Non-linear MHD convective flow of Carreau nanofluid over an exponentially stretching surface with activation energy and viscous dissipation

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Abstract

Numerical approach for a non-linear mixed convective magnetohydrodynamic two-dimensional Carreau nanofluid through an exponentially permeable stretching surface with viscous dissipation and velocity slip under the influence of Arrhenius activation energy in chemical reaction is reported. The effects of thermophoresis and Brownian motion are considered. The governing nonlinear equations of this model are transmuted into ODE's through similarity variables and solved them with a shooting method based on R-K 4th order. Responses of fluid velocity, transfer rates (heat and mass) versus pertinent parameters of the problem for suitable values are obtained and the computational calculations for friction coefficient, Nusselt number and Sherwood number for the both suction and injections regions are presented in plots and tables. It is found that fluid velocity is an increasing function of Weissenberg number. Momentum boundary layer thickness is depressed by magnetic field impact. Increasing trend in Carreau fluid temperature is noticed due to larger values of thermophoresis and Brownian motion effects. Concentration field is a decreasing function of Brownian motion but an increasing function of thermophoresis. Activation energy augments the concentration curves and lowered by Schmidt number. Comparison of the results is made with already published results and we got good agreement.

Keywords Nonlinear mixed convection · MHD · Exponential stretching · Carreau nanofluid · Activation energy

1 Introduction

The studies of boundary layer flow and transfer of heat and mass energies over stretching and/or shrinking surfaces are extensively presented by many researchers due to their vast technological applications such as design of electronic chips, preparation of metallic and plastic sheets. The basic work on those boundaries is made by Crane [1]. Later several investigations done to study the behaviors of flow and heat and mass transfer over linear, non-linear, stretching surfaces with variable thickness, exponential stretching surfaces etc. Few of such studies are mentioned here. Khedr et al. [2] presented the characteristics of MHD

flow of a micropolar fluid past a stretching permeable surface along with heat generation or absorption. An analytical investigation is made by Magyari and Chamkha [3] to discuss the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flow via a permeable stretched surface. Seyadi et al. [4] studied numerically the behavior of heat and mass transfer of MHD Eyring–Powell flow in a stretching channel along with the effects of radiation, chemical reaction and heat generation/absorption. They used a high accuracy numerical method based on the Galerkin method with Multi-wavelet basis (MWGM) to obtain the solutions of coupled differential equations. A similar investigation is

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reported to study the combined effect of heat generation or absorption and first-order chemical reaction on natural convective micropolar fluid flow past a uniformly stretching permeable surface by Damseh et al. [5]. Takhar and Chamkha [6] studied the unsteady laminar boundary-layer flow of a viscous electrically conducting motion of a fluid over an impulsive stretching of a flat surface numerically by using implicit finite-difference scheme. Swati Mukhopadhyay [7] discussed the velocity and thermal slip effects on MHD boundary layer flow towards a porous exponentially stretching sheet with suction/blowing and thermal radiation. Hsiao [8, 9] discussed the behavior of nanofluids over stretching surfaces with different physical aspects.

In recent years, developing tendency in nanotechnology and nanoscience due to their importance in the fields of science and engineering that reveals synthesis and progressions of distinct nanomaterials. Enormous studies on nanomaterials and nanofluids have been carried by many scientists and engineers. Nanofluids are attracted by several researchers due to their absorbing and thermal transport behaviors in distinct fields. As nanofluids have high thermal conductivity, they got constancy to prevent rapid settling and obstacle besides the boundaries of heat transmission regions. Involvement of nanofluids happen in micro-manufacturing processes, cells of fuel, cooling processes, chips of microprocessor designs and their usage, nuclear industries, the apparatus of chemical and mechanical fields and in several designs of devices etc. The introduction of nanoparticles is made through Choi [10]. He depicted that thermal properties of fluid is improved by the involvement of nanoparticles. Main advantage noted in the usage of nanoparticles is that they able to work to gain specific determinations like the decline of friction, the heat transfer coefficient and thermal energy storage. The convective transport phenomena for nanofluids is discussed by Buongiorno [11] through a mathematical model in which nanofluid flow comprising of Brownian motion and thermophoretic dispersion of the nanoparticles is presented. In literature so much of noteworthy works are found on nanofluids by the engineers and scientists. Daniel et al. [12] studied the thermophoretic and Brownian motion effects of stratified electro magnetohydrodynamic flow under convective heat and mass conditions. Hayat et al. [13] discussed the characteristics of Carreau fluid flow and the heat transfer over a permeable stretched sheet using Homotopy method. Reddy and Sandeep [14] have jointly worked on the Carreau liquid flow and transfer of heat with cross diffusion effects via a permeable stretchable surface along with convective slip boundaries and thermal radiation. With the help of Cattaneo-Christov heat flux modelling, Ramadevi et al. [15] reported the magnetohydrodynamic flow corresponding to Carreau fluid over a melting surface. They discussed the results

for Newtonian fluid and non-Newtonian fluid behaviors in their study. Two-dimensional magnetohydrodynamic flow of Carreau liquid through a variable stretched sheet is an investigation of Khan et al. [16]. Farroq et al. [17] also obtained the local similarity solutions for Carreau stagnation point flow past a non-linear stretched surface along with double stratification effects. A numerical work also due to homogeneous-heterogeneous reactions of Carreau fluid flow through stretchable cylinder has been discussed by Khan et al. [18]. Eid et al. [19] reported a numerical work on Carreau nanofluid flow over the permeable nonlinear stretchable surface. A numerical analysis due to Arrhenius activation energy along with binary chemical reaction for a magnetohydrodynamic Carreau fluid flow under Cattaneo-Christov heat flux model is a study of Kumar et al. [20]. Sulochana et al. [21] reported a Stagnation-point Carreau nanofluid flow past a permeable surface with transpiration effect, thermophoresis and Brownian motion. A non-Darcy magnetohydrodynamic Carreau fluid flow over permeable stretching surface is another similar investigation by Kala et al. [22]. Mamatha et al. [23] depicted the impact of convective boundary conditions applied for Carreau magnetohydrodynamic dusty fluid under stretching surface with a heat source. Waqas et al. [24] discussed numerically the effects of Brownian motion and thermophoresis on magnetohydrodynamic stretchable flow of convected Carreau nanoliquid over an exponentially stretching surface. Khan et al. [25] presented the unsteady heat and mass transfer mechanisms using Buongiorno's model in a MHD Carreau nanofluid flow past a permeable stretching surface. Rama Subba Reddy Gorla and Ali Chamkha [26] analyzed the natural convection via a non-isothermal vertical plate in a porous medium saturated by a nanofluid with thermophoresis and Brownian motion effects. Chamkha and Aly [27] discussed the magnetohydrodynamic free convective flow of a nanofluid over a vertical plate with heat generation / absorption effects.

Chemical reaction effects on steady/unsteady motion of fluids through porous medium along with behaviors of physical aspects discussed by many authors. A few of them are presented here. Chamkha et al. [28] presented the effects of chemical reaction on unsteady heat and mass transfer via a permeable stretching surface. They have shown the similarity solutions for the both suction/injection cases. Hsiao [29] has been reported an article to study the radiative electrical MHD activation energy thermal extrusion manufacturing system efficiency with the help of Carreau-Nanofluid and parameters control method. Arrhenius activation energy in binary chemical reactions in the studies of non-Newtonian fluids is also found in literature with different physical effects. Irfan et al. [30] examined the three dimensional Carreau nanofluid along with the effects of nonlinear mixed convection

and thermal radiation influenced by Arrhenius activation energy in chemical reaction. Irfan et al. [31] also discussed the dual solutions of a magnetohydrodynamic nonlinear radiative time dependent Carreau fluid along with the influence of activation energy in binary chemical reaction. Recently Alsaadi et al. [32] reported on Arrhenius activation energy and the effects of nonlinear radiative MHD Williamson fluid flow through a surface which is convectively heated.

In the present article, a novel modeling for a steady two-dimensional nonlinear convective flow, heat and mass transfer of Carreau nanofluid over an exponentially stretching permeable surface with viscous dissipation, applied magnetic field, Arrhenius activation energy involved in binary chemical reaction, convective boundary

2 Mathematical formulation

A steady two-dimensional magnetohydrodynamic boundary layer flow of an incompressible Carreau nanofluid over an exponentially stretching permeable surface with convective heat and mass conditions are considered. The effects of Brownian motion and thermophoresis are taken into consideration. Temperature, Concentration distributions and applied magnetic field are treated to be functions of exponential variation. Here x-axis is parallel to the surface and the y-axis is perpendicular to it (see Fig. 1).

The equations which are pertaining to the problem are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \left[1 + \Gamma^2 \left(\frac{\partial u}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} + \nu(n-1) \Gamma^2 \frac{\partial^2 u}{\partial y^2} \left(\frac{\partial u}{\partial y} \right)^2 \left[1 + \Gamma^2 \left(\frac{\partial u}{\partial y} \right)^2 \right]^{\frac{n-3}{2}} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) + g\beta_{1T}(T - T_\infty)^2 + g\beta_{1C}(C - C_\infty)^2 - \frac{\sigma B^2(x)}{\rho} u \quad (2)$$

conditions is studied numerically and the outcomes are reported. Effects of Brownian motion and thermophoresis are taken into consideration. Authors strongly believe that no such modeling was attempted earlier. Governing equations of the present model are converted into a set of ordinary differential equations using similarity transformation and then solved by shooting technique based on R-K 4th order. The responses of fluid velocity, both temperature and concentration domains for different parameters of the problem are presented through plots. Also, the values of Skin friction, Nusselt number and Sherwood number are displayed in tables. Comparison of the present results with the published results is discussed.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^m \exp \left(-\frac{E_a}{k^* T} \right) \quad (4)$$

and accompanying conditions are

$$\begin{aligned} u &= U_w(x) + N(x)v \frac{\partial u}{\partial y}, v = -V_w(x), T = T_w(x), C = C_w(x), \\ -k \frac{\partial T}{\partial y} &= h_1(T_0 - T), -D_B \frac{\partial C}{\partial y} = h_2(C_0 - C) \quad \text{at } y = 0 \\ u = 0, T &\rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (5)$$

Fig. 1 Schematic diagram for flow pattern

Here h_1, h_2 are taken to be convective heat transfer and mass transfer coefficients respectively, k denotes thermal conductivity, $N(x) = N_1 e^{-x/2L}$ is denoted as velocity slip factor and N_1 is the initial value of velocity slip factor. Here $V_w(x) = V_0 e^{x/2L}$ is a velocity at the wall, where V_0 is the initial strength of suction. Here $V_w(x) > 0$ indicates suction $V_w(x) < 0$ indicates injection (see reference [7]).

The stretching velocity U_w , the temperature and concentration distributions at the wall, applied magnetic field are described below.

$$U_w(x) = U_0 e^{x/L}; T_w(x) = T_\infty + (T_0 - T_\infty) e^{(2x/L)}; C_w(x) = C_\infty + (C_0 - C_\infty) e^{(2x/L)}, B(x) = B_0 e^{x/2L}$$

The similarity variables are

$$\eta = y \sqrt{\frac{U_0}{2Lv}} e^{x/2L}, \psi(x, y) = \sqrt{2LU_0 v} e^{x/2L} f(\eta) \\ T(x, y) = T_\infty + (T_0 - T_\infty) e^{(2x/L)} \theta(\eta), C(x, y) = C_\infty + (C_0 - C_\infty) e^{(2x/L)} \phi(\eta) \quad (6)$$

Through the stream function ψ , we defined $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Using (6) in the Eqs. (2)–(4), we got

$$(1 + We^2 f'^2)^{\frac{n-3}{2}} (1 + nWe^2 f'^2) f'''' + ff'' - 2f'^2 + 2\lambda e^{2X} [\theta(1 + \gamma_t \theta e^{2X}) + M^* \phi(1 + \gamma_c \phi e^{2X})] - 2Mf' = 0 \quad (7)$$

$$\theta'' + Pr(f\theta' - 4f'\theta) + Ec.Pr.f'^2 + Pr.e^{2X} [N_t \theta'^2 + N_b \theta' \phi'] = 0 \quad (8)$$

$$\phi'' + Sc(f\phi' - 4f'\phi) + \left(\frac{N_t}{N_b}\right) \theta'' - Sc\Lambda e^{-X} \phi(1 + \Lambda^* e^{2X} \theta)^m \exp\left[\frac{-E}{1 + \Lambda^* e^{2X} \theta}\right] = 0 \quad (9)$$

Also the reduced conditions are

$$f = S, f' = 1 + Pf'', \theta = \left(\frac{1}{Bi_T}\right) \theta' + e^{-2X}, \\ \phi = \left(\frac{1}{Bi_C}\right) \phi' + e^{-2X} \quad \text{at } \eta = 0 \\ f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (10)$$

The parameters used in this problem are listed below:

$X = x/L$ is dimensionless coordinate for the plate; $P = N_1 \sqrt{\frac{U_0 v}{2L}}$ is velocity slip parameter; $S = V_0 \sqrt{\frac{2L}{U_0 v}}$ is the suction/injection parameter; $We = \sqrt{\frac{\Gamma^2 U_0^3 e^{(3X/L)}}{2vL}}$ is the local

Weissenberg number; $\lambda = \frac{Gr_T}{Re^2} = \frac{g\beta_T(T_w - T_\infty)L^3}{\nu^2}$ is mixed convection parameter; $M^* = \frac{Gr_C}{Gr_T}$ is a ratio of the concentration to the thermal buoyancy forces; $\gamma_t = \frac{\beta_{1T}(T_w - T_\infty)}{\beta_T}$ is the nonlinear convection parameter caused by temperature; $\gamma_c = \frac{\beta_{1C}(C_w - C_\infty)}{\beta_C}$ is the nonlinear convection parameter caused by concentration; $N_b = \frac{\tau_{D_B}(C_w - C_\infty)}{\nu}$ is a parameter due to Brownian motion; $N_t = \frac{\tau_{D_T}(T_w - T_\infty)}{\nu T_\infty}$ is a parameter due to thermophoresis; $Pr = \nu/\alpha$ is Prandtl number; $Ec = \frac{U_0^2}{c_p(T_0 - T_\infty)}$ is Eckert number; $Sc = \frac{\nu}{D_B}$ is the Schmidt number; $\Lambda = \frac{2LK^2}{U_0}$ is chemical reaction rate; $\Lambda^* = \frac{T_0 - T_\infty}{T_\infty}$ is temperature ratio parameter; $E = \frac{E_a}{k^* T_\infty}$ is the non-dimensional Arrhenius chemical energy; $M = \frac{Ha^2}{Re} = \frac{\sigma B_0^2 L}{\rho U_0}$ is magnetic field parameter; $Bi_T = \frac{h_1 y}{\eta k}$, $Bi_C = \frac{h_2 y}{\eta k}$ are the Biot numbers. Here $Ha = \sqrt{\sigma B_0^2 L^2 / \rho \nu}$ is the Hartmann number; $Re = \frac{U_0 L}{\nu}$ is Reynolds number. Also 'n' is the power-law index and 'm' is the fitted rate constant.

The physical parameters of engineering interest, friction factor C_{fx} , local Nusselt and Sherwood numbers i.e., Nu_x and Sh_x are specified as

$$C_{fx} = \frac{\tau_w}{\rho U_w^2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xj_w}{D_B(C_w - C_\infty)} \quad (11)$$

Here τ_w , q_w and j_w stands for wall shear stress, heat and mass flux correspondingly, which are specified below.

$$\tau_w = \mu \frac{\partial u}{\partial y} \left[1 + \Gamma^2 \left(\frac{\partial u}{\partial y} \right)^2 \right]^{n-1/2} \bigg|_{y=0}, \\ q_w = -k \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=0}, j_w = -D_B \left(\frac{\partial C}{\partial y} \right) \bigg|_{y=0} \quad (12)$$

In non-dimensional form they prescribed as

$$\sqrt{Re_x} C_{fx} = \sqrt{2f''(0)} [1 + We(f''(0))^2]^{(n-1)/2}, \\ \frac{Nu_x}{\sqrt{X/2}\sqrt{Re_x}} = -\theta'(0), \frac{Sh_x}{\sqrt{X/2}\sqrt{Re_x}} = -\phi'(0) \quad (13)$$

Local Reynold's number, $Re_x = \frac{xU_w(x)}{\nu}$.

3 Numerical approach

Numerical Solutions for converted system of Eqs. (7)–(9) along with conditions (10) are achieved through shooting technique based on R–K 4th order procedure. For this process, equations are altered into a set of first order ordinary differential equations using the meaningful replacements as mentioned below:

$$f_1 = f,$$

$$f_2 = \frac{df_1}{d\eta} \text{ or } \frac{df}{d\eta},$$

$$f_3 = \frac{df_2}{d\eta} \text{ or } \frac{d^2f}{d\eta^2},$$

$$f_4 = \theta(\eta),$$

$$f_5 = \frac{df_4}{d\eta} \text{ or } \frac{d\theta}{d\eta},$$

$$f_6 = \phi(\eta),$$

$$f_7 = \frac{df_6}{d\eta} \text{ or } \frac{d\phi}{d\eta}.$$

Making use of above, we write

$$\frac{df_1}{d\eta} = f_2,$$

$$\frac{df_2}{d\eta} = f_3,$$

$$\frac{df_3}{d\eta} = \frac{2(f')^2 - ff'' - 2\lambda e^{2X}[\theta(1 + \gamma_t e^{2X}\theta) + M^*\phi(1 + \gamma_c e^{2X}\phi)] + 2Mf'}{(1 + We^2(f'')^2)^{(n-3)/2}(1 + nWe^2(f'')^2)},$$

$$\frac{df_4}{d\eta} = f_5,$$

$$\frac{df_5}{d\eta} = Pr(4f'\theta - f\theta') - Ec \cdot Pr \cdot (f'')^2 - Pre^{2X}[N_t\theta'^2 + N_b\theta'\phi'],$$

$$\frac{df_6}{d\eta} = f_7,$$

$$\frac{df_7}{d\eta} = Sc(4f'\phi - f\phi') - \left(\frac{N_t}{N_b}\right)\theta'' + Sc \cdot \Lambda \cdot e^{-X}\phi(1 + \Lambda^*e^{2X}\theta)^m \exp\left[\frac{-E}{1 + \Lambda^*e^{2X}\theta}\right],$$

The corresponding initial conditions will be

$$f_1(0) = S, f_2(0) = 1 + Pf_3(0), f_3(0) = k_1, f_4(0) = k_2, \\ f_5(0) = Bi_T(k_2 - e^{-2X}) f_6(0) = k_3 f_7(0) = Bi_C(k_3 - e^{-2X})$$

Table 1 Numerical values of skin friction for various values of We, M, λ, n and S

We	M	λ	n	C_{fx}	
				$S=0.5$	$S=-0.5$
0.2	1.0	1.0	4.0	-2.553236	-1.764808
0.5	–	–	–	-1.662398	-1.328209
0.7	–	–	–	-1.391724	-1.156398
0.5	1.0	1.0	4.0	-1.662867	-1.328476
–	2.0	–	–	-1.891976	-1.641021
–	3.0	–	–	-2.057223	-1.851347
0.5	1.0	1.0	4.0	-1.662867	-1.328476
–	–	1.5	–	-1.533758	-1.118731
–	–	2.0	–	-1.386599	-0.847425
0.5	1.0	1.0	2.0	-2.909279	-1.817405
–	–	–	3.0	-1.999158	-1.498077
–	–	–	4.0	-1.662398	-1.328209

For the responses of the system, we employed a shooting technique depending on RK 4th order method. For one set of values of the parameters, values of k_1, k_2 and k_3 are assumed and the system of first order ordi-

nary differential equations is solved as an initial value problem from $\eta = 0$ to $\eta = \eta_\infty$ (η_∞ is a large value of η and η_∞ : 4–12) using RK method. The values of f', θ and ϕ at η_∞ are compared with their expected values and

accordingly k_1 , k_2 and k_3 are modified and the system of equations again solved as an initial value problem. The procedure is repeated a number of times till the values of f' , θ and ϕ at η_∞ are very close to the expected values. By the above mentioned procedure we can get values of k_1 , k_2 and k_3 for one set of values of the parameters of the study. Similarly we determined the values of k_1 , k_2 and

k_3 for all possible values of the parameters of the study. Using the initial conditions, the equations are solved to get the velocity, temperature and concentration distributions and also the quantities like skin friction, heat and mass transfer rates. During this process, the step size and convergence criteria are maintained as 0.001 and 10^{-6} in all cases.

Table 2 Numerical values of Nusselt number for various values of M , λ , Bi_T , Ec , N_t , N_b and S

M	λ	Bi_T	Ec	N_t	N_b	Nu_x	
						$S=0.5$	$S=-0.5$
1.0	1.0	2.0	0.002	0.1	0.1	0.332810	0.297811
2.0	–	–	–	–	–	0.327676	0.289923
3.0	–	–	–	–	–	0.320621	0.283866
1.0	1.0	2.0	0.002	0.1	0.1	0.332810	0.297811
–	1.5	–	–	–	–	0.336314	0.301954
–	2.0	–	–	–	–	0.341251	0.307400
1.0	1.0	0.3	0.002	0.1	0.1	0.092932	0.089648
–	–	0.6	–	–	–	0.161132	0.151918
–	–	0.9	–	–	–	0.213482	0.197984
1.0	1.0	2.0	0.02	0.1	0.1	0.327230	0.294277
–	–	–	0.03	–	–	0.324192	0.292350
–	–	–	0.04	–	–	0.321197	0.290450
1.0	1.0	2.0	0.002	0.1	0.1	0.288544	0.261534
–	–	–	–	0.2	–	0.285902	0.259746
–	–	–	–	0.3	–	0.282513	0.257759
1.0	1.0	2.0	0.002	0.1	0.1	0.225668	0.208063
–	–	–	–	–	0.2	0.223436	0.205943
–	–	–	–	–	0.3	0.220393	0.203077

Table 3 Numerical values of Sherwood number for various values of Sc , E , Bi_C , N_t , N_b and S

Sc	E	Bi_C	N_t	N_b	Sh_x	
					$S=0.5$	$S=-0.5$
0.2	1.0	2.0	0.1	0.1	0.078805	0.100919
0.3	–	–	–	–	0.108272	0.125119
0.4	–	–	–	–	0.133868	0.146053
0.2	0.2	2.0	0.1	0.1	0.034257	0.062879
–	0.4	–	–	–	0.025694	0.055891
–	0.6	–	–	–	0.017647	0.049443
0.2	1.0	0.3	0.1	0.1	0.042391	0.045222
–	–	0.6	–	–	0.064985	0.068882
–	–	0.9	–	–	0.078973	0.083399
0.2	1.0	2.0	0.05	0.1	0.127499	0.133026
–	–	–	0.07	–	0.099310	0.109764
–	–	–	0.09	–	0.071007	0.086473
0.2	1.0	2.0	0.1	0.1	0.171196	0.169345
–	–	–	–	0.2	0.182362	0.178659
–	–	–	–	0.3	0.189058	0.184275

Table 4 Variations in $f''(0)$ for different values of M when $n = We = S = \lambda = 0$

M	$f''(0)$	$f''(0)$
	Ref. [24]	Present work
0.0	-1.281828	-1.281832
0.2	-1.313294	-1.313286
0.4	-1.403028	-1.403045

Table 5 Variations in $-\theta'(0)$ for different values of Pr when $We = S = M = N_t = N_b = Ec = X = 0$ and $n = 1$

Pr	$-\theta'(0)$	$-\theta'(0)$
	Ref. [25]	Present work
0.72	1.088915	1.088920
1.00	1.333333	1.333328
3.00	2.509698	2.509689
10.00	4.796853	4.796845

Fig. 2 Effect of Weissenberg number on velocity

4 Results and discussion

After employing shooting method (based on RK 4th order) on the converted system of Eqs. (7)–(9) including the conditions prescribed in (10), the responses of fluid velocity, temperature and concentration domains for distinct important physical parameters in both suction ($S > 0$) and injection ($S < 0$) cases are deliberated through plots. Contour plots and streamline plots are also displayed. The friction coefficient, Nusselt number and Sherwood number computational values are presented in Tables 1, 2 and 3 for suction and injection regions. Validity of the present study (Tables 4 and 5) is discussed at the end of the discussion

Fig. 3 Effect of magnetic parameter on velocity **Fig. 4** Effect of mixed convection on velocity **Fig. 5** Effect of power-law index on velocity

a **b** **c**

Fig. 6 Contour plot and stream lines (for values of f). **a** [$Pr = 0.7$], **b** $\lambda = M = E = \Lambda = \Lambda^* = 1$, $N_t = N_b = 0.1$, $S = 0.5$, $Sc = 0.2$, $X = 1$. **c** $We = \gamma_t = \gamma_c = 0.5$, $n = 4$, $m = 2$, $Ec = 0.002$, $P = 0.1$, $Bi_T = Bi_C = 2$, $M^* = 0.1$]

of the results. The results are calculated for a particular set of values of the parameters which are mentioned in figures. If $n > 1$, it indicates the shear thickening case and if $0 < n < 1$, it indicates the shear thinning case. Results are presented for shear thickening case ($n = 4$) in the presence of suction/injection.

4.1 Velocity profile

The consequences in fluid velocity with the parameters We , M , λ and n for both suction and injection regions are displayed in Figs. 2, 3, 4 and 5. Figure 2 reveals the influence of Weissenberg number (We) on velocity. Increased values of We heightens the profiles of velocity in shear thickening fluids. As per the definition of Weissenberg number it is the ratio of the relaxation time of the fluid and a specific process time and in the case of shear thickening fluids ($n > 1$), due to thickness of the fluid is diminished, so that the fluid velocity is slightly enhanced. The magnetic field impact on fluid velocity is shown in Fig. 3. It reveals that the larger values of magnetic parameter depreciate the velocity. Also the magnetic parameter depresses the momentum boundary layer thickness. Since the magnetic parameter is the ratio of electromagnetic force to the viscous force and so the increased magnetic effect leads to occurrence of Lorentz forces, as a result motion of the fluid gets obstructed and hence velocity is decreased. Variations in velocity of the fluid due to mixed convection parameter shown in Fig. 4. The enhancement in mixed convection leads to the intensification of fluid velocity. This happens due to larger values of mixed convection contributes enrichment of buoyancy forces, as a result velocity increased. Figure 5 is plotted to show the effect of n (power law index) on velocity of the fluid. It was noticed that, the velocity drops almost exponentially with fixed values of n . But varying ' n ', the velocity profiles are seen to be heightened with the same rate. Since the non-Newtonian aspects decreases with a rise in n and as a result the velocity is enhanced. Contour plots and Streamlines are displayed in Fig. 6a–c to observe the flow pattern.

4.2 Temperature profile

Temperature variations corresponding to the parameters M , λ , Ec , Bi_T , N_t and N_b are presented through the Figs. 7, 8, 9, 11 and 12. Mixed convection parameter versus temperature profile is shown in Fig. 7. Enhanced mixed convection

causes decrement in temperature. When we increase the mixed convection parameter, the difference between temperature at the wall and free stream temperature also intensified. So temperature decreases. Figure 8 reveals the change in temperature due to magnetic effect. In this case also thermal boundary layer thickness is intensified with an enhancement in magnetic parameter. So that increment in M causes the improvement in temperature profiles. Eckert number (Ec) versus temperature of the fluid is exhibited in Fig. 9. As ' Ec ' generates heat inside the flow region, fluid temperature gets elevated. In Fig. 10, the variations in temperature due to thermal Biot number (Bi_T) are shown. In physical point of view, an upgrade in Bi_T , makes more grounded convection which prompts the elevated temperature. So that bigger values of Bi_T causes an increment in thermal boundary layer as result, the profiles of temperature are heightened. The impact of thermophoresis parameter (N_t) on the dimensionless temperature in shear thickening case is displayed in Fig. 11. The enhancing trend in temperature is noticed with specified values of N_t . As, thermophoretic force is affected by the temperature gradient, the heated particles dragged away from hot to cold surface which causes raise in temperature field. In this case, vast amount of nanoparticles is transmuted from the intense surface which intensifies the temperature of Carreau fluid. The variations of temperature in response to a change in Brownian motion parameter (N_b) is displayed in Fig. 12. Temperature enhances by uplifting the Brownian motion parameter. Moreover, thermal boundary layer thickness is a rising function of the Brownian motion parameter in shear thickening ($n > 1$) fluid. As per the definition of Brownian motion, increment in N_b , causes the larger kinetic energy of the nanoparticles and as result nanofluid temperature is rises.

4.3 Concentration profile

The changes in concentration profile with the parameters Sc , E , Bi_C , N_t and N_b are exhibited in Figs. 13, 14, 15, 16 and 17. Figure 13 displays the concentration field for the changing values of Schmidt number (Sc). From this figure, we see that the concentration of the fluid is decreased with bigger values of Sc . The rate of decrease in concentration is larger for high values of Sc . Since the larger values of Schmidt number are equivalent to smaller mass diffusivity. So the reducing trend in concentration profiles is observed. Figure 14 is drafted to scrutinize the consequence of shear thickening liquids for activation energy parameter (E) on concentration field. Enhancing values of ' E ' causes a decline in the field of concentration. As a physical aspect, advanced values of ' E ' falling-off the modified

Fig. 7 Effect of magnetic parameter on temperature

Fig. 9 Effect of Eckert number on temperature

Fig. 8 Effect of mixed convection on temperature

Fig. 10 Effect of thermal Biot number on temperature

Arrhenius function which stimulates the generative chemical reaction quantity. So that, the concentration of Carreau liquid is augmented. The behavior of concentration profiles with mass transfer Biot number (Bi_C) is displayed in Fig. 15. In both suction and injection regions, we noticed that the rising values of this Biot number, intensifies the concentration of fluid. As per the definition of Biot number, through convection the transferred mass will be distributed over the surface. So that nanofluid concentration profiles are intensified. Figures 16 and 17 are portrayed to know the impacts of both the parameters N_t and N_b on concentration field. For both the situations ($S > 1$ and $S < 1$), the concentration of Carreau nanofluid far away from the wall boost up for growing values of N_t (Fig. 16). The variation of concentration domain due to N_b is visualized in Fig. 17. We

observed from this figure that far away from the plate, concentration is diminished with a rising values of N_b in both suction and injection regions. Physically, augmentation in N_b , the collision of macroscopic particles of a liquid intensifies due to the instance that N_b reduces the concentration field far away from the wall.

Tables 1, 2 and 3 shows the computational values for friction coefficient, Nusselt number and Sherwood number for different values of parameters in both suction and injection regions. Growth in skin friction coefficient is noticed due to increment in local Weissenberg number. High magnetic effect decreases the friction coefficient and heat transfer rate. Enhanced mixed convection elevates the friction coefficient and transfer of heat. Skin friction is also intensified by high power-law index values. Increased

Fig. 11 Effect of thermophoresis on temperature

Fig. 13 Effect of schmidt number on concentration

Fig. 12 Effect of Brownian motion on temperature

Fig. 14 Effect of activation energy on concentration

Brownian motion and thermophoresis elevates the heat transfer rate. Also Nusselt number is lowered by an increment in Eckert number but it is elevated by enhanced mixed convection and thermal Biot number. Sherwood number is lowered by a rise in thermophoresis effect and enhanced by improved Brownian motion. Larger values of Schmidt number and concentration Biot number results the growth in Sherwood number and it seen to be diminished by bigger values of activation energy parameter.

4.4 Validity of the result

Comparison of the numerical computations in this study with the published results in references [24, 25] is

displayed through the tables as presented below. Table 4 describes the comparison in skin friction coefficient for varying values of the magnetic parameter ' M ' and the values of other parameters are taken to be zero. The comparison in Nusselt number for various values of Prandtl number ' Pr ' for ($n = 1$) and in the absence of other parameters is displayed in Table 5. These Computations are in good agreement with published results.

5 Conclusions

Numerical investigation on chemically reacting two-dimensional steady incompressible and electrically conducting Carreau nanofluid through an exponentially

Fig. 15 Effect of mass transfer Biot number on concentration

Fig. 17 Effect of Brownian motion on concentration

Fig. 16 Effect of thermophoresis on concentration

stretching permeable surface under chemical reaction with activation energy, viscous dissipation, slip velocity, nonlinear mixed convection has been carried out. Shooting method based on RK-method is used to solve the reduced system of equations with corresponding boundary conditions. Emphasis is on theoretical analysis only. This type of modeling is not found in literature. The model may be suitable for different fluids also. In general, if $n > 1$ it is shear thickening case and if $0 < n < 1$, it is shear thinning case. Solutions are obtained for both suction and injection regions for the case ($n = 4$) for distinct values of

other parameters. For shear thinning case, solutions do not exist.

From this analysis, it can be concluded that the fluid velocity is an increasing function of local Weissenberg number. Momentum boundary layer thickness is depressed by magnetic field impact. Increasing trend in Carreau fluid temperature is noticed due to larger values of thermophoresis and Brownian motion effects. Concentration field is a decreasing function of Brownian motion but an increasing function of thermophoresis. Activation energy augments the concentration profile but Schmidt number lowers the same. Nusselt number is elevated by Brownian motion and thermophoresis effects. Heat transfer rate is lowered by an increment in Eckert number but it is elevated by enhanced mixed convection and thermal Biot number. Sherwood number is lowered by a rise in thermophoresis effect and enhanced by improved Brownian motion effect.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no “conflict of interest”.

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