Fuzzy logic-based tuning of PID controller to control flexible manipulators

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Abstract

Flexible manipulators are widely used in overhead cranes for industrial purposes and accurate control of the tip point of such a system is very difficult. Control of a 6DOF flexible manipulator is attempted in this study. Model of the manipulator is built in sim-mechanics environment of MATLAB and simulated. Joints of the manipulator are made flexible with the use of joint spring and damper. The behavior of the system was found to be unstable and a controller was used to bring the stability. For this purpose, a conventional PID controller was used and its gain values are tuned using different approaches in this study. Finally, a manually designed fuzzy logic-based approach was also used to tune the PID gains and compared with that of the conventional tuning approaches. In order to apply controller to the system, equivalent linearized transfer function of the model is derived in MATLAB. Among all the conventional tuned PID controllers applied, RRT-based method has performed best in terms of phase margin and gain margin. Also, the performance of fuzzy logic-tuned PID controller was found to be better compared to the conventional PID tuned controllers.

Keywords Sim-mechanics · Joint flexibility · Fuzzy logic · SISO design toolbox

List of symbols

(X_i, Y_i)	X and Y coordinate
O_i	Center of (X, Y) coordinate
L	Link length
M	Link mass
J	Joint
K_p	Proportional gain
K_d	Derivative gain
K_i	Integral gain
e	Error
$\dot{e}$	Time derivative of error
num	Numerator
den	Denominator

GM	Gain margin
PM	Phase margin

Abbreviations

JSD	Joint spring damper
JS	Joint sensor
RJ	Revolute joint

1 Introduction

In real world, almost all the manipulators are flexible in nature. Besides, manipulators with the incorporation of reasonable flexibility become more advantageous than the rigid manipulators because of its ability to carry higher work load with low inertia and precise high speed operation. Therefore, development of such manipulators is increasing day by day. Kinematics, dynamics, motion planning and control of rigid manipulators have been discovered well so far. But, research on dynamic modeling of flexible manipulator has not been analyzed that much. The reason behind it may be the higher complexity involved in formulating such dynamic system. The overall motion of the flexible manipulator composed of two different

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motion component. One motion comes from considering the rigid part of the manipulator motion and the other comes from considering the flexible part of the same manipulator motion. Thus, two different kind of motion element can be observed in case of the complete motion of the flexible manipulator. One is rigid body motion and the other is the motion arising from the flexibility. Due to the participation of these two different motions in the formation of final manipulator motion, the resulting dynamic motion equation of the flexible manipulator system becomes highly complex and often it becomes very difficult to formulate. This initiates more challenging control task for flexible manipulator compared to the rigid one. Here, both the flexible manipulator model considering the joint flexibility only and the rigid manipulator model with rigid joint and link have been constructed in sim-mechanics. The simulation results obtained from both the models have been noticed carefully. This directly points out to the greater tolerable stability of the flexible manipulator. For this reason, an effective control scheme should be adopted for the purpose of controlling flexible manipulator system that will be based on properly developed dynamic models of the said system. Main aim of this current research work has been concentrated on the use of classical PID-type control technique in order to control such flexible manipulator. This needs proper tuning of PID gain values. In this work, the sim-mechanics model of flexible manipulator has been developed as multi-arm manipulator for a single-link manipulator. The tuning of PID controller using SISO design toolbox and fuzzy logic-based approach have been presented and discussed.

Theodore and Ghoshal [1] have developed a two-stage control algorithm for multi-link flexible manipulator applying model-based joint inversion control law and impedance control using end point sensing. But, in this work, they have addressed only the problem of active damping of flexural vibrations of the robot links. Shan et al. [2] have introduced a dynamic model of a single-link flexible manipulator using Euler–Bernoulli beam theory and assumed mode method-based deflection analysis with tip mass and bonded PZT actuators. But here, they have developed a combined scheme of PD feedback controller for AC servo motor and PPF controller for PZT actuators to suppress multi-mode vibration. Although, the authors have tried to use classical PD controller but ended up with more complex controller design. A dynamic modeling of robots with elastic joints have been introduced by Luca et al. [3] where a PD control law has been developed using motor variables with on-line gravity compensation for robot manipulators with elastic joints. But in their work, they have concentrated mainly on the designing of controller rather than the development of system dynamic model. Feliu et al. [4] have designed discrete feed forward

controller (DFC) to motivate optimal mechanical design of a single-link manipulator. In this study, the authors have not considered multi-link flexible manipulator. Thus, the systems nonlinearities arise from the multi-link system are avoided. Vakil et al. [5] have developed the dynamic model of a multi-link flexible manipulator (MLFM) using the combination of assumed mode shape method and Lagrange equation. Here, they have designed a state feedback controller according to gain scheduling technique, where an observer has been used to get direct measurement of the states. But, from the developed model simulation it can be seen that the faster the maneuver, the higher the tracking error. A new hybrid fuzzy neural controller (HFNC) has been developed by Subudhi and Morris [6] for a very complex flexible multivariable system in order to achieve effective tip regulation performance. In this case, although the multivariable hybrid controller performs well considering the coupling effect, but the practical implementation is very difficult because of the large number of rules and the number of scaling factors to be tuned. Mehrez and Badawy [7] have investigated the effect of the manipulator joint inertia on the response of the arm of a single-link flexible manipulator. But they have avoided the modeling complexity arises from multi-link manipulator system. Ahmad et al. [8] have developed hybrid intelligent control schemes for the trajectory tracking and vibration control of a flexible-joint manipulator. In this scheme, they have incorporated a non-collocated fuzzy logic controller, a non-collocated PID and an input shaping scheme for the vibration reduction in the flexible-joint system. But they have not given much on the dynamic modeling of such flexible manipulator. Ramalingam et al. [9] have utilized a lightweight hybrid composite material for single-link robotic manipulator purpose to reduce its weight. Here, the numerical analysis has been carried out for competing and replacement of the rigid one. Here also, the authors have neglected the complexity of multi-link manipulator. A coupling dynamic model has been formulated by Wang et al. [10] for a flexible manipulator system with harmonic drive using experimental identification method. In this work, parameters of the driven model of the harmonic joint and parameters of coupling vibration model of the flexible manipulator have been identified. Accordingly, coupling dynamic models of the proposed system have been obtained. Coulomb friction of the joint has been identified by step current excitation and uniform rotation experiments at a low speed. Then, the transfer function model of the harmonic joint has been established and identified by a pseudorandom binary sequence excitation. But, with their proposed system model, the control task becomes more complex. Liu et al. [11] have presented the adaptive boundary control problem of a flexible manipulator in the presence of input saturation and external

disturbances. Here, the composite control task is decomposed into two independent tasks which make the control system more complex. A robust PD control of a two-link flexible manipulator based on the LMI approach has been presented by Mohamed et al. [12]. But in their work, the authors have shown that laborious tuning effort for each loading condition is needed for a PID controller. An investigation into the development of a closed-loop vibration control strategy for flexible manipulator systems has been presented by Shaheed et al. [13]. But in this study, the authors have considered only a single-link manipulator. Kiang et al. [14] have presented a survey of the sensor systems and control methods for FLMs. Here, they have shown that many research works have been focusing on single- or two-link flexible manipulators. Also there has not been many works that are based on flexible manipulators with large elastic deflections. Karagulle et al. [15] have addressed the vibration control of a two-link flexible manipulator. Here, the Newmark solution is used to solve the mathematical model which consists of time-dependent mass, stiffness and force matrices. But in this study, the authors have not considered multi-link flexible manipulator model. Ahmad et al. [16] have developed a PD-type composite fuzzy logic controller for the trajectory tracking and vibration suppression of a flexible-joint manipulator. But, in their work also the authors have considered only single-link model. The Lagrange mechanics and the assumed mode method have been used by Shawky et al. [17] to derive a proposed dynamic model of a single-link flexible manipulator having a revolute joint. Here also, the authors have avoided the complexity arises due to multi-link flexible manipulator. Sun et al. [18] have derived the model for a flexible manipulator system by employing the lumped spring-mass method. But, in their model, the control task becomes more complex considering multi-link manipulator. Sang-Myeong Kim [19] has presented an LES to represent the dynamics of a PD-controlled flexible manipulator in a unique and exact manner. But in his study, the author has considered only a single degree of freedom manipulator. The dynamic equations, which indicate strong nonlinear coupling for a 3D flexible manipulator, are derived, and then used by Liu et al. [20] for the design of the boundary control. But, the presented work lack of proper justification of the proposed model. Gao et al. [21] have presented the n -dimensional discretized model of the two-link flexible via the AMM which realizes the conversion from PDE to ODE and facilitates the analysis of dynamic uncertainty and the design of controller. In this work, the authors have neglected the complexity in designing the controller for multi-link manipulator. The dynamic equations of multi-link multi-DOF flexible manipulator have been established by Meng et al. [22]. Here, vibration responses of the tip under different elastic

modulus, damping and joint stiffness were studied, and vibration characteristics of the tip with both link and joint were also analyzed. But, in their study, the authors have not dealt with control issues. Wei et al. [23] have proposed the global mode method (GMM) to obtain a reduced-order analytical dynamic model for a signal flexible-link flexible-joint (SFF) manipulator. But in the study also, the authors have not discussed the control issues. Meng et al. [24] have established a mathematical model of a flexible-base space robot system. Based on the model, the dynamic coupling equation between the appendages and the end-effector motion is then derived. But the authors have not considered the control issues. Yang et al. [25] have developed the closed-form dynamics and adaptive robust tracking control of a space robot with two-link flexible manipulators under unknown disturbances. But in this work, the authors have not considered the mathematical complexity of multi-link manipulator. Mostafa Sayahkarajy [26] has presented a finite element (FE) modeling, modal analysis, linearization, and control of a two-link flexible manipulator (TLFM) based on the global vibration modes. In this study, the author has avoided the complexity in formulating multi-link manipulator model. Kumar et al. [27] have presented the theoretical studies to determine the modal parameters and investigate the dynamic instability of a harmonically driven planar two-link manipulator with flexible joints. But the authors have not discussed the control issues in their work. Lochan et al. [28] have attempted for the robust tip trajectory synchronization between assumed modes method modeled two-link flexible manipulators. But in their model, it can be observed that the control methodology is relatively complex.

However, all the developed models have their own limitations on considering both link as well as joint flexibilities, gravitational and frictional effects and deriving dynamic solution of large number of DOFs. It is also certain to say that accurate control of such systems become very difficult until or unless system dynamic equations are properly known/derived. Therefore, there is still a need to develop a robust controller for attenuating flexible manipulators. That is why, in this research work, a flexible manipulator considering flexible joints has been modeled with the help of sim-mechanics. By means of six revolute joints, six serially connected links have been used to construct the overall manipulator system in order to add extra flexibility to the manipulator. Finally, a classical PID controller has been used to control the said flexible manipulator. In order to obtain satisfactory controller performance, different controller gain tuning methods have been studied and most efficient gain tuning method has been adopted to control the overall manipulator system.

The structure of the paper has been arranged in the following way: In Sect. 2, the construction of a six-link

manipulator model in sim-mechanics environment has been described with joint flexibility, and the conventional PID controller-based controlling action on the manipulator system has been demonstrated. Results and discussions are elaborated in Sect. 3. Here, both the SISO design tool-based and fuzzy logic-based tuning of the PID controller gain values has been introduced and compared. In the end, a few remarks regarding the conclusion are given in Sect. 4.

2 Model development and control of a six-link manipulator considering the joint flexibility

In this research work, a novel approach has been taken for the purpose of modeling a six-link six-joint manipulator system considering flexible joints in sim-mechanics environment. Figure 1 illustrates the physical structural of the overall manipulator system.

Here, (X_i, Y_i) is the coordinate frame attached to each link body and " O_i " implies the center of each coordinate frame. In this case, considered manipulator system is having six different links connected in a serial manner. The first link connection of the manipulator has been made with the base frame, and the sixth link of the manipulator has been utilized for serving the purpose of material handling by considering a predefined given weight. Each link length has been considered as 0.5 m for the purpose of simplification. First link mass has been considered as 0.5 kg which is followed by the subsequent increment in the mass of 0.5 kg for each of the successive links. Thus, the final link mass has been taken as 3 kg. The sim-mechanics model of the above-mentioned six-link flexible manipulator is shown in Fig. 2.

Here, the first-link body has been connected to the ground by means of revolute joint. Thus, each subsequent link body has been connected with the previous one by different revolute joints. The joint spring damper block

has been attached to each joint in order to introduce joint flexibility to the overall system. The coordinate system attached to each link body defines the geometrical position of the respective link considering the ground as origin (0, 0). In each joint block, "B" indicates the base and "F" indicates the follower. Two different joint sensors have been attached to the first and sixth link of the system. Through these sensors, the first and sixth joint angular position and acceleration can be extracted from the scope box placed to the other end of the sensor. The two coefficients arising from the damping and stiffness property of the material have been considered for the six revolute joints, respectively, that have been used to construct the six-link flexible serial manipulator. Here, the joint spring-damper serves the purpose for an actuator, which calculates the damped torque and applies the same to the body of the link. Furthermore, the position of the center of gravity of the six link bodies have been changed a little from the points where the geometric center of the link body lies in order to address greater complex behavior of the above said problem. In order to justify the proposed sim-mechanics model, only a single-link manipulator has been considered first. This acts as a single pendulum falling under the action of gravitational force. Then, the simulation results have been compared to the known real pendulum. The joint angular position and motion built in sim-mechanics perfectly matches the results obtained from real-life behavior of the same pendulum model. The simulated view of the proposed flexible manipulator dynamic model built in sim-mechanics environment is shown in Fig. 3.

2.1 Control methodology and the analysis of stability of the six-link manipulator with joint flexibility

It has been noticed that with the incorporation of flexibility, the manipulators become more advantageous than its completely rigid counterparts. From the available resources, one can easily get the kinematic as well as

Fig. 1 Model of flexible manipulator having six links connected with six joints

Fig. 2 Six-link six-joint flexible manipulator system model constructed in the environment of sim-mechanics

Fig. 3 Simulated view of the proposed sim-mechanics model

dynamic equations for the rigidity-based multi-link manipulators. But, in order to establish the dynamic model for the flexible manipulators, an efficient analytical method should be adopted properly due to its higher degree of complexity. For this reason, a serially connected six-link

manipulator model with joint flexibility has been constructed with the help of sim-mechanics. Our basic aim is to control the deflection of the free end of the manipulator applying actuating forces at the base. A controller can be designed knowing the transfer function. However,

deriving the exhaustive mathematical function relating the angular position of the free end to that of the actuating forces applied at the base is highly complex. On the other hand, once Simulink model is built and working, transfer function for the system can be derived using the standard function namely “dlinmod” or “linearize.” Here, transfer function is derived using SISO toolbox of MATLAB. This derived transfer function is given below and provided similar response under step and frequency input with the sim-mechanics model. Internally, it identifies the system and derives the transfer function depending on the dynamics of the driving signal and the system.

Angular position of the free - end of the manipulator/actuating force at joint 1 =

$$\frac{3.41e^{-13}s^9 - 0.58s^8 + 1089s^7 - 3.4e^5s^6 + 1.24e^7s^5 - 1.003e^7s^4 - 7.34e^5s^3 - 1.94e^4s^2 - 227.2s - 0.99}{s^{10} + 601.6s^9 + 1.05e^5s^8 + 5.76e^6s^7 + 7.3e^7s^6 + 9.12e^7s^5 + 7.65e^6s^4 + 2.64e^5s^3 + 4588s^2 + 39.9s + 0.14}$$

The graphical results from the system transfer function are shown in Fig. 4.

From the bode plot, it has been noticed that the manipulator system produces such kind of open-loop bode which clearly indicates the system instability. For this reason, the requirement of the system stability has been found essential. In order to serve for the above purpose, the controlling method used in a conventional PID controller has been adopted here.

Often, it has been found that the classical PID-type control strategy is very useful for such cases. The PID control technique is dependent on three different gain parameters for its control action. So, the proper tuning of those gain

parameters is required to achieve effective performance of the PID controller. In this research work, SISO design-based and fuzzy logic-based PID controller tuning approaches have been introduced to tune the PID controller gains.

3 Results and discussion

The prime objective of this kind of research work is the development of a manipulator with single-link that will dictate the behavior of a utopian rod or pendulum. Here, two differently constructed models of the manipulator

system have been introduced. For the first case, a rigidly constructed single-link body has been divided to six equal length links which connects each other in a serial manner by means revolute joints. For second model, the joint damper elements have been introduced to all the six joints. Simulation processes for these two different models have been conducted with the help of Simulink. Table 1 clearly indicates the positional movement taken in angle and the angular velocity of first and sixth joint for those two differently modeled manipulators at various time zones.

The movements of the said manipulator with flexibility have been noticed carefully which shows greater smooth

Fig. 4 Graphical results from the extracted state space manipulator system model

Table 1 Comparison between the angular displacements along with velocity for the first, sixth joining element in case for flexible and rigid system

Instances (S)	System with joint flexibility				Rigidly constructed system			
	Displacement (°)		Velocity (°/S)		Displacement (°)		Velocity (°/S)	
	J (I)	J (VI)	J (I)	J (VI)	J (I)	J (VI)	J (I)	J (VI)
0	0	0	0	0	0	0	0	0
5	−3.9	36.8	−10	18.7	99	42.8	−89	79
10	−5.8	50.8	−5.9	20.5	57	136.4	−811	−738
15	−3.6	65.6	10.7	36.5	−88	109	−61	−117
20	0	93	2.4	54	146	92	558	−309
25	1.2	115.2	3.1	24	−117	75	−472	−792
30	3.1	116	3.9	−8.6	−39	134.7	1348	−496

action compared to the rigidly constructed manipulator. Furthermore, lesser time has been taken by the flexible manipulator in order to settle down. That is why further analysis has been performed for the manipulator having flexibility.

Both the flexible and rigid manipulators are unstable and effective controlling action is needed to attain the stability. PID controller is one of the widely used controllers in this scenario and one can get it easily in Simulink.

3.1 Tuning of controller gain parameters

In this work, the activities regarding the control issues have been introduced to the system by taking help from the SISO design tool box of MATLAB 7.8. Conventional-type controlling method like PID controller has been taken into consideration in the present work. The toolbox of SISO design tool is having a provision to tune the PID controller gain values automatically and accordingly it generates tuned controller gain values in an automatic manner. At first, the compensator parameters are tuned for a given step input and maximum possible stable performance of the system has been observed. These corresponding compensator parameters have been further used to control the overall flexible system model.

The sim-mechanics model for the above said purpose is shown in Fig. 5.

Table 2 illustrates the gain and phase margin values of four differently configured controllers where the tuning has been done automatically with the help of three different methods.

The tuning process based on Ziegler Nichols (ZN) method is widely used and this tuning method has been adopted in this case. Two different response-based ZN tuning processes have been studied here. Controller gain values tuned with the help of frequency response-based ZN method gives greater margin of stability among the

prescribed controller gain tuning methods. The controller gains values that have been tuned with the help of robust response time (RRT) method give the most fruitful result as long as the stable performance is concerned. The most stable performance has been obtained using the compensator $\frac{-0.0045(1+20s)}{s}$, which is based on the RRT method. The values of the compensator gains have been found to be ($K_p = -0.089$, $K_D = 0$, $K_I = -0.0045$), and this can be noted down for future use. The negative gain values indicate the action of directly acting controller. This means an increased control action is needed for the increased control variable (measurement). Or else, for a closed-loop system the positive feedback loop should have to be considered.

The angular position of the first joint of the manipulator is shown in Fig. 6.

In the second approach, the tuning of gain parameters has been done using a mamdani-type fuzzy logic-based tuning process. The control action of a fuzzy logic greatly depends on the construction of its fuzzy knowledge base. In order to tune the PID controller based on fuzzy logic, it is required to construct properly the rule base and data base of the fuzzy knowledge base. In the present work, the fuzzy data base has been constructed with two input variables, i.e., e , $\dot{e}$ and three output variables, i.e., K_p , K_d , K_i . Here, the triangular membership function has been considered for all the variables. Three membership functions have been used for each variable. The ranges of all the variable are tabulated in Table 3.

Figure 7 illustrates the sim-mechanics model diagram for the fuzzy logic-based tuning of the controller gains. Here, step input has been given to the system and the acceptable controller performance has been noticed in order to achieve stable system performance. The rule base of the fuzzy logic is a collection of rules that correlate the input variables to the output variables according

Fig. 5 Tuning of PID controller gains for given step input

Table 2 Gain margin, phase margins for the model under various controlling action

Controlling action	Controlling action tune following Z–N method				Controlling action tune Following R–R–T method	
	Frequency based		Step based			
	GM (DB)	PM (°)	GM (DB)	PM (°)	GM (DB)	PM (°)
P	5.90	43	2.68	29.8	8.97	60
PI	5.96	22.7	2.95	15.8	34.9	60
PID	4.78	25.6	1.20	35.7	35.8	60
PID with derivative filter	4.21	25.8	0.594	31.9	35.6	60

to the selected rule. The manual selection of nine rules has been introduced here to achieve the acceptable performance of the fuzzy logic-based tuning approach. The selected rule base is given in Table 4.

The angular position of the first joint of the manipulator with fuzzy logic-based PID controller tuning is shown in Fig. 8.

This clearly indicates greater acceptability of the second approach. From the simulation results shown here, it

can be noted that although both the approaches produce more or less stable performance of the system, but the second approach gives more stability to the flexible system by minimizing the overall error (Table 5).

Thus, it can be said that the classical PID-type controller with fuzzy logic-based gain tuning approach can be well-implemented to control a multi-DOF flexible manipulator system.

Fig. 6 Position of the first joint—RRT method-based PID tuning **Table 3** Ranges of input and output variables of the fuzzy data base

Input variable	Range	Output variable	Range
e	$[-10 \ 10]$	K_p	$[0.01 \ 20.01]$
edot	$[-100 \ 100]$	K_d	$[0.01 \ 8.01]$
		K_i	$[0.01 \ 4.01]$

Table 4 Rule base of the fuzzy logic system

e	edot	K_p	K_d	K_i
NL	NB	L	L	L
NL	Z	L	M	L
NL	PB	L	M	H
Z	NB	L	M	L
Z	Z	M	M	M
Z	PB	M	H	M
PL	NB	H	L	H
PL	Z	M	L	H
PL	PB	H	H	H

Here, the PID controller has been introduced to the system model to control the flexible manipulator system. The controller gain values have been chosen as the most suitably tuned gain values.

4 Conclusions

In this work, model of a flexible manipulator is made using sim-mechanics and controlled with the help of a PID controller. Behavior of the flexible manipulator changes with time and demand an adaptive time-dependent controller. Therefore, two tuning methods (classical Ziegler Nichols technique and fuzzy logic-based method) for tuning the controller gains are adopted in this study. Following observations are made from the study.

- Deriving the exact mathematical model of the flexible manipulator is difficult. Sim-mechanics model has performed well with payload.
- Derived system transfer function has approximated the sim-mechanics model and generated similar performance under step and frequency inputs.
- Both the tuning methods adopted in this study provided stable solutions. However, fuzzy logic-tuned gains generated low steady state error and settled quickly. On the contrary, the RRT-tuned gains resulted in low percentage overshoot and low rise time. Since the settling time is very low with the fuzzy logic-

Fig. 7 Tuning of PID controller gains using fuzzy logic controller

Fig. 8 Position of the first joint—fuzzy logic-based PID tuning

Table 5 Performance comparison between RRT method and fuzzy logic-tuned PID controller

Parameters	PID controller gain tuning method	
	RRT based	Fuzzy logic based
Rise time (S)	0.1	0.15
Settling time (S)	110	60
Maximum overshoot (°)	160	175
Steady state error (°)	85	75

tuned gains, this can be adopted for flexible manipulator control.

There are many ways the present work can be extended starting from the detailed mathematical modeling, dynamic analysis, and controller design in the presence of payload.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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