Tensor sparse PCA and face recognition: a novel approach

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Abstract

Face recognition is the important field in machine learning and pattern recognition research area. It has a lot of applications in military, finance, public security, to name a few. In this paper, the combination of the tensor sparse PCA with the nearest-neighbor method (and with the kernel ridge regression method) will be proposed and applied to the face dataset. Experimental results show that the combination of the tensor sparse PCA with any classification system does not always reach the best accuracy performance measures. However, the accuracy of the combination of the sparse PCA method and one specific classification system is always better than the accuracy of the combination of the PCA method and one specific classification system and is always better than the accuracy of the classification system itself.

Keywords Tensor · Sparse PCA · Kernel ridge regression · Face recognition · Nearest-neighbor

1 Introduction

Face recognition is the important field in machine learning and pattern recognition research area. It has a lot of applications in military, finance, public security, to name a few. Computer scientists have worked in face recognition research area for almost three decades. In the early 1990, the Eigenface [1] technique has been employed to recognize faces and it can be considered the first approach used to recognize faces. The Eigenface [1] used the principle component analysis (PCA) which is one of the most popular dimensional reduction methods [2, 3] to reduce the dimensions of the faces. This PCA technique can also be used in other pattern recognition problems such as speech recognition [4, 5]. After using PCA to reduce the dimensions of the faces, the Eigenface use nearest-neighbor method [6] to recognize or to classify faces.

To classify the faces, a graph (i.e., kernel) which is the natural model of relationship between faces can also be employed. In this model, the nodes represent faces. The edges represent for the possible interactions between

nodes. Then, machine learning methods such as support vector machine [7], kernel ridge regression [8], artificial neural networks [9] or the graph-based semi-supervised learning methods [10–12] can be applied to this graph to classify or to recognize the faces. The artificial neural networks, support vector machine, and kernel ridge regression are supervised learning methods. The graph-based semi-supervised learning methods are the semi-supervised learning methods. Please note that the kernel ridge regression method is the simplest form of the support vector machine method.

In the last two decades, the SVM learning method has successfully been applied to some specific classification tasks such as digit recognition, text classification, and protein function prediction and face recognition problem [7]. However, the kernel ridge regression method (i.e., the simplest form of the SVM method) has not been applied to any practical applications. Hence, in this paper, in addition to the nearest-neighbor method, we will also use the kernel ridge regression method applied to the face recognition problem.

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Last but not least, in this literature review part, we would like to discuss a little bit about deep learning approach in general and about the deep convolution neural network in specific. In recent years, the deep convolution neural network can be considered the current state-of-the-art method for image classification problems [13–19] and it has been applied to solve a lot of big data classification problems in various fields such as natural language processing field and computer vision field. However, in our paper, the dataset that we used contains only hundreds of face images. Thus, it is not appropriate and suitable to employ the deep convolutional neural network to classify such a tiny image dataset. In the other words, we just employ the classic nearest neighbor method and the classic kernel ridge regression method to classify our images.

Next, we will introduce the principle component analysis. Principle component analysis (i.e., PCA) is one of the most popular dimensionality reduction techniques [2]. It has several applications in many areas such as pattern recognition, computer vision, statistics, and data analysis. It employs the eigenvectors of the covariance matrix of the feature data to project on a lower dimensional subspace. This will lead to the reduction in noises and redundant features in the data and the low time complexity of the nearest-neighbor and the kernel ridge regression approach solving face recognition problem.

However, the PCA has two major disadvantages which are the lack of sparsity of the loading vectors and each principle component is the linear combination of all variables. From data analysis viewpoint, sparsity is necessary for reduced computational time and better generalization performance. From modeling viewpoint, although the interpretability of linear combinations is usually easy for low-dimensional data, it could become much harder when the number of variables becomes large. To overcome this hardness and to introduce sparsity, many methods have been proposed such as [20–23]. In this paper, we will introduce new approach for sparse PCA using alternating direction method of multipliers (i.e., ADMM method) [24]. Then, we will try to combine the sparse PCA dimensional reduction method with the nearest-neighbor method (and with the kernel ridge regression method) and apply these combinations to the face recognition problem. This work, to the best of our knowledge, has not been investigated.

Last but not least, we will also develop the novel tensor sparse PCA method and apply this dimensional reduction method to the face database (i.e., tensor) in order to reduce the dimensions of the face database. Then, we will apply the nearest-neighbor method or the kernel ridge regression method to recognize the transformed faces.

We will organize the paper as follows: Sect. 2 will present the alternating direction method of multipliers.

Section 3 will derive the sparse PCA method using the ADMM method in detail. Section 4 will present the sparse PCA algorithm. Section 5 will present the detailed version of the tensor sparse PCA algorithm. In Sect. 6, we will apply the combination of tensor sparse PCA algorithm with the nearest-neighbor algorithm and with the kernel ridge regression algorithm to faces in the dataset available from [25]. Section 7 will conclude this paper and discuss the future directions of researches of this face recognition problem.

2 Alternating direction method of multipliers

In this section, we will introduce the alternating direction method of multipliers. The detailed information about the alternating direction method of multipliers can be found in [24]. First, assume that we want to solve the following problem

$$\begin{aligned} &\text{minimize } f(x) + g(z) \\ &\text{subject to } Ax + Bz = c \end{aligned}$$

with variables $x \in R^n$ and $z \in R^m$, where $A \in R^{p \times n}$, $B \in R^{p \times m}$
Next, we will form the augmented Lagrangian

$$\begin{aligned} L_\rho(x, z, y) = & f(x) + g(z) + y^T(Ax + Bz - c) \\ & + \frac{\rho}{2} \|Ax + Bz - c\|_2^2 \end{aligned}$$

Finally, x^{k+1} , z^{k+1} and y^{k+1} can be solved as the followings

$$\begin{aligned} x^{k+1} = & \operatorname{argmin}_x L_\rho(x, z^k, y^k) \\ z^{k+1} = & \operatorname{argmin}_z L_\rho(x^{k+1}, z, y^k) \\ y^{k+1} = & y^k + \rho(Ax^{k+1} + Bz^{k+1} - c), \end{aligned}$$

where $\rho > 0$ is the positive penalty parameter.

3 Sparse principle component analysis derivation

Assume that we are given the data matrix $D \in R^{n \times p}$ (n is the number of samples and p is the number of features). Next, we will formulate our sparse PCA problem. This problem is in fact the following optimization problem.

$$\begin{aligned} &\text{minimize}_{x, z} - \|\tilde{D}x\|_2^2 + \lambda \|z\|_1 \\ &\text{such that } x = z \quad \text{and} \quad \|z\| \leq 1. \end{aligned}$$

Our objective is to find the sparse vector x . Please note that

$$\tilde{D} = \begin{bmatrix} d_1 - \mu \\ d_2 - \mu \\ \vdots \\ d_n - \mu \end{bmatrix},$$

where $\mu = \frac{1}{n} \sum_{i=1}^n d_i$ be the mean vector of all row vectors $d_1, d_2, \dots, d_n$ of D .

First, the augmented Lagrangian of the above optimization problem can be derived as the following:

$$L_p(x, z, y) = -\|\tilde{D}x\|_2^2 + \lambda\|z\|_1 + y^T(x - z) + \frac{\rho}{2}\|x - z\|_2^2$$

Then, x^{k+1} , z^{k+1} , and y^{k+1} can be solved as the followings:

$$x^{k+1} = \operatorname{argmin}_x L_p(x, z^k, y^k)$$

Hence,

$$\begin{aligned} \frac{dx^{k+1}}{dx} &= \frac{d}{dx} \left(-\|\tilde{D}x\|_2^2 + y^{kT}(x - z^k) + \frac{\rho}{2}\|x - z^k\|_2^2 \right) \\ &= \frac{d}{dx} \left(-x^T \tilde{D}^T \tilde{D} x + y^{kT}(x - z^k) + \frac{\rho}{2}\|x - z^k\|_2^2 \right) \\ &= -2\tilde{D}^T \tilde{D} x + y^k + \rho(x - z^k) \end{aligned}$$

Next, we solve

$$\frac{dx^{k+1}}{dx} = 0 \Leftrightarrow (-2\tilde{D}^T \tilde{D} + \rho I)x = -y^k + \rho z^k$$

Thus,

$$x^{k+1} = (-2\tilde{D}^T \tilde{D} + \rho I)^{-1}(-y^k + \rho z^k)$$

Next, we have

$$z^{k+1} = \operatorname{argmin}_z L_p(x^{k+1}, z, y^k)$$

Hence,

$$\begin{aligned} \frac{dz^{k+1}}{dz} &= \frac{d}{dz} \left(\lambda\|z\|_1 + y^{kT}(x^{k+1} - z) + \frac{\rho}{2}\|x^{k+1} - z\|_2^2 \right) \\ &= \lambda\xi - y^k + \rho(x^{k+1} - z)(-1) \\ &= \lambda\xi - y^k + \rho(z - x^{k+1}), \end{aligned}$$

where

$$\xi_i = \begin{cases} 1 & \text{if } z_i > 0 \\ [-1, 1] & \text{if } z_i = 0 \\ -1 & \text{if } z_i < 0 \end{cases} \quad (1)$$

Solve $\frac{dz^{k+1}}{dz} = 0$, we have

$$z_i^{k+1} = x_i^{k+1} + \frac{1}{\rho}y_i^k - \frac{\lambda}{\rho}\xi_i$$

if $z_i^{k+1} > 0$, $\xi_i = 1$, then

$$x_i^{k+1} + \frac{1}{\rho}y_i^k - \frac{\lambda}{\rho} > 0 \Rightarrow x_i^{k+1} + \frac{1}{\rho}y_i^k > \frac{\lambda}{\rho}$$

if $z_i^{k+1} < 0$, $\xi_i = -1$, then

$$x_i^{k+1} + \frac{1}{\rho}y_i^k + \frac{\lambda}{\rho} < 0 \Rightarrow x_i^{k+1} + \frac{1}{\rho}y_i^k < -\frac{\lambda}{\rho}$$

if $z_i^{k+1} = 0$, then

$$-\frac{\lambda}{\rho} \leq x_i^{k+1} + \frac{1}{\rho}y_i^k \leq \frac{\lambda}{\rho}$$

Thus,

$$z_i = \operatorname{sign} \left(x_i^{k+1} + \frac{1}{\rho}y_i^k \right) \max \left(|x_i^{k+1} + \frac{1}{\rho}y_i^k| - \frac{\lambda}{\rho}, 0 \right)$$

Finally, we have

$$y^{k+1} = y^k + \rho(x^{k+1} - z^{k+1})$$

4 Sparse principle component analysis algorithm

In this section, we will present the sparse PCA algorithm

Algorithm 1: Sparse PCA algorithm

```

1 Input: The dataset  $D \in R^{n \times p}$ , where  $p$  is the dimension of the dataset and  $n$  is the
   total number of observations in the dataset
2 Compute  $\tilde{D} = \begin{bmatrix} d_1 - \mu \\ d_2 - \mu \\ \vdots \\ d_n - \mu \end{bmatrix}$  Where  $\mu = \frac{1}{n} \sum_{i=1}^n d_i$  be the mean vector of all row vectors
    $d_1, d_2, d_3, \dots, d_n$  of  $D$ 
3 Randomly select parameters  $\rho, \lambda$ .
4 Set  $B = eye(p)$ 
5 Set  $X = zeros(p, dim)$ 
6 for  $i = 1 : dim$  do
7     i. Compute  $L_p(x, z, y)$ 
8     ii. Initialize  $x^0, z^0, y^0$ 
9     iii. Set  $k = 0$ 
10    iv. do
11        a. Compute  $x^{k+1} = argmin_x L_p(x, z^k, y^k)$ 
12        b. Compute  $z^{k+1} = argmin_z L_p(x^{k+1}, z, y^k)$ 
13        c. Compute  $y^{k+1} = y^k + \rho(x^{k+1} - z^{k+1})$ 
14        d.  $k = k + 1$ 
15    v. While  $\|x^{k+1} - x^k\| > 10^{-10}$ 
16        vi.  $x = \frac{x^{k+1}}{norm(x^{k+1})}$ 
17        vii.  $X(:, i) = x$ 
18        viii.  $\tilde{D} = \tilde{D}(I - xx^T)$ 
19 End
20 Output: The matrix  $\tilde{D}X$ 

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5 Tensor sparse principle component analysis algorithm

In this section, we will present tensor sparse PCA method.

Suppose Y is a three mode tensor with dimensionalities N_1, N_2 , and N_3 , where N_1 is the height of the image, N_2 is the width of the image, $N_3 = n_1 + \dots + n_p$ is the total number of images in the dataset and P is the total number

of people in the dataset. Then, the first mode unfolding of Y is written as $Y_{(1)}$ and is the matrix $N_2 N_3 * N_1$. The second mode unfolding of Y is written as $Y_{(2)}$ and is the matrix $N_1 N_3 * N_2$. The third mode unfolding of Y is written as $Y_{(3)}$ and is the matrix $N_1 N_2 * N_3$.

The refolding is the reverse operation of the unfolding operation. Thus, the tensor sparse PCA can be represented as follows

Algorithm 2: Tensor Sparse PCA algorithm

```

1 Input: The three mode tensor  $Y$ 
2 for  $i = 1 : 2$  do
3     i. Unfold the three mode tensor  $Y$  to  $Y_{(i)}$ 
4     ii. Apply the sparse PCA algorithm (Algorithm 1) to  $Y_{(i)}$  and get the output
         $Z_{(i)}$ 
5     iii. Refold the matrix  $Z_{(i)}$  to the three mode tensor  $Y$ 
6     End
7 for  $i = 1 : P$  do
8     i. Unfold the three mode sub-tensor  $Y^{(i)}$  to  $Y_{(3)}^i$ 
9     ii. Apply the sparse PCA algorithm (Algorithm 1) to  $Y_{(3)}^i$  and get the output
         $Z_{(3)}^i$ 
10    iii. Refold the matrix  $Z_{(3)}^i$  to the three mode tensor  $Y^{(i)}$ 
11    End
12 Merge all three mode sub-tensor  $Y^{(i)}$  to form the final three mode tensor  $Y$ 
13 Output: The tensor  $C = Y$ .

```

6 Experiments and results

In this paper, we use the images from the Yale dataset [25] to test our approaches. This dataset contains 165 grayscale images in GIF format of 15 people. There are 11 images per person, one per different facial expression: center-light, with glasses, happy, left-light, etc. Then, the set of 120 face samples recorded of 15 different people (8 face samples per person) are used for training. Then, another set of 45 face samples of these people (3 face samples per person) are used for testing the accuracy measure. The description of this dataset and this dataset itself are available from [25]. Each face sample (i.e., the image) is the $R^{(32 \times 32)}$ matrix. Then, we will merge all rows of the face sample (i.e., the matrix) sequentially from the first row to the last row into a single big row which is the $R^{(1 \times 1024)}$ row vector (i.e., $32 \times 32 = 1024$). These row vectors will be used as the feature vectors of the nearest-neighbor method and the kernel ridge regression method.

Next, the PCA and the sparse PCA algorithms will be applied to face samples in the training set and the testing set to reduce the dimensions of the face samples in

the dataset. Please note that the PCA algorithm is also the dimensional reduction method. Information about PCA algorithm can be found in [1]. After applying the PCA and the sparse PCA algorithms to the face samples in the training set and the testing set, the number of dimensions of these transformed feature vectors of the Yale face dataset will be 200, 300, 400, 500, and 600. Then, the nearest-neighbor method and the kernel ridge regression method will be applied to these new transformed feature vectors.

At the very beginning, each face sample is the $R^{(32 \times 32)}$ matrix. There are 120 face samples in the training set. So, the training set is in fact the $R^{(32 \times 32 \times 120)}$ tensor. There are 45 face samples in the testing set. Hence, the testing set is in fact the $R^{(32 \times 32 \times 45)}$ tensor. Then, we apply the tensor sparse PCA algorithm to the training set and the testing set to reduce the dimensions of the face dataset. The tensor sparse PCA algorithm is described clearly in Sect. 5, algorithm 2. Now, the training set is the $R^{(25 \times 25 \times 75)}$ tensor and the testing set is the $R^{(25 \times 25 \times 45)}$ tensor. Then, we will merge all rows of the face sample (i.e., the new matrix) into a single big row which is the $R^{(1 \times 625)}$ row vector. Finally, the nearest-neighbor method and the kernel ridge regression method will be applied to the new transformed dataset.

Table 1 Accuracies of the nearest-neighbor method, the combination of PCA method and the nearest-neighbor method, the combination of sparse PCA method and the nearest-neighbor method, and the combination of tensor sparse PCA method and the nearest-neighbor method (%)

The nearest-neighbor method	90.81
PCA ($d = 200$) + The nearest-neighbor method	91.11
PCA ($d = 300$) + The nearest-neighbor method	91.11
PCA ($d = 400$) + The nearest-neighbor method	91.11
PCA ($d = 500$) + The nearest-neighbor method	91.11
PCA ($d = 600$) + The nearest-neighbor method	91.11
Sparse PCA ($d = 200$) + The nearest-neighbor method	91.70 (89.02 $\pm$ 2.42)
Sparse PCA ($d = 300$) + The nearest-neighbor method	91.70 (90.40 $\pm$ 1.31)
Sparse PCA ($d = 400$) + The nearest-neighbor method	91.41 (88.23 $\pm$ 2.25)
Sparse PCA ($d = 500$) + The nearest-neighbor method	91.70 (87.57 $\pm$ 3.18)
Sparse PCA ($d = 600$) + The nearest-neighbor method	91.11 (89.02 $\pm$ 1.13)
Tensor sparse PCA + The nearest-neighbor method	92 (89.59 $\pm$ 1.42)

Bold indicate those methods reach the highest accuracy

Table 2 Accuracies of the kernel ridge regression method, the combination of PCA method and the kernel ridge regression method, the combination of sparse PCA method and the kernel ridge regression method, and the combination of tensor sparse PCA method and the kernel ridge regression method

The kernel ridge regression method	95.85
PCA ($d = 200$) + The kernel ridge regression method	96.15
PCA ($d = 300$) + The kernel ridge regression method	96.15
PCA ($d = 400$) + The kernel ridge regression method	96.15
PCA ($d = 500$) + The kernel ridge regression method	96.15
PCA ($d = 600$) + The kernel ridge regression method	96.15
Sparse PCA ($d = 200$) + The kernel ridge regression method	96.15 (92.84 $\pm$ 3.19)
Sparse PCA ($d = 300$) + The kernel ridge regression method	96.44 (91.01 $\pm$ 4.02)
Sparse PCA ($d = 400$) + The kernel ridge regression method	96.44 (93.10 $\pm$ 2.27)
Sparse PCA ($d = 500$) + The kernel ridge regression method	96.15 (91.90 $\pm$ 3.14)
Sparse PCA ($d = 600$) + The kernel ridge regression method	96.74 (94.42 $\pm$ 1.31)
Tensor sparse PCA + The kernel ridge regression method	87.85 (85.63 $\pm$ 1.18)

Bold indicate those methods reach the highest accuracy

In this section, we experiment with the above nearest-neighbor method and kernel ridge regression method in terms of accuracy measure. The accuracy measure Q is given as follows:

$$Q = \frac{\text{True Positive} + \text{True Negative}}{\text{True Positive} + \text{True Negative} + \text{False Positive} + \text{False Negative}}$$

All experiments were implemented in Matlab 6.5 on virtual machine. For the nearest neighbor method, the number of neighbors that is used is 23. We run each method 20 times, and the best accuracy performance measure of each method is recorded. In recent years, the deep convolutional neural network can be considered the current state-of-the-art method for image classification problem. However, it just has been applied to big data only. In our experiments, our used dataset just contains only hundreds of face images. Thus, it is not appropriate and not suitable to use the deep convolutional neural network in this paper, due to lack of time and lack of space. The accuracy performance measures of the above proposed methods are given in the following Tables 1 and 2.

From the above tables, we recognize that the combination of the tensor sparse PCA method and one specific classification system does not always reach the best accuracy performance measure. This accuracy depends on the classification system. However, the accuracy of the combination of the sparse PCA method and one specific classification system is always better than the accuracy of the combination of the PCA method and one specific classification system and is always better than the accuracy of the classification system itself. The reasons are given in Sect. 1.

The time complexities of the tensor sparse PCA and the sparse PCA is not recorded here since these two methods employ the ADMM optimization technique which is the very fast optimization technique. Information about the time complexity of ADMM optimization technique can be found in [24].

7 Conclusion

In this paper, the detailed versions of the sparse PCA method and the tensor sparse PCA method have been proposed. The experimental results show that the combination of the tensor sparse PCA with any specific classification system does not always reach the best accuracy performance measure. However, the accuracy of the combination of the sparse PCA method and one specific classification system is always better than the accuracy of the combination of the PCA method and one specific classification system and is always better than the accuracy of the classification system itself.

In the future, we will test the accuracies of the combination of the tensor sparse PCA method with a lot of classification systems, for, e.g., the SVM method or deep neural network methods.

In this paper, we use the alternating direction method of multipliers (i.e., the ADMM method) to solve the sparse PCA problem. In the near future, we will use the other optimization methods such as the proximal gradient method to solve the sparse PCA problem. Then, we will compare the accuracies and the run time complexities among these optimization techniques. To the best of our knowledge, this work has not been investigated up to now.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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