Simulation approach on turbulent thermal performance factor of Al_2O_3 -Cu/water hybrid nanofluid in circular and non-circular ducts

Ing Jiat Kendrick Wong¹ · Ngieng Tze Angnes Tiong¹ Received: 20 July 2020 / Accepted: 1 February 2021 / Published online: 15 February 2021
© The Author(s) 2021

Abstract

This paper presents the numerical study of thermal performance factor of Al_2O_3 -Cu/water hybrid nanofluid in circular and non-circular ducts (square and rectangular). Turbulent regime is studied with the Reynolds number ranges from 10000 to 100000. The heat transfer performance and flow behaviour of hybrid nanofluid are investigated, considering the nanofluid volume concentration between 0.1 and 2%. The thermal performance factor of hybrid nanofluid is evaluated in terms of performance evaluation criteria (PEC). This present numerical results are successfully validated with the data from the literature. The results indicate that the heat transfer coefficient and Nusselt number of Al_2O_3 -Cu/water hybrid nanofluid are higher than those of Al_2O_3 /water nanofluid and pure water. However, this heat transfer enhancement is achieved at the expense of an increased pressure drop. The heat transfer coefficient of 2% hybrid nanofluid is approximately 58.6% larger than the value of pure water at the Reynolds number of 10000. For the same concentration and Reynolds number, the pressure drop of hybrid nanofluid is 4.79 times higher than the pressure drop of water. The heat transfer performance is the best in the circular pipe compared to the non-circular ducts, but its pressure drop increment is also the largest. The hybrid nanofluid helps to improve the problem of low heat transfer characteristic in the non-circular ducts. In overall, the hybrid nanofluid flow in circular and non-circular ducts are reported to possess better thermal performance factor than that of water. The maximum attainable PEC is obtained by 2% hybrid nanofluid in the square duct at the Reynolds Number of 60000. This study can help to determine which geometry is efficient for the heat transfer application of hybrid nanofluid.

Keywords Hybrid nanofluid · Heat transfer · Pressure drop · Performance evaluation criteria

Abbreviations

CFD	Computational fluid dynamic
PEC	Performance evaluation criteria
SIMPLE	Semi Implicit Method for Pressure Linked Equations

improving through progressive research. Extensive efforts are made for heat transfer enhancement purposes, but the performance is limited by the poor thermal conductivity of conventional heat transfer fluids, such as water, oil and ethylene glycol [1]. The advancement in material engineering has made the creation of nano-sized particles (average particle size less than 100 nm) possible, leading to the development of a new class of heat transfer fluid. The term “nanofluid” was introduced by Choi and Eastman [2] to represent this heat transfer fluid which contains nano-sized metallic or non-metallic particles. The most commonly used

1 Introduction

The development of industrial activities in the past few decades has created a substantial demand of energy sources. Heat transfer technologies are constantly

✉ Ngieng Tze Angnes Tiong, angnesnt@curtin.edu.my | ¹Department of Chemical Engineering, Curtin Malaysia, CDT 250, 98000 Miri, Sarawak, Malaysia.

nanoparticle includes oxide ceramic (Al_2O_3 , CuO), metal carbides (SiC), nitrides (AlN , SiN), metal (Al , Cu), carbon in different form (diamond, fullerene), and non-metal (graphite) [3]. Nanofluid is found to enhance the poor thermal properties of conventional heat transfer fluids. Dispersing nanoparticle into fluid can greatly augment the thermal conductivity of the base fluid, which subsequently increases the convective heat transfer rate [4, 5]. This heat transfer ability is also found to improve with the increase of nanoparticle loading. It is postulated that the increment of particle loading limits the mean free path between particles within the fluid. This causes the nanoparticles to collide more aggressively, forming compactly packed thermal interfaces and chain between particles in fluid, thus improving the thermal conductivity. Moreover, with the decrement of the mean free path, the Brownian motion of particles is amplified, and hence the heat transfer rate is augmented [6, 7].

Although the introduction of nanofluid is proven to increase the heat transfer rate of the base fluid by raising its thermal properties and delay the development of thermal boundary, the heat transfer augmentation is still considered low. Therefore, new studies carried out are focusing on hybrid nanofluid. Hybrid nanofluid is synthesized by dispersing two or more different nanoparticles into the base fluid. Most of the research claims that hybrid nanofluid possesses better thermal properties compared to nanofluid with single nanoparticles [8, 9]. Hameed et al. [8] had compared the laminar heat transfer and flow characteristics of three different nanofluids, which one of it was Al_2O_3 /water nanofluid, while the other two were hybrids, namely Al_2O_3 - Cu /water and Al_2O_3 -CNT/water nanofluids. From their experiments, it was revealed that the heat transfer enhancement of Al_2O_3 -CNT hybrid nanofluid was the greatest, followed by Al_2O_3 - Cu /water and Al_2O_3 /water nanofluids. They also remarked an average increment of Nusselt number from 13.66 to 26.62% when the nanoparticle loading of Al_2O_3 -CNT/water nanofluid was increased from 0.1 to 0.3 vol.%.

Furthermore, Takabi and Shokouhmand [9] investigated the heat transfer characteristic of Al_2O_3 /water nanofluid and Al_2O_3 - CuO /water hybrid nanofluid with the Reynolds number ranged from 10000 to 100000. 1% Al_2O_3 - CuO /water hybrid nanofluid was reported to increase the Nusselt number by 32.07% when compared to that of water while Al_2O_3 /water mono nanofluid with the same volume concentration increased the Nusselt number by merely 13.68%. The heat transfer potential of hybrid nanofluid was higher than water and mono nanofluid due to the attribution of hybrid nano-composite, which possessed high crystalline nature and surface functionalized, thus allowing the development of effective thermal interfaces within the fluid medium [6].

Moreover, in terms of flow behaviour, the mono nanofluid and hybrid nanofluid could increase the friction factor and pressure drop of the host fluid. These friction factor and pressure drop increment of nanofluid were attributed to the increased effective viscosity of the fluid [10, 11]. When the volume concentration of nanofluid was raised, the viscosity increased and caused the pressure drop to increase, which unfortunately increased the penalty of pumping power [10, 12, 13]. Moghadassi et al. [10] studied the laminar flow behavior of 0.1% Al_2O_3 /water mono nanofluid and Al_2O_3 - Cu /water hybrid nanofluid. The friction factor of 0.1% Al_2O_3 - Cu /water hybrid nanofluid was discovered to be 6.93% and 15.53% higher than those of Al_2O_3 /water mono nanofluid and water, respectively.

Research of hybrid nanofluid has been carried out extensively in circular duct [8–10, 12–19]. Labib et al. [12] performed a numerical study on the effect of nanoparticle loading and Reynolds number on hybrid nanofluid consisting of Al_2O_3 nanoparticles and carbon nanotubes (CNTs). 22.8% augmentation in heat transfer coefficient was reported for (0.05 vol% CNTs + 1.6 vol% Al_2O_3)-water hybrid nanofluid in comparison to 0.05 vol% CNTs-water nanofluid. Suresh et al. [13] examined experimentally the characteristic of Al_2O_3 - Cu /water hybrid nanofluid in laminar circular flow. An average Nusselt number increment of 10.94% was observed for 0.1% Al_2O_3 - Cu /water hybrid nanofluid when comparison was made with the pure water. This indicated that significant heat transfer enhancement could be achieved by incorporating a small amount of copper nanoparticles into the alumina matrix. On the other hand, Suresh et al. [14] also analyzed the turbulent behaviour of Al_2O_3 - Cu /water hybrid nanofluid in a uniformly heated circular tube. The average heat transfer enhancement of 0.1% Al_2O_3 - Cu /water hybrid nanofluid was claimed to be approximately 8% in comparison to water. Besides, the friction factor of 0.1% hybrid nanofluid was slightly higher than that of 0.1% Al_2O_3 /water nanofluid. Gupta et al. [15] scrutinized the heat transfer and flow behaviour of $\text{Zn-Fe}_2\text{O}_4$ /water hybrid nanofluid in a circular duct. From their results, 0.5 wt% of $\text{Zn-Fe}_2\text{O}_4$ /water hybrid nanofluid could enhance the heat transfer coefficient by 43% at the Reynolds number of 2200. At the same Reynolds number and nanofluid concentration, the pressure drop increment was recorded to be approximately 72%. Rahman et al. [16] stated that the dominance nanoparticles of Al_2O_3 - Cu /water hybrid nanofluid had a stronger effect on the thermal characteristic of hybrid nanofluid. Moreover, Ramadhan et al. [19] studied experimentally and numerically the convective heat transfer of TiO_2 - SiO_2 / (ethylene-glycol/water) hybrid nanofluid. At the Reynolds number of 11000 and volume concentration of 3%, a heat transfer coefficient increment of 36% was obtained in comparison to the base fluid. The friction

factor of the hybrid nanofluid was also stated to be larger than the base fluid.

If compared to the work done on the circular duct, less studies were conducted on hybrid nanofluid flowing in plain ducts with non-circular cross-section [20–22]. Besides circular pipe, non-circular ducts such as square and rectangular ducts are worth to be scrutinized as they have gained popularity in thermal application in industries like compact heat exchangers, electronics and aerospace due to their lower pressure drop when compared to the circular duct [23, 24]. Tekir et al. [20] investigated numerically the forced convection of Al_2O_3 -Cu/water hybrid nanofluid in a square duct under the turbulent regime. They noticed an increment of 53% in heat transfer coefficient for 2% hybrid nanofluid. Concentration of hybrid nanofluid was also discovered to have insignificant effect on the friction factor. Benkhedda et al. [21] examined the laminar mixed convection of Ag-TiO₂/water hybrid nanofluid in a horizontal annulus. They declared that the Ag-TiO₂/water hybrid nanofluid contributed to better thermal and dynamic performance compared to pure mono nanofluid and water. Huminic and Huminic [22] conducted numerical study on the laminar heat transfer and flow behaviour of graphene oxide (GO)-cobalt oxide (Co₃O₄)/water hybrid nanofluid in ducts with different cross-section (circular, flat and elliptical ducts). The range of volume concentration they considered was between 0.1 and 0.2%. They reported that the type of geometry had a more significant impact on flow performance and heat transfer if compared to the volume concentration of hybrid nanofluid. The thermal performance factor of hybrid nanofluid were lower than that of water for all volume concentration and geometries studied.

According to the review above, there is limited information on the comparison of the thermal performance factor of hybrid nanofluid in plain ducts with different cross-section (circular, square and rectangular ducts). Therefore, it is the aim of this study to address this research gap. It is strongly motivated to study and compare the thermal performance factor of hybrid nanofluid in different geometries as the thermal performance factor is a way to assess the effectiveness of hybrid nanofluid in terms of its heat transfer and flow performance. The pressure drop enhancement of hybrid nanofluid was claimed to be insignificant to their augmented convective heat transfer ability [14, 20]. However, some researchers also pointed out that the friction factor enhancement might exceed the thermal enhancement [7, 11, 15, 22, 25, 26]. Hence, it is vital and necessary to evaluate the thermal performance factor of the usage of hybrid nanofluid. The analysis from this study can contribute in determining the type of geometry which is more efficient for the utilization of hybrid nanofluid for heat transfer application.

This manuscript is arranged into different sections. Section 2 provides the methodology employed for this numerical study, including the computation domain of the geometries, the solved governing equations, the thermo-physical properties of hybrid nanofluid, the boundary conditions and numerical solution strategy applied, and lastly the equations adopted for the analysis. The grid independence study is presented in Sect. 3. Section 3 is followed by the results and discussion in Sect. 4. The main findings of the study are then concluded in Sect. 5.

2 CFD modeling

2.1 Geometry and computational domain

In this study, the geometries investigated consisted of circular and non-circular (rectangular and square) ducts. Fig. 1 shows a quarter of the computational grids for the geometries. All these geometries shared the same hydraulic diameter and length. The dimension and sizing of all the ducts studied are listed in Table 1. The length of the tubes was set to be 3 m to ensure the flow was hydrodynamically and thermally fully developed.

2.2 Governing equations

The numerical study of nanofluid was carried out using the single-phase model. The single-phase model considers the mixture of base fluid and nanoparticles to be a single homogeneous fluid. The nanoparticles are assumed to be well dispersed within the base fluid. The single-phase approach was used for this present study due to its simplicity and some researchers [9, 20–22] claimed that the single-phase approach was able to model the hybrid nanofluid. The present study focused only on the turbulent flow regime. The range of Reynolds number studied was from 10000 to 100000. The assumptions incorporated for the simulations were:

1. The flow was in three-dimensional steady state condition.
2. Hybrid nanofluid was incompressible.
3. The nanoparticles and the base fluid were in thermal equilibrium and they had the same velocity.
4. Heat losses to the surrounding were negligible.
5. The pipes were smooth with zero roughness.
6. Natural convection was negligible.
7. The thermo-physical properties of hybrid nanofluid were constant and temperature independent.

Fig. 1 Computational grid: **a** circular, **b** square and **c** rectangular ducts

Table 1 Dimensions of circular and non-circular ducts

Geometry	Hydraulic Diameter, D_h (m)	Length, L (m)	Cross-section (m ²)
Circular	0.01	3	–
Square	0.01	3	0.01×0.01
Rectangular	0.01	3	0.02×0.00667

Based on the assumptions made, the governing equations of mass, momentum and energy are expressed as follows [27]:

$$\nabla \cdot (\rho \vec{v}) = 0 \quad (1)$$

$$\frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_l}{\partial x_l} \right) \right] + \frac{\partial}{\partial x_j} (-\rho u'_i u'_j) \quad (2)$$

$$\nabla \cdot \left[\vec{v} \left(\rho \left(h - \frac{P}{\rho} + \frac{v^2}{2} \right) + P \right) \right] = \nabla \cdot ((k + k_t) \nabla T) \quad (3)$$

where ρ is the fluid density, v and u are the velocity component, P is the static pressure, μ is the viscosity, δ_{ij} is the

Kronecker delta, h is the sensible enthalpy, $(k + k_t)$ is the effective thermal conductivity, k_t is the turbulent thermal conductivity, and T is the temperature. Standard k - ϵ turbulence model with enhanced wall treatment was selected to solve the turbulent kinetic energy, k and dissipation rate, ϵ . The term of $\frac{\partial}{\partial x_j} (-\rho u'_i u'_j)$ in Eq. (2), and k_t in Eq. (3) were also estimated with the turbulence model. Enhanced wall treatment was adopted to obtain accurate result regards the temperature and pressure difference, especially near the wall of ducts. The wall y^+ value was checked to be smaller than or close to one. The transport equations of k - ϵ turbulence model are shown in the following equations [27]. The Eddy viscosity, μ_t and turbulence intensity, I of fluid were calculated using Eq. (6) and Eq. (7) [27].

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left(\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right) + G_k - \rho \epsilon \quad (4)$$

$$\frac{\partial}{\partial t} (\rho \epsilon) + \frac{\partial}{\partial x_i} (\rho \epsilon u_i) = \frac{\partial}{\partial x_j} \left(\left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right) + C_{1\epsilon} \frac{\epsilon}{k} G_k - C_{2\epsilon} \rho \frac{\epsilon^2}{k} \quad (5)$$

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \quad (6)$$

$$l = 0.16Re_D^{-\frac{1}{8}} \quad (7)$$

In Eq. (4) and Eq. (5), t is the time, u_i is the velocity component, μ is the viscosity, σ_k and σ_ϵ are the turbulent Prandtl Number for turbulent kinetic energy (k) and dissipation rate (ϵ), respectively, G_k is the turbulent kinetic energy generation due to mean velocity gradient, and Re_D is the Reynolds number. The constants, C used for the transport equations [Eq. (5) and Eq. (6)] are 1.44 for $C_{1\epsilon}$, 1.92 for $C_{2\epsilon}$ and 0.09 for C_μ [27].

These governing and transport equations were solved using CFD software ANSYS Fluent 19.1 to obtain the fluid temperature and pressure drop along the ducts. These data were then used to study the heat transfer, pressure drop and thermal performance factor of hybrid nanofluid.

2.3 Thermo-physical properties

The hybrid nanofluid considered in this study was alumina oxide-copper/water (Al_2O_3 -Cu/water) hybrid nanofluid. Al_2O_3 -Cu/water was selected as it was widely studied and proven to increase the heat transfer rate with the increase of particle loading. The heat transfer rate of some other hybrid nanofluids would reduce at a high volume concentration due to the agglomeration of particles [20]. This hybrid nanofluid was also chosen due to non-reactivity of Al_2O_3 and high thermal conductivity of Cu. In addition, the dispersion of nanoparticles in Al_2O_3 -Cu/water hybrid nanofluid and the stability of the hybrid nanofluid were found to be good. The volume fraction of hybrid nanoparticles was specified within the range of 0.1–2%. The weight-percentage ratio of hybrid nanoparticle, Al_2O_3 -Cu used for the simulations was 9:1.

The density (ρ) and heat capacity (C_p) of hybrid nanofluid were estimated using Eqs. (8) to (11) as listed [9, 20]. The base fluid (water), hybrid nanofluid and single-particle nanofluid are denoted by the subscript bf , hnf and nf , respectively. In Eq. (12), φ is the hybrid nanofluid volume fraction. In addition, the thermal conductivity (k) and dynamic viscosity (μ) for various hybrid nanofluid concentration were determined using the experimental data of

Al_2O_3 -Cu/water hybrid nanofluid obtained by Suresh et al. [28], which was applicable for the concentration range from 0–2%.

$$\rho_{nf} = (1 - \varphi)\rho_{bf} + \varphi\rho_{np} \quad (8)$$

$$\rho_{hnf} = (1 - \varphi)\rho_{bf} + \varphi_{Cu}\rho_{Cu} + \varphi_{Al_2O_3}\rho_{Al_2O_3} \quad (9)$$

$$(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_{bf} + \varphi(\rho C_p)_{np} \quad (10)$$

$$(\rho C_p)_{hnf} = (1 - \varphi)(\rho C_p)_{bf} + \varphi_{Cu}(\rho C_p)_{Cu} + \varphi_{Al_2O_3}(\rho C_p)_{Al_2O_3} \quad (11)$$

$$\varphi = \varphi_{Cu} + \varphi_{Al_2O_3} \quad (12)$$

The properties of water, aluminium oxide and copper nanoparticles, as well as hybrid nanofluid with different volume concentration are shown in Table 2 below [20].

2.4 Boundary conditions

The governing equations were solved based on the defined boundary condition. The fluid entered the domain boundary with a uniform velocity profile with the flow normal to the boundary. The axial velocity of the nanofluid was calculated based on the Reynolds number used. An inlet temperature of 300 K was imposed. The outflow boundary condition was applied at the outlet boundary. No slip boundary condition was enforced at the tube walls. A constant heat flux of 20 kW/m² along the tube wall was considered.

2.5 Numerical solution strategy

The Computational Fluid Dynamic (CFD) solver ANSYS FLUENT version 19.1 was utilized in this present study. The governing equations with the boundary conditions were solved iteratively using a segregated, pressure-based solver. The double precision solver was turned on to adjust the heat transfer rate in the computational domain.

Table 2 Properties of water, nanoparticles and hybrid nanofluid

Material	Density, ρ (kg/m ³)	Heat capacity, C_p (J/kg K)	Thermal conductivity, k (W/m K)	Dynamic viscosity, μ (kg/m s) $\times 10^{-3}$
Water	995.70	4178.00	0.615	0.800
Aluminium Oxide	3970.00	779.20	40.000	–
Copper	8940.00	390.00	400.000	–
0.1% Hybrid Nanofluid	998.91	4163.53	0.619	0.900
0.5% Hybrid Nanofluid	1011.74	4106.58	0.633	1.200
1% Hybrid Nanofluid	1027.78	4037.40	0.652	1.602
2% Hybrid Nanofluid	1059.86	3905.30	0.685	1.935

Besides, the Semi Implicit Method for Pressure Linked Equations (SIMPLE) algorithm was employed to resolve the problem of velocity and pressure coupling. For the spatial discretization, the least squares cell-based gradient was adopted. The standard scheme was implemented for the pressure term while the second order upwind scheme was applied for the discretization of momentum and energy terms. Moreover, the under relaxation factors for the converged solutions were fixed at their default values. The convergence criterion was based on the residual values of calculated variables of mass, velocity and energy components. The residual values were set to 10^{-6} for all the variables in the calculations for this present numerical computation. The simulation was monitored to converge when the sum of normalized residuals of each equation and variable was less than the set residual values. Furthermore, the convergence was also judged by the mass flux report, where the net mass imbalance must be smaller than 1% of the inlet mass flux.

2.6 Data reduction

The average convective heat transfer coefficient (h_{av}) and Nusselt number (Nu) were determined using Eq. (13) and Eq. (15), respectively [20]. The Reynolds number (Re) of the fluid was computed using Eq. (16) [20].

$$h_{av} = \frac{q''}{(T_w - T_f)} \quad (13)$$

$$T_f = \frac{T_{out} + T_{in}}{2} \quad (14)$$

$$Nu = \frac{h_{av} D_h}{k} \quad (15)$$

$$Re = \frac{\rho v D_h}{\mu} \quad (16)$$

$$D_h = \frac{4A}{P} \quad (17)$$

where q'' is the heat flux, T_w is the wall temperature, T_f is the film temperature, T_{in} is the inlet temperature, T_{out} is the outlet temperature, A is the cross-sectional area and P is the perimeter. For flow behaviour of hybrid nanofluid, the friction factor (f) and pressure drop (ΔP) were calculated using Eq. (18) and Eq. (19), respectively [20]. Moreover, the thermal performance factor of hybrid nanofluid was evaluated in terms of performance evaluation criteria (PEC) as in Eq. (20) [20].

$$f = \frac{2\Delta P D}{\rho v^2 L} \quad (18)$$

$$\Delta P = P_{in} - P_{out} \quad (19)$$

$$PEC = \frac{Nu_{enhanced}/Nu_{bf}}{(f_{enhanced}/f_{bf})^{1/3}} \quad (20)$$

For validation purposes, the Dittus-Boelter correlation [Eq. (21)] was used to compute the Nusselt number of water while Eq. (23) was applied for friction factor of water [20]. The validity ranges of the Dittus-Boelter correlation are: $0.6 \leq Pr \leq 160$, $Re \geq 10000$ and $\frac{L}{D_h} > 10$ while for Eq. (23), it is valid for $3000 \leq Re \leq 5 \times 10^6$ [29].

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (21)$$

$$Pr = \frac{C_p \mu}{k} \quad (22)$$

$$f = (0.790 \ln Re - 1.64)^{-2} \quad (23)$$

3 Grid independence study

Prior to performing the simulations, the grid independence studies were conducted to find the optimum mesh size. The mesh needs to be grid independent to ensure the accuracy of the simulations. The effect of mesh size on the Nusselt number and pressure drop was examined. The error rate decreases as the mesh number increases. The computational time will only increase after this certain mesh number. From Fig. 2, it can be noticed that the Nusselt number and pressure drop for water flow in the square duct remain nearly unchanged after a certain number of mesh nodes. Hence, it is concluded that the mesh has reached its grid independence. These are also true for circular and rectangular ducts. Therefore, mesh elements of 501832, 507828, and 502272 have been used as a base for other simulations for square, rectangular, and circular ducts, respectively. The maximum skewness of the mesh chosen for all the ducts is smaller than 0.8, which indicates a good mesh quality.

Fig. 2 Variation of **a** Nusselt number and **b** pressure drop of water with mesh element for square duct

4 Results

4.1 Result validation

Before the simulations for hybrid nanofluid, the numerical heat transfer and pressure drop results of water in the circular and non-circular ducts were validated against the values from Eq. (21) and Eq. (23), respectively. This was to ensure the reliability and accuracy of the present numerical model and method. The deviation between the Nusselt number and pressure drop from simulation and those of correlations for the square duct is shown in Table 3. From the table, the numerical results compare well with the values from correlations, such that the deviation is lower than 10%. A remarkable agreement between the Nusselt number and pressure drop is also observed for the circular

and rectangular ducts. The deviation is all below 15%. This indicates that the simulation results are reliable such that the simulation technique and model selected as well as the geometries and mesh generation are suitable for simulations that are going to be performed.

To ensure the validity of the simulation for hybrid nanofluid, the simulated results of 1% and 2% hybrid nanofluid ($\text{Al}_2\text{O}_3\text{-Cu/H}_2\text{O}$) in the square duct were compared with numerical results obtained by Tekir et al. [20]. The square duct considered in this present study has the same dimension as the one used by Tekir et al. [20]. Besides, the amount of heat flux applied for constant heat flux condition was also made the same as that of Tekir et al. [20] for comparison purposes. The comparison results are tabulated in Table 4. The highest deviation between the present numerical results with those of Tekir et al. [20] is approximately 4%. In addition, the result of 1% hybrid nanofluid from this present numerical study for circular duct was also validated against the numerical data obtained by Takabi and Shokouhmand [9] as depicted in Fig. 3. As observed from Fig. 3, the present results are in good agreement with that of Takabi and Shokouhmand [9].

Furthermore, the simulated results of this present study for 0.1% $\text{Al}_2\text{O}_3\text{-Cu/H}_2\text{O}$ hybrid nanofluid in circular duct were also validated with the experimental result of Suresh et al. [14] and the comparison is depicted in Fig. 4. From Fig. 4a, the Nusselt number of 0.1% $\text{Al}_2\text{O}_3\text{-Cu/H}_2\text{O}$ hybrid nanofluid obtained from this present study agrees well with that of Suresh et al. [14] with the maximum deviation of 12.6%. The largest deviation between the friction factor of 0.1% $\text{Al}_2\text{O}_3\text{-Cu/H}_2\text{O}$ hybrid nanofluid in the circular duct of this present result and Suresh et al. [14] is approximately 9%. From the validation done for hybrid nanofluid in circular and square duct, it was justified that the assumptions, the applied turbulence model, thermo-physical properties of hybrid nanofluid used, numerical setting, and numerical solution strategy have acceptable reliability and accuracy.

4.2 Effect of hybrid nanofluid concentration and Reynolds number

The effect of Reynolds number on the Nusselt number of mono nanofluid and hybrid nanofluid is depicted in Fig. 5.

Table 3 Deviation of simulated results from those of correlations for water flow in square duct

Re	Nu from Eq. (21)	Simulated Nu	Nu deviation (%)	ΔP from Eq. (23) (Pa)	Simulated ΔP (Pa)	ΔP deviation (%)
10000	73.92	71.15	3.75	3460.32	3741.91	8.14
20000	128.70	126.42	1.77	11498.46	12000.54	4.37
40000	224.08	224.92	0.38	38815.11	39256.95	1.14
60000	309.94	317.74	2.52	79580.01	79585.86	0.01
100000	466.40	505.55	8.39	197771.72	196860.20	0.46

Table 4 Comparison of simulated Nusselt number and friction factor with literature data for Al_2O_3 -Cu/ H_2O hybrid nanofluid in square duct

Re	Nu from [20]	Simulated Nu	Nu deviation (%)	f from [20]	Simulated f	f deviation (%)
For 1% Al_2O_3 -Cu/ H_2O nanofluid						
10000	89.00	91.06	2.32	0.0320	0.0322	0.57
20000	157.00	162.98	3.81	0.0260	0.0259	0.21
40000	291.00	292.65	0.57	0.0215	0.0218	1.40
60000	409.00	416.60	1.86	0.0194	0.0196	0.99
100000	637.00	659.95	3.60	0.0172	0.0177	2.81
For 2% Al_2O_3 -Cu/ H_2O nanofluid						
10000	95.00	94.14	0.90	0.0320	0.0322	0.57
20000	169.00	168.68	0.19	0.0260	0.0259	0.21
40000	303.00	303.43	0.14	0.0215	0.0212	1.36
60000	429.00	433.81	1.12	0.0194	0.0192	1.18
100000	682.00	682.80	0.12	0.0180	0.0173	3.75

As observed in Fig. 5, for the Reynolds number of 100000, the Nusselt number enhancement of 1% Al_2O_3 -Cu/water hybrid nanofluid in the circular duct are approximately 29% and 17% when compared to the Nusselt number of water and 1% Al_2O_3 nanofluid, respectively. For the square duct, the Nusselt number of 1% Al_2O_3 -Cu/water hybrid nanofluid is approximately 33.5% higher than the value of 1% Al_2O_3 nanofluid at the Reynolds number of 100000. Besides, it can be noticed from Fig. 5 that the Nusselt number slope for Al_2O_3 -Cu/ H_2O hybrid nanofluid is steeper than that of pure water and Al_2O_3 nanofluid of the same nanoparticle loading. Incorporation of a small amount of copper nanoparticles into Al_2O_3 nanofluid can improve the thermal properties considerably. The thermal conductivity of Al_2O_3 -Cu/water hybrid nanofluid is higher than Al_2O_3 /water due to the contribution of higher thermal conductivity from metallic copper nanoparticles. Therefore, the application of hybrid nanofluid is able to enhance the Nusselt number significantly when compared to the base

Fig. 3 Comparison of average Nusselt number between present result and literature data for 1% Al_2O_3 -Cu/ H_2O hybrid nanofluid in circular duct

fluid and mono nanofluid, especially at higher Reynolds number.

As of Fig. 6, when the concentration of Al_2O_3 -Cu/ H_2O hybrid nanofluid increases from 0.1% to 2%, the Nusselt

Fig. 4 Comparison of **a** Nusselt number and **b** friction factor between present result and literature data for 0.1% Al_2O_3 -Cu/ H_2O hybrid nanofluid in circular duct

Fig. 5 Comparison of Nusselt number of hybrid nanofluid, mono nanofluid and pure water in **a** circular and **b** square ducts

number for circular duct flow increases. Similar trends are also observed for the square and rectangular ducts. The Nusselt number enhancement of 2% hybrid nanofluid is the highest, followed by 1%, 0.5% and 0.1%. The Nusselt number of 2% hybrid nanofluid is 37.47% higher than the value of water at the Reynolds number of 10000. In addition, the Nusselt number of hybrid nanofluid also increases with the increase of the Reynolds number. This is due to the increment of fluid inlet velocity when the Reynolds number increases (Table 5).

From Fig. 7, for the Reynolds number of 10000, the maximum convective heat transfer coefficient of 58.6% higher than that of water is achieved by 2% hybrid nanofluid. Similar to Nusselt number, the convective heat transfer coefficient ratio (h_{hnf}/h_{bf}) is also improving with the increase of hybrid nanofluid concentration. These occur because when the concentration of hybrid nanofluid increases, the fluid properties such as thermal conductivity and dynamic viscosity increase. The high thermal conductivity of nanofluid allows it to capture, gather and release heat energy easier to the surrounding fluid, hence enhancing the heat

convection rate. These findings are also reported by Takabi and Shokouhmand [9].

Moreover, when the dynamic viscosity increases with the concentration increment, the velocity of fluid at a particular Reynolds number increases. This can be proven from the velocity profile of hybrid nanofluid at the outlet of the circular duct ($Re=100000$) shown in Fig. 8. The x-axis is the radial distance between the tube centre and tube wall, in which 0 mm refers to the exact point at the tube center and 5 mm refers to the point at the tube wall. Higher velocity enhances the energy transfer rate within the fluid. It can also be observed that despite the outlet velocities of hybrid nanofluid are different for different concentration, they share the same trend. The fluid velocity is higher at the centre. This trend is reported for the square and rectangular ducts as well. The increased random motion of nanoparticles at the centre of the duct would induce temperature profile flattening, contributing to steeper temperature gradient between fluid and tube wall. This can reduce the thermal boundary layer thickness, delay the boundary layer development process and hence enhance the heat transfer rate [12].

Table 6 shows the pressure drop of hybrid nanofluid with respect to that of the base fluid ($\Delta P_{hnf}/\Delta P_{bf}$) for the circular duct. The pressure drop of hybrid nanofluid is observed to be larger than the pressure drop of pure water. The trend of pressure drop is similar for all the geometries studied. The highest pressure drop enhancement in circular duct is attained by 2% hybrid nanofluid. The pressure drop of 2% Al₂O₃-Cu/H₂O hybrid nanofluid is 4.785 times larger with respect to that of pure water. The increment in pressure drop of hybrid nanofluid is mainly due to the increment of viscosity when concentration increases. The increment in fluid viscosity leads to the augmentation of wall shear stress (Fig. 9). Higher wall shear stress is an indicator for higher pressure drop. Furthermore, it could be noticed that the pressure drop enhancement of hybrid nanofluid of a particular concentration decreases with a Reynolds number increment. These occur as the increment of fluid velocity due to increasing Reynolds number reduces the thermal and velocity boundary layer thickness, thus restricting the frictional effect caused by hybrid nanofluid when flowing across the tube. Similar finding was also reported by Tekir et al. [20].

As compared to results of Takabi and Shokouhmand [9], they claimed that the friction factor enhancement of 1% Al₂O₃-Cu/water hybrid nanofluid and Al₂O₃/water nanofluid for circular flow were approximately 8.45% and 6.45%, respectively at the Reynolds number of 20000. The friction factor enhancement of Al₂O₃-Cu/water hybrid nanofluid of this present study is 7.92%, which is close to that of Takabi and Shokouhmand [9] under the same

Fig. 6 Nusselt number of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in circular duct

Table 5 Fluid inlet velocities of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid

Reynolds number	Volume concentration (%)			
	0.1	0.5	1	2
	Velocity (m/s)			
10000	0.90	1.19	1.56	1.83
20000	1.80	2.37	3.12	3.65
40000	3.60	4.74	6.23	7.30
60000	5.41	7.12	9.35	10.95
100000	9.01	11.86	15.59	18.26

Fig. 7 Variation of convective heat transfer coefficient ratio (h_{hnf}/h_{bf}) with concentration in circular duct ($Re=10000$)

condition. Therefore, it can be deduced that hybrid nanofluid does consist of higher frictional effect as compared to mono nanofluid.

Fig. 8 Outlet velocity profile of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid for circular duct (at $Re=100000$)

Table 6 Variation of pressure drop ratio ($\Delta P_{hnf}/\Delta P_{bf}$) with concentration and Reynolds number in circular duct

Re	Volume concentration (%)			
	0.1	0.5	1	2
10000	1.098	1.928	3.382	4.785
20000	1.019	1.789	3.138	4.440
40000	1.057	1.856	3.256	4.606
60000	1.060	1.861	3.264	4.618
100000	1.063	1.866	3.274	4.632

4.3 Effect of geometry

The Nusselt number of 1% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in different geometries is depicted in Fig. 10. As observed in Fig. 10, the circular duct possesses better heat transfer performance, followed by rectangular duct and square duct. At the Reynolds number of 100000, the Nusselt number of 1% hybrid nanofluid are 681.89, 673.3 and 659.95 for circular, rectangular and square ducts, respectively. These occur as circular duct consists of a smooth curve body which allows particle to collide and transfer heat and energy equally, while rectangular and square ducts consist of sharp edges which may restrict the flow and hence reduce the heat transfer ability of the fluid. Figure 11 shows the temperature contours of 2% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid at the outlet of different geometries at the Reynolds number of 10000. It can be noticed from Fig. 11 that the temperature is more distributed in the circular duct when compared to those of the non-circular ducts. Besides, higher temperature regions (hot spots) are observed near the corners of the square and rectangular

Fig. 9 Wall shear stress of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in circular duct at $Re = 100,000$

Fig. 10 Nusselt number of 1% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in different geometries

ducts. It is postulated that the flow near the sharp edges (or corners) of the non-circular ducts is suppressed by the bulk flow field due to its higher velocity. Interaction between the fluid at the corner and the main stream is poor. As a result, the heat from the heated walls cannot be distributed effectively by the flow to the bulk fluid and the heat transfer performance of the non-circular ducts is reduced to a certain extent if compared to the circular duct. Whereas, the flow in the circular duct is distributed more uniformly throughout the pipe due to the absence of sharp edges and hence, higher velocity can be attained at the near-wall region. This contributes to a more uniform heat transfer between the tube wall and the bulk fluid. For a more uniform heat transfer, the temperature difference between the tube wall and the bulk fluid is smaller. According to Eq. (13), the heat transfer coefficient is greater with smaller difference between the wall and bulk temperature, hence, heat transfer rate is also greater. Similar findings are obtained by other researchers like Kumar et al. [30], Obot

and Adu-Wusu [31], Ray and Misra [32], and Daschiel et al. [33].

Figure 12 shows the pressure drop of 1% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in different geometries. It can be clearly seen from Fig. 12 that the pressure drop of hybrid nanofluid in circular duct is larger than those of the non-circular ducts while the pressure drop in both the non-circular ducts are comparable. The pressure drop across the circular duct for 1% hybrid nanofluid is 696 kPa at Reynolds number of 100,000 while they are 661 kPa and 653 kPa for the square and rectangular ducts, respectively, at the same Reynolds number and concentration. The large pressure drop experienced in the circular duct is a result of its high wall shear stress. The wall shear stress of 1% hybrid nanofluid at the Reynolds number of 100,000 are 570 Pa, 551 Pa and 536 Pa for the circular, square and rectangular ducts, correspondingly. The high wall shear stress in the circular duct is induced by its high near-wall velocity gradient [29]. The velocity gradient of 1% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in the circular, square and rectangular ducts are 3169.4/s, 2270.1/s, and 1064.8/s, respectively, at the Reynolds number 100,000. Besides, the difference in pressure drop for the geometries studied becomes more significant when the Reynolds number increases. This is because the pressure drop of hybrid nanofluid increases as the Reynolds number increases, as discussed in Sect. 4.2. These present numerical results agree with the one in literature stating that the ducts with non-circular cross section has lower heat transfer and pressure drop compared to circular pipe [24, 34–36].

4.4 Thermal performance factor

It was proven from the previous section that the use of hybrid nanofluid in circular and non-circular tubes can significantly enhance the heat transfer performance. However, it is achieved at the expense of higher pressure drop across the tubes, especially utilizing hybrid nanofluid with high volume concentration. As the penalty from the increased pressure drop might affect the promising benefit offered by hybrid nanofluid in heat transfer application, it would be necessary to evaluate its thermal performance factor. Thermal performance factor is analyzed in terms of performance evaluation criteria (PEC) which is used to determine whether the heat transfer performance of hybrid nanofluid is able to surpass the increased pressure drop penalty imposed. The PEC is a ratio between the effect of heat transfer enhancement and friction factor increment and it can be determined from Eq. (20). The PEC ratio of water at different Reynolds number are equal to one, hence hybrid nanofluid with PEC ratio greater than 1 indicates the dominance of overall heat transfer

Fig. 11 Temperature contours of 2% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid at the outlet of a quarter of **a** circular, **b** rectangular and **c** square ducts at $Re = 10000$

performance of fluid over the flow resistance. This implies better thermal performance factor compared to water.

Table 7 shows the PEC of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in different geometries. In general, the PEC of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid is all greater than one for the Reynolds number and concentration studied, except for 0.1–1% of $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid flowing in the rectangular duct at the Reynolds number of 10000. The flow resistance or friction factor dominates the heat transfer enhancement, hence the overall thermal hydraulic performance is lower than water when PEC is lower than 1 for hybrid nanofluid flow in the rectangular duct. The PEC ratio for all ducts are discovered to typically increase with the increment of concentration. This is attributed to higher heat transfer augmentation than the friction factor increment as the concentration of hybrid nanofluid increases.

Fig. 12 Pressure drop of 1% $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ hybrid nanofluid in different geometries

Table 7 PEC of Al_2O_3 -Cu/ H_2O hybrid nanofluid in different geometries

Reynolds number	Volume concentration (%)			
	0.1	0.5	1	2
	PEC			
<i>Circular duct</i>				
10000	1.13	1.23	1.34	1.38
20000	1.11	1.21	1.32	1.36
40000	1.09	1.20	1.31	1.35
60000	1.09	1.20	1.31	1.36
100000	1.09	1.20	1.31	1.36
<i>Square duct</i>				
10000	1.09	1.20	1.30	1.35
20000	1.09	1.20	1.31	1.36
40000	1.08	1.23	1.31	1.37
60000	1.09	1.20	1.32	1.39
100000	1.08	1.21	1.31	1.37
<i>Rectangular duct</i>				
10000	0.87	0.91	0.78	1.33
20000	1.08	1.18	1.31	1.37
40000	1.07	1.18	1.30	1.35
60000	1.09	1.20	1.29	1.33
100000	1.10	1.24	1.39	1.35

In addition, the PEC ratio obtained for all the geometries is comparable. The highest PEC of 1.39 is achieved by 2% Al_2O_3 -Cu/ H_2O hybrid nanofluid flow in the square duct at the Reynolds number of 60000. Greater PEC value indicates that the heat transfer performance of hybrid nanofluid in the square duct is able to surpass the increased pressure drop penalty imposed. The pressure drop of a non-circular ducts like square duct is lower in comparison to circular duct, but due to that, the heat transfer performance is also low [34–36]. The simulation results from this present study shows that the dispersion of hybrid nanoparticles in water can help to augment the heat transfer characteristic in the non-circular ducts to the level that is comparable to circular pipe. Besides the benefit of having a lower pressure drop, this heat transfer enhancement offered by hybrid nanofluid can provide more opportunity for the use of non-circular geometries for heat transfer applications in industries.

5 Conclusion

In this present study, turbulent thermal performance factor of Al_2O_3 -Cu/water hybrid nanofluid in ducts with various cross-section (circular, square and rectangular ducts) was numerically investigated. The following conclusions can be outlined based on the present numerical results:

- The Al_2O_3 -Cu/water hybrid nanofluid possesses greater heat transfer capability compared to Al_2O_3 /water nanofluid and pure water. This heat transfer enhancement of hybrid nanofluid is postulated to be owing to the higher thermal conductivity of hybrid nanofluid when compared to those of mono nanofluid and pure water.
- The maximum heat transfer coefficient and Nusselt number enhancement are attained by 2% hybrid nanofluid. At the Reynolds number of 10000, the Nusselt number and heat transfer coefficient of 2% hybrid nanofluid are approximately 37.47% and 58.6% larger than the values of pure water, respectively.
- The pressure drop of Al_2O_3 -Cu/water hybrid nanofluid is found to be larger than the pressure drop of water. The pressure drop of hybrid nanofluid in the circular duct can increase to be 4.79 times larger than the value of water. The pressure drop increment of hybrid nanofluid is caused by the viscosity increment of the hybrid nanofluid.
- Circular pipe is discovered to have better heat transfer performance, but higher pressure drop than the non-circular ducts. The absence of sharp edges on circular pipe provided smoother flow path, thus promoting the heat transfer rate.
- The hybrid nanofluid is efficient and possessed superior thermal performance factor than water. The maximum PEC is determined to be 1.39 which was obtained by 2% hybrid nanofluid in the square duct at the Reynolds Number of 60000.

In overall, the adoption of hybrid nanofluid aids to resolve the poor heat transfer performance in non-circular ducts which have the advantage of low pressure drop, leading to more future industrial heat transfer application. This study is limited to single-phase model and temperature-independent properties for Al_2O_3 -Cu/water hybrid nanofluid. Therefore, it is recommended to consider the temperature-dependent hybrid nanofluid properties and the multiphase model in the future numerical study of heat transfer and flow behaviour of hybrid nanofluid for different geometries.

Acknowledgements The authors acknowledge the Department of Chemical Engineering, Curtin University Malaysia for providing all the necessary resources and facilities in conducting this study.

Authors' contributions NTAT designed, coordinated this research and drafted the manuscript. IJKW carried out simulation, data analysis and manuscript drafting. The authors read and approved the final manuscript.

Compliance with ethical standards

Conflict interest The authors declare that they have no conflict of interest

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Naser A, Teixeira JA, Addali A (2018) A review on nanofluids: fabrication, stability, and thermophysical properties. *J Nanomater* 2018:6978130
- Choi SUS, Eastman J (1995) Enhancing thermal conductivity of fluids with nanoparticles. Paper presented at the ASME International Mechanical Engineering Congress and Exposition, San Francisco, CA
- Choi SUS (2008) Nanofluids: a new field of scientific research and innovative applications. *Heat Trans Eng* 29:429–431
- Yu W, Xie H (2012) A review on nanofluids: preparation, stability mechanisms, and applications. *J Nanomater* 2012:435873
- Ghadimi A, Saidur R, Metselaar HSC (2011) A review of nanofluid stability properties and characterization in stationary conditions. *Int J Heat Mass Trans* 54:4051–4068
- Madhesh D, Kalaiselvam S (2014) Experimental analysis of hybrid nanofluid as a coolant. *Proced Eng* 97:1667–1675
- Yarmand H, Gharehkhani S, Ahmadi G, Shirazi SFS, Baradaran S, Montazer E, Zubir MNM, Alehashem MS, Kazi SN, Dahari M (2015) Graphene nanoplatelets–silver hybrid nanofluids for enhanced heat transfer. *Energy Convers Manag* 100:419–428
- Hameed MS, Suresh S, Singh RK (2019) Comparative study of heat transfer and friction characteristics of water-based alumina–copper and alumina–CNT hybrid nanofluids in laminar flow through pipes. *J Therm Anal Calorim* 136:243–253
- Takabi B, Shokouhmand H (2015) Effects of Al_2O_3 -Cu/water hybrid nanofluid on heat transfer and flow characteristics in turbulent regime. *Int J Mod Phys C* 26:1550047
- Moghadassi A, Ghomi E, Parvizi F (2015) A numerical study of water based Al_2O_3 and Al_2O_3 -Cu hybrid nanofluid effect on forced convective heat transfer. *Int J Therm Sci* 92:50–57
- Sidik S, Adamu IM, Jamil MM, Kefayati GHR, Mamat R, Najafi G (2016) Recent progress on hybrid nanofluids in heat transfer applications: a comprehensive review. *Int Commun Heat Mass Trans* 78:68–79
- Labib MN, Nine MJ, Afrianto H, Chung H, Jeong H (2013) Numerical investigation on effect of base fluids and hybrid nanofluid in forced convective heat transfer. *Int J Therm Sci* 71:163–171
- Suresh S, Venkitaraj KP, Selvakumar P, Chandrasekar M (2012) Effect of Al_2O_3 -Cu/water hybrid nanofluid in heat transfer. *Exp Therm Fluid Sci* 38:54–60
- Suresh S, Venkitaraj KP, Hameed MS, Sarangan J (2014) Turbulent heat transfer and pressure drop characteristics of dilute water based Al_2O_3 -Cu hybrid nanofluids. *J Nanosci Nanotechnol* 14:2563–2572
- Gupta M, Singh V, Said Z (2020) Heat transfer analysis using zinc ferrite/water (hybrid) nanofluids in a circular tube: An experimental investigation and development of new correlations for thermophysical and heat transfer properties. *Sustain Energy Technol Assess* 39:100720
- Rahman MRA, Leong KY, Idris AC, Saad MR, Anwar M (2017) Numerical analysis of the forced convective heat transfer on Al_2O_3 -Cu/water hybrid nanofluid. *Heat Mass Trans* 53:1835–1842
- Balla HH, Abdullah S, MohdFaizal WM, Zulkifli R, Sopian K (2013) Numerical study of the enhancement of heat transfer for hybrid CuO-Cu nanofluids flowing in a circular pipe. *J Oleo Sci* 62:533–539
- Balla HH, Abdullah S, Zulkifli R, MohdFaizal WM, Sopian K (2012) Effect of oxides nanoparticle materials on the pressure loss and heat transfer of nanofluids in circular pipes. *J Appl Sci* 12:1396–1401
- Ramadhan AI, Azmi WH, Mamat R, Hamid KA (2020) Experimental and numerical study of heat transfer and friction factor of plain tube with hybrid nanofluids. *Case Stud Therm Eng* 22:100782
- Tekir M, Ekiciler R, Arslan K (2017) Numerical simulation of hybrid nanofluid flow in a square cross-sectioned horizontal duct. Paper presented at the 18th Symposium on Thermal Science and Engineering of Serbia, Sokobanja, Serbia
- Benkhedda M, Boufendi T, Touahri S (2018) Laminar mixed convective heat transfer enhancement by using Ag-TiO₂-water hybrid nanofluid in a heated horizontal annulus. *Heat Mass Trans* 54:2799–2814
- Huminic G, Huminic A (2020) A numerical approach on hybrid nanofluid behavior in laminar duct flow with various cross sections. *J Therm Anal Calorim* 140:2097–2110
- Heris ZS, Nassan TH, Noie SH, Sardarabadi H, Sardarabadi M (2013) Laminar convective heat transfer of Al_2O_3 /water nanofluid through square cross-sectional duct. *Int J Heat Fluid Flow* 44:375–382
- Ting HH, Hou SS (2016) Numerical study of laminar flow and convective heat transfer utilizing nanofluids in equilateral triangular ducts with constant heat flux. *Materials* 9:576
- Hussein A, Noor MM, Kadrigama K, Devarajan R, Rahman MM (2017) Heat transfer enhancement using hybrid nanoparticles in ethylene glycol through a horizontal heated tube. *Int J Automot Mech Eng* 14:4183–4195
- Nimmagadda R, Venkatasubbaiah K (2015) Conjugate heat transfer analysis of micro-channel using novel hybrid nanofluids (Al_2O_3 +Ag/water). *Eur J Mech - B/Fluids* 52:19–27
- Ansys Fluent 17.0 Theory Guide. (2016) ANSYS Inc., USA
- Suresh S, Venkitaraj KP, Selvakumar P, Chandrasekar M (2011) Synthesis of Al_2O_3 -Cu/water hybrid nanofluids using two step method and its thermo physical properties. *Coll Surf A: Physicochem Eng Asp* 388:41–48
- Incropera FP, DeWitt DP, Bergman TL, Lavine AS (2007) Fundamentals of heat and mass transfer. John Wiley and Sons, New York
- Kumar R, Kumar A, Goel V (2018) Effect of rounded corners on heat transfer and fluid flow through triangular duct. *J Heat Trans* 140:121701
- Obot NT, Adu-Wusu K (1985) The flow pattern in a scalene triangular duct having two rounded corners. *J Fluids Eng* 107:455–459
- Ray S, Misra D (2010) Laminar fully developed flow through square and equilateral triangular ducts with rounded corners

- subjected to H1 and H2 boundary conditions. *Int J Therm Sci* 49:1763–1775
33. Daschiel G, Frohnäpfel B, Jovanović J (2013) Numerical investigation of flow through a triangular duct: the coexistence of laminar and turbulent flow. *Int J Heat Fluid Flow* 41:27–33
34. Heris ZS, Noie SH, Talaii E, Sargolzaei J (2011) Numerical investigation of Al_2O_3 /water nanofluid laminar convective heat transfer through triangular ducts. *Nanoscale Res Lett* 6:179
35. Heris ZS, Talaii E, Noie S (2012) CuO/ water nanofluid heat transfer through triangular ducts. *Iran J Chem Eng* 9:23–32
36. Mehrjou B, Heris SZ, Mohamadifard K (2015) Experimental study of CuO/water nanofluid turbulent convective heat transfer in square cross-section duct. *Exp Heat Trans* 28:282–297

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.