Nature-inspired and hybrid optimization algorithms on interval Type-2 fuzzy controller for servo processes: a comparative performance study

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Abstract

In this paper, performance evaluations of six well-known nature-inspired algorithms have been reported containing genetic algorithm, cuckoo search, particle swarm optimization, differential evolution, bee colony, and combined particle swarm optimization and differential evolution (CPSODE) algorithms. Based on these optimization algorithms, input and output scaling factors of an interval Type-2 fuzzy PID controller (IT2-FLC) are chosen for closed-loop servo tracking. Optimal values of the scaling factors are chosen by minimization of the objective function which is defined based on the closed-loop controller performance criteria. A detailed comparative analysis is reported based on the simulation and experimental results. Performance analysis reveals that improved performance, reliability, robustness, and lesser noise sensitivity are reported by IT2-FLC with the optimal values obtained by the hybrid algorithm CPSODE.

Keywords Genetic algorithm (GA) · Particle swarm optimization (PSO) · Differential evolution (DE) · Cuckoo search (CS) · Bee colony (BC) · Combined particle swarm optimization and differential evolution (CPSODE) algorithms · Interval Type-2 fuzzy controller · Servo tracking process

1 Introduction

Over the last decade, various nature-inspired algorithms have been reported to optimize a wide range of real-world engineering problems [1–5]. Considerable success of these algorithms remains in their capability to produce high-quality solution within reasonable time. Problem solving through optimizations usually becomes complex with the size of optimization parameters involved, space complicity, length of execution time, and many other factors. Genetic algorithm (GA) [6, 7], particle swarm optimization (PSO) [8], differential evolution (DE) [9], bee colony (BC) [10], cuckoo search (CS) [11], and recently developed hybrid algorithm like combined PSO and DE (CPSODE) [12] are some of the most popular and very recent nature-inspired algorithms

which are quite well accepted by the engineering community. In the reported study, these optimization algorithms have been exercised toward performance enhancement of a servo tracking process.

DC servo system positioning is widely used in various motion control applications for robotic manipulators [13, 14]. However, nonlinearity of a motor and uncertainty have significant role in influencing the closed-loop performance where load changes abruptly. Thus to achieve superior closed-loop response in such applications, interval Type-2 fuzzy logic controller [15–20] is considered to be an appropriate choice. Literature survey reveals that IT2-FLC [21–26] has been successfully implemented and found to be more competent than classical controllers in various applications [13, 14, 24, 25, 27, 28]. To encompass

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the uncertainties in a systematic manner, Zadeh [15] introduced the concept of fuzzy sets, and especially for Type-2 fuzzy sets, footprint of uncertainty (FOU) in membership function (MF) [18–20] has the ability for managing the vagueness and the uncertainty present in the process model [18–20, 22, 24]. This fact has been substantiated by a number of works reported in the literature where Type-2 fuzzy sets [21–26, 28, 29] are found to be more capable to perform satisfactorily with higher degree of uncertainties compared to Type-1 fuzzy sets [27]. The only limitation remains with its computational complicity; however, it can be reduced considerably by choosing center-of-sets (COS)-type reduction method [20].

Here, well-known double-input-type fuzzy PID (FPID) [30–32] controller is considered where the choice of scaling factors (SFs) has an important role toward desired closed-loop performance. Usually, the values of the scaling factors are chosen by trial-and-error method. However, in the proposed work for proper designing of the IT2-FLC popular heuristic search algorithms like GA [6, 7], PSO [8], DE [9], BC [10], CS [11], and CPSODE [12] are exercised to obtain the optimal values of the input–output SFs. A comparative study has also been reported depicting the performance comparison among the different reported algorithms under load disturbance, model uncertainty, and measurement noise. Subsequent section, i.e., Section 2, briefly describes the prior mentioned optimization algorithms. Section 3 includes the details of the interval Type-2 fuzzy controller (IT2-FLC). Section 4 describes the experimental results related to servo tracking involving SRV02 [33] and QUBE-Servo-2 [34] along with the optimal scaling factors for the concerned IT2-FLC. A comparative study has also been made based on the simulation and experimental results, and finally, the conclusion is provided at the end.

2 Nature-inspired optimization algorithms

2.1 Genetic algorithm (GA)

GA is the most widely accepted optimization algorithm, primarily designed by Jon Holland [6]. GA [7] employs multi-directional search by initiating the process through a population of possible solutions, and it does not suffer from local optima as it has higher probability of escaping it. This algorithm starts with a group of chromosome known as population where the population size is related to the complexity of the problem. The initial choice of population is done randomly, and this algorithm employs

three operations (selection, crossover, and mutation) on the population to reach toward the optimal solution.

‘Selection’ procedure includes the picking up of members of the mating pool from the current population with the help of proper selection procedure. The purpose of this action is to improve the solution over successive generations. ‘Crossover’ operation is helpful to share information among the chromosomes and propagate to the new generation of population as offspring of parent chromosome. Hence, the ‘crossover’ operation makes GA to enlarge the search space. The crossover probability ρ_c is the probability of crossover that takes place between two parent chromosomes where ρ_c has some predefined value. If $\rho_c \geq r$ (where r is a random number, $r \in [0, 1]$), then crossover occurs; otherwise, the parent chromosome is directly copied to the mating pool. For crossover, a crossover point is randomly generated integer in the range character length of the chromosome. The crossover operation consists of swapping the segments of the parent chromosomes from the one crossover point or multi-point crossover. Here, we have done crossover using swapping of the segments of parent chromosome. ‘Mutation’ operation imparts a small change at a random position within a chromosome. The intention of mutation in GA is to salvage from local optima and explore adequately every region of the search space. The mutation probability (ρ_m) is intentionally kept low; otherwise, it will disturb the search space too much and have difficulty in convergence.

2.2 Particle swarm optimization (PSO)

PSO technique is inspired by fish and bird swarm intelligence, and it was originally reported by Kennedy et al. [8]. In PSO algorithm, an optimization problem under investigation is presented by the position of a particle vector. Each particle has a position as well as a current velocity, which represents the magnitude and direction toward achieving a better solution. In each iterative cycle, the fitness of each particle is evaluated based on an objective function $f(x)$. At every iteration, velocity ($v(t)$) for each particle is updated based on its current velocity and local as well as global swarm information as given by Eq. 1a. Subsequently, new position for each particle is updated using Eq. 1b:

$$v(t+1) = w \cdot v(t) + (c_1 \cdot r_1(p(t) - x(t))) + (c_2 \cdot r_2(g(t) - x(t))) \quad (1a)$$

$$x(t+1) = x(t) + v(t+1) \quad (1b)$$

In Eq. 1a, present velocity is obtained based on the weighted velocity of the previous sampling instant. The other parameters of Eq. 1a are the local ($p(t)$) and global ($g(t)$) positional information of the particle. Here, c_1 and c_2 are the cognitive components related to the local and global weight, respectively, and both of them are considered to be two ($c_1 = c_2 = 2$). In Eq. 1a, r_1 and r_2 are the random variables and their values are in the range $[0, 1]$ and w is the inertia weight whose value decreases from 0.9 to 0.4 with the number of iteration. Such technique of choosing inertia weight offers better performance compared to random choice of inertia weight. Based on some predefined criteria related to the convergence of fitness value defined as the reciprocal of the objective function ($f(x)$), optimal particle position vector is achieved after a number of iterations.

2.3 Cuckoo Search (CS)

Cuckoo Search (CS) is one of the most popular nature-inspired metaheuristic algorithms, developed by Yang and Deb [11]. It incorporates Lévy flight method rather than simple isotropic random walks which makes CS potentially efficient than GA. As per the CS algorithm, cuckoo lays one egg at a time and dumps it in a random nest. Secondly, the best nests with high quality will be transferred to the next generation. Thirdly, the available host nest number remains fixed, and the eggs laid are discovered by the host bird with a probability (ρ_a) within 0 and 1. In such case, the host bird can determine either to get rid of the egg or simply abandon the nest and build a new nest (with new random solutions). For minimization problem, the fitness function is reciprocal of the objective function.

The Lévy flight method used while generating a new solution for i th cuckoo is given by Eq. 2:

$$x_i(t+1) = x_i(t) + \alpha \oplus \text{Lévy}(\lambda) \quad (2)$$

Here, Lévy flight ($1 < \lambda \leq 3$) is a random walk phenomenon, while the random step length is drawn from a Lévy's distribution, which has an infinite variance with an infinite mean. For this reason, CS has the global convergence property stronger than PSO. As a result, PSO may converge prematurely to local optima, whereas CS may converge to global optima. In Eq. 2, x_i is the chosen nest by the i th cuckoo and the value of step size (α) is considered to be unity.

2.4 Differential evolution (DE)

DE is a stochastic, population-based optimization algorithm introduced by Storn and Price in 1996 [9]. This algorithm has been successfully used in different areas of

science and engineering. The success and appreciation of DE algorithm crucially depend on appropriate selection of trial vector generation strategies and their control parameters. It mainly involves trial-and-error scheme for searching. In DE, three crucial control parameters are population size (NP), scaling factor (F), and crossover rate (CR). These parameters need to be properly tuned to successfully solve an optimization problem. This algorithm undergoes initialization of the population, subsequently mutation, followed by recombination. After recombination, it again goes through selection process and if the termination criterion is not reached, then same steps are repeated for the next generation with the selected candidates of the population.

DE algorithm involves a population NP with d -dimensional individual vectors, $X_{i,G} = (x_{i1,G} + x_{i2,G} + \dots + x_{id,G})$ where $i = 1, 2, \dots, NP$, from one generation to the next. X_i is initialized, and then at each generation G, each target vector employs mutation and forms mutant vector by their weighted difference

$$V_{i,G} = \{v_{i1,G}, v_{i2,G}, \dots, v_{iNP,G}\} \quad \text{and}$$

$$V_{i,G+1} = X_{ir1,G} + F(X_{ir2,G} - X_{ir3,G}). \quad (3a)$$

In Eq. 3a, F is the scaling factor lies in between 0 and 1. $X_{ir1,G}, X_{ir2,G}, X_{ir3,G}$ are randomly selected $r = 1, 2, \dots, NP$. After mutation and crossover with each pair of mutation vector, the trial vector $U_{i,G+1}$ for each individual $X_{i,G+1}$ is formed:

$$u_{ij,G+1} = \begin{cases} v_{ij,G+1} & \text{if } \text{rand}_j(0, 1) \leq \text{CR or } j = j_{\text{rand}} \\ x_{ij,G+1} & \text{otherwise} \end{cases} \quad (3b)$$

where $i = 1, 2, \dots, NP$ and $j = 1, 2, \dots, d$. j_{rand} is a random number $[1, d]$, and CR is the crossover rate. Subsequently, to decide whether to select or not $U_{i,G+1}$ for the next generation fitness function $f(U_{i,G+1})$ and $f(X_{i,G+1})$ are compared:

$$X_{i,G+1} = \begin{cases} U_{i,G} & \text{if } f(U_{i,G}) < f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \quad (3c)$$

The success and appreciation of DE algorithm crucially depend on appropriate selection of trial vector generation and the control parameters.

2.5 Bee colony (BC)

The bee colony (BC) algorithm is proposed by Karaboga in 2005 [10] which simulates the foraging behavior of bee colony. This algorithm is based on the characteristics observed in nature by bees' colony like it can extend itself over in multiple direction and over long distances (more than 10 km) to avail a large number of food sources, and have the capacity of memorization, learning and

transmission of information in colony, and thus forming the swarm intelligence.

In BC algorithm for the minimal model of swarm-intelligent forages selection in a honey bee colony, it consists of three kinds of bees: employed bees (n_e), onlooker bees (n_o), and scout bees (n_s). Half of the colony consists of employed bees, and the other half includes onlooker bees. The responsibility of the employed bees is to exploit the nectar sources explored before and convey information to the waiting onlooker bees in the hive about the quality of the food source sites which they are exploiting. Based on the shared information of the employed bee, the onlooker bees wait in the hive and decide on a food source to exploit. If the source is abandoned, she becomes a scout and starts to randomly search the environment in order to find a new food source.

2.6 Hybrid algorithm CPSODE

To minimize the limitations involved with PSO [8] such as trapping at local minima and premature convergence, PSO is combined with DE [9] and it results CPSODE [12]. This algorithm has a novel feature which incorporates cooperation mechanism into the evolution of search agents. To obtain the minima, total population is divided into two groups. One group employs PSO evolution strategy, and the other adopts DE algorithm. In this algorithm, each particle is evolved based on cooperation information sharing of other particles. A certain threshold of trapped particles is there, i.e., if threshold S is reached, the trapped particles will be replaced by new particles from the other evolution group.

In CPSODE, the PSO group is denoted by G_{PSO} and the DE group by G_{DE} . Each particle position can be denoted as M -dimensional search space with P number of particles in G_{PSO} and D number of particles in G_{DE} . Initialization of each particle by individual DE and PSO strategy are done. Next, i th particle position can be represented as for PSO group, $P_i = (p_{i1}, p_{i2}, \dots, p_{im})$, $i = 1, 2, \dots, P$. Each particle contains its previous best position, $B_i = (b_{i1}, b_{i2}, \dots, b_{im})$, and velocity, $V_i = (v_{i1}, v_{i2}, \dots, v_{im})$, $i = 1, 2, \dots, P$. B_g represents the best solution over the whole population. After each iteration step, the particle updates by velocity and position by Eqs. 4a and 4b:

$$V_{im}(t+1) = wV_{im}(t) + c_1 \times r_1 \times (B_{im} - P_{im}) + c_2 \times r_2 \times (B_{gm} - P_{im}) \quad (4a)$$

$$P_{im}(t+1) = P_{im}(t) + V_{im}(t+1) \quad (4b)$$

where B_{im} is the m th vector for the best position of particle i and B_{gm} is the m th vector for the best position of the current population. Here, $r_1, r_2 \in [0, 1]$ are uniformly

generated random number, and c_1, c_2 are the acceleration coefficients.

After certain number of iterations, some stagnant individuals in the population G_{PSO} and G_{DE} can be created. They are counted by

$$\text{Cnt}(t+1) = \begin{cases} 1 & \text{if } (f(X_i^t) \leq f(X_i^{t-1}) \leq f(X_i^{t-2})) \\ 0 & \text{otherwise} \end{cases} \quad (4c)$$

The number of trapped particles in G_{PSO} and G_{DE} can be calculated by

$$\text{Cnt}_{\text{PSO}} = \sum_{i=1}^P \text{Cnt}_i, i \in G_{\text{PSO}}; \text{Cnt}_{\text{DE}} = \sum_{j=1}^P \text{Cnt}_j, j \in G_{\text{DE}} \quad (4d)$$

When Cnt_{PSO} or Cnt_{DE} reaches stopping criteria S , then the trapped particles will be replaced by the filial generation of two particles from the other group. The mutation strategy is similar to DE. The new particle for the trapped particle becomes:

$$P_i^{t+1} = P_{\min} + \text{rand}(0, 1) \times (P_{\max} - P_{\min}) \quad (4e)$$

where $(P_{\max}, P_{\min})$ denotes the range of the other population. The counter of particles i will be reset to zero, $\text{Cnt}_i(t+1) = 0$.

Here, CPSODE algorithm is utilized for proper designing of the Mamdani-type interval Type-2 fuzzy logic controller designing for servo tracking applications. Here, the optimized values for the scaling factors are searched out for designing of a Type-2 fuzzy controller which is capable to provide superior closed-loop performance.

3 Interval Type-2 Fuzzy Logic controller

The concept of Type-2 fuzzy set was introduced by Zadeh [15] in 1975. Further discussions of Type-2 fuzzy sets are elaborated in [18–20]. Literature survey reveals that most of the reported works [29, 30] related to Type-2 fuzzy logic controller designing involve Type-2 fuzzy sets where all points in the FOU have unity secondary membership grade. The use of interval Type-2 fuzzy set reduces computational complexity which is the prominent drawback of Type-2 FLC compared to Type-1 FLC [27]. Here, the designing optimal interval Type-2 FLC is shown in Fig. 1 where the crisp inputs from the input sensors are first fuzzified into input MFs which are Type-2 fuzzy sets. Inference engine with the help of rule base and data base produces output MFs which are also Type-2 fuzzy sets. Interval Type-2 FLC has the similar inference engine and rule base as with Type-1, only the input and output MFs are of interval Type-2 fuzzy sets. The inference engine combines the fired rules, and it maps the input Type-2 sets to output Type-2

Fig. 1 Block diagram of T2-FPID

fuzzy sets. Type reducer is used to convert the Type-2 fuzzy output sets into Type-1 fuzzy sets. Popular type reduction methods are KM method [18, 20], COS method [20], and uncertainty bound method [18]. After the type reduction, Type-1 output fuzzy sets are defuzzified [20] and the crisp output is obtained [18, 19].

In the literature, different types of Type-2 fuzzy controllers [30–32] are reported. Here, a simple but effective fuzzy PI + fuzzy PD-type controller structure is considered with a two-dimensional linear rule base and Mamdani-type inferencing [31] as shown in Fig. 1. For simplicity and ease of implementation, triangular MFs for error (e), change of error (Δe), and U (control action) with five fuzzy sets are employed as shown in Fig. 2. Observe that Fig. 2 shows

Fig. 2 Membership function of Type-2 fuzzy sets for e , Δe , u , Δu

fuzzy partitions for the same universe of discourse of input (e , Δe) and output (u) linguistic variables. The controller output u is determined by rules of the form: If e is E and Δe is ΔE , then u is U ,

$$\text{where } e(k) = r - y(k) \quad (5)$$

$$\Delta e(k) = e(k) - e(k - 1) \quad (6)$$

The input variables (e) and $\Delta(e)$ are partitioned by N number of membership functions, and assuming that for each possible combination of the N sets, there is a rule for this two domain inputs X_1, X_2 , and the rule base comprises N^2 rules. The change in controller output u is determined by the n th rule:

If e is $\tilde{X}_1$ and Δe is $\tilde{X}_2$, then u^n is U^n ($i, j = 1, 2, \dots, N$) and U^n the distinct Type-2 fuzzy set and $[u^n, \bar{u}^n]$ an interval. Next, we compute the membership of input variable $\tilde{X}_1, \tilde{X}_2$ as $[\mu_{\tilde{X}_1}^-(e), \mu_{\tilde{X}_1}^+(e)]$ and $[\mu_{\tilde{X}_2}^-(\Delta e), \mu_{\tilde{X}_2}^+(\Delta e)]$ for $i, j = 1, 2, \dots, N$. $\mu_{\tilde{X}_1}^-(e)$ and $\mu_{\tilde{X}_1}^+(e)$ are the upper and lower membership grades of the interval firing set $\mu_{\tilde{X}_1}^-(e)$. Similarly, $\mu_{\tilde{X}_2}^-(\Delta e)$ and $\mu_{\tilde{X}_2}^+(\Delta e)$ are the lower and upper membership grades of input variable $\mu_{\tilde{X}_2}^-(\Delta e)$.

Next, we compute the firing interval of the n th rule:

$$F^n(e, \Delta e) = [f^n(e, \Delta e), \bar{f}^n(e, \Delta e)], \text{ where } n = 1, \dots, N^2. \quad (7)$$

$$= \left[\mu_{\tilde{X}_1}^-(e) \times \mu_{\tilde{X}_2}^-(\Delta e), \mu_{\tilde{X}_1}^+(e) \times \mu_{\tilde{X}_2}^+(\Delta e) \right], \quad (8)$$

$i = 1, 2, \dots, N$ and $j = 1, 2, \dots, N$

With the $F^n(e, \Delta e)$ and the corresponding rule consequents, we perform the center-of-sets-type reduction (COS):

$$U_{\text{COS}(e, \Delta e)} = \bigcup_{\substack{f^n \in F^n(e, \Delta e) \\ u^n \in U^n}} \frac{\sum_{n=1}^N f^n u^n}{\sum_{n=1}^N f^n} = [u_l, u_r] \quad (9a)$$

$$u_l = \frac{\sum_{n=1}^L \bar{u}^n \bar{f}^n + \sum_{n=L+1}^N \underline{u}^n \bar{f}^n}{\sum_{n=1}^L \bar{f}^n + \sum_{n=L+1}^N \underline{f}^n}; u_r = \frac{\sum_{n=1}^R \bar{u}^n \bar{f}^n + \sum_{n=R+1}^N \underline{u}^n \bar{f}^n}{\sum_{n=1}^R \bar{f}^n + \sum_{n=R+1}^N \underline{f}^n} \quad (9b)$$

The switching points L and R are determined by Karnik and Mendel (KM) [20] algorithm such that $\underline{u}^L \leq u_l \leq \underline{u}^{L+1}$ and $\bar{u}^R \leq u_r \leq \bar{u}^{R+1}$. Finally, computation of the defuzzified output is

Table 1 Fuzzy control rule base

$\Delta e/e$	NB	NM	ZE	PM	PB
NB	NB	NB	NB	NM	ZE
NM	NB	NB	NM	ZE	PM
ZE	NB	NM	ZE	PM	PB
PM	NM	ZE	PM	PB	PB
PB	ZE	PM	PB	PB	PB

$$U = \frac{u_l + u_r}{2}, \quad (10)$$

The rule base consisting of 25 rules for computing u is shown in Table 1, which is a commonly used rule base designed in sliding mode principle [35]. From Fig. 1, we find that the final controller output U is obtained by the relation:

$$U = G_{PD} \cdot u + G_{PI} \cdot \Delta u \quad (11)$$

In this IT2-FLC of fuzzy PID [30, 31] type, the values of the actual inputs e and Δe are mapped to the interval $[-1, 1]$ by the input SFs, G_e , and $G_{\Delta e}$, respectively. The defuzzified output u_N is translated into the actual output u by the output SF, G_{PI} , and G_{PD} . Initial settings with suitable values of G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} are found based on the knowledge about the process to be controlled and sometimes through trial. Here in this paper, we have used some heuristic algorithms to find the optimal values of the two input scaling factors for error G_e , change of error $G_{\Delta e}$ and two output scaling factor G_{PI} , and G_{PD} .

4 Results and settings

In order to understand the effect of optimization on the scaling factors of the interval Type-2 fuzzy PID controller, both simulation and real-time experimentation have been done. The improvement of the IT2-FLC without optimization and with optimization is demonstrated in terms of different performance indices—percentage overshoot (% O.S.), rise time (t_r), settling time (t_s), integral absolute error (IAE), integral time absolute error (ITAE). Noise sensitivity of the IT2-FLC and optimal IT2-FLC is also evaluated in the presence of measurement noise.

4.1 Optimization of parameters

In this paper for the optimal value selection of the four parameters G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} of the interval Type-2 fuzzy controller by the above-mentioned optimization algorithms CS, DE, PSO, GA, CPSODE, and BCA [6–12], a minimum value of the objective function is required. To achieve an optimal solution for set-point tracking and load

Table 2 Tuning values of the parameters for PSO, GA, CS, DE, and BCA

Algorithm	Parameter	Value
PSO	c_1, c_2 , population, Max. iteration	2, 2, 50, 100
GA	ρ_c, ρ_m , selection operator, total chromosomes, Max. iteration	6.667, 0.2, 50, 100
CS	ρ_a , nest, Max. iteration	0.25, 50, 100
DE	F, CR, population, Max iteration	0.45, 0.5, 50, 100
BCA	Colony size, n_e, n_o, n_s , limit	50, 50% of the colony, 50% of the colony, 1, 2.
CPSODE	$c_1, c_2, w_{\min}, w_{\max}$, F, CR, population, Max. iteration	2, 2, 0.4, 0.9, 0.8, 0.5, 40, 100

rejection phases, the following objective function $f(x)$ is considered involving absolute value of overshoot (%OS), settling time (t_s), and integral absolute error (IAE):

$$\text{Objective function} = \text{abs}(\%OS) + t_s + \text{IAE} \quad (12)$$

Each optimization algorithm is carried for 100 iterations and executed for 20 runs by proper setting of the parameters which are given in Table 2.

The optimization algorithms have been used to achieve the minimum value of the objective function which is based on the closed-loop performance using the Type-2 fuzzy controller in servo system. In this paper, using the above-mentioned optimization algorithms proper selection of the four parameters G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} , for the interval Type-2 fuzzy controller in the servo system: (i) nominal model and two experimental setup Quanser, Canada made (ii) SRV02, and (iii) QUBE-Servo-2, has been successively done. The optimal results of PSO, GA, BCA, CPSODE, DE, and CS for the different systems are given in Tables 3, 4, and 5. The servo system descriptions on which the experiments are performed are discussed in the following section.

4.2 System description

Quanser SRV02 In this section, we present the Quanser, Canada made SRV02 [33] rotary servo plant, pictured in Fig. 3 consisting of a DC motor enclosed in a solid aluminum frame and equipped with a planetary gear box. The motor has its own internal gearbox that drives external gears. The potentiometer and encoders measure the angular position of the load gear, and the tachometer measures its velocity. The schematic diagram of SRV02 is shown in Fig. 3. The electrical equation for the SRV02 servo system can be given by Eq. (13) as follows:

Table 3 Optimization results of parameters G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} for model in (42)

Algorithm used	G_e	$G_{\Delta e}$	G_{PI}	G_{PD}	Fitness value	Execution time (s)
CS	0.5168	27.9218	0.0228	0.12967	18.9036	70.849
DE	1.4812	47.2963	0.0998	1.00	9.1783	367.972
GA	0.7000	36.0000	0.0160	0.1000	25.3465	340.4465
PSO	0.500	20.0000	0.03075	0.0100	11.7096	185.7556
BCA	1.2779	31.7102	0.08306	0.4859	14.3742	173.974
CPSODE	0.500	37.0737	0.0100	0.1000	98.0392	780.976

Table 4 Optimization results of parameters G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} for SRV02 system

Algorithm used	G_e	$G_{\Delta e}$	G_{PI}	G_{PD}	Fitness value	Execution time (s)
CS	0.74091	41.4513	0.08705	0.3533	0.6281	455.261
DE	0.51765	48.4522	0.06546	0.3357	0.5177	1495.980
GA	0.9000	50.0000	0.06400	0.6400	0.7572	2421.351
PSO	0.9715	50.0000	0.06431	0.3030	0.4890	1566.669
BCA	1.2815	45.4603	0.0506	0.64479	0.5683	12890.162

Table 5 Optimization results of parameters G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} for QUBE-Servo-2

Algorithm used	G_e	$G_{\Delta e}$	G_{PI}	G_{PD}	Fitness value	Execution time(s)
CS	0.57145	27.2746	0.01483	0.4473	0.3761	147.9788
DE	0.7541	35.4677	0.0100	0.8497	0.4390	381.5669
GA	0.8333	44.0000	0.0820	0.7000	0.4141	1104.08
PSO	0.9054	46.7083	0.1000	0.0100	0.45513	286.9098
BCA	0.443	49.4319	0.0160	0.2757	0.46795	677.1070

$$e_b(t) = K_m w_m(t) \quad (13)$$

Here, e_b is the back-emf (electromotive) voltage, w_m the speed of the motor shaft and K_m the back-emf constant of the motor. Then by using KVL, we can write

$$V_m(t) - R_m I_m - L_m \frac{dI_m(t)}{dt} - K_m w_m(t) = 0 \quad (14)$$

$$I_m(t) = \frac{V_m(t) - K_m w_m}{R_m} \quad (15)$$

The mechanical equation of the system can also be derived. Equation of motion describing the speed of the load shaft is w_L and τ_m the applied motor torque. Since SRV02 is a one-degree-of-freedom rotary system, Newton's second law of motion can be written as:

$$J \cdot \alpha = \tau \quad (16)$$

where J is the moment of inertia of the body (about its center mass), α the angular acceleration of the system, and τ sum of the torques being applied to the body. According to Fig. 3, the SRV02 gear train along with the viscous

friction acting are the motor shaft B_m and the load shaft B_l . Then, the load equation is as follows:

$$J_l \frac{dw_L(t)}{dt} + B_l w_L(t) = \tau_l(t) \quad (17)$$

where J_l is the moment of inertia of the load and τ_l the total torque applied on the load.

Again, the motor shaft equation is expressed as:

$$J_m \frac{dw_m(t)}{dt} + B_m w_m(t) + \tau_{ml}(t) = \tau_m(t) \quad (18)$$

where J_m is the motor shaft moment of inertia, and τ_{ml} is the resulting torque acting on the motor shaft from the load torque.

Thus, the resulting torque at the load shaft from an applied motor torque can be written as:

$$\tau_l(t) = n_g K_g \tau_{ml}(t) \quad (19)$$

K_g is the gear ratio, and n_g is the gearbox efficiency.

The planetary gearbox is directly mounted on the SRV02 motor and is represented by N_1 and N_2 gears, and the gear ratio is:

Fig. 3 SRV02 DC motor armature circuit and gear train

$$K_{gi} = \frac{N_2}{N_1} \quad (20)$$

Next is the internal gear ratio. The motor gear N_3 and the load gear N_4 are directly meshed together and are visible from outside as

$$K_{ge} = \frac{N_4}{N_3} \quad (21)$$

The gear ratio of the SRV02 gear train is given by:

$$K_g = K_{ge} K_{gi} \quad (22)$$

Thus, the torque seen at the motor shaft is:

$$\tau_{ml}(t) = \frac{\tau_l(t)}{n_g K_g} \quad (23)$$

where n_g is the gearbox efficiency and K_E the gear ratio. Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution:

$$\theta_m(t) = K_g \theta_l(t) \quad (24)$$

The relationship between the angular speed of the motor shaft $w_m(t)$ and the angular speed of the load shaft $w_l(t)$ by taking time derivative is:

$$w_m(t) = K_g w_l(t) \quad (25)$$

Resultant differential equation that describes the motion of the load shaft with respect to an applied motor torque is given below:

$$J_m K_E \frac{dw_l(t)}{dt} + B_m K_g w_l(t) + \frac{J_l \frac{dw_l(t)}{dt} + B_l w_l(t)}{n_g K_g} = \tau_m(t) \quad (26)$$

Collecting the coefficients in terms of the load, shaft velocity, and acceleration gives:

$$(n_g J_m K_g^2 + J_l) \frac{dw_l(t)}{dt} + (n_g B_m K_g^2 + B_l) w_l(t) = n_g K_g \tau_m(t) \quad (27)$$

Defining the following terms:

$J_{eq} = n_g J_m K_g^2 + J_l$, $B_{eq} = n_g B_m K_g^2 + B_l$, Eq. (22) reduces to

$$J_{eq} \frac{dw_l(t)}{dt} + B_{eq} w_l(t) = n_g K_g \tau_m(t) \quad (28)$$

Combining the electrical and mechanical equation, an expression has been derived combining the effect of load shaft speed in terms of the applied motor voltage. The motor torque is proportional to the following:

$$\tau_m(t) = n_m k_t I_m(t) \quad (29)$$

where k_t is the current–torque constant, n_m the motor efficiency, and I_m the armature current. We can express the motor torque with respect to the input voltage, $V_m(t)$, and load shaft speed $w_m(t)$ by substituting the motor armature current:

$$\tau_m(t) = n_m k_t \frac{(V_m(t) - K_m w_m)}{R_m} \quad (30)$$

Again, we can express the above equation in terms of V_m and w_l by inserting the motor load–shaft speed equation:

$$\tau_m(t) = n_m k_t \frac{(V_m(t) - K_m K_g w_l(t))}{R_m} \quad (31)$$

Substituting Eq. (31) in Eq. (28), we get

$$J_{eq} \frac{dw_l(t)}{dt} + B_{eq} w_l(t) = n_g K_g n_m k_t \frac{(V_m(t) - K_m K_g w_l(t))}{R_m}$$

$$J_{eq} \frac{dw_l(t)}{dt} + \left(B_{eq} + \frac{n_g K_g^2 n_m k_t}{R_m} \right) w_l(t) = \frac{n_g K_g n_m k_t V_m(t)}{R_m}$$

$$J_{eq} \frac{dw_I(t)}{dt} + B_{eq,v} w_I(t) = A_m V_m(t) \quad (32)$$

where $B_{eq,v} = B_{eq} + \frac{n_g K_g^2 n_m k_t}{R_m}$ and $A_m = \frac{n_g K_g n_m k_t}{R_m}$. The final derived Eq. (32) is a first-order system.

Here, $R_m = 2.6 \Omega$, the motor armature resistance; $n_g = 0.9$, the gear box efficiency; $K_g = 70$, the high-gear total gear ratio; $n_m = 0.69$, the motor efficiency; $k_t = 7.68 \times 10^{-3} \text{ N} \cdot \text{m/A}$, the motor current-torque constant; and $B_{eq} = 0.015 \text{ Nm} \cdot (\text{rad/s})$, the high-gear equivalent viscous damping coefficient; $J_{eq} = 2.087 \times 10^{-3} \text{ Kg} \cdot \text{m}^2$, high-gear equivalent moment of inertia without external load. Then, evaluate J_{eq} assuming disk load is attached to the high gear. Next, theoretically evaluate $B_{eq,v}$ and A_m . Performing the bump test in the laboratory, we find the value of open-loop gain, $K = 1.53 \text{ rad/s/V}$, and by gain at cutoff frequency = 0.0254 s.

Hence, the open-loop transfer function for position control application by the SRV02 is given by

$$G(s) = \frac{1.53}{s(0.0254s + 1)}. \quad (33)$$

QUBE-Servo-2 As per the discussion given in the user manual [34], the QUBE-Servo-2 has a brushed DC motor that is connected to a PWM amplifier. Encoders are used here to measure angular position. There are many types of encoders, but we have used here in this experiment the rotary incremental optical encoder, and the angle they measure depends on the last position and when it was last powered. The Quanser is a direct-drive rotary servo system as shown in Fig. 4. As the encoder resolution is 512 lines per revolution and goes up to 2048 in quadrature mode, to get a measurement in degrees we need a gain of $360^\circ/2048 \text{ cnts} = 0.1758^\circ/\text{cnts}$.

The motor armature circuit schematic of QUBE-Servo-2 is shown in Fig. 10, and the electrical and mechanical parameters are terminal resistance, $R_m = 8.4 \Omega$, torque constant, $K_t = 0.042 \text{ N} \cdot \text{m/A}$, motor back-emf constant, $K_m = 0.042 \text{ V}/(\text{rad/s})$, rotor inertia, $J_m = 4.0 \times 10^{-6} \text{ kg} \cdot \text{m}^2$, load hub inertia, $J_h = 0.6 \times 10^{-6} \text{ kg} \cdot \text{m}^2$, load hub radius, $r_h = 0.0111 \text{ m}$, load hub mass, $m_h = 0.0106 \text{ kg}$, rotor

inductance, $L_m = 1.16 \text{ mH}$, mass of load disk, $M_d = 0.053 \text{ kg}$, and radius of disk load, $r_d = 0.0248 \text{ m}$. The DC motor shaft is connected to a load hub. The hub is a metal disk used to mount the disk or rotary pendulum and has a moment of inertia of J_h . A disk load is attached to the output shaft with a moment of inertia of J_d .

Here, in QUBE-Servo-2 the back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, w_m , and the back-emf constant of the motor, k_m . It opposes the current flow. The electrical equation of this system is similar to the electrical equation of SRV02. The back-emf voltage is same as Eq. 12. Also, using Kirchhoff's voltage law, the equation is same as in Eq. 13 and the value of $I_m(t)$ is similar to Eq. 14.

Next, the motor shaft equation is expressed as

$$J_{eq} w_m(t) = \tau_m(t) \quad (34)$$

where J_{eq} is total moment of inertia of a disk about its pivot, with mass m and radius r , and is given by

$$J = \frac{1}{2} m r^2 \quad (35)$$

Based on the model, we have designed a model that applies a voltage and reads the position using the encoder. Based on the parameters, we can calculate the total moment of inertia acting on the motor shaft:

$$J_{eq} = J_m + J_h + J_d \quad (36)$$

The moment of inertia of the hub and disk load is:

$$J_h = \frac{1}{2} m_h r_h^2 \quad (37)$$

$$J_d = \frac{1}{2} m_d r_d^2 \quad (38)$$

$$\text{Therefore, } J_{eq} = 2.09 \times 10^{-5} \quad (39)$$

From the bump test modeling which is based on step response of a stable system, the resulting steady-state gain is: $K = 22.7 \text{ rad}/(\text{Vs})$ and the time constant is: $\tau = 0.158 \text{ s}$ and the voltage-to-speed transfer function of the identified model becomes

Fig. 4 QUBE-Servo 2 DC motor and load

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{22.7}{0.158s + 1} \quad (40)$$

And the voltage-to-position transfer function is

$$P(s) = \frac{\theta_m(s)}{V_m(s)} = \frac{22.7}{s(0.158s + 1)} \quad (41)$$

4.3 Simulation results

As discussed in the above section from Eqs. 33 and 41, DC servo position control system may be given by first-order integrating plus delay (FOIPD) model:

$$G_p(s) = \frac{Ke^{-\theta_d s}}{s(\tau s + 1)} \quad (42)$$

Here, we consider time constant, $\tau = 1$ s, dead time $\theta_d = 0.1$ s, and open-loop gain, $K = 1$. Performance of IT2-FLC is compared with GA-IT2-FLC, DE-IT2-FLC, PSO-IT2-FLC, CS-IT2-FLC, BCA-IT2-FLC, and CPSODE-IT2-FLC. The fitness value, execution time, and the values of G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} after repeated runs are given in Table 3. Robustness of the IT2-FLC is also evaluated for 50% increased value of dead time (i.e., $\theta_d = 0.15$ s), along with the optimal

IT2-FLC. Responses for the nominal mode of the controller are given in Fig. 5. Performance indices for the nominal and perturbed models (with dead time 50% increased) are given in Table 6. The performance indices are found to be improved for CS-IT2-FLC, DE-IT2-FLC, PSO-IT2-FLC,

Table 6 Comparison performances for model in (42)

θ_d	Controller	%OS	t_s (s)	IAE	ITAE
0.1	IT2-FLC	20.1	46.8	14.64	531.9
	CS-IT2-FLC	1.4	17.1	7.268	151.5
	DE-IT2-FLC	00.7	13.9	3.518	29.98
	GA-IT2-FLC	1.9	18.8	7.842	180.6
	PSO-IT2-FLC	19.3	5.5	8.094	260.0
	BCA-IT2-FLC	0.2	10	3.303	48.48
	CPSODE-IT2-FLC	1.62	19.1	11.69	300.7
0.15	IT2-FLC	20.2	46.3	14.70	535.2
	CS-IT2-FLC	1.5	19.2	7.275	152
	DE-IT2-FLC	0.03	21.2	3.513	34.36
	GA-IT2-FLC	1.8	15.00	6.587	168.4
	PSO-IT2-FLC	1.4	23.4	8.397	279.6
	BCA-IT2-FLC	0.02	14.00	3.41	55.92
	CPSODE-IT2-FLC	2.7	29.2	11.67	299.7

Fig. 5 Responses of IT2-FLC using different optimizations for model in (42) with dead time 0.15

Fig. 6 Responses of IT2-FLC using different optimizations for model in (42) with noise

Table 7 Performance analysis for model in (42) with noise power 0.02

θ_d	Controller	IAE	ITAE
0.1	IT2-FLC	18.82	625.6
	CS-IT2-FLC	17.92	688.9
	DE-IT2-FLC	8.687	413.4
	GA-IT2-FLC	16.12	598.5
	PSO-IT2-FLC	11.89	424.1
	BCA-IT2-FLC	8.404	374.4
	CPSODE-IT2-FLC	17.11	527.1
0.15	IT2-FLC	16.99	534
	CS-IT2-FLC	13.59	536.8
	DE-IT2-FLC	8.66	418.5
	GA-IT2-FLC	16.19	600.3
	PSO-IT2-FLC	12.07	434.1
	BCA-IT2-FLC	8.503	378.8
	CPSODE-IT2-FLC	17.28	539.1

GA-IT2-FLC, BCA-IT2-FLC, and CPSODE-IT2-FLC compared to IT2-FLC. Similarly, the responses with noise power 0.02 are depicted in Fig. 6 and performance indices in Table 7.

DE-IT2-FLC performance is found to be superior compared to GA, BCA, PSO and BCA, and CPSODE-optimized IT2-FLC (Tables 6 and 7).

4.4 Experimental results

Efficiency of the IT2-FLC and optimized IT2-FLC is experimentally verified on Quanser, Canada SRV02 and QUBE-Servo-2 position control system. The snapshot of actual experimental setup of Quanser SRV02 is given in Fig. 7 and QUBE-Servo-2 in Fig. 8. For both the systems, the desired angular position (set value) is given through PC. Quanser SRV02 and QUBE-Servo-2 position control are identified in Sect. 4.2 as first-order integrating plus time delay (FOIPD) model and derived as

$$\text{for SRV02, } G_{\text{SRV02}}(s) = \frac{1.53}{s(0.0254s + 1)}. \quad (43)$$

$$\text{and QUBE-Servo-2, } G_{\text{QUBE}}(s) = \frac{22.7}{s(0.158s + 1)}. \quad (44)$$

Fig. 7 Experimental setup of SRV02

Fig. 8 Experimental setup of QUBE-Servo 2

With the QUARC and MATLAB–Simulink, reported controllers are implemented and their performances are evaluated. Performance study of set-point tracking and load regulation with IT2-FLC, CS-IT2-FLC, DE-IT2-FLC, PSO-IT2-FLC, GA-IT2-FLC, BCA-IT2-FLC, and CPSODE-IT2-FLC is shown in Fig. 9 for SRV02 and Fig. 10 for QUBE-Servo-2. The comparative study for the controllers is depicted in Tables 8 and 9.

Performance of SRV02 and QUBE-Servo-2 is also studied in the presence of measurement noise for IT2-FLC and

with optimal IT2-FLC (Figs. 9, 10, 11, 12). Here, noise power applied for SRV02 with sample time 0.002 is 10^{-6} , and similarly, noise power 0.02 for QUBE-Servo-2 with sample time is 0.01. The related responses with measurement noise are given in Fig. 11 for SRV02 and Fig. 12 for QUBE-Servo-2, and analysis of performance indices is given in Tables 10 and 11.

Here based on the response-related performance indices, it is clear that applying optimization algorithms in determining the value of G_e , $G_{\Delta e}$, G_{PI} , and G_{PD} of the IT2-FLC there is immense improvement in close-loop response under model uncertainty and in the presence of measurement noise. Moreover, comparative study shows performance applying CPSODE optimization algorithm in Type-2 fuzzy controllers for practical systems is more superior to all the other heuristic algorithms GA, PSO, CS, DE, and BCA.

5 Conclusion

This paper present a comparative study of six types of nature-inspired optimization algorithms: the cuckoo search (CS) algorithm, the differential evolution (DE) algorithm, the particle swarm optimization (PSO), the genetic algorithm (GA), the bee colony algorithm (BCA), and the combined PSO and DE algorithm (CPSODE) to perform the input and output scaling parameters of the interval Type-2 fuzzy PID controller for servo systems. The comparison performances are in terms of the performance indices of systems responses under set-point tracking, load disturbances, and measurement noise. The analysis of simulation demonstrates a high performance of the differential evolution (DE) algorithm producing low percentage overshoot (% O.S.), IAE, and ITAE for simulation. The experimental result with two servo systems SRV02 and QUBE-Servo-2 depicts CPSODE algorithm has superior performance despite the fact that the execution time for computation of the optimal parameter values is high compared to other algorithms.

Fig. 9 Responses of IT2-FLC using different optimizations on SRV02

Fig. 10 Responses of IT2-FLC using different optimizations on Servo-2 QUBE

Table 8 Comparison performances for SRV02 process

Controller	%OS	t_s (s)	IAE	ITAE
IT2-FLC	23.59	4.416	0.7774	3.270
CS-IT2-FLC	13.00	2.380	0.3499	1.236
DE-IT2-FLC	8.40	2.732	0.4352	1.625
GA-IT2-FLC	12.15	2.732	0.3479	1.200
PSO-IT2-FLC	14.21	2.414	0.4073	1.537
BCA-IT2-FLC	18.02	3.282	0.4067	1.481
CPSODE-IT2-FLC	28.23	2.698	0.3000	0.7387

Table 9 Comparison performances for QUBE-Servo-2

Controller	%OS	t_s (s)	IAE	ITAE
IT2-FLC	6.7	3.62	0.6847	1.017
CS-IT2-FLC	0.2	1.84	0.7553	0.4804
DE-IT2-FLC	0.0	2.56	0.7086	0.5053
GA-IT2-FLC	0.0	2.79	0.6145	0.6623
PSO-IT2-FLC	1.4	2.52	0.5413	0.4703
BCA-IT2-FLC	0.8	2.73	0.6419	0.3827
CPSODE-IT2-FLC	0.1	2.39	0.5776	0.7306

Fig. 11 Responses of IT2-FLC using different optimizations on SRV02 with noise (noise factor 10^{-6})

Fig. 12 Responses of IT2-FLC using different optimizations on Servo-2 QUBE with noise (noise factor 0.02)

Table 10 Performance analysis for SRV02 process with noise

Controller	IAE	ITAE
IT2-FLC	0.9280	3.208
CS-IT2-FLC	0.4915	1.732
DE-IT2-FLC	0.8924	3.062
GA-IT2-FLC	0.6955	2.835
PSO-IT2-FLC	0.6456	2.739
BCA-IT2-FLC	0.5166	1.646
CPSODE-IT2-FLC	0.4848	1.4333

Table 11 Performance analysis for QUBE-Servo-2 with noise

Controller	IAE	ITAE
IT2-FLC	1.074	2.139
CS-IT2-FLC	1.101	2.290
DE-IT2-FLC	0.899	1.849
GA-IT2-FLC	0.780	2.597
PSO-IT2-FLC	0.869	2.742
BCA-IT2-FLC	0.862	2.509
CPSODE-IT2-FLC	0.7311	2.251

Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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