Unless mentioned otherwise, Re based drag was used for uniform sparger (A1) and Tsuchiya drag with the lift was used for the rest of the sparger for all further simulations.

The results obtained from the simulations of uniform (A1 sparger with 100% opening) and non-uniform spargers

(A4 and A7 with opening of 69% and 18%, respectively) with $U_G=2.9$ cm/s are shown in Fig. 6 using Re based drag and the drag proposed by Tsuchiya et al. [19]. The snapshots of gas volume fraction distributions, gas, and liquid velocity vectors were compared against the experimental results reported by Julia et al. [6] and the results were found to be in good agreement.

3.2 Influence of different parameters on bubble column hydrodynamics

3.2.1 Flow structure with different sparger type

To understand the effect of aeration pattern on gas–liquid flow, seven types of spargers have been used for the simulation. The detailed dimension of the spargers is given in Table 2. For this aspect, simulations were performed with a constant inlet superficial gas velocity ($U_G=2.9$ cm/sec). Snapshots of gas volume fraction distribution and simulated gas velocity vector and liquid velocity vector for different spargers are shown in Fig. 7(i), (ii) and (iii), respectively. It can be noticed that for the sparger A1 with 100% opening, the gas phase is rising uniformly from the bottom of the column, filling the entire column radially. At this condition, the flow pattern shows all the features

Fig. 5 Effect of different drag and lift closures on time-averaged vertical liquid velocity for **a** A1 (100%), A4 (69%) and **c** A7 (18%) sparger at $U_G = 2.9$ cm/s (*drag used by Monahan and Fox)

of homogeneous flow. As the sparger was changed from A1 to A2 by blocking some portions of the sparging area (reduced from 100 to 95%) next to the wall, gas-phase appeared to be detached from the wall up to a limited height while moving upward. This height and blockage area of the sparger formed a space filled with continuous phase around the gas plume near the bottom of the column. This space continuously enlarged as the blockage

portion increased by changing the spargers from A2 to A7 in order.

It can also be seen that the velocity vectors of the gas phase where the vectors are significantly smaller near the detachable area of the gas from the wall. It is also interesting to see that the liquid phase circulation is happening for all spargers except the uniform sparger (A1). The non-aeration regions are introduced from both sides of the

Fig. 6 Simulated **a** gas volume fraction distribution, **b** gas velocity vector and **c** liquid velocity vector using (i) Re based drag for A1 (100%) and (ii and iii) Tsuchiya drag for A4 (69%) and A7 (18%) ($U_G = 2.9$ cm/s)

wall radially and that increases toward the center of the column. As a result, the liquid phase circulations are developed at both sides of the bottom corner in the form of twin vortices. The formed vortices pushed the incoming gas toward the center of the column and the neck zone expands as the non-sparging region increases (see Fig. 7 (ii)b–f). The local sparger (A7), where the non-sparging region is at the extreme position, the vortices started forming periodically at both walls of the column as seen in Fig. 7 (iii)g. The gas-phase pushes the liquid in both the

directions radially creating the periodic oscillation of the liquid phase.

Further, the effect of sparger on flow pattern was studied by comparing the simulated time-averaged vertical liquid velocity with the measurements of Julia et al. [6] and illustrated in Fig. 8. The solid lines represent the simulated results whereas the symbols indicate the measured data. The predicted velocity profiles were compared at different heights of the bubble column. The velocity profiles predicted at $y = 0.594$ (extreme top position) agreed well with the measurements for all aeration patterns.

Most of the velocity profiles showed variation at the center of the column and the down flow of liquid phase occurred near the side walls, which is because of the decreased gas volume fraction distribution near the neck region. The predicted velocity profiles are mostly uniform for spargers A3–A7 (see Fig. 8c–g), but over predicts the magnitude at $y = 0.0792$ m, 0.1584 m and 0.3168 m. It is observed that the velocity magnitude predicted for A7 agreed well with the measurements. Harteveld [17] has stated that the overall volume containing a lower gas volume fraction is larger in the 2D-bubble column than in a cylindrical column. The high circulation in the column led to the high upward velocity at the column center and downward flow near the side walls. The main disagreement between some predicted results and the measurements is attributed to the drag coefficient (C_D) chosen for the simulations. As discussed earlier, the drag coefficient needs to be adjusted for a better prediction with the measurements.

Even for more quantitative analysis and validation of the model, Fig. 9 is presented in terms of vertical velocity fluctuations. At this juncture, it is noted that the velocity fluctuation time series was tested with the Re based drag and Tsuchiya drag for A1 and A7, respectively (results are not shown). The Re based drag was found to be satisfactory for A1 whereas Tsuchiya drag was for A7 compared to the measurements [6]. As it is seen from Fig. 9a, the liquid velocity values are quite uniform for sparger A1 which was even evident from the previous figure (see Figs. 7a (i) and 8a). Unlike the measurements [6], the predicted signals from A4 sparger showed a lot of fluctuations (see Fig. 9d). This is probably because of the existence of a wide range of bubble size distribution inside the column. In particular, the smaller bubbles oscillate in an irregular and chaotic manner [22]. In case of the local sparger (A7), where the disabled aerated portion is much higher compared to the rest of the sparger, the oscillating behavior is much stronger. All spargers show some kind of oscillation except A1 which means the flow pattern for those spargers is not in the exact homogeneous regime but in a transition regime.

Fig. 7 Simulated (i) gas volume fraction distribution, (ii) gas velocity vectors and (iii) liquid velocity vector for different spargers **a** A1 (100%), **b** A2 (95%), **c** A3 (85%), **d** A4 (69%), **e** A5 (59%), **f** A6 (44%) and **g** A7 (18%) ($U_G = 2.9$ cm/s)

A comparison of power spectra of liquid velocity fluctuations time series predicted for all spargers is shown in Fig. 10. The power spectra obtained for A7 was higher ($f_d = 0.174$ Hz) compared to other spargers. The lowest dominant frequency observed was 0.011 Hz for A1 as the gas as well as liquid velocity is uniform and seen in the previous figure. The power spectra for spargers A5,

A6 and A7 (see Fig. 10e–g) showed high frequencies and that may be because of dense gas regions and high interactions within the gas phase. However, Singh et al. [23] explained in their experiments that the dominant frequency was observed because of the bubble swarm effect and frequent bubble–bubble interactions in the column.

Fig. 8 Effect of sparger type on time-averaged vertical liquid velocity **a** A1 (100%), **b** A2 (95%), **c** A3 (85%), **d** A4 (69%), **e** A5 (59%), **f** A6 (44%) and **g** A7 (18%) ($U_G = 2.9$ cm/s)

Fig. 9 Vertical liquid velocity fluctuation time series in the column ($y=0.1584$, $X_n=0.5$) for different sparger type **a** A1 (100%), **b** A2 (95%), **c** A3 (85%), **d** A4 (69%), **e** A5 (59%), **f** A6 (44%) and **g** A7 (18%) ($U_G=2.9$ cm/s)

Fig. 10 Power spectra of liquid velocity fluctuations in the column ($y=0.1584$, $X_n=0.5$) for different sparger type **a** A1 (100%), **b** A2 (95%), **c** A3 (85%), **d** A4 (69%), **e** A5 (59%), **f** A6 (44%) and **g** A7 (18%) ($U_G=2.9$ cm/s)

Fig. 11 Simulated **a** gas volume fraction distribution, **b** gas velocity vectors and **(c)** liquid velocity vectors for uniform sparger A1 (100%) at (i) 4.3 cm/s and (ii) 5.8 cm/s using Re based drag

3.2.2 Flow structure with varying gas velocity

One of the major parameters which changes the flow structure, i.e., homogeneous to other regimes is the incoming gas velocity. To see the effect of superficial gas velocity (U_G), simulations are performed with three different velocities (2.9, 4.3 and 5.8 cm/s) using uniform sparger A1 (100% opening) and Re based drag. Figures 6(i) and 11 shows typical simulated snapshots of meandering bubble plume and velocity vectors of gas and liquid phases in the column at different U_G which is consistent with that of Julia et al. [6]. It is observed that at $U_G = 2.9$ cm/s (see Fig. 6(ii)), the velocity field for the gas phase is uniform so as the liquid phase led to homogeneous flow. However, increasing the gas velocity to 4.3 cm/s (Fig. 11(i)) or even higher to 5.8 cm/s (Fig. 11(ii)), the plume starts oscillating. Therefore, the corresponding velocity vectors show similar oscillating behavior. At low U_G , the bubbles travel uniformly leaving the column with full of gas volume fraction distribution, whereas at high velocity the bubbles were detached from the wall and started moving periodically. Also, at high gas velocity, the gas phase began gathering near the sparger for a shorter period, then spread over the entire column

covering the wider region near the upper liquid level. The liquid phase which is stagnant inside the column circulated alongside the gas phase. The constant movement of the gas phase to the periodical movement is because of the change in gas velocity as the aeration pattern (sparger) kept constant throughout.

It is also interesting to observe the effect of gas velocity in terms of time-averaged parameters such as vertical liquid velocity and gas volume fraction distributions as a function of column height. Figure 12(i), which signifies the time-averaged vertical liquid velocity, the solid lines indicate the predicted data from simulations whereas the symbols are the measured data obtained from Julia et al. [6]. The predictions are compared with the measured data [6] and a fair agreement was observed. At low velocity ($U_G = 2.9$ cm/s), the two-fluid model which records the time-averaged liquid velocity profiles at lower sections of the bubble column ($y = 0.0792$ and 0.1584 m), under-predicts the value compared to the measurements whereas those at the upper section ($y = 0.3168$ and 0.594 m) agreed well with the measurements. As the velocity increases to 4.3 and 5.8 cm/s (Fig. 12(i)b–c), the velocity magnitude at the upper portion of the column appears to be under-predicted whereas profiles at the lower portion over-predicts compared to the measured data. The difference between both the results attributed to the way the aeration was done. In the simulation, the whole column area was used as the gas sparging area whereas needles are used in the experiments [6] for controlled aeration. Also, Re based drag was used to carry out those simulations and was found to be the best among other drag closures for A1. It is important to note that, the prediction can be improved by adjusting the drag coefficient (C_D) in the two-fluid model implemented in OpenFOAM. The corresponding time-averaged gas volume fraction distribution is shown in Fig. 12(ii). As it was confirmed that the regime at gas velocity of 2.9 cm/s for A1 was a homogeneous regime, the gas distribution throughout the width of the column is a flat profile irrespective of the height (see Fig. 12(ii) (a)). As the velocity increases, the magnitude of the time-averaged volume fraction distribution increases. The only difference in the magnitude of the gas distribution at $y = 0.0792$ m which is very near to the sparger and kept lower and is shown in Fig. 12(ii)b–c.

3.2.3 Influence of bubble size

Uniform size bubbles are obtained in homogeneous regime inside the bubble column and much research has been done to predict such flow regimes in the bubble column through CFD simulations. Mostly, Eulerian–Eulerian (EE) models are used to tackle those gas–liquid flow problems successfully [e.g., 24,25]. Bubble size inside

Fig. 12 Time-averaged (i) vertical liquid velocity and (ii) gas volume fraction distribution at different superficial gas velocity **a** 2.9 cm/s, **b** 4.3 cm/s and **c** 5.8 cm/s

Fig. 13 Computational snapshots of gas volume fraction distribution and corresponding liquid and gas velocity flow field for different bubble size at different sparger type
a A1 (100%), 2 mm bubble;
b A4 (69%), 2 mm bubble,
c A7 (18%), 2 mm bubble,
d A1 (100%), 8 mm bubble;
e A4 (69%), 8 mm bubble,
f A7 (18%), 8 mm bubble
($U_G = 2.9$ cm/s)

the column generally varies depending upon the flow conditions (i.e. flow rate, aeration pattern, etc.) and fluid properties.

Apart from the superficial velocity of the gas and aeration pattern (sparger type), bubble size (d_b) too influences the hydrodynamics of the flow. The implementation of the two-fluid EE model was done with an account of constant bubble size for particular cases, while different constant bubble size was included in the model

when necessary for different simulations. All simulations were performed with d_b ranging from 2 to 8 mm to see the effect of bubble size on the flow pattern. The snapshots of organic phase volume fraction, gas, and liquid velocity vectors are shown in Fig. 13 at U_G of 2.9 cm/s for sparger opening 100% (A1), 69% (A4) and 18% (A7) spargers. It is to be noted that the lift coefficient (C_L) takes a positive value for small (< 4 mm) bubbles and negative value for large (> 5.8 mm) bubbles [20].

Fig. 14 Effect of bubble size on time-averaged vertical liquid velocity for **a** A1 (100%), 2 mm bubble; **b** A4 (69%), 2 mm bubble, **c** A7 (18%), 2 mm bubble, **d** A1 (100%), 8 mm bubble; **e** A4 (69%), 8 mm bubble, **f** A7 (18%), 8 mm bubble ($U_G = 2.9$ cm/s)

Fig. 15 Vertical liquid velocity fluctuation time series in the column ($y=0.1584$, $X_n=0.5$) for different bubble size at different sparger type **a** A1 (100%), 2 mm bubble; **b** A4 (69%), 2 mm bubble; **c** A7

(18%), 2 mm bubble, **d** A1 (100%), 8 mm bubble; **e** A4 (69%), 8 mm bubble, **f** A7 (18%), 8 mm bubble ($U_G=2.9$ cm/s)

Therefore, C_L is controlled by bubble Reynolds number when $d_b < 4$ mm whereas it is governed by Eotvos number when $d_b > 4.4$ mm. Therefore, the phase interaction forces such as Re based drag (part of Tsuchiya correlation), Tsuchiya drag which is both Re and Eotvos number based [19] and lift forces [20] in the two-fluid CFD model used for the prediction of the results. In Fig. 13, as moving from A1 to A7 sparger (see Fig. 13a–c), the gas phase detached from the bottom of the column which was seen in earlier cases. However, because of the smaller bubble size ($d_b = 2$ mm), significant gas volume fraction distribution was observed near the wall [20]. It is seen from Fig. 13d–f where $d_b = 8$ mm was considered in the simulations, the gas volume fraction distributions were seen near the center of the column and it was well mentioned by Tomiyama et al. [20] for bigger bubbles. However, the particular trend using $d_b = 8$ mm for A1 sparger is not visible due to uniformly aerated sparger.

The predicted time-averaged vertical liquid velocity was compared with the measurements at different heights of the bubble column (see Fig. 14). For A1 (Fig. 14a), it is observed that the velocity distribution is different at different heights of the column using a constant bubble size of 2 mm and agreed well with the measurements except for the profile at $y = 0.3168$ m. There is a high magnitude of the liquid velocity near the side walls of the column for $y = 0.3168$ and 0.594 m, respectively. It is because the liquid phase attends the maximum velocity near the side walls and suppress at the top middle region of the column. The difference between both the predicted results for A1 arise from the different constant bubble size considered in the two-fluid model irrespective of the usage of similar drag model. In the case of aeration pattern A7, the use of constant d_b of 8 mm predicts the velocity profile well compared to d_b of 2 mm and can be seen in Fig. 14c, f. The profiles are seen to be downward near the wall as the gas phase pushes the liquid near the center which intern creates a recirculation of liquid near the wall.

For further quantitative comparisons, the vertical liquid velocity fluctuation time series at superficial velocity of 2.9 cm/s was prepared from the same set of simulations described in Fig. 15 at $y = 0.1584$ m and $X_n = 0.5$. As seen earlier, the flow regime is homogeneous at A1 for 2.9 cm/s, therefore the comparison between Fig. 15a, b signifies that the velocity fluctuations are uniform using d_b of 6 mm rather than using 2 mm. Similarly, the velocity magnitude is more using 8 mm bubbles at A7 (see Fig. 15b) compared to that of 2 mm size (see Fig. 15d) in the simulation and it is already evident from the time-averaged velocity plot in Fig. 14. Therefore, it is observed that the oscillation is more in the case of small bubble size compared to the large one with the same operating conditions in the simulation.

Alike the previous studies, a comparison of power spectra of liquid velocity fluctuations time series predicted (see Fig. 16) for the same set of bubble size (d_b) studied in Fig. 15. As observed earlier, decreasing the aerated area, the dominant frequency (f_d) found to increase. A similar trend was observed at $d_b = 2$ mm from sparger A1 to A7. The power spectra obtained for A1 using $d_b = 8$ mm has multiple dominant frequencies compared to $d_b = 2$ mm ($f_d = 0.045$ Hz). The lower dominant frequency was observed using a smaller bubble size (2 mm) compared to the bubble size of 8 mm for spargers A4 and A7 whereas multiple dominant frequencies were observed for uniform sparger (A1). Hence, from multiple analysis, it was observed that the column hydrodynamics changes significantly with bubble size.

3.3 Influence of multiple parameters on flow transition

From the above results, it was noticed that variation in sparger opening, superficial gas velocity and bubble size affect the flow structure. To visualize the flow regime with a variation of two parameters simultaneously, flow structure maps have been generated. Such maps are frequently used to visualize flow instabilities during the hot deformation of metals [26]. The flow structure maps shown in Fig. 17 have been generated using dominant frequency (f_d) obtained from the simulations. Figure 17a shows the flow structure map in the sparger opening-superficial gas velocity window while Fig. 17b shows the flow structure map in the sparger opening-bubble size map. Contours in the maps indicate the dominant frequency (f_d). It is noticed from the results that in most of the cases the flow becomes re-circulatory when $f_d \approx 1$. Therefore, in this work, the dominant frequency, $f_d = 1$ has been taken as the boundary that delineates the re-circulatory regime from the homogeneous regime. The colored region shows the re-circulatory regime whereas the rest of the region indicates the homogeneous regime.

From Fig. 17a, it can be seen that with an increase in superficial gas velocity, spargers with larger openings are needed to keep the flow homogeneous. The transition from homogeneous to re-circulatory flow regime is highly sensitive to superficial gas velocity. On the other hand, it can be seen from Fig. 17b that the transition from homogeneous to re-circulatory flow regime is not so sensitive to bubble size. These maps can be used as a tool to find a suitable flow regime for a set of operating parameters.

Fig. 16 Power spectra of vertical liquid velocity fluctuation in the column ($y=0.1584$, $X_{T1}=0.5$) for different bubble size at different sparger type **a** A1 (100%), 2 mm bubble; **b** A4 (69%), 2 mm bubble,

c A7 (18%), 2 mm bubble, **d** A1 (100%), 8 mm bubble; **e** A4 (69%), 8 mm bubble, **f** A7 (18%), 8 mm bubble ($U_G=2.9$ cm/s)

Fig. 17 Flow regime map for **a** gas superficial velocity–sparger opening and **b** bubble size–sparger opening

4 Concluding remarks

Gas–liquid flow in a 2D-rectangular bubble column was studied to understand different parameters which changes the flow regime. The combined effect of the parameters such as gas superficial velocity, sparger opening and bubble size were rigorously analysed. The three-dimensional simulations were performed using OpenFOAM with the developed *twoPhaseEulerFoam* module. The predicted time-averaged velocity and gas volume fraction profiles were validated with the available experimental data.

It was found that for a given gas velocity, a decrease in the opening area of the sparger, the flow regime changes from homogeneous to re-circulatory. The re-circulatory flow led to a non-uniform gas volume fraction distribution across the height of the bubble column. In addition, asymmetric liquid velocity and gas volume fraction distribution profiles were observed using a uniformly aerated sparger compared to all other spargers. After evaluating several drag models, *Re* based drag model was chosen for uniform sparger (with 100% opening), whereas Tsuchiya drag coefficient model which contains both Reynolds and Eotvos number was implemented for the spargers with opening 18–95%. The velocity profiles were highly over-predicted with no lift force using different drag closures in the model. The implementation of the lift model with Tsuchiya drag model improved the time-averaged liquid velocity profile quite well for all spargers except the uniform sparger. Therefore, Tsuchiya drag with the lift model (for selective cases) predicts all the behavior quite well compared to other drag closures.

The flow regime map shows that spargers with a minimum opening area of 80% are recommended for obtaining a homogeneous regime with superficial velocity of 2.9 cm/s for a bubble size of 3.76 mm. For higher superficial velocities, spargers with larger opening areas need to be chosen to sustain a homogeneous flow regime. The transition from homogeneous to re-circulatory flow regime is highly sensitive to superficial gas velocity. The use of different bubble sizes affect the dominant frequency as well as hydrodynamic inside the column. However, the influence of bubble size on shifting the regime from homogeneous to re-circulatory is insignificant.

It is concluded that few additional drag closures with/without lift closures need to be thoroughly analyzed and to be implemented in CFD model for accurate prediction of dynamics inside the column. Furthermore, drag correction factors need to be tested for the improvement of the numerical predictions. A constant average bubble size was used in the Eulerian-Eulerian (EE) model to calculate the interphase momentum exchange terms. The model does not account for the change in the bubble size distribution. Therefore, a considerable difference between the measured and predicted time-averaged profiles was observed for few cases. In order to improve the predictions, it is important to perform multi-fluid simulations integrated with the population balance model which accounts for the coalescence and breakage phenomena between bubbles in the bubble column using bubble size distributions.

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Declarations

Conflict of interest The author declares that there is no conflict of interest.

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