Hall current effects on a magnetic nanofluid layer under temperature gradient

Jaimala Bishnoi¹ · Shubham Kumar¹ · Reema Singh¹

Received: 20 August 2020 / Accepted: 20 January 2021 / Published online: 26 January 2021
© The Author(s) 2021

Abstract

In the Bénard convection governed nanofluid confined between two horizontal infinite free-free boundaries, the effect of Hall current is investigated when it is subjected to the constant vertical magnetic field. The Buongiorno mathematical model for hydromagnetic flow with Hall currents has been considered. The Brownian motion effects and thermophoresis of nanoparticles have been included in the energy equation. A realistic boundary condition based on zero nanoparticle mass flux at the walls is incorporated for the analysis. The impacts of Brownian motion, convective heat transfer, thermophoresis of nanoparticles and zero mass flux conditions are also deliberated through the behaviour of the related parameters. The parameters representing the model incorporate the consequences of newly introduced physically realistic boundary conditions and Hall currents. For comprehensive physical interpretation the embedded parameters have been plotted and deliberated graphically. It is found that Hall current is responsible to enhances the instability of the system and sets the convection earlier. A rigorous comparison has been made with the existing results. The results are shown graphically and verified numerically.

Keywords Nanofluid · Hall current · Brownian motion · Transverse magnetic field

Abbreviations

$(\rho c)_f$	Heat capacity of the nanofluid
$(\rho c)_p$	Heat capacity of the nanoparticles
D_B	Brownian diffusion coefficient
D_T	Thermophoresis diffusion coefficient
H	Magnetic field
k	Thermal conductivity of nanofluid
L_e	Lewis number
N_A	Modified diffusivity ratio
N_B	Modified particle density increment
P	Pressure
P_r	Prandtl number
P_{rM}	Magnetic Prandtl number
Q	Nanofluid magnetic number
R_a	Rayleigh number
R_m	Basic-density Rayleigh number
R_n	Concentration Rayleigh number
t	Time (dimensionless)

T	Nanofluid temperature (dimensionless)
(u, v, w)	Velocity components (dimensionless)
v	Nanofluid velocity (dimensionless)
(x, y, z)	Cartesian coordinate (dimensionless)

Greek symbols

ρ_p	Nanoparticle density
ρ_f	Base fluid density
ρ_0	Reference density of nanofluid
μ_e	Magnetic permeability
μ	Viscosity
ϕ	Dimensionless volume fraction

✉ Reema Singh, reemamalik28@gmail.com; Jaimala Bishnoi, jaimalaccsu1@gmail.com; Shubham Kumar, shubhammzn17@gmail.com |
¹Department of Mathematics, Chaudhary Charan Singh University, Meerut, UP 250004, India.

1 Introduction

The poor thermal conductivity was a big impediment of the fluids used as cooling stimulants in heat transfer equipment, radiators and electronic cooling systems used in different fields of importance like medicine, defense, transportation etc. Choi and his counterparts [1–3] performed several experiments by suspending various metallic and metal oxide nanoparticles in different types of fluids and inferred that the thermal conductivity of the base fluid can be increased to many folds by adding low concentration of nanosized particles of the materials having higher thermal conductivity than the base fluid. These are two phase fluids where a liquid or gas can act as a host phase and nanosized particles or fibers are included as foreign guests. The particles added are larger than atomic dimensions but small enough (between 1 and 100 nm) to exhibit Brownian motion. Another pioneer in the field of nanofluids is Buongiorno [4], who discussed the various slip mechanism important in nanofluids like Brownian diffusion, Magnus effect, inertia, thermophoresis, fluid drainage, diffusionphoresis and gravity setting and recognized that in the absence of turbulent eddies, these are Brownian diffusion and thermophoresis which dominate the other slip mechanisms. It has also been established that the behavior of nanofluid is basically governed by the material of nanoparticles, their volume fraction, size, shape, spatial distribution, pH value, temperature and the type of the base-fluid. The inclusion of the nanoparticles with a surrounding interfacial layer disturbs the properties of the host fluid in a positive and interesting way and the thermal conductivity of the fluid increases to many folds with an increase in the volume fraction of the nanoparticles. Consequently, the rate of heat transfer is also increased. Thus, the introduction of the nanofluid started a revolution in the fields which were in an urgent need of such type of fluids like transportation, nuclear reactors, medicine, electronics and food [5–8].

Due to temperature-induced differences in buoyancy, free or natural heat convection termed as Bénard convection occurs in fluids. In free convection, the mass transfer process is governed by the variations of the density gradient generating the buoyancy force of the flow. The Bénard convection in nanofluids in the presence of Lorentz force are providing promising results through the progressively ironic and varied technologies used for cancer therapy, solar cell augmentation, pharmacology, flow-meters, pumps, magnetic storage, magnetohydrodynamic generators, accelerators, cooling the electronic equipment, magnetic field sensors, geothermal reservoirs, insulators, heating and ventilation control in building design etc. [9–13].

The Hall effect is nothing but an extension of the Lorentz force, which describes the force exerted on a charged particle moving through a magnetic field. When a sufficiently high strength magnetic field is applied in the direction perpendicular to the electric field, an electric potential is generated across the conductor, which is perpendicular to both the magnetic as well as electric field. This phenomenon is produced due to the induced Lorentz force and is called Hall Effect. It is a measurable transverse voltage discovered by Hall [14]. In fluids specifically in ionized gases or liquids if subjected to a strong magnetic field and even a small electron density, the Hall effects are so effective that these cause the magnetic orientation to change the flow outline of the fluid. Hall current effective IC switch or sensor is exceptionally useful to detect the magnetic field and provides digital signal for on and off in nuclear power reactors, Hall accelerators, flight magnetogasdynamics and magnetohydrodynamic generators. Since nanofluids are performing an observant role in diversified fields, the Hall current effective hydromagnetic flows of nanofluids have been studied by many authors. Recently, Yadav et al. [15] discussed the convection of nanofluids under Hall currents saturated in a Brinkman porous medium. Das et al. [16] studied the Hall effects for natural convection of a nanofluid with thermal radiation. Wahed [17] investigated the flow over a stretching plate embedded in a ferromagnetic fluid suspended with Fe_3O_4 nanoparticles in the presence of hall current, nonlinear thermal radiation and joule heating. Abbasi et al. [18] investigated the effect of Hall currents on a nanofluid with variable thermal conductivity. Shah et al. [19] discussed the flow on a micropolar nanofluid saturated between two horizontal parallel plates in a rotating reference frame under the influence of Hall current. Krishna and Chamkha [20] analyzed the Hall effects on the flow of a water-based nanofluid using Galerkin optimal homotopy asymptotic method. Further, ion slip and hall effect is seen by Krishna and Chamkha for Ag and TiO_2 nanofluids using perturbation technique [21]. Shah et al. [22] studied the impact of Hall effect in the presence of Cattaneo-Christov heat flux and rotation on the squeezing 3-D flow of nanofluid between parallel plates. Sharma and Gupta [23] discussed the thermal instability of nanofluids under Hall currents subjected to the local thermal non-equilibrium (LTNE) effects.

The above-mentioned literature does not conduct the Hall currents in nanofluid with passive management of nanoparticles at the boundary walls. The present study focuses to explore the role of Hall current to the thermal convection of nanofluid under constant transversal magnetic field considering the realistic boundary conditions motivated by the prominence passive management of temperature at the boundaries suggested by Stefan [24]. The Galerkin-type weighted residuals method is used. To

see the flow pattern of nanofluid, comparative graphs are also given. Using MATLAB software, the stationary convection is discussed analytically as well as graphically.

2 Mathematical modeling

Consider a layer of electrically conducting incompressible fluid comprised with nanoparticles. A Cartesian co-ordinate system (x, y, z) is chosen where z axis is taken at right angle to boundaries. The fluid layer is lying between infinitely extended horizontal plates situated at $z^* = 0$ and $z^* = L$. The system is heated from below such that the temperature T_0^* at the lower boundary, is greater than the temperature T_1^* at the upper boundary. Further consider that the transversal vertical magnetic field $\mathbf{H}^* = (0, 0, H_0^*)$ acts on the system.

The mathematical equations relevant to the physical configuration are taken from Yadav et al. [15, 25]. The mass-conservation equation for the nanofluid and the nanoparticle are respectively taken as

$$\nabla \cdot \mathbf{v}^* = 0 \quad (1)$$

and

$$\left(\frac{\partial}{\partial t^*} + (\mathbf{v}^* \cdot \nabla) \right) \phi^* = D_B \nabla^{*2} \phi^* + \left(\frac{D_T}{T_1^*} \right) \nabla^{*2} T^{*2}, \quad (2)$$

The energy equation and momentum conservation equation are considered as

$$(\rho c)_f \left(\frac{\partial}{\partial t^*} + (\mathbf{v}^* \cdot \nabla) \right) T^* = k \nabla^{*2} T^* + (\rho c)_p \left[D_B \nabla^* \phi^* \cdot \nabla^* T^* + \frac{D_T}{T_1^*} \nabla^* T^* \cdot \nabla^* T^* \right], \quad (3)$$

$$\rho_0 \left(\frac{\partial}{\partial t^*} + (\mathbf{v}^* \cdot \nabla) \right) \mathbf{v}^* = -\nabla^* p^* + \mu \nabla^{*2} \mathbf{v}^* + [\phi^* \rho_p + (1 - \phi^*) \rho_0 \{1 - \beta(T^* - T_1^*)\}] \mathbf{g} + \frac{\mu_e}{4\pi} (\nabla^* \times \mathbf{H}^*) \times \mathbf{H}^*. \quad (4)$$

The modified Maxwell equations under the influence of Hall currents are taken as

$$\left(\frac{\partial}{\partial t^*} + (\mathbf{v}^* \cdot \nabla) \right) \mathbf{H}^* = (\mathbf{H}^* \cdot \nabla^*) \mathbf{v}^* + \eta \nabla^{*2} \mathbf{H}^* - \frac{\xi}{4\pi eN} \nabla^* \times [(\nabla^* \times \mathbf{H}^*) \times \mathbf{H}^*], \quad (5)$$

$$\nabla^* \cdot \mathbf{H}^* = 0, \quad (6)$$

where $\mathbf{v}^*$ is the nanofluid velocity, ϕ^* is the volume fraction of nanoparticles (ϕ_0^* is the reference value), t^* is the time, p^* is the pressure, ρ is the density of the base fluid, ρ_p is the density of nanoparticles, ρ_0 is the reference density of nanofluid, $\mathbf{g}$ is the gravitational acceleration, $(\rho c)_f$ is the heat capacity of the nanofluid, $(\rho c)_p$ is the heat capacity of the nanoparticles, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T_1^* is the reference temperature, k is the thermal conductivity, β is thermal expansions coefficient, μ is viscosity, η is electrical resistivity, ξ is the velocity of light, e is charge of electron and N is the electron number density.

On the boundaries, the temperature is taken as constant while the nanoparticle flux is assumed to be zero [25] and therefore, we assume

$$w^* = 0, \quad T^* = T_0^*, \quad D_B \frac{d\phi^*}{dz^*} + \frac{D_T}{T_1^*} \frac{dT^*}{dz^*} = 0 \quad \text{at } z^* = 0, \quad (7)$$

$$w^* = 0, \quad T^* = T_1^*, \quad D_B \frac{d\phi^*}{dz^*} + \frac{D_T}{T_1^*} \frac{dT^*}{dz^*} = 0 \quad \text{at } z^* = L, \quad (8)$$

Introducing non-dimensional parameters as

$$(x, y, z) = \left(\frac{x^*, y^*, z^*}{L} \right), \quad (u, v, w) = \left(\frac{u^*, v^*, w^*}{\alpha} \right) L, \quad (H_x, H_y, H_z) = \left(\frac{H_x^*, H_y^*, H_z^*}{H_0^*} \right), \\ t = \frac{t^*}{L^2} \alpha, \quad p = \frac{p^*}{\mu \alpha} L^2, \quad \phi = \frac{\phi^* - \phi_0^*}{\phi_0^*}, \quad T = \frac{T^* - T_1^*}{T_0^* - T_1^*}, \quad (9)$$

Equations (1)–(6) are obtained in non-dimensional form

as follows:

$$\nabla \cdot \mathbf{v} = 0 \quad (10)$$

$$\frac{1}{Pr} \left[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right] \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v} - R_m \hat{\mathbf{e}}_z + R_a T \hat{\mathbf{e}}_z - R_n \phi \hat{\mathbf{e}}_z + Q \frac{Pr}{Pr_m} (\nabla \times \mathbf{H}) \times \mathbf{H}, \quad (11)$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right] T = \nabla^2 T + \frac{N_B}{L_e} (\nabla T \cdot \nabla \phi) + \frac{N_A N_B}{L_e} (\nabla T \cdot \nabla T), \quad \frac{d\phi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad (12)$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right] \phi = \frac{N_A}{L_e} \nabla^2 T + \frac{1}{L_e} \nabla^2 \phi, \quad \frac{d^2 T_b}{dz^2} = 0. \quad (13)$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right] \mathbf{H} = (\mathbf{H} \cdot \nabla) \mathbf{v} + \frac{P_r}{P_{rM}} \nabla^2 \mathbf{H} - M \frac{P_r}{P_{rM}} \nabla \times [(\nabla \times \mathbf{H}) \times \mathbf{H}], \quad \text{Equations (19) and (20) provide the following steady state solutions compatible with the boundary conditions:} \quad (14)$$

$$\nabla \cdot \mathbf{H} = 0. \quad T_b = 1 - z, \quad (15)$$

The non-dimensional form of boundary conditions is obtained as

$$w = 0, \quad T = 1, \quad \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0 \quad \text{at } z = 0, \quad (16)$$

$$w = 0, \quad T = 0, \quad \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0 \quad \text{at } z = 1. \quad (17)$$

In the above process the following non-dimensional parameters are obtained:

$$\begin{aligned} L_e &= \frac{\alpha}{D} \text{ (Nanofluid Lewis number),} \\ N_A &= \frac{\beta_T(T_0^* - T_1^*)}{D_B T_0^* \phi_0^*} \text{ (Modified diffusivity ratio number),} \\ N_B &= \frac{(\rho C)_p \phi_0^*}{(\rho C)_f} \text{ (Modified particle density increment number),} \\ P_r &= \frac{\mu}{\rho_0 \alpha} \text{ (Nanofluid Prandtl number),} \\ P_{rM} &= \frac{\mu}{\rho_0 \eta} \text{ (Magnetic Prandtl number),} \\ R_a &= \frac{\rho g \beta L^3 (T_0^* - T_1^*)}{\mu} \text{ (Rayleigh number),} \\ R_m &= \frac{[\rho_p \phi_0^* + \rho_0 (1 - \phi_0^*)] g L^3}{\mu} \text{ (Basic density Rayleigh number),} \\ R_n &= \frac{(\rho_p - \rho_0) \phi_0^* g L^3}{\mu} \text{ (Concentration Rayleigh number),} \\ Q &= \frac{\mu_e H_0^2 L^{\mu\alpha}}{4\pi \rho_0 \nu \eta} \text{ (Nanofluid magnetic number),} \\ \text{and } M &= \frac{H_0^2 \xi}{4\pi e N \eta} \text{ (Hall Current number).} \end{aligned}$$

Following [4] density of nanoparticles is assumed to be greater than the density of the base fluid ($\rho_p > \rho$). Thus, throughout the analysis to follow R_n is taken as positive.

2.1 Conduction state

Assume that the basic state is time independent and is given by

$$\mathbf{v} = 0, \quad T = T_b(z), \quad \phi = \phi_b(z), \quad \rho = \rho_b(z), \quad p = p_b(z), \quad \mathbf{H} = \hat{\mathbf{e}}_z. \quad (18)$$

Substituting (18) in the flow governing equations and performing certain mathematical calculations, we get

$$\frac{d\phi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad (19)$$

and

$$\frac{d^2 T_b}{dz^2} = 0. \quad (20)$$

Equations (19) and (20) provide the following steady state solutions compatible with the boundary conditions:

$$T_b = 1 - z, \quad (21)$$

$$\phi_b = \phi_0 + z N_A. \quad (22)$$

2.2 Perturbed state

Imposing the space and time dependent perturbations onto the basic state, we get

$$\mathbf{v} = \mathbf{v}', \quad T = T_b(z) + T', \quad \phi = \phi_b(z) + \phi', \quad \rho = \rho_b(z) + \rho', \quad \mathbf{H} = \hat{\mathbf{e}}_z + \mathbf{H}'. \quad (23)$$

where prime (') denotes the perturbations /disturbances.

Substituting Eq. (23) into Eqs. (10)–(15) and under linear stability theory neglecting the product of prime quantities, we get

$$\nabla \cdot \mathbf{v}' = 0, \quad (24)$$

$$\frac{1}{P_r} \frac{\partial \mathbf{v}'}{\partial t} = -\nabla p' + \nabla^2 \mathbf{v}' + R_a T' \hat{\mathbf{e}}_z - R_n \phi'_z \hat{\mathbf{e}}_z + Q \frac{P_r}{P_{rM}} [(\nabla \times \mathbf{H}') \times \hat{\mathbf{e}}_z], \quad (25)$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{L_e} \left[N_A \frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z} \right] - 2 \frac{N_A N_B}{L_e} \frac{\partial T'}{\partial z}, \quad (26)$$

$$\frac{\partial \phi'}{\partial t} + N_A w' = \frac{1}{L_e} \nabla^2 \phi' + \frac{N_A}{L_e} \nabla^2 T', \quad (27)$$

$$\frac{\partial \mathbf{H}'}{\partial t} = \frac{\partial w'}{\partial z} \hat{\mathbf{e}}_z + \frac{P_r}{P_{rM}} \nabla^2 \mathbf{H}' - M \frac{P_r}{P_{rM}} \nabla \times [(\nabla \times \mathbf{H}') \times \hat{\mathbf{e}}_z], \quad (28)$$

$$\nabla \cdot \mathbf{H}' = 0, \quad (29)$$

Operate $\hat{\mathbf{e}}_z \cdot \text{curl curl}$, on Eq. (28), and use the identity $\text{curl curl} = \text{grad (div)} - \nabla^2$. The resulting equation and Eq. (29), help to provide the z-component of the momentum equation given by Eq. (25) in the following form:

$$\left[\nabla^4 - \frac{1}{P_r} \frac{\partial}{\partial t} \nabla^2 \right] w' + R_a \nabla_p^2 T' - R_n \nabla_p^2 \phi' + Q \frac{P_r}{P_{rM}} \left(\frac{\partial \mathbf{H}'}{\partial z} \right) = 0, \quad (30)$$

Solving Eq. (28) we get

$$\frac{\partial H'_x}{\partial t} = \frac{P_r}{P_{rM}} \nabla^2 H'_x - M \frac{P_r}{P_{rM}} \frac{\partial \zeta'_x}{\partial z}, \quad (31)$$

$$\frac{\partial H'_y}{\partial t} = \frac{P_r}{P_{rM}} \nabla^2 H'_y - M \frac{P_r}{P_{rM}} \frac{\partial \zeta'_y}{\partial z}, \quad (32)$$

$$\frac{\partial H'_z}{\partial t} = \frac{P_r}{P_{rM}} \nabla^2 H'_z - M \frac{P_r}{P_{rM}} \frac{\partial \zeta'_z}{\partial z}. \quad (33)$$

Using Eqs. (31) to (33), we get

$$\left(\nabla^4 - \frac{1}{P_r} \nabla^2 \frac{\partial}{\partial t} \right) \Gamma_2 w' + R_a \nabla^2 \Gamma_2 T' - R_n \nabla^2 \Gamma_2 \phi' - Q \frac{P_r}{P_{rM}} \nabla^2 \Gamma_1 \frac{\partial^2 w'}{\partial z^2} = 0, \quad (34)$$

where the operators Γ_1 and Γ_2 are given by

$$\Gamma_1 = \left(\frac{P_r}{P_{rM}} \nabla^2 - \frac{\partial}{\partial t} \right), \quad (35)$$

$$\Gamma_2 = \left[\Gamma_1^2 + \left(M \frac{P_r}{P_{rM}} \right)^2 \nabla^2 \frac{\partial^2}{\partial z^2} \right]. \quad (36)$$

3 Linear stability analysis

Assume the small perturbations in the form of wave and decompose them as follows:

$$(w', T', \phi') = [W(z), \Theta(z), \Phi(z)] \exp [ik_x x + ik_y y + \sigma t], \quad (37)$$

where k_x and k_y are the wave numbers along the x and y directions respectively, and $a = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number.

Using (37) in Eqs. (26), (27) and (34) the resulting equations governing the stability or instability of the system are given by

$$W + \left[D^2 - a^2 - \frac{N_A N_B}{L_e} - \sigma \right] \Theta - \frac{N_B}{L_e} D \Phi = 0, \quad (38)$$

$$N_A W - \frac{N_A}{L_e} [D^2 - a^2] \Theta + \left[\sigma - \frac{1}{L_e} (D^2 - a^2) \right] \Phi = 0, \quad (39)$$

$$\begin{aligned} (D^2 - a^2) \left[(D^2 - a^2) - \frac{\sigma}{P_r} \right] \Gamma_4 W - R_a a^2 \Gamma_4 \Theta \\ + R_n a^2 \Gamma_4 \Phi - \left[Q \frac{P_r}{P_{rM}} (D^2 - a^2) \right] \Gamma_3 D^2 W = 0, \end{aligned} \quad (40)$$

where

$$\Gamma_3 = \left[\frac{P_r}{P_{rM}} (D^2 - a^2) - \sigma \right], \quad (41)$$

$$\Gamma_4 = \left[(\Gamma_3)^2 + \left(M \frac{P_r}{P_{rM}} \right)^2 (D^2 - a^2) D^2 \right]. \quad (42)$$

Use Galerkin weighted residual technique and assume the approximate solutions compatible with the boundary conditions as

$$W = W_0 \sin \pi z, \quad T = \Theta_0 \sin \pi z, \quad \Phi = -N_A \Phi_0 \sin \pi z. \quad (43)$$

Substituting (43) in Eq. (38)–(40), and integrating the resulting equations with respect to z from $z=0$ to $z=1$, we have

$$W_0 - (J + \sigma) \Theta_0 = 0, \quad (44)$$

$$W_0 + \frac{J}{L_e} \Theta_0 - \left(\sigma + \frac{J}{L_e} \right) \Phi_0 = 0, \quad (45)$$

$$\begin{aligned} \left[\frac{P_r^2}{P_{rM}^2} J^4 + \frac{\sigma}{P_{rM}} J^3 + \frac{\sigma^3}{P_r} + \frac{\sigma^2}{P_{rM}} J + \frac{M^2 \sigma P_r \pi^2 J}{P_{rM}^2} + \sigma^2 J^2 + \frac{2 P_r \sigma J^3}{P_{rM}} + \frac{M^2 P_r^2 \pi^2 J^3}{P_{rM}^2} + \frac{Q P_r}{P_{rM}} \left(\frac{P_r \pi^2 J^2}{P_{rM}} + \sigma \pi^2 J \right) \right] W_0 \\ - (R_a a^2 \Gamma_5) \Theta_0 - (N_A R_n a^2 \Gamma_5) \Phi_0 = 0, \end{aligned} \quad (46)$$

where

$$\Gamma_5 = \left\{ \frac{P_r^2 J^2}{P_{rM}^2} + \sigma^2 + \frac{2\sigma P_r J}{P_{rM}} + \frac{M^2 \pi P_r^2 J}{P_{rM}^2} \right\} \text{ and } J = \pi^2 + a^2. \quad (47)$$

For a non-trivial solution of the system of equations in W_0 , Θ_0 and Φ_0 given by (44)–(46), the determinant of coefficients should be zero i.e.

$$\begin{vmatrix} \left[\frac{P_r^2 J^4}{P_{rM}^2} + \frac{\sigma}{P_{rM}} J^3 + \frac{\sigma^3}{P_r} + \frac{\sigma^2}{P_{rM}} J + \frac{M^2 \sigma P_r \pi^2 J}{P_{rM}^2} + \sigma^2 J^2 \right] & -R_a a^2 \{\Gamma_5\} & -N_A R_n a^2 \{\Gamma_5\} \\ + \frac{2P_r \sigma J^3}{P_{rM}} + \frac{M^2 P_r^2 \pi^2 J^3}{P_{rM}^2} + \frac{QP_r}{P_{rM}} \left(\frac{P_r \pi^2 J^2}{P_{rM}} + \sigma \pi^2 J \right) & & \\ -1 & (J + \sigma) & 0 \\ 1 & \frac{J}{L_e} & -\frac{J}{L_e} - \sigma \end{vmatrix} = 0. \quad (48)$$

3.1 Stationary Convection

In order to analyze stationary convection at the marginal state we put $\sigma=0$ in Eq. (48) and get the Rayleigh number as

$$R_a = \frac{J^2(J^2 + Q\pi^2 + M^2\pi^2 J)}{(J + M^2\pi)a^2} - N_A R_n(1 + L_e). \quad (49)$$

Substituting $a^2 = \pi^2 x$ in J , Eq. (49) provides

$$R_a = \frac{\pi^3(1+x)^2[\pi^2(1+x)^2 + Q + M^2\pi(1+x)]}{[(1+x)\pi + M^2]x} - N_A R_n(1 + L_e). \quad (50)$$

It is important to mention that the Rayleigh number given by (49) is governed by the parameters introduced due to Hall currents. All the parameters are also modified according to the new boundary conditions. Equation (50) says that if there is a regular fluid ($N_A = 0$), then the Rayleigh number in the presence of magnetic field is increased showing that the presence of nanoparticles will exaggerate the convection earlier.

Further, if we put $N_A = 0$, $R_n = 0$, $L_e = 0$ and $M = 0$, the Rayleigh number for the regular fluids [27] has been recovered.

In order to obtain the critical wave number, we put $\frac{dR_a}{dx} = 0$. We get the following fourth-degree equation in $x_c (= \pi x^{\frac{1}{2}})$:

$$(2\pi^3)x_c^4 + (5\pi^3 + 4M^2\pi^2)x_c^3 + (3\pi^3 + 6M^2\pi^2 + 2M^4\pi)x_c^2 - (\pi^3 - M^4\pi - QM^2 + \pi Q)x_c - (\pi^3 + 2M^2\pi^2 + M^4\pi + \pi Q + QM^2) = 0. \quad (51)$$

The critical wave number for stationary convection may be obtained from Eq. (51). Here it has been confirmed that the Hall current and magnetic field register their effects by modifying the critical wave number which has been obtained for a fluid without magnetic field.

To investigate the stability or instability of the flow we calculate the variations in Rayleigh number R_a against the magnetic number Q , Hall current parameter M , Lewis

number L_e , modified diffusivity ratio number N_A and concentration Rayleigh number R_n .

Calculating the derivative of R_a with respect to Q we get

$$\frac{dR_a}{dQ} = \frac{(a^2 + \pi^2)^3 \pi^2}{(a^2 + \pi^2 + M^2\pi)a^2}, \quad (52)$$

which is clearly positive i.e., the Rayleigh number for stationary convection is increasing on increasing the magnetic field parameter Q . Thus, the magnetic field helps to suppress the instability of the nanofluid.

$$\frac{dR_a}{dM} = \frac{2M\pi(a^2 + \pi^2)^2\{(\pi - 1)(a^2 + \pi^2)^2 - Q\pi^2\}}{(a^2 + \pi^2 + M^2\pi)a^2} \quad (53)$$

conveys that the hall current parameter M stabilizes the system only for $Q < \frac{(\pi-1)(a^2+\pi^2)^2}{\pi^2}$ and when $Q > \frac{(\pi-1)(a^2+\pi^2)^2}{\pi^2}$ the system is unstable with increasing M .

$$\frac{dR_a}{dL_e} = -N_A R_n \quad (54)$$

predicts that the nanofluid Lewis number is crashing the stability of the system.

From Eq. (49), we get

$$\frac{dR_a}{dN_A} = -(1 + L_e)R_n, \quad (55)$$

and

$$\frac{dR_a}{dR_n} = -(1 + L_e)N_A. \quad (56)$$

Equations (55) and (56) show that N_A and R_n both individually have destabilizing effect.

3.2 Oscillatory Convection

Let us assume that at the marginal state, the disturbances provoke or die as oscillatory modes, i.e., $\sigma = i\omega$ ($\omega > 0$), then from (48) Rayleigh number is obtained as

$$R_a = \frac{(AC + BD) + i\omega(BC - AD)}{(C^2 + D^2)}, \quad (57)$$

where A , B , C and D are given as

$$A = \left\{ \begin{aligned} & \frac{P_r^2 J^6}{P_{rM}^2 L_e} - \frac{\omega^2 J^3}{P_{rM} L_e} - \frac{\omega^2 J^4}{L_e} + \frac{M^2 P_r^2 \pi^2 J^5}{L_e} + \frac{Q P_r^2 \pi^2 J^4}{P_{rM}^2 L_e} - \frac{J^4 \omega^2}{P_{rM} L_e} + \frac{J \omega^4}{P_r L_e} - \frac{M^2 P_r \pi^2 J^2 \omega^2}{P_{rM}^2 L_e} \\ & - \frac{2\omega^2 P_r J^4}{P_{rM} L_e} - \frac{Q P_r \pi^2 J^2 \omega^2}{P_{rM} L_e} - \frac{J^4 \omega^2}{P_{rM}} + \frac{J \omega^4}{P_r} - \frac{M^2 P_r \pi^2 J^2 \omega^2}{P_{rM}^2} - \frac{2\omega^2 P_r J^4}{P_{rM}} - \frac{Q P_r \pi^2 J^2 \omega^2}{P_{rM}} \\ & - \frac{P_r^2 J^4 \omega^2}{P_{rM}^2} + \frac{J \omega^4}{P_r} + \omega^4 J - \frac{M^2 P_r^2 \pi^2 J^3 \omega^2}{P_{rM}^2} - \frac{Q P_r^2 \pi^2 J^2 \omega^2}{P_{rM}^2} - \frac{N_A R_n P_r^2 J^3 a^2}{P_{rM}^2 L_e} - \frac{N_A R_n J \omega^2 a^2}{L_e} \\ & - \frac{N_A R_n M^2 P_r^2 \pi^2 J^2 a^2}{P_{rM}^2 L_e} - \frac{N_A R_n P_r^2 J^3 a^2}{P_{rM}^2} + N_A R_n a^2 \omega^2 J - \frac{N_A R_n M^2 P_r^2 \pi^2 J^2 a^2}{P_{rM}^2} + \frac{2N_A R_n P_r \omega^2 J a^2}{P_{rM}} \end{aligned} \right\}, \quad (58)$$

$$B = \left\{ \begin{aligned} & \frac{J^5}{P_{rM} L_e} - \frac{\omega^2 J^2}{P_r L_e} + \frac{M^2 P_r \pi^2 J^3}{P_{rM}^3 L_e} + \frac{2P_r J^4}{P_{rM} L_e} + \frac{Q P_r \pi^2 J^3}{P_{rM} L_e} + \frac{P_r^2 J^5}{P_{rM}^2 L_e} - \frac{J^2 \omega^2}{P_{rM} L_e} \\ & - \frac{J^3 \omega^2}{L_e} + \frac{M^2 P_r^2 \pi^2 J^4}{P_{rM}^2 L_e} + \frac{Q P_r^2 \pi^2 J^3}{P_{rM}^2 L_e} + \frac{P_r^2 J^5}{P_{rM}^2} - \frac{J^2 \omega}{P_{rM}} - \omega^2 J^2 + \frac{M^2 P_r^2 \pi^2 J^4}{P_{rM}^2} \\ & + \frac{Q P_r^2 \pi^2 J^3}{P_{rM}^2} - \frac{J^3 \omega^2}{P_{rM}} - \frac{\omega^4}{P_r} - \frac{M^2 P_r \pi^2 J \omega^2}{P_{rM}} - \frac{2\omega^2 P_r J^3}{P_{rM}} - \frac{Q P_r \pi^2 J \omega^2}{P_{rM}} \\ & - \frac{2N_A R_n P_r J^2 a^2}{P_{rM} L_e} - \frac{2N_A R_n P_r J^2 a^2}{P_{rM}} - \frac{N_A R_n P_r^2 J^2 a^2}{P_{rM}^2} - \frac{2N_A R_n P_r^2 \pi J a^2}{P_{rM}^2} + 2N_A R_n a^2 \omega^2 \end{aligned} \right\}, \quad (59)$$

$$C = \left\{ \frac{P_r^2 J^3}{P_{rM}^2 L_e} - \frac{\omega^2 J}{L_e} + \frac{M^2 P_r^2 \pi^2 J^2}{P_{rM}^2 L_e} + \frac{2\omega^2 P_r J}{P_{rM}} \right\} a^2 \quad (60)$$

and

$$D = \left\{ \frac{P_r^2 J^2}{P_{rM}^2} - \omega^2 + \frac{M^2 P_r^2 \pi J}{P_{rM}^2} - \frac{2P_r J^2}{P_{rM} L_e} \right\} a^2. \quad (61)$$

Fig. 1 Physical configuration

Since the Rayleigh number R_a is always a real physical quantity, Eq. (57) says that we should necessarily have

$$BC - AD = 0 \quad (62)$$

or

$$F_1 \lambda^3 + F_2 \lambda^2 + F_3 \lambda + F_4 = 0, \quad (63)$$

where $\lambda = \omega^2$,

Fig. 2 R_a^{st} vs. Q for concentration Rayleigh number R_n

Fig. 5 R_a^{st} vs. Q for Hall current parameter M

Fig. 3 R_a^{st} vs. Q for modified diffusivity ratio N_A

Fig. 6 Comparative study of convection with and without Hall current

Fig. 4 R_a^{st} vs. Q for Lewis number L_e

Fig. 7 Comparative graph for convection in a porous medium and a continuous medium

$$F_1 = \frac{Ja^2}{P_r P_{rM} L_e} \{P_r P_{rM} L_e + 2P_{rM} L_e + 2P_{rM} - 2P_r^2\},$$

$$F_2 = \frac{Ja^2}{P_r P_{rM}^2 L_e^2} \left\{ \begin{aligned} &-2P_r^2 L_e J^2 - 2M^2 P_r^2 L_e \pi^2 J - M^2 P_r^2 L_e \pi J + 4P_r^2 L_e J - 2P_r^2 L_e J^2 - 2P_r^3 L_e J^2 \\ &-2M^2 P_r^3 L_e \pi J - 2P_r^2 L_e^2 J^2 - 2P_r^2 L_e^2 J^3 - 2M^2 P_r^3 L_e^2 \pi^2 J - 5P_r^3 L_e^2 J^3 - 3QP_r^3 L_e^2 \pi^2 J \\ &-M^2 P_r^2 L_e^2 \pi^2 J - P_r^3 L_e^2 J^2 - M^2 P_r^3 L_e^2 \pi J - M^2 P_r^3 L_e^2 \pi^2 J^2 + P_r J^2 + M^2 P_{rM} P_r^2 L_e \pi J \\ &+ 2P_r^2 P_{rM} L_e J^2 - 4P_r^2 P_{rM} L_e^2 J^2 + 6N_A R_n P_r P_{rM} L_e a^2 - P_r P_{rM} L_e^2 J^3 - 2P_r^2 P_{rM} L_e^2 J^3 \\ &-QP_r^2 P_{rM} L_e^2 \pi^2 J - P_r^2 J^2 + P_r P_{rM}^2 J^3 - 3N_A R_n P_r P_{rM}^2 L_e a^2 + N_A R_n P_r P_{rM}^2 L_e^2 a^2 \end{aligned} \right\}.$$

The lengthy expressions of F_3 and F_4 are not required in the analysis of over-stability, so we can skip them here.

It is to be noted that Eq. (63) is a cubic equation in $\lambda (= \omega^2)$, therefore it has three roots /values which are necessarily to be real and positive. The sum of the roots of Eq. (63) is $-F_2/F_1$, therefore for oscillatory convection to occur, either F_1 or F_2 has to be negative. If both of them are of the same symbol positive or negative, the chances of oscillatory convection are excluded.

It is absorbing to note that following Buongiorno [4] and Yadav et al. [15], the parameters are assigned the following values relevant for alumina water nanofluid suspensions:

$$\phi_0^* = 0.01, \quad \rho_{f0} = 1000 \text{ kg/m}^3, \quad \mu = 10^{-3} \text{ Pas}, \quad \rho_p = 4 \times 10^3 \text{ kg/m}^3, \quad D_B = 4 \times 10^{-11} \text{ m}^2/\text{s}$$

$$D_T = 6 \times 10^{-11} \text{ m}^2/\text{s}, \quad \alpha_f = 2.0 \times 10^{-7} \text{ m}^2/\text{s}, \quad (\rho c)_p = 3.1 \times 10^{-6} \text{ J/m}^3$$

$$(\rho c)_f = 4 \times 10^6 \text{ J/m}^3, \quad \beta = 3.4 \times 10^{-3} \text{ 1/K}, \quad T_0^* - T_1^* = 1 \text{ K}, \quad T_1^* = 300 \text{ K}$$

$$L_e = 500, P_r = 5, P_{rM} = 1, N_A = 5, N_B = 7.5 \times 10^{-4}, R_n = 0.122 \text{ and } M = 0.5$$

and it is found F_1 is positive and F_2 is negative making $-F_2/F_1$ positive, a requirement for the existence of oscillatory convection (Fig. 1). Thus, the oscillatory convection exists for alumina-water nanofluid which was nonexistent when it was saturated in a porous medium [15].

4 Results and discussion

To discuss the effect of various parameters, we plot the stationary convection for fixed values of the remaining parameters other than the one whose effect is to be analyzed. The stationary curves have been drawn in Figs. 2, 3, 4, 5, 6 and 7 for the critical Rayleigh number R_a^{st} verses the nanofluid magnetic number Q .

The behaviour of the concentration Rayleigh number R_n is shown in Fig. 2. It is drawn for $L_e = 200$, $N_A = 5$ and $M = 0.5$ for different values of R_n . It is seen that the Rayleigh

number decreases with the increasing concentration Rayleigh number. Thus, the parameter R_n has destabilizing effect. It is simply a realistic situation because the Brownian motion of nanoparticles increases as volumetric ratio of nanoparticles increases.

In Fig. 3, R_a vs. Q curves are drawn for different values of modified diffusivity ratio N_A for fixed $L_e = 200$, $R_n = 0.1$ and $M = 0.5$. In analogy with the concentration Rayleigh number the increasing values of N_A are decreasing the value of Rayleigh number R_a . Hence, the modified diffusivity ratio N_A advances the onset of convection.

The effect of nanofluid Lewis number L_e has been

unveiled in Fig. 4. The critical Rayleigh number R_a is plotted against nanofluid magnetic number Q for different values of L_e for fixed $N_A = 5$, $R_n = 0.1$, $M = 0.5$. It is observed that as the value of nanofluid Lewis number L_e increases, the value of critical Rayleigh number R_a decreases. Hence, L_e inhibits the stability.

Figure 5 portrays that the behaviour of Hall current parameter M for $R_n = 0.1$, $L_e = 200$ and $N_A = 5$. It shows that increment in the value of Hall current parameter M leads to decrease the value of critical Rayleigh number R_a . Hence, the Hall current parameter M hastens the onset of convection.

An analytic comparison of the stationary convection of a nanofluid with and without Hall currents is shown in Fig. 6. It clearly shows that in the presence of Hall currents the value of Rayleigh number falls largely. Hence, the convection sets in earlier.

Yadav et al. [15] have discussed the Hall current effect convection when the nanofluid is saturated in a horizontal porous medium. A comparison of stationary convection for the two types of medium has been shown in Fig. 7. It is important to mention that in both the cases Rayleigh number is increasing with an increase in magnetic number but rate of increment is down when the layer is saturated in a porous medium. Thus, the presence of porous medium enhances the convection and stability is achieved in a continuous medium.

5 Conclusion

Bénard convection in a continuous flow medium in the presence of horizontal constant magnetic field and Hall currents has been investigated for a Newtonian nanofluid. Nanoparticle flux at the boundaries is taken to be nil. It is observed that the presence of Hall current destabilizes the stationary convection and provokes the convection. The effect of other parameters like concentration Rayleigh number, modified diffusivity ratio and nanofluid Lewis number is also destabilizing.

The comparison of the convection with the one when there are no Hall currents clearly predicts that the Hall currents are responsible to exhibit the instability of the system.

The comparison of the present configuration with the one in a porous medium says that the porous medium sets the convection earlier.

Finally, the conditional existence of oscillatory convection has been obtained. Its existence has been verified for alumina water nanofluid.

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest amongst them.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Choi SUS, Eastman JA (1995) Enhancing thermal conductivity of fluids with nanoparticles. *ASME Fed* 66:99–105
- Yu HW, France DM, Routbort JL, Choi SUS (2009) Review and comparison of nanofluid thermal conductivity and heat transfer enhancements. *J Heat Transf Eng* 29:432–460. <https://doi.org/10.1080/01457630701850851>
- Eastman JA, Choi SUS, Li S, Yu HW, Thompson LJ (2001) Anomalous increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Appl Phys Lett* 78:718–720. <https://doi.org/10.1063/1.1341218>
- Buongiorno J (2006) Convective transport in nanofluids. *ASME J Heat Transfer* 128:240–250. <https://doi.org/10.1115/1.2150834>
- Das SK, Choi SUS, Yu W, Pradeep T (2007) Nanofluids: science and technology. Wiley J & Sons, Saulnierville
- Michaelides, Efstathios ES (2014) Nanofluidics: thermodynamic and transport properties. ISBN 978-3-319-05621-0
- Tripathi D, Bég OA (2014) A study on peristaltic flow of nanofluids: Application in drug delivery systems. *Int J Heat Mass Transf* 70:61–70. <https://doi.org/10.1016/j.ijheatmasstransfer.2013.10.044>
- Wong KV, Leon OD (2010) Applications of nanofluids: current and future. *Adv Mech Eng* 2:1. <https://doi.org/10.1155/2010/519659>
- Tzou DY (2008) Thermal instability of nanofluids in natural convection. *Int J Heat Mass Transf* 51:2967–2979. <https://doi.org/10.1016/2007.09.014>
- Heris ZS, Salehi H, Noie HS (2012) The effect of magnetic field and Nanofluid on thermal performance of two-phase closed thermosiphon. *Int J Phys Sci* 7:534–543. <https://doi.org/10.5897/IJPS11.1019>
- Hamada MAA, Pop I, Ismail MdMA (2011) Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate. *Nonlinear Anal Real World Appl* 12:1338–1346. <https://doi.org/10.1016/j.nonrwa.2010.09.014>
- Ghasemi B, Aminossadati MS, Raisi A (2011) Magnetic field effect on natural convection in a nanofluid-filled square enclosure. *Int J Therm Sci* 50:1748–1756. <https://doi.org/10.1016/j.ijthermalsci.2011.04.010>
- Karcher C, Kühndel J (2016) Control of heat transfer in engine coolers by Lorentz forces. *J Phys Conf Ser* 745:032050. <https://doi.org/10.1088/1742-6596/745/3/032050>
- Hall EH (1879) On a new action of the magnet on electric current. *Am J Math* 2:287–292. <https://doi.org/10.2307/2369245>
- Yadav D, Mohamed RA, Cho H, Lee J (2015) Effect of Hall current on the onset of MHD convection in a porous medium layer saturated by a nanofluid. *J Appl Fluid Mech* 9:2379–2389. <https://doi.org/10.18869/acadpub.jafm.68.236.25048>
- Das S, Jana RN, Makinde OD (2016) Magnetohydrodynamic free convective flow of nanofluid past an oscillating porous flat plate in a rotating system with thermal radiation and Hall effects. *J Mech Eng* 32:197–210. <https://doi.org/10.1017/jmech.2015.49>
- Wahed A, Mohamed S (2017) Rotating ferro-nanofluid over stretching plate under the effect of hall current and joule heating. *J Magn Magn Mater* 429:287–293. <https://doi.org/10.1016/j.jmmm.2017.01.032>
- Abbasi FM, Maimoona G, Shehzad SA (2018) Hall effects on peristalsis of boron nitride-ethylene glycol nanofluid with temperature dependent thermal conductivity. *Physica E Low Dimens Syst Nanostruct* 99:275–284. <https://doi.org/10.1016/j.physe.2018.02.006>
- Shah Z, Islam S, Gul T, Bonyah E, Khan MA (2018) The electrical MHD and Hall current impact on micropolar nanofluid flow between rotating parallel plates. *Results Phys* 9:1201–1214. <https://doi.org/10.1016/j.rinp.2018.01.064>

20. Krishna MV, Chamkha AJ (2018) Hall effects on MHD squeezing flow of a water-based nanofluid between two parallel disks. *J Porous Media* 22:209–223. <https://doi.org/10.1615/JPorMedia.2018028721>
21. Krishna MV, Chamkha AJ (2019) Hall and ion slip effects on MHD rotating boundary layer flow of nanofluid past an infinite vertical plate embedded in a porous medium. *Results Phys* 15:102652. <https://doi.org/10.1016/j.rinp.2019.102652>
22. Shah Z, Alzahrani EO, Alghamdi W, Ullah MZ (2020) Influences of electrical MHD and Hall current on squeezing nanofluid flow inside rotating porous plates with viscous and joule dissipation effects. *J Therm Anal Calorim* 140:1215–1227. <https://doi.org/10.1007/s10973-019-09176-7>
23. Sharma J, Gupta U (2020) Nanofluid convection under Hall currents and LTNE effects. *J Mater Proces* 26:3369–3377. <https://doi.org/10.1016/j.matpr.2019.10.149>
24. Baehr HD, Stephan K (2011) *Heat and mass transfer*, 3rd edn. Springer, New York
25. Yadav D, Bhagrava R, Aggarwal GS (2013) Thermal instability in a nanofluid layer with vertical magnetic field. *J Eng Math* 80:147–164. <https://doi.org/10.1007/s10665-012-9598-1>
26. Nield DA, Kuznetsov AV (2014) Thermal instability in a porous medium layer saturated by a nanofluid: a revised model. *Int J Heat Mass Transf* 68:211–214. <https://doi.org/10.1016/j.ijheatmasstransfer.2013.09.026>
27. Chandrasekhar S (1961) *Hydrodynamic and Hydromagnetic stability*. Oxford University Press, Oxford

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.