Research Article

Multi-objective optimization of thermophysical properties of f-Al₂O₃ nano-dispersions in heat transfer oil

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Abstract

Nanofluids are proven to be the next-generation smart fluids with tunable thermal and viscous properties. Nanomaterial concentration plays a vital role in determining the heat transfer and viscous transport characteristics. An optimum concentration is generally required to regulate a feasible and economical heat transfer operation. This research involves the modeling and optimizing different temperature-dependent thermal and viscous parameters for varying concentrations of nanofluids. The nanofluids consist of functionalized alumina (f-Al₂O₃) nano-dispersions in thermal oil (highly refined mineral oil). The experimentally measured temperature-dependent nanofluids' properties are used to optimize thermophysical parameters using Response Surface Methodology. Two case studies/scenarios are considered in the present research, where the primary objective is to maximize thermal conductivity for heat transfer applications and minimize nanoparticle loadings for economical operation. The input parameters include temperature and nanoparticle loadings. The output parameters or response include thermal conductivity, viscosity, density, and specific heat of nanofluids. For case study 1, the optimal findings for the thermal conductivity, viscosity, density, and specific heat are 0.146061 W/m °C, 0.031889 Pa.s, 838.529 kg/m³ and 1533.9 J/kg °C, respectively. For case study 2, the optimal findings for thermal conductivity, viscosity, density, and specific heat are 0.13476 W/m °C, 0.0226062 Pa.s, 831.071 kg/m³ and 1791.14 J/kg °C, respectively. Although the optimal value for thermal conductivity decreased in case study 2, the nanoparticle weight % was reduced from 1 to 0.322473%.

Keywords Functionalized alumina · Heat transfer oil · Nanofluid · Optimization · RSM · Thermal conductivity

1 Introduction

Nanofluids have significant potential for the next-generation cooling media with advanced and tunable temperature-dependent properties. The enhancement in heat transfer characteristics of conventional heat transfer fluids has received extensive attention from researchers in recent decades [1, 2]. The typical conventional heat transfer fluids include water, oil, ethylene glycol, etc. The lower thermal behavior of conventional heat transfer fluids is a

crucial limitation in the heat transfer operation [3]. The dispersion of nano-sized solid particles (ranging between 1 and 100 nm) in conventional heat transfer fluids has produced promising results in heat transfer applications by increasing the system's effective thermal conductivity [4]. Nanofluids can be utilized in different heat generation, energy storage, and energy conversion applications such as nuclear cooling, microelectronics, power plants, paints and pigments, foods and beverages, etc. [5, 6].

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Optimization is one of the most important engineering tools for addressing the issues related to the increased cost of material and energy [7, 8]. The dispersion of nanomaterials in conventional liquids enhances thermal conductivity and alters heat capacity, effective viscosity, and density, which are important parameters in designing the heat transfer equipment. These parameters are highly influenced by the nanoparticle loadings and temperature of the system [9]. The thermal conductivity is the primary parameter in increasing the heat transfer performance and is highly dependent on the temperature and nanomaterial loadings. The optimum temperature and loadings of nanoparticles can provide a feasible and economical solution to the heat transfer industrial sector. The nanofluid system can become uneconomical at high nanoparticle concentrations due to the high price of nanomaterials. At present, nanofluids are expensive, and their applications in the industry seem uneconomical. However, the uneconomical factor does not necessarily depend on the material capital cost, and there are other operational factors involved, which makes nanofluid technology expensive [10]. Therefore, it is essential to determine the optimum conditions of temperature, nanomaterial concentration, which is the focus of this research study. The literature review suggests that Response Surface Methodology (RSM) can be an effective tool for optimizing process variables and provide a feasible solution [11].

Design of Experiment (DOE) is an optimizing tool, which is generally utilized to plan the experimental studies. This tool's primary function is to estimate the impact of different variables simultaneously or separately, or in combination [12]. In contrast, RSM is a statistical technique to model and analyze the association between multiple inputs and output parameters [13]. Different studies have been performed in nanofluids to optimize several parameters and provide statistical analysis and relationships among various objective functions. The contributions of Esfe and co-workers [14, 15] to optimize viscosity and thermal conductivity for different nanofluids using RSM methods are considered notable. Iranmanesh et al. [16] used the RSM model based on a 2-level factorial Central Composite Design (CCD) to optimize viscosity and thermal conductivity of nanofluids containing graphene-nanoplatelets. Hitami et al. [17] used the RSM method to determine optimum T-shaped cavity geometry. Shirvan et al. [18] used the RSM model to investigate the statistical analysis of three parameters of Cu-water nanofluid consisting of Rayleigh number, particle loadings, and shape of the nanoparticles. A numerical study was performed, and the application was focused on the free convective heat transfer and entropy generation inside a wavy cavity. The optimum status of the parameters was determined using numerical results. In another study [19], RSM sensitivity

analysis was applied to optimize pressure drop and heat transfer in a nanofluid-filled solar heat exchanger. In another study [20], optimization of heat transfer coefficient and the pressure drop was determined during forced convection operation of functionalized nanotubes-based nanofluids via RSM (multi-objective algorithm). The two-target optimizing method showed high precision and more than 0.9 regression coefficient.

In a recent study by Jing et al. [21], three types of non-Newtonian nanofluid systems were utilized in a rectangular cavity under a magnetic field with varying fin geometries. The RSM method was used to investigate the influence of fin geometry, particle loading, particle type, and Hartmann number on nanofluids' thermal characteristics such as Bejan number, Nusselt number, and entropy. The optimized conditions for the minimized Bejan number and maximized Nusselt number were determined. Chan et al. [22] utilized a multifactorial optimization and modeling design method to study the relationship between nanofluid velocity and temperature on the thermal behavior of Al_2O_3 -based non-aqueous nanofluids. In another recent study by Nasirzadehroshenin et al. [23], titania-alumina-based hybrid nanofluids were utilized to investigate thermophysical properties at varying nanoparticle loadings, sizes, and temperatures. It was concluded that all parameters significantly influence the thermal conductivity and viscosity of nanofluids. Two modeling techniques were employed, i.e., Artificial Neural Network (ANN) and RSM- Central Composite Design (CCD), to optimize and accurately predict and estimate optimum viscosity and thermal conductivity.

An earlier study by LotfizadehDehkordi et al. [24] implemented the Box–Behnken Design (BBD) model to study the impact of ultrasonic thermal conductivity parameters and the stability of nanofluids. TiO_2 water-based nanofluids were utilized to optimize the parameters (power and time) using the RSM model combined with 3-level BBD. It was suggested that the increase in thermal conductivity and stability could be simultaneously pursued to obtain optimized thermal performance of the application. Saidollah et al. [25] optimized and predicted different parameters affecting titania-based nanofluids' stability using RSM and ANN, respectively. The optimized parameters included zeta-potential and nanoparticle size using process parameters, including surfactant particle loading and nanofluids' pH. It was concluded from RSM results that the surfactant particle loadings had a significant effect on the zeta-potential. However, a substantial impact was observed by the pH of nanofluids on the size of TiO_2 nanoparticles. Montazer et al. [26] employed the quadratic model of RSM to investigate the design variables of the effective density of functionalized nanotubes and graphene nanoplatelets-based nanofluids. Nanoparticle loading and the

temperature were determined as performance factors with reasonable accuracy. A similar study [27] was performed to optimize the effective density of ZnO- and SiO₂-based nanofluids using RSM. The outcomes of the investigation showed a trained net with a good agreement between predictions and experiments.

Esfe and Motallebi [28] used RSM and ANN techniques to optimize and model heat capacity, viscosity, thermal conductivity, and heat transfer coefficient of aluminum-oil nanofluid. It was concluded that the optimum values of thermophysical properties were obtained at maximum values of temperature. Sonawane and Juwar [29] optimized thermal conductivity and viscosity of Fe₃O₄-ethylene glycol nanofluid using a full-factorial RSM. The obtained optimal conditions revealed ultrasonic time of 3.6 h, nanoparticle loading of 0.8 vol.%, and 80 °C temperature.

This research aims to find the optimum conditions of input parameters that affect thermal conductivity, density, and viscosity. The thermophysical properties of novel nanofluids, i.e., functionalized alumina (f-Al₂O₃) nano-dispersions in heat transfer oil, are experimentally measured in previous work [30]. The optimized values of the temperature, nanoparticle loadings, thermal conductivity, density, and viscosity are obtained using the desirability approach for multi-objective optimization. The optimization study for this nanofluid system has not been found in the literature to our knowledge. Two case studies are included in this research: (a) maximizing thermal conductivity focusing on the advanced thermal applications and (b) minimizing nanoparticle concentration focusing on the economic and feasible aspect of nanofluids.

The manuscript is organized into four primary sections. Section 1 (introduction) describes the significance of this research regarding recent trends in the optimization of nanofluids' thermophysical properties. Section 2 (Materials and Methodology) presents the composition of nanofluid, thermophysical properties, and design of experiment details. Section 3 (Results and Discussion) presents empirical model development of the four considered properties, adequacy, and verification tests. Two optimization case studies are considered detailed in Sect. 3.3. The concluding remarks, along with the future direction, is presented in Sect. 4.

2 Methodology

Functionalized alumina (40 nm) nanoparticles and heat transfer oil (C15–C50) based nanofluids are studied in this research. The detailed characterizations, functionalization procedure, and stability analysis can be found in previous work [30]. The highly stable nanofluids were obtained after applying a combination of functionalization techniques

and ultrasonication. A total of four nanofluid concentrations, i.e., 0 (pure base fluid), 0.5, 0.75, 1, and 3 weight %, were utilized to measure thermophysical properties such as dynamic viscosity, thermal conductivity, and density. The temperature range was selected from 25 to 55 °C. The detailed results and analysis of the thermophysical properties can be found in our previous work [30].

The design of the experiment (DoE) was used to conduct the experiments. The response or results of the experiments depends on the several inputs and their interactions. Therefore, the impact of two input parameters, such as temperature and nanoparticle concentration, were studied on several responses, i.e., thermal conductivity, viscosity, density, and specific heat, using a Face-centered central composite design. After obtaining the DoE results, the next step involved developing an empirical correlation between inputs, responses, and statistical analysis by using RSM [11, 31].

The RSM examines the influence of most and least dominant input process parameters on the thermophysical properties, which is also called the response. It is a straightforward and powerful tool to analyze multiple inputs and responses for modeling and optimization [32]. The first step involves the development of a second-order polynomial equation. RSM-based second-order polynomial Eq. 1 provides the interaction information between the input parameters and responses. The second step involves the optimization of process parameters with specific goals, either minimization or maximization of both response and input.

$$Response = \beta_0 + \sum_{i=1}^p \beta_i X_i + \sum_{i=1}^p \beta_{ii} X_i^2 + \sum_{i=1}^{p-1} \sum_{j=1}^p \beta_{ij} X_i X_j \quad (1)$$

3 Results and discussion

The DoE results using the face-centered CCD with two input parameters, i.e., temperature and nanoparticle weight percent, and their respective responses are given in Table 1.

3.1 Empirical model development for thermal conductivity (k), viscosity (μ), density (ρ), and specific heat (cp)

RSM was used to develop a quadratic polynomial empirical relation between inputs and their respective response. The analysis of variance (ANOVA) was used for optimization in the next step. It was applied to the quadratic polynomial by examining the model's significance and all the input parameters. The ANOVA developed the model's degree of

Table 1 DoE according to face-centered CCD

Sr. No	Temperature (°C)	Weight percent (%)	Thermal conductivity (W/m °C)	Density (kg/m ³)	Viscosity (Pa.s)	Specific heat (J/kg °C)
1	25	1	0.142	0.0585	853.4	1387.41
2	25	0.5	0.137	0.05597	850.7	1537.68
3	40	0.5	0.1375	0.0402	841.2	1591.308
4	55	0.5	0.138	0.022545	831.9	1670.361
5	55	0	0.13	0.021075	829.6	2052.526
6	25	0	0.134	0.05224	848.3	1819.453
7	40	0.5	0.1367	0.04241	841.2	1592.3
8	40	0.5	0.1383	0.04008	841.2	1592
9	40	0.5	0.1372	0.0411	841.2	1590.2
10	40	0.5	0.1369	0.04172	841.2	1591.5
11	40	1	0.145	0.04097	843.5	1489.836
12	40	0	0.132	0.03741	839	1933.604
13	55	1	0.147	0.023255	834.2	1591.452

freedom, the sum of squared deviations, and the mean square for each input. The input is considered significant if the model's value or p -value is less than 0.05. The ANOVA results for the thermal conductivity are provided in Table 2. The input (A, B), the square of input (A^2 , B^2), their interactions (AB), and the developed model's significance can be analyzed from Table 2.

The model's F value is 198.9, and the p -value is less than 0.05, which shows that the model is significant. The predictive model for the thermal conductivity is given by Eq. 2. The thermal conductivity is represented by k , while A and B represent the temperature and weight percent.

$$k = -0.00011A - 0.0032B - 1.2E(-07)A^2 + 0.0039B^2 + 0.13693333 \quad (2)$$

The ANOVA results of the viscosity are given in Table 3. The results show that the model is significant because the model's F value is 432.7078, and the p -value is less than

0.05. The predictive model is given in Eq. 3, where v represents the viscosity, while A and B represent the temperature and weight percent, respectively.

$$v = -0.00062A + 0.0143B - 5.1E(-06)A^2 - 0.0049B^2 + 0.0711 \quad (3)$$

The ANOVA results of the density are given in Table 4. The results show that the model is significant because the model's F value is 16351.61, and the p -value is less than 0.05.

The predictive model for density is given in Eq. 4, where d represents the density, while A and B represent the temperature and weight percent.

$$d = -0.6615A + 5.151B + 0.00049A^2 + 0.2483B^2 - 0.0167AB + 864.5 \quad (4)$$

Table 2 The model and inputs result for thermal conductivity using ANOVA

Source	Sum of squares	Degree of freedom	Mean squares	F values	p values
Model	0.000264	5	5.29E-05	198.9	< 0.0001
A	6.67E-07	1	6.67E-07	2.5	0.1574
B	0.00024	1	0.000241	904.6	< 0.0001
A^2	2.1E-09	1	2.1E-09	0.0079	0.9317
B^2	2.61E-06	1	2.61E-06	9.81	0.0165
AB	2.025E-05	1	2.03E-05	76.12	< 0.0001
Residual	1.86E-06	7	2.66E-07	—	—
Lack of fit	2.94E-07	3	9.81E-08	0.2501	0.8579
Pure error	1.56E-06	4	3.92E-07	—	—
Total	0.00026	12	—	—	—

Table 5 shows the ANOVA results for specific heat. The model's F value is 217.36, while the corresponding p -value is less than 0.05.

The predictive model is given in Eq. 5, where cp represents the specific heat, while A and B represent the temperature and weight percent, respectively.

$$cp = 4.335A - 865.61B + 0.030955A^2 + 458.69B^2 - 0.9677AB + 1707.54 \quad (5)$$

Table 3 The model and inputs result for viscosity using ANOVA

Source	Sum of squares	Degree of freedom	Mean squares	F values	p values
Model	0.001702011	5	0.00034	432.7078	< 0.0001
A	0.001661171	1	0.001661	2111.625	< 0.0001
B	0.000024	1	0.000024	30.50799	0.0009
A^2	3.70373E-06	1	3.7E-06	4.70805	0.0666
B^2	4.14808E-06	1	4.15E-06	5.272905	0.0553
AB	4.1616E-06	1	4.16E-06	5.290086	0.0550
Residual	5.50675E-06	7	7.87E-07	–	–
Lack of fit	1.55587E-06	3	5.19E-07	0.525073	0.6882
Pure error	3.95088E-06	4	9.88E-07	–	–
Total	0.001707518	12	–	–	–

Table 4 The model and inputs result for density using ANOVA

Source	Sum of squares	Degree of freedom	Mean squares	F values	p values
Model	569.5543	5	113.9109	16,351.61	< 0.0001
A	535.815	1	535.815	76,914.87	< 0.0001
B	33.60667	1	33.60667	4824.151	< 0.0001
A^2	0.034688	1	0.034688	4.979375	0.0608
B^2	0.01064	1	0.01064	1.527401	0.2564
AB	0.0625	1	0.0625	8.971715	0.0201
Residual	0.048764	7	0.006966		
Lack of Fit	0.048764	3	0.016255		
Pure Error	0	4	0		
Total	569.6031	12			

Table 5 The model and inputs result for specific heat using ANOVA

Source	Sum of squares	Degree of freedom	Mean squares	F values	p values
Model	3.968E+05	5	79,361.74	217.36	< 0.0001
A	54,111.25	1	54,111.25	148.21	< 0.0001
B	2.979E+05	1	2.979E+05	815.86	< 0.0001
A^2	134.33	1	134.33	0.3679	0.5633
B^2	36,319.03	1	36,319.03	99.47	< 0.0001
AB	210.70	1	210.70	0.5771	0.4723
Residual	2555.77	7	365.11		
Lack of fit	2553.16	3	851.05	1304.55	< 0.0001
Pure error	2.61	4	0.6524		
Total	3.994E+05	12			

3.2 Adequacy and verification tests of the developed models

The next step after the model's development was to check for the adequacy test using normal plots for residuals. An adequate model must not follow any trend or sequence, and the points must fall close to the straight line. The residual plots for the model of the (a) thermal conductivity, (b) viscosity, (c) density, (d) specific heat are shown in Fig. 1. It can be observed from Fig. 1 that the experimental data is close to the straight line, distributed randomly, and are not following any pattern. These results indicate that the model is reliable for optimization and prediction. After the adequacy test, the

model was checked using the outlier plot. The plots of thermal conductivity, viscosity, and density are shown in Fig. 2a, b, c, and d, respectively. Those data points which are outside the permissible range (± 3.5) will be considered abnormal. All data points lie within the permissible range for the developed model, as illustrated in Fig. 2.

The detailed analysis and verification tests showed that the present model could be used for the prediction. The effect of input parameters on response can be best described and understood with a 3D surface shown in Fig. 3. There are four 3D surface plots presented in Fig. 3: thermal conductivity, viscosity, density, and specific heat. Figure 3a shows that thermal conductivity significantly increases with the increase in weight percent

Fig. 1 Normal plot of residual of **a** thermal conductivity; **b** viscosity; **c** density; **d** specific heat

Fig. 2 Normal plot of residual of **a** thermal conductivity; **b** viscosity; **c** density; **d** specific heat

compared to temperature. Figure 3b illustrates the opposite behavior: by increasing weight %, the viscosity increases and decreases with the temperature increase. Figure 3c is drawn between density and input parameters. It can be observed from Fig. 3c that by increasing the temperature, the density substantially decreases due to the expansion of nanofluid volume with constant mass. The density of nanofluid increased upon increasing the weight percent, which is due to the considerable difference in nanoparticle and base fluid densities. The specific heat significantly increases with the increase in weight %.

After the verification tests, the developed model was validated using experimental data. The developed models (Eqs. 2, 3, 4, and 5) were validated using experimental values. The comparison between experimental and predicted results is shown in Fig. 4, and it is evident from the results that both are in good agreement.

3.3 Optimization cases

The present research included two case studies based on the different applications of nanofluid. Both case studies are non-dominated with respect to each other, which

Fig. 3 3D response surface plot of **a** thermal conductivity; **b** viscosity; **c** density; **d** specific heat

means each case study is neither best nor worse than another. The decision-maker needs to analyze the solution according to their requirements, including design, operation, and the final product specification.

The first scenario involves the maximization of the thermal conductivity of nanofluids. Generally, nanomaterials in the fluid increase the fluid's overall thermal conductivity, which is beneficial for different efficient heat transfer applications. However, it also increases the fluid's overall density and viscosity, which has an adverse impact on the heat transfer applications, especially convective heat transfer processes. Therefore, the combined effect of all properties is necessary to be considered to understand the

thermal and viscous transport of nanofluids. In case study 1, the emphasis was given on nanofluids' application in different heat transfer operations. Therefore, the primary objective, in this case, was to maximize the thermal conductivity of nanofluids. The potential applications for this case study include heat exchangers (plate, tubular, helical, double pipe, multichannel, microchannel, shell, and tube), electronic cooling devices, double-pane windows, thermosyphon, natural convection processes, engine cooling management, nuclear cooling, drilling fluids, reactor cooling jackets, nano lubricants, etc. The second scenario (case study 2) corresponds to the minimizing of nanoparticle concentration. Nanoparticles are expensive materials, and

Fig. 4 Validation of the model for **a** thermal conductivity, **b** viscosity, **c** density, and **d** specific heat

nanoparticles' preparation (either top-down or bottom-up technique) is still costly to date. This is one of the main challenges in the commercialization of nanofluids. Therefore, optimizing the quantity of nanomaterials in the nanofluid system is vital for economical and feasible operation and potential commercialization at an industrial scale.

3.3.1 Case study 1

Case study 1 comprises the computation of the optimum condition of input parameters to maximize thermal conductivity and specific heat while minimizing viscosity and density. The optimization goals, upper and lower

limit of input parameters, and responses are given in Table 6. Multi-objective optimization offers a unique solution in which input parameters and their interaction provide the best results for the multiple responses. It is the special focus of this research. The combined optimization solutions for case study 1 are summarized in Table 7.

The best solution with the highest desirability (0.527) was selected from Table 7. The ramp function for inputs and all responses is shown in Fig. 5. The 3D plot of temperature, weight %, and desirability, along with the bar graph for case study 1, is presented in Fig. 6. The optimum values for the input parameters

Table 6 Optimization goals, factors, and ranges for case study 1

Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
A:temperature	Minimize	25	55	1	1	3
B:weight %	Maximize	0	1	1	1	3
Thermal conductivity	Maximize	0.13	0.147	1	1	3
Density	Minimize	829.6	853.4	1	1	3
Viscosity	Minimize	0.02107	0.0585	1	1	3
Specific heat	Maximize	1387.41	2052.53	1	1	3

Table 7 Combined optimization solutions

Number	Temperature (°C)	Weight percent (%)	Thermal conductivity (W/m °C)	Density (kg/m ³)	Viscosity (Pa.s)	Specific heat (j/kg °C)	Desirability
1	48.035	1.000	0.146	838.529	0.032	1533.899	0.527
2	47.842	1.000	0.146	838.650	0.032	1532.678	0.526
3	47.163	1.000	0.146	839.078	0.033	1528.392	0.526
4	49.119	1.000	0.146	837.846	0.031	1540.812	0.526
5	46.142	1.000	0.146	839.724	0.034	1521.997	0.524
6	50.859	1.000	0.147	836.752	0.028	1552.068	0.516

Fig. 5 Ramp function the optimized solution case study 1

such as temperature and weight % are 48.0347 °C and 1%, respectively. The optimum response value for thermal conductivity is 0.146061 W/m °C, viscosity is 0.031889 Pa.s, density is 838.529 kg/m³ and specific heat is 1533.9 j/kg °C.

3.3.2 Case study 2

The second optimization case study focuses on minimizing weight %, viscosity, and density while maximizing the thermal conductivity, specific heat, and input

temperature. The optimization goals, upper and lower limit of input parameters, and responses are given in Table 8. The combined optimization solution is tabulated in Table 9.

The best solution with the highest desirability (0.685) is selected from Table 9.

The ramp function for inputs and all responses are shown in Fig. 7. The 3D plot of temperature, weight %, and desirability, along with the bar graph for case study 2, is shown in Fig. 8. The optimum values for the input parameters such as temperature and weight % are 55 °C and

Fig. 6 3D desirability plot and bar graph for case study 1

Table 8 Optimization goals, factors, and ranges for case study 2

Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
A:temperature	Maximize	25	55	1	1	3
B:weight %	Minimize	0	1	1	1	3
Thermal conductivity	Maximize	0.13	0.147	1	1	3
Density	Minimize	829.6	853.4	1	1	3
Viscosity	Minimize	0.021075	0.0585	1	1	3
Specific heat	Maximize	1387.41	2052.53	1	1	3

Table 9 Combined optimization solutions

Number	Temperature (°C)	Weight percent (%)	Thermal conductivity (W/m °C)	Density (kg/m ³)	Viscosity (Pa.s)	Specific heat (J/kg °C)	Desirability
1	55.000	0.322	0.135	831.071	0.023	1791.141	0.685
2	55.000	0.326	0.135	831.085	0.023	1789.059	0.685
3	55.000	0.332	0.135	831.115	0.023	1784.955	0.685
4	55.000	0.347	0.135	831.178	0.023	1776.207	0.685

0.322473%, respectively. The optimum response values for thermal conductivity, viscosity, density, and specific heat are 0.13476 W/m °C, 0.0226062 Pa.s, 831.071 kg/m³ 1791.14 J/kg °C, respectively.

It is evident from the results that the optimum value of thermal conductivity is significantly low compared to case study 1. Meanwhile, the nanoparticle's weight % is also low in case study 2, which is a tradeoff. For the application where a comparatively low thermal conductivity of nanofluid is required, the case study 2 optimum conditions are suitable due to less nanoparticle quantity, which could positively impact overall economics.

4 Conclusions

RSM based optimization study was conducted for the thermophysical properties of f-Al₂O₃ nanoparticle's dispersion in heat transfer oil. Two input parameters, such as nanofluid concentration and temperature, were used to study the effect on optimization. An empirical model was developed and validated using experimental results, and good agreement was found. For different applications of nanofluids, two optimization case studies have been investigated. For case study 1, the optimal input parameter, such as temperature and weight

Fig. 7 Ramp function the optimized solution case study 2

Fig. 8 3D plot and bar graph for the optimum solution for the combined case study 2

%, are 48.0347 °C and 1%, respectively. The corresponding optimum response values for thermal conductivity, viscosity, density, and specific heat are estimated as 0.146061 W/m °C, 0.031889 Pa.s, 838.529 kg/m³, and 1533.9 j/kg °C. For case study 2, the optimum temperature and weight % values are 55 °C and 0.322473%, respectively. The optimum response values for thermal conductivity, viscosity, density, and specific heat are 0.13476 W/m °C, 0.0226062 Pa.s 831.071 kg/m³ and 1791.14 j/kg °C, respectively.

The multi-objective optimization of nanofluids' thermophysical properties using RSM is effective in determining the feasible range of conditions for the heat transfer

operation. However, a wide range of the experimental input data should be considered in future investigations to get a clearer and precise perspective on real conditions' applicability. The authors intend to expand this research with more objective functions and consider nanofluids' applications other than heat transfer. It is recommended to include more parameters in the optimization process that may affect nanofluid operation.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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