Buckling and vibration analysis of shape memory laminated composite beams under axially heterogeneous in-plane loads in the glass transition temperature region

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Abstract

Buckling and vibration study of the shape memory polymer composites (SMPC) across the glass transition temperature under heterogeneous loading conditions are presented. Finite element analysis based on C° continuity equation through the higher order shear deformation theory (HSDT) is employed considering non linear Von Karman approach to estimate critical buckling and vibration for the temperature span from 273 to 373 K. Extensive numerical investigations are presented to understand the effect of temperature, boundary conditions, aspect ratio, fiber orientations, laminate stacking and modes of phenomenon on the buckling and vibration behavior of SMPC beam along with the validation and convergence study. Effect of thermal conditions, particularly in the glass transition region of the shape memory polymer, is considerable and presents cohesive relation between dynamic modulus properties with magnitude of critical buckling and vibration. Moreover, it has also been inferred that type of axial loading condition along with the corresponding boundary conditions significantly affect the buckling and vibration load across the glass transition region.

Keywords SMP · SMPC · Buckling · Vibration · Glass transition region · HSDT

1 Introduction

Shape memory polymers (SMP) possess the feature to deform under the application of stress and regain its original orientation in the vicinity of a stimulus such as electricity, magnetic field, heat, chemical or moisture [1]. Present study focuses on the on the heat sensitive thermoset polymer, which has better thermomechanical properties as it has improved energy storage and robust intermolecular crosslinks [2]. SMPs have the wide span of distinct characteristics of resistance to the harsh environment and shape controllability but has low tensile strength at elevated temperature [3]. The overall mechanical properties can be enhanced with the incorporation of fiber in shape memory polymer composites (SMPC). Hence, new horizons are

created for its application such as aerospace [4], electronics [5], biomedical engineering [6] and textile [7].

Laminated composites are in extensive application in civil, aerospace, military and aeronautical avenues because of the enhanced strength, stiffness and lightness which has encouraged numerous investigators to present theories to estimate the mechanical behavior of laminated composites beams (LCB) under various deformations. Among these, Higher order shear deformation theory (HSDT) considers higher order axial distortion variation along thickness of a beam, which results in prediction of behavior for laminated composite beam with improved precision than the classical beam theory (CBT) and First order shear deformation theory [8].

In the last few decades, with advent of some novel applications, laminated SMPCs have also been identified

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with great potentials in numerous areas of smart structural applications. Thus, SMPC beams are constantly subjected to numerous elastic instabilities, among which buckling and vibration would be focus of analysis in this paper. In this context, a brief literature review of buckling and vibration study of LCB is summarized. Mathew et al. [9] presented the thermal buckling behavior of LCB through FSDT based finite element method (FEM). Lee [10] evaluated the thermal buckling behavior based on layer-wise theory. Employing, Euler–Bernoulli beam theory, Emam and Nayfeh [11] estimated the post buckling behavior of LCB through Euler–Bernoulli beam theory. Aydogdu [12, 13] presented the mechanical and thermal buckling behavior of the LCB for different conditions through Ritz method. Vo and Thai [14] studied buckling and vibration of LCB through HSDT with the consideration of bending and shear components along transverse displacement. Modifying the previous models with consideration of shearing, Eltaher et al. [15, 16] presented bending, vibration and buckling behavior of thick nanobeams under the mechanical loading. Nguyen et al. [17] analyzed the thermo-mechanical vibrational and buckling behavior of LCB through novel shape function based on Ritz method. It must be noticed that, buckling can even appear in the ambience of non-uniform variable in plane loading conditions. Some literatures can be found for the buckling and vibration analysis of isotropic structures subjected to non-uniform loadings [18–21]. However, there are few works that focus on the elastic buckling and buckling behavior of laminated composites under influence of non-uniform axially variable in-plane loadings.

Beam structures are exposed to heterogenous in plane loading in real life applications such as aircrafts fins, stiffened plates on marine structures and beam of skyscrapers [22]. Thus, it is significant to study the buckling behavior of beams under non-uniform axially variable loadings. Nemeth [23] analyzed the buckling behavior of LCB under varying in-plane edge load and presented solutions for different fiber orientations and loading conditions. Lopatin and Morozov [24, 25] evaluated the buckling parameters of laminated composite plates subjected to varying in-plane loading. Zhong and Gu [26] analyzed the buckling phenomenon for crossply laminates for variable in plane edge loading through FSDT. Cagdas and Adali [27] optimized the layer thickness for various fiber angle and boundary conditions for the estimation of maximum load resistant to buckling. Based on analytically derived parametric study, Kalyan and Bhaskar [28] performed buckling analysis of orthotropic plate. Kumar et al. [29] presented buckling parameters for skew composite plate under the influence of variable in-plane loading. Mallela and Upadhyay [30]

formulated parametric correlations to design composite panel subjected to varying in-plane loading and predicted its buckling behavior. Kim and Lee [31] modelled the buckling behavior of thin LCB subjected to varying loading.

Buckling and vibration study to estimate post-buckling behavior of LCB has rarely been attempted for nonlinear axially heterogenous in-plane loading. Except for some of the works in recent times, most of analysis assumes uniform compressive load throughout the beam or plate. Karamanli and Aydogdu [8] studied the buckling phenomenon in LCB and sandwich under various axially varying in-plane loading through HSDT with Ritz solution. Eltaher et al. [32] presented a model to predict static stability of LCB under varying axial loads. As per the available literature, the study the buckling and vibration phenomenon with their mode-shapes for SMPC beams under the axially heterogeneous in-plane loads has rarely been studied. Based on the inspiration, in the present study, buckling and vibration analysis of SMPC laminated beams subjected to axially varying in-plane load has been investigated. The paper presents buckling and vibrations analysis of SMPC beam for different boundary conditions, aspect ratio, fiber orientation, laminate stacking and modes of phenomenon.

The article is organized as following: Sect. 2 discusses theory of shape memory effect in SMPC and mathematical formulation incorporated in finite element analysis. Section 3 presents the results obtained through the evaluation of finite element analysis (FEA), which include convergence and validation study followed with the analysis of influence of different factors on the buckling and vibration on SMPC beam. Section 4 describes the conclusion inferred through the numerical study.

2 Theory and formulation

The theory to estimate the dynamic mechanical parameters of smart polymer matrix with the incorporation of carbon fiber in the composite is discussed followed with the formulation of buckling and vibration analysis through FEA.

2.1 Modelling of shape memory effect

The material characteristics of the temperature regulated polymer is referred from an experimental work in Ref. [33]. Analytical correlation in terms of Eq. (1) is formulated though curve fitting of experimental data, to estimate storage modulus $E(T)$ of SMP for the varying temperature condition across T_g [34].

$$E(T) = (E_1 - E_2) \cdot \exp\left(-\left(\frac{T}{T_\beta}\right)^{m_1}\right) + (E_2 - E_3) \cdot \exp\left(-\left(\frac{T}{T_g}\right)^{m_2}\right) + E_3 \cdot \exp\left(-\left(\frac{T}{T_f}\right)^{m_3}\right) \quad (1)$$

where T is the span of temperature considered for analysis. E_1, E_2 and E_3 are SMP's moduli at the commencement of glass transition temperature (T_β), glass transition temperature (T_g) and flow region temperature (T_f) respectively. m_1, m_2 and m_3 are Weibull exponents for optimized curve-fitting. These values to be substituted in Eq. (1) has been obtained from the work by Gu et al. [41], summarized in Table 1. Experimentally determined values of parameter were extracted through Dynamic mechanical Analysis (DMA). Data of experiments and the subsequent confirmation by Eq. (1) is depicted in Fig. 1a, which highlights considerable convergence. Storage modulus has been assumed as Elastic modulus in the following stages of

analysis, due to negligible magnitude of loss modulus than the storage modulus [35]. Temperature governed Poisson's ratio (μ) is evaluated by phase transition model for the matrix [36].

$$\mu = \mu_g v_g + \mu_r (1 - v_g) \quad (2)$$

$$v_g = 1 - \frac{1}{1 + \exp[-(T - T_m)/Z]} \quad (3)$$

where μ_g and μ_r are Poisson's ratio of the frozen and active phase respectively. v_g is the volume fraction of the frozen phase. T_m is reference temperature. Z is parameter

Table 1 Values of parameters in model for SMP [35]

Parameter	T_g	T_m	T_β	T_f	E_1	E_2	E_3	m_1, m_2, m_3	μ_g	μ_r	Z
Values	305 K	300.5 K	295.2 K	415.5 K	2.552 GPa	1.876 GPa	5 MPa	19.3, 58.4, 177.6	0.35	0.499	7

Fig. 1 Material properties of SMP with respect to temperature (T). **a** Simulation of Storage modulus with experiment. **b** Poisson's ratio. **c** Shear modulus

Fig. 2 Influence of variation of temperature on modulus of SMPC: **a** longitudinal, **b** transverse, **c** shear modulus and **d** Poissons ratio of SMPC

characterizing the width of the phase transition zone. These material parameters are utilized in the following stage to evaluate the moduli of the SMPC beam under the influence of varying temperature in the vicinity of transition region of polymer matrix. Temperature dependent Poisson's ratio and shear modulus are described in Fig. 2.

The properties of SMPC is estimated by the theory of volume averaging [37] through Eqs. (4–8), which are successively utilized to analyze the behavior of the laminate under different loadings.

$$E_{c1} = E_{f1}V_f + E_mV_m \quad (4)$$

$$E_{c2} = (1 - C)E_{c2}^1 + CE_{c2}^2 \quad (5)$$

$$\mu_{c21} = (1 - C)\mu_{c21}^1 + C\mu_{c21}^2 \quad (6)$$

$$\mu_{c12} = \mu_{c21} \frac{E_{c2}}{E_{c1}} \quad (7)$$

$$G_{c12} = (1 - C)G_{c12}^1 + CG_{c12}^2 \quad (8)$$

where E_{c1} and E_{c2} are longitudinal and transverse modulus of composite respectively and E_{f1} and E_{f2} are the corresponding values for the fiber. G_{f12} and G_{c12} are the shear modulus along plane 1-2. For fiber and composite respectively. μ_{c12} and μ_{c21} are Poisson's ratio of composites in 1-2 and 2-1 plane. The superscript 1 and 2 indicate values of constants in series and parallel respectively and corresponding properties are calculated by Eqs. (9–12), and respectively substituted to evaluate the properties of

Table 2 Material parameters of carbon fiber [37]

Parameters	E_{f1}	E_{f2}	G_{f12}	μ_f	C
Magnitude	2.30×10^5 MPa	8.20×10^3 MPa	2.73×10^4 MPa	0.25	0.20

SMPC. The properties of fiber reinforcement are summarized in Table 2. These temperature regulated properties of SMPC are presented in Fig. 2.

$$E_{c2}^1 = \frac{E_{f2}E_m}{E_{f2}V_m + E_mV_f} \quad (9)$$

$$E_{c2}^2 = E_{f2}V_f + E_mV_m \quad (10)$$

$$\mu_{c21}^1 = \mu_fV_f + \mu_mV_m \quad (11)$$

$$\mu_{c21}^2 = \frac{\mu_fE_{f2}V_f + \mu_mE_mV_m}{E_{f2}V_f + E_mV_m} \quad (12)$$

$$G_{c12}^1 = \frac{G_{f12}G_m}{G_{f12}V_m + G_mV_f} \quad (13)$$

$$G_{c12}^2 = G_{f12}V_f + G_mV_m \quad (14)$$

2.2 Mathematical formulation

The mathematical formation of FEM is based on the governing equation presented in Ref. [38]. One dimensional LCB is employed in the present study subjected to varying in-plane heterogeneous load. LCB has the length, breadth and thickness as 'a', 'b' and 't' respectively (Fig. 3). Displacement

field equations in 'x' and 'z' coordinate through the HSDT considering C° continuity equation is given as [38],

$$\bar{u}(x, z) = \mathbf{u} + (C_1z - C_2z^3)\Psi_x + (-C_2z^3)\vartheta_x; \bar{w}(x, z) = \mathbf{w}. \quad (15)$$

where $\mathbf{u}$, $\mathbf{w}$, and $\vartheta = \partial w / \partial x$ are displacement along mid-plane axis, displacement along transverse direction, rotation of normal to the mid-plane along 'y'-axis and slope along 'x'-axis, respectively, with $C_1 = 1$, $C_2 = \frac{4}{3}h^2$.

Displacement vector for the modified C^0 continuous model with four degrees of freedom can be expressed as [38]

$$\{\mathbf{q}\} = [u \quad w \quad \vartheta_x \quad \Psi_x]^T \quad (16)$$

2.2.1 Stress, strain and displacement relation

Here, total strain is the combination of linear $\{\bar{\epsilon}^L\}$ and nonlinear strain vector $\{\bar{\epsilon}^{NL}\}$ with von Karman nonlinearity, stated as [39]

$$\{\bar{\epsilon}\} = [B]\{\mathbf{q}\} + \frac{1}{2}[A_{nl}]\{\vartheta_{nl}\} \quad (17)$$

[B] is the geometrical matrix given in A1. Similarly, plane stress and total strain relationship can be expressed as,

Fig. 3 Geometrical configuration of composite beam [8]

$$\left\{ \begin{array}{c} \overline{\sigma_x} \\ \overline{\tau_{xz}} \end{array} \right\} = \begin{bmatrix} \overline{Q_{11}} & 0 \\ 0 & \overline{Q_{55}} \end{bmatrix} \left\{ \begin{array}{c} \overline{\epsilon} \\ \overline{\gamma} \end{array} \right\} \quad (18)$$

In above expression, transformed reduce matrix denoted is denoted by $\{\overline{Q_{ij}}\}$ [39].

2.2.2 FEA of the composite beam

C^0 continuity equation employing 1-D Hermitian beam element with four degrees of freedom per node is considered [38]. Rewritten as the following type of beam element geometry and the displacement vector.

$$\{\mathbf{q}\} = \sum_{i=1}^{NN} N_i \{\mathbf{q}\}_i; \mathbf{x} = \sum_{i=1}^{NN} N_i x_i; \quad (19)$$

In order to calculate transverse displacement and slope, linear interpolation for axial displacement and rotation of normal and Hermite cubic interpolation functions are considered. Using FEM, Eq. (19) can be remodeled as,

$$\boldsymbol{\Pi}_1 = \sum_{e=1}^{NE} \boldsymbol{\Pi}_a^{(e)} = \sum_{e=1}^{NE} \left(\mathbf{u}_L^{(e)} + \mathbf{u}_{NL}^{(e)} \right) \quad (20)$$

where NE and (e) denote the number of elements and elemental, respectively.

Equation (20) can be further evaluated as,

$$\boldsymbol{\Pi}_1 = \frac{1}{2} \sum_{e=1}^{NE} \left[\{\mathbf{q}\}^{T(e)} [K_I + K_{nl}]^e \{\mathbf{q}\}^{(e)} \right] = \{\mathbf{q}\}^T [K_I + K_{nl}] \{\mathbf{q}\} \quad (21)$$

where $[K_I]$ is global linear matrix, $[K_{nl1}]$, $[K_{nl2}]$ and $[K_{nl3}]$ are non-linear stiffness matrices and $\{\mathbf{q}\}$ is global displacement vector.

Work done due to in-plane variable non-uniform loading can be represented as follows [38],

$$\boldsymbol{\Pi}_2 = \frac{1}{2} \iint N_n^x \left\{ \frac{\partial u}{\partial x} \right\}^2 dA \quad \text{where,} \quad N_n^x = N \left[P_1 \left(x + \frac{a}{z} \right)^2 + P_2 \left(x + \frac{a}{z} \right) + P_3 \right] \quad (22)$$

P_1 , P_2 and P_3 are the in-plane load varying coefficients as shown in the Fig. 3.

Kinetic energy of vibrating sandwich composite beam can be expressed as [40],

$$\boldsymbol{\Pi}_3 = \frac{1}{2} \iiint \rho \{\dot{\mathbf{u}}\}^T \{\dot{\mathbf{u}}\} dv \quad (23)$$

ρ is the density of the beam, $\{\dot{\mathbf{u}}\}$ is the velocity vector. Mechanically induced buckling and vibrational analysis of the in-plane variable heterogenous loading over laminated composite beam evaluated by the minimization of

first variation of $(\boldsymbol{\Pi}_1 - \boldsymbol{\Pi}_2 + \boldsymbol{\Pi}_3)$ with respect to generalized displacement vector is given by,

$$\{ [K_L + K_{NL}] - \lambda [K_G] \} \{\mathbf{q}\} = 0 \quad (24)$$

$$[K] \{\mathbf{q}\} = \lambda [K_G] \{\mathbf{q}\} \quad (25)$$

$$[K] \{\mathbf{q}\} = \omega^2 [M] \{\mathbf{q}\} \quad (26)$$

$$[K] = \{ [K_I] + [K_{nl}] - [K_G] \} \quad (27)$$

where $[K_G]$ is the geometric stiffness matrix given in A3, λ is the buckling load parameters, ω is the natural frequency of the beam, $[M]$ is the global consistent mass matrix and $\{q\}$ is the transverse deflection. The solutions of the above standard equations are obtained by Newton Raphson method which is the most popular solution method due to its quick convergence.

3 Result and discussions

FEA analysis of SMPC beam has been performed through a MATLAB program, through the mathematical modelling described in Sect. 2. The results evaluated by means of this program are presented in this section which is distributed in three sub-sections. The first sub-section will compile the validation and comparison of numerical analysis with the previous works. In the second subsection, buckling analysis of SMPC laminated beam will be performed for the axially varying in-plane load. Effect of BCs, orientation of fiber, aspect ratio and laminate ply-staking on buckling behavior of SMPC beam would be presented in this sub-section in the glass transition region of SMPC. Buckling shape mode analysis are also incorporated to visualize effects of dif-

ferent BCs and dynamic thermal property of SMPC. Third subsection presents vibration analysis of SMPC beam.

3.1 Convergence and validation

The number of elements (neL), adopted for FEA, is based on the convergence study presented in Fig. 4a. Convergence study are performed for three boundary conditions (BCs) of clamped (C-C), clamped-free (C-F), hinged (H-H) and clamped-hinged (C-H) with the consideration of two aspect ratios (a/h)—5 and 10. Nondimensional

Fig. 4 Convergence and validation of NCBL. **a** NCBL with respect to neL for different aspect ratios and BCs. **b** NCBL for different axial loadings

Critical Buckling Load (NCBL) inversely proportional to neL from 10 to 18, where the slope is considerably higher. In the range of 18–26, the trajectory stabilizes, with minimal deviation after 30. Thus, neL = 30 has been selected for FEA, to obtain results with the minimum number of elements for reducing computation without diluting the accuracy.

The results of buckling analysis for different axial loading conditions are indicated in Fig. 4b, with the corresponding comparison with results indicated in [8]. Evaluation of NCBL has been done for simply supported (S-S) and clamped (C-C) BCs, for 5 different axial loading conditions with unidirectional [0°/0°] and asymmetric [0°/45°] fiber orientation. The analysis presents considerable convergence with the prior works where the highest NCBL for N_4^m type axial loading. The range of deviation between the results by current analysis and [8] ranges from 4% for N_1^m type loading to 31% for N_4^m type loading. NCBL can be calculated from Eq. (28) with the consideration of material properties as $E_1 = 25E_2$, $G_{12} = 0.5E_2 = G_{13}$, $G_{23} = 0.2E_2$, $\mu_{12} = 0.25$ [8].

$$NCBL = \frac{\lambda L^2}{E_1 b h^3} \quad (28)$$

Results of vibration analysis, with its comparison with the previous works [41], are compiled in Table 3. The parameters assumed in this study are: $E_1 = 21 \times 10^6$; $E_2 = 1.4 \times 10^6$; $G_{12} = 0.6 \times 10^6$; $\mu_{12} = 0.3$; $\rho = 0.13 \times 10^{-3}$; $G_{23} = 0.5 \times 10^6$; $G_{13} = G_{12}$ and $\mu_{12} = 0.3 \times E_2/E_1$. Evaluated values are in congruence with previous results, as indicated in Table 3. Nondimensional Critical frequency (ω_{cr}) of beam with aspect ratio of 15 and fiber orientation of [0°/90°/90°/0°] are estimated by Eq. 29 as,

$$\omega_{cr} = \frac{\omega a^2}{\rho/4E_1 h^2} \quad (29)$$

3.2 Buckling analysis

Parametric study of buckling behavior of smart LCB is discussed in this section. The values of NCBL is stable before the initiation of glass transition, i.e. $T = T_\beta$, then

Table 3 Comparison of evaluated critical frequency with [41]

BCs	Present	Reference [41]	Present	Reference [41]	Present	Reference [41]	Present	Reference [41]
	ω_1		ω_2		ω_3		ω_4	
CC	4.910	4.59	11.080	10.291	18.38	16.966	26.20	24.041
CS	3.55	3.525	9.6	9.442	16.78	16.384	24.42	23.685
CF	0.925	0.9241	4.94	4.893	11.67	11.44	17.257	18.697
SS	2.50	2.50	8.545	8.545	15.95	15.85	23.72	23.71

it abruptly declines to regain stability after 313 K. Buckling behavior of SMPC is in coherence with the results of Fig. 1. Decrease in NCBL of 52.94%, 50.02%, 49.93%, 37.51% and 48.04% has been observed for N_1^m , N_2^m , N_3^m , N_4^m and N_5^m types of load distribution respectively for clamped (C-C) BC. For clamped free (C-F) BC, decline in NCBL by 16.67%, 15.08%, 16.22%, 16.39% and 64.28% are noticed in corresponding N_1^m , N_2^m , N_3^m , N_4^m and N_5^m types loading. Similar drastic slump in has also been observed in other four BCs, which is coherence with the results of dynamic mechanical analysis in Fig. 1.

Effects of BCs of SMPC laminated composite is depicted in Fig. 5 for clamped (C-C), clamped free (C-F), clamped hinged (C-H), hinged (H-H), clamped simply supported (C-S) and simply supported (S-S) boundary conditions. Aspect ratio (a/h) of 5 with fiber orientation $[0^\circ/90^\circ/0^\circ]$ has been considered for the buckling analysis due to varying in-plane loading condition across glass transition temperature of SMPC. Results depict the effect of different loads across the temperature range from 273 to 373 K in terms non-dimensional critical buckling load (NCBL), which is highest in case of clamped boundary condition. Among all the load distributions, N_3^m and N_4^m types load distribution have higher critical buckling load as compared to uniform distribution (N_1^m), while N_2^m and N_5^m load distributions have lower critical buckling load than N_1^m . This observation holds true for all the six BCs considered in the analysis. Clamped (CC) BC has maximum critical buckling load, whereas clamped-free (C-F) has the minimum critical buckling load. Ascending order of NCBL in terms of BCs is arranged as C-F, S-S, H-H, C-S, C-H and C-C, which is indicated in Table 4.

Temperature is also be noticed to govern the buckling behavior of SMPC for all the BCs. Critical buckling load is stable before the glass transition region of SMPC, that is, 295 K. However, NCBL suddenly declines after 295 K and stabilizes after 313 K, which is after the glass transition region. Thus, DMA can be referred to understand the volatile behavior in the glass transition region for buckling analysis of SMPC, as indicated by trajectory in Fig. 5 which fundamentally seems to be govern by results in Fig. 1.

Effect of aspect ratio from 5 to 50 on the critical buckling load under varying load for the six BCs at T_g is described in Table 4. In the case of C-C, NCBL increases by 147.90%, 58.56% and 19.61% when aspect ratio increases from 5 to 10, 10 to 20 and 20 to 50 respectively for all the varying loading conditions, indicating stability of buckling behavior with the increase of aspect ratio. Similar behaviors are noticed in C-S and S-S BCs. 99.73%, 33.77% and 10.47% increase of NCBL is noticed with the respective increase of aspect ratio from 5 to 10, 10 to 20 and 20 to 50 in C-S and S-S boundary conditions for all considered loading

conditions. However, for the C-F, C-H and H-H BCs, an uneven behavior in the variation of critical buckling load is observed. Difference of 107.67%, 51.53% and 56.36% for aspect ratio from 5 to 10, 10 to 20 and 20 to 50 respectively are calculated in C-H boundary condition, while 66.77%, 44.57% and 90.39% variations in the similar aspect ratio range of H-H boundary condition for all the loading types. Dynamic temperature condition also affects the behavior of SMPC, as evident in Fig. 4 with the constant aspect ratio of 5, indicating a corresponding regulating influence of critical buckling load for all BCs.

Downward trends are observed as the angle of fiber orientation increases from 0° to 90° , as presented in Table 5. NCBL are reported for four layered symmetric SMPC clamped with aspect ratio of 20. The decrease of 5.97%, 17.47%, 29.11%, 41.11%, 50.83% and 37.59 is observed for all loading condition across the glass transition region, when fiber orientation is increased from 0° to 90° with the interval of 15° . Temperature also play a vital on buckling behavior of SMPC laminate at different fiber orientations. During the shift from pre-glass transition to T_g , decrease in NCBL by 6.54%, 6.17%, 5.36%, 5.37%, 9.72%, 27.24% and 50.52% is noted for interval of 15° from 0° to 90° . Corresponding decrease of NCBL from pre-glass transition to post-glass transition is 15.27%, 14.56%, 12.911%, 12.38%, 19.30%, 49.94% and 91.58%. Thus, temperature also affect the buckling behavior at different fiber orientations for all the considered loading conditions.

Influence of laminate stacking is presented in Table 6 for four BCs under N_1^m type of loading condition. Among the considered sequences, unidirectional laminate $[0^\circ/0^\circ/0^\circ/0^\circ]$, has the maximum NCBL for all the BCs. It is observed that when the outer layers have ply angle of 0° , NCBL is on a higher magnitude but, as the value proceed towards 90° , NCBL decreases.

Effect of glass transition region of SMPC and different types of buckling modes at glass transition temperature is discussed through Table 7 and Fig. 6, in this section. NCBL for four buckling modes at three temperatures of $T=T_\beta$, T_g and T_f with the consideration of linearity and non-linearity are presented in Table 7. Prior to the beginning of glass transition, NCBL decreases by 0.14% and 0.42% in Mode-1, 0.09% and 0.29% in Mode-2, 0.13% and 0.41% in Mode-3 and 0.13% and 0.41% in Mode-4 with the assumption of nonlinearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity. At glass transition temperature, T_g , NCBL decreases by 0.18% and 0.55% in Mode-1, 0.12% and 0.38% in Mode-2, 0.19% and 0.58% in Mode-3 and 0.19% and 0.59% in Mode-4 with the assumption of nonlinearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity. Similarly, after glass transition, NCBL reduces by 0.26% and 0.75% in Mode-1, 0.17% and 0.55% in Mode-2, 0.28% and 0.87% in Mode-3 and 0.29% and 0.91% in Mode-4 with

Fig. 5 Non-dimensional critical buckling load (NCBL) under various axial loads with respect to temperature at different BCs

Table 4 NCBL of SMPC beam with aspect ratio for various loading conditions at six BCs ($T = T_g$)

BCs	a/h	N_1^m	N_2^m	N_3^m	N_4^m	N_5^m
C-C	5	11.7685	7.8456	14.7106	24.5177	8.1725
	10	29.1744	19.4496	36.4681	60.7802	20.26
	20	46.2587	30.8391	57.8234	96.3723	32.1241
	50	55.3278	36.8852	69.1597	115.2662	38.4220
C-F	5	2.7178	1.8118	3.3972	5.6621	1.8873
	10	2.2816	2.1026	3.9424	6.5706	6.5706
	20	3.2855	3.2855	4.1069	6.8449	2.2816
	50	3.3244	2.2162	4.1555	6.9258	2.3086
C-H	5	9.0783	6.0522	11.3479	18.9132	6.3044
	10	18.8533	12.5688	23.5666	39.2777	13.0925
	20	28.5688	19.0458	35.7110	59.5183	19.8394
	50	44.6704	29.7802	55.8380	93.0634	31.0211
H-H	5	7.3193	4.8795	9.1491	15.2485	5.0828
	10	12.2063	8.1375	15.2579	25.4298	8.4766
	20	17.6465	11.7643	22.0581	36.7635	12.2545
	50	33.5985	22.3990	41.9982	69.9970	23.3323
C-S	5	8.7090	5.8060	10.8863	18.1439	6.0479
	10	17.3944	11.5962	21.7430	36.2383	12.0794
	20	23.2691	15.5127	29.0864	48.4774	16.1591
	50	25.7048	17.1365	32.1311	53.5518	17.8506
S-S	5	8.7090	5.8060	10.8863	18.1439	6.0479
	10	17.3944	11.5962	21.7430	36.2383	12.0794
	20	23.2691	15.5127	29.0864	48.4774	16.1591
	50	25.7048	17.1365	32.1311	53.5518	17.8506

Table 5 NCBL of clamped SMPC beam with different fiber orientations under various types of loading across glass transition region

Temperature	θ°	N_1^m	N_2^m	N_3^m	N_4^m	N_5^m
$T=T_g$ [$\theta^\circ/\theta^\circ/\theta^\circ/\theta^\circ$]	0°	50.9473	33.9648	63.6841	106.140	35.3800
	15°	47.9034	31.9356	59.8793	99.7988	33.2662
	30°	39.5360	26.3573	49.420	82.3666	27.4555
	45°	28.0270	18.6847	35.0338	58.3897	19.4632
	60°	16.5042	11.0028	20.6303	34.3838	11.4612
	75°	8.1156	5.4104	10.1446	16.9076	5.6358
	90°	5.0646	3.3764	6.3308	10.5513	3.5171
$T=T_g$ [$\theta^\circ/\theta^\circ/\theta^\circ/\theta^\circ$]	0°	47.6178	31.7452	59.5223	99.203	33.0679
	15°	44.9489	29.9659	56.1861	93.6436	31.2145
	30°	37.4141	24.9427	46.7676	77.9460	25.9820
	45°	26.5218	17.6812	33.1522	55.2538	18.4179
	60°	14.8993	9.9329	18.6241	31.0403	10.3467
	75°	5.9047	3.9364	7.3808	12.3014	4.1004
	90°	2.5061	1.6707	3.1326	5.2211	1.7403
$T=T_f$ [$\theta^\circ/\theta^\circ/\theta^\circ/\theta^\circ$]	0°	43.1655	28.7770	53.9569	89.9281	29.9760
	15°	40.9260	27.2840	51.1576	85.2626	28.4208
	30°	34.4312	22.9541	43.0390	71.7318	23.9106
	45°	24.5576	16.3717	30.6970	51.1617	17.0539
	60°	13.3188	8.8792	16.6485	27.7475	9.2491
	75°	4.0624	2.7082	5.0780	8.4633	2.8211
	90°	0.4262	0.2841	0.5328	0.8880	0.2960

Table 6 NCBL of SMPC beam with $a/h=20$ for various laminate orientations at four BCs under N_1^m type loading ($T = T_g$)

Ply orientation	C-C	C-H	C-S	S-S
90/0/90/0	27.4618	18.0427	13.2625	6.9988
90/0/0/90	9.28607	7.43574	4.3180	2.2231
90/45/45/90	5.80351	4.9975	2.67959	1.37342
45/0/0/45	29.4122	19.1232	14.2635	7.5482
0/0/0/0	47.6178	29.3481	24.0251	13.0714
0/15/15/0	47.2883	29.1588	23.8413	12.9646
0/30/30/0	46.3998	28.6495	23.3474	12.6780
15/30/30/15	44.0383	27.3026	22.0439	11.9255
30/45/45/30	36.1146	22.8447	17.7677	9.4959
30/15/30/15	41.2539	25.7261	20.5244	11.0554

Table 7 NCBL of clamped SMPC beam across glass transition region considering linearity and nonlinearity for different buckling mode shapes under

Temperature	W_{max}/h	Mode 1	Mode 2	Mode 3	Mode 4
$T=T_\beta$	0	15.6785	17.2363	19.7854	20.1986
	0.1	15.6566	17.2198	19.7587	20.1717
	0.2	15.5911	17.1693	19.6776	20.0897
$T=T_g$	0	11.8336	12.6093	14.0886	14.3071
	0.1	11.8116	12.5936	14.0618	14.2792
	0.2	11.7468	12.5447	13.9796	14.1940
$T=T_f$	0	8.4944	8.8446	9.6372	9.7520
	0.1	8.4725	8.8292	9.6103	9.7232
	0.2	8.4089	8.7803	9.5268	9.6351

the assumption of nonlinearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity.

The decline of $\sim 24.5\%$, 26.8% , 28.8% and 29.2% in Mode1, 2, 3 and 4 respectively during the increase of temperature from pre-glass transition condition to T_g . Moreover, from T_g to post-transition, decrease in NCBL is $\sim 28.3\%$, 30% , 31.5% and 32% are noticed in Mode1, 2, 3 and 4 respectively. The drastic drop experienced in the NCBL in the different modes of buckling is due to the volatile modulus of SMPC in the transition region, as depicted in Fig. 1. Buckling mode shapes in Mode1, 2, 3 and 4 are indicated in Fig. 6 for N_1^m , N_2^m , N_3^m , N_4^m and N_5^m types loading conditions on SMPC laminate beam clamped at T_g . The difference in the effect of loading condition along the unit length for N_2^m , N_3^m , N_4^m and N_5^m types of loading conditions are negligible, however for N_1^m type loading, buckling behavior of Mode-1 is reverse as compared to other loadings.

Some other cases of buckling such as Self buckling of SMPC beam due to its own weight or micro-buckling of the SMPC may occur under the influence of large bending strain at the elevated temperatures after the glass transition temperature of shape memory polymer [42], however in the current work the focus is on the buckling under in plane axial loading.

3.3 Vibration analysis

Effect of temperature on the vibrational behavior of the SMPC is depicted in Fig. 7 and Table 8. Sudden drop in the nondimensional frequency is observed within glass transition region from 295 to 330 K, for various loading condition. The values of nondimensional frequency are stable prior and after the glass transition region for all the

Fig. 6 Buckling mode shapes of symmetric hinged SMPC beam with $a/h=20$ for various loading conditions

Fig. 7 Non-dimensional frequency under various axial loads with respect to temperature at different BCs

Table 8 Nondimensional critical frequency of clamped SMPC beam across glass transition region considering linearity and nonlinearity for different buckling mode shapes under

Temperature	W_{max}/h	Mode 1	Mode 2	Mode 3	Mode 4
$T=T_g$	0	2.3497	5.3205	8.7824	12.0267
	0.1	2.3496	5.3202	8.7817	12.0277
	0.2	2.3492	5.3195	8.7798	12.0290
$T=T_f$	0	2.0310	4.5412	7.4495	10.4467
	0.1	2.0309	4.5409	7.4488	10.4462
	0.2	2.0304	4.5402	7.4469	10.4443
$T=T_r$	0	1.7304	3.8119	6.2109	8.6679
	0.1	1.7302	3.8116	6.2102	8.6672
	0.2	1.7297	3.8108	6.2081	8.6650

BCs, due to the constant modulus in the corresponding temperature. This has also been highlighted in results of buckling analysis discussed in the previous section. Slump in the critical frequency during the glass transition region, is pertinent in all the BCs, although it is magnified in C-F boundary condition. Decrease in critical frequency of 39.5%, 72.7%, 37.5%, 28.1% and 66.2% has been noticed for $N_1^m, N_2^m, N_3^m, N_4^m$ and N_5^m types of load distribution respectively for clamped free (C-F) BC. Moreover, the decline in the frequency before and following the transition region can be similarly observed in other BCs, presented in Fig. 1.

Influence of BCs and temperature on nondimensional frequency of SMPC under varying loads is indicated in Fig. 7, for the temperature range from 273 to 373 K. Aspect ratio (a/h) of four layered beam is considered to be 5 with fiber orientation of $[90^\circ/0^\circ/0^\circ/90^\circ]$ for C-C, C-F, C-H, C-S and S-S boundary conditions. Critical frequency is maximum in C-C boundary condition, whereas it is lowest for C-F, for all the varying loading conditions which is in coherence with the different ideal loading conditions applied on the composite beam. Ascending order of the Ascending order of nondimensional frequency, in terms of BCs is arranged as C-F, S-S, C-S, C-H and C-C. Among all the BCs, temperature can be observed to significantly regulate the vibrational behavior of SMPC beam in the glass transition region which is coherent to the results in Fig. 1.

Influence of glass transition temperature can be elaborated with the effect of temperature on different modes of vibrations, as stated in Table 8. Nondimensional frequency for four modes of vibrations are compiled in Table 8, for temperature, $T=T_g, T_f$ and T_r , with respect to linearity and non-linearity. Before the glass transition temperature, at $T=T_g$, critical frequency decreases by 0.004% and 0.02% in Mode-1, 0.005% and 0.02% in Mode-2, 0.008% and 0.03% in Mode-3 and 0.008% and 0.02% in Mode-4 with consideration of nonlinearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity. At glass transition temperature, T_g ,

critical frequency decreases by 0.005% and 0.03% in Mode-1, 0.006% and 0.02% in Mode-2, 0.009% and 0.03% in Mode-3 and 0.005% and 0.02% in Mode-4 with consideration of non-linearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity. Similarly, after glass transition, $T=T_f$, critical frequency reduces by 0.01% and 0.04% in Mode-1, 0.008% and 0.03% in Mode-2, 0.01% and 0.05% in Mode-3 and 0.008% and 0.03% in Mode-4 with consideration of nonlinearity of $W_{max}/h = 0.1$ and 0.2 respectively as compared to linearity.

Decrement of critical frequency 15.7%, 17.2%, 17.9% and 15.1% in Mode 1, 2, 3 and 4 respectively during the increase of temperature from pre-glass transition condition to T_g . Moreover, from T_g to post-transition, decrease in critical frequency is 14.8%, 16.1%, 16.6% and 17% are noticed in Mode1, 2, 3 and 4 respectively. The sudden decline observed in the critical frequency in the different modes of buckling is due to the detrimental modulus properties of SMPC in the glass transition region, as depicted in Fig. 1.

The application of frequency may lead to the alteration of inherent material properties [43] but in the current study primary focus is on the variation of material properties in the glass transition region of SMPC with the constant frequency.

4 Conclusion

Buckling and vibration behavior of SMPC beam subjected to axially heterogeneous in-plane loading conditions are analyzed in this paper through C° continuity based HSST using Von Karman non-linearity. It was noticed that temperature significantly controls the buckling and vibrations of the SMPC, particularly in the glass transition region of the matrix, because of its volatile behavior.

Based on the thermal dependence and other analysis, it was evident that boundary conditions have pronounced effects on buckling and vibration behavior of SMPC beam. Magnitude of NCBL and critical frequency are maximum for the clamped BC, whereas, it is minimum for clamped free BC, among the considered boundary conditions for all loading conditions. Among all the considered loadings, deflection of buckling and vibrations are maximum for N4m type loading condition and minimum for N2m type loading. Effect of aspect ratio was considerable during the initial increment from 5 to 20, but stabilized after 50. Aspect ratio has a uniform effect on buckling behavior under C-C, CS and S-S boundary conditions, however it is uneven under C-F, C-H and H-H. Effect of fiber orientations and laminate stacking is significant on the buckling behavior of the SMPC beam, with maximum values for 0° fiber orientation and minimum for 90° orientation. Modes of buckling and vibrations presents more clarity on the behavior of beam

under different loading conditions and temperature range in which all the modes under different loading have similar trajectory except for N1m type of loading condition.

The research presents a comprehensive study of the heterogeneous loading conditions on the buckling and vibrations of the SMPC, which would aid the designers for the efficient development of smart structures such as smart antennas, sensors and actuators. These new experimental developments may present new challenges that will motivate the researchers for novel analysis and applications.

Authors' contributions Nilesh Tiwari: Conceptualization, Drafting, Visualization, Editing; A. A. Shaikh: Review, Rewriting.

Availability of data and material Not applicable.

Code availability Not applicable.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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Appendix 1

$$[B] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & -C_4 \frac{\partial}{\partial x} & -C_2 \frac{\partial}{\partial x} & 0 & 0 & -C_4 \frac{\partial}{\partial x} & -C_2 \frac{\partial}{\partial x} & 0 \\ 0 & C_1 \frac{\partial}{\partial x} & 0 & C_1 & 0 & C_1 \frac{\partial}{\partial x} & 0 & C_1 \\ 0 & 0 & -3C_4 & -3C_4 & 0 & 0 & -3C_4 & -3C_4 \end{bmatrix} \quad (A1)$$

$$K_g^{(e)} = \frac{1}{2} \int_{A^e} B_g^{(e)T} [N_n^x] B_g^{(e)} dA \quad (A2)$$

$$D = \int_{-\frac{h}{2}}^{\frac{h}{2}} [T]^T [\bar{Q}] [T] dz \quad (A3)$$

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