A snail-shaped chaotic system with large bandwidth: dynamical analysis, synchronization and secure communication scheme

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Received: 15 October 2019 / Accepted: 29 April 2020 / Published online: 9 May 2020
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Abstract

In this paper, a snail-shaped chaotic system with large bandwidth is first introduced, it contains two quadratic, one cubic and two quartic nonlinear terms. The new snail-shaped autonomous system can exhibit periodic, quasi-periodic and chaotic behaviors with the variations of its parameters. The major properties of the proposed model are discussed using equilibrium points, Kaplan–Yorke dimension, Lyapunov exponents spectrum and bifurcation diagrams. The feasibility of the snail-shaped system is verified by implementing an electronic circuit via Multisim software. The obtained results proving the complex chaotic behavior of the new system, which make it very desirable to use in many fields of engineering especially in secure communication. Also, synchronization between the snail-shaped chaotic system and the Rossler hyperchaotic system is achieved. Finally, a new simple secure communication scheme is developed based on the proposed system and using drive response synchronization in order to prove the success of the new system to complete the encryption/decryption process.

Keywords Chaotic system · Dynamical analysis · Lyapunov exponent · Bifurcation diagram · Synchronization · Circuit design

1 Introduction

In recent years, the study of chaotic systems become a very interest area of research because of its wide applications in many fields of science such as secure communication and cryptography [1–3].

Many chaotic systems have been proposed [4–6] since the first model introduced by the American meteorologist Edward Lorenz in 1963 [7]. The great technological development, especially in secure transmission making the designing of new chaotic systems a necessary need, but there is no uniform method to construct or to predict a chaotic behavior, For this reason all new systems are proposed as a mathematical models and verified using theoretical analysis and numerical simulations.

The main theoretical tool to identify a chaotic system is that of Lyapunov exponents, which are a quantity that characterizes the rate of divergence of close trajectories, a positive Lyapunov exponent indicate that the system generates a chaotic behavior. In addition, the real feasibility of the proposed systems is verified by designing its electronic circuits using simulation software or physical implementations.

In this work, a new snail-shaped chaotic system is first introduced which is very complex and has a large bandwidth; it contains eleven-terms with five nonlinearities making it able to generate rich dynamical behaviors. Dynamical properties are discussed theoretically and numerically. In addition, a comparative study between the proposed system and 30 recently proposed systems

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is introduced. Then, an electronic circuit is implemented using Multisim software, which validate the theoretical analysis and proved the feasibility of the new system. Also, a synchronization between the new system and the classical Rossler hyperchaotic system is achieved using active control method. Finally, a secure communication scheme is developed using the drive response synchronization and the four signals generated by the new system. The obtained results confirm the effectiveness of the new snail-shaped chaotic system to complete the encryption/decryption process in a secure communication scheme.

2 The new chaotic attractor

2.1 Algebraic structure of the new system

In this section, we announce a new 4D chaotic system with eleven terms, including five nonlinearities. The new system has four parameters and it is described by the following differential equations:

$$\begin{cases} \dot{x} = a(y - x - zw) \\ \dot{y} = bx - xz^3 - w^3 \\ \dot{z} = x^2 - z + w \\ \dot{w} = cxyzw - dw \end{cases} \quad (1)$$

The new system generates a chaotic behavior for these values of parameters:

$$a = 4, \quad b = 4.8, \quad c = 0.5, \quad d = 0.5 \quad (2)$$

Some famous chaotic attractors have specific names according to their shapes, among them: Butterfly-shaped attractor [8], torus-shaped attractor [9], funnel-shaped attractor [10] and so on. Our new system (1) exhibits a snail-shaped attractor as depicted in Fig. 1 for the

parameters values as in (2). We can see that the new attractor in Fig. 1b has a same shape as a snail that is shown in Fig. 2a.

2.2 Sensitivity to initial conditions

As we know, the main property of the chaotic systems is that of the sensitivity about the initial values. The high sensitivity to the initial conditions implies the existence of chaos. Figure 2 shows the high sensitivity of the proposed system (1) to a very little change in the initial values ($\pm 10^{-10}$).

2.3 Lyapunov exponents and Kaplan–Yorke dimension

The two most important tools to characterize a chaotic behavior of a system are the Lyapunov exponents and the Kaplan–Yorke dimension.

Fig. 2 Time series of the x variable for $x_0 = 0.2$ and $x_0 = 0.2000000001$

Fig. 1 **a** A snail. **b** Chaotic snail-shaped attractor of the new system (1)

(a)

(b)

As it is known, the Lyapunov exponent is a measure of exponential rates of convergence and divergence for an uncertainty on the trajectories initial points. When it is positive the uncertainty increases, which means divergence of trajectories (chaos). On the contrary, a negative exponent indicates convergence of the dynamical system trajectories [11].

Hence, a continuous dynamical system is chaotic, if it has at least one positive Lyapunov exponent.

The Lyapunov exponents of the proposed model (1) are shown in Fig. 3 as:

$$LE_1 = 0.518, LE_2 = 0.000, LE_3 = -0.795, LE_4 = -4.722 \quad (3)$$

So, system (1) has one positive Lyapunov exponents. Hence, the new system is chaotic.

In addition, a chaotic attractor should have fractional dimension. For our proposed system (1), we used the Kaplan–Yorke dimension, which is a useful tool in order to determine the fractal dimension of the corresponding attractor [12], and it can be calculated by using the Lyapunov exponents as the following:

$$KY = 2 + \frac{LE_1 + LE_2}{|LE_3|} \quad (4)$$

$$KY = 2 + \frac{0.518}{|-0.795|} = 2.652$$

Since the Kaplan–Yorke dimension is fractal. So, the new model generates a complex chaotic behavior.

Fig. 3 Lyapunov exponents of the new system (1)

2.4 Frequency spectrum and bandwidth

Average normalised frequency spectrum of the new system (1) second state y is depicted in Fig. 4. The bandwidth (BWT) of the new snail-shaped chaotic system is approximately equal to 15, which is more than at least 30 chaotic systems reported in literature, as shown in Table 1.

2.5 Comparison between the proposed chaotic system and 30 reported systems

As we know:

1. Kaplan–Yorke dimension (KYD) is an index for characterizing the complexity and the unpredictability of a chaotic system, higher Kaplan–Yorke dimension implies higher complex chaotic behaviour [12].
2. The bandwidth (BWT) of a carrier signal generated by a chaotic system should be high in order to completely cover the masked information transmitted in a secure transmission scheme [13].

In this subsection, a comparison between the proposed snail-shaped chaotic system with 30 famous and recently reported chaotic systems is introduced using Kaplan–Yorke dimension and bandwidth. This comparison showed that the Kaplan–Yorke dimension of our system is higher than the most famous and novel chaotic systems. In addition, we can see from Table 1 that the bandwidth of system (1) is much higher than the other proposed systems. Therefore, the new snail-shaped chaotic system is more complex and more useful for secure communication than at least 30 reported systems as explained in Table 1.

Fig. 4 Average power spectrum of the second state signal y

Table 1 Kaplan–Yorke dimension and bandwidth of 30 reported systems

No.	System	KYD	BWT
1	Lorenz [7]	2.062	4
2	Chen [14]	2.175	6
3	Su [15]	2.044	5
4	Akgul [16]	2.024	5
5	Zhang [17]	2.025	5
6	Gholamin et al. [18]	2.055	6
7	Lai et al. [19]	2.196	6
8	Tuna et al. [20]	2.191	4
9	Wu et al. [21]	2.015	7
10	Volos et al. [22]	2.175	2.5
11	Jay et al. [23]	2.160	13
12	Sambas et al. [24]	2.072	1.5
13	Yang et al. [25]	2.112	4
14	Lassoued et al. [26]	2.124	2
15	Vaidyanathan et al. [27]	2.162	9
16	Lien et al. [28]	2.163	3
17	Kapitaniak et al. [29]	2.107	3
18	Xu et al. [30]	2.074	2
19	Idowu et al. [31]	2.236	2
20	Jinjie et al. [32]	2.185	9
21	Lai et al. [33]	2.091	8
22	Sahin et al. [34]	2.579	2
23	Zhou et al. [35]	2.062	4
24	Vaidyanathan et al. [36]	2.063	5
25	Tasikmalaya et al. [37]	2.066	7
26	Wang et al. [38]	2.032	4
27	Vaidyanathan et al. [39]	2.213	2
28	Yanmin et al. [40]	2.276	10
29	Zheng [41]	2.594	2
30	Benkouider et al. [42]	2.280	13
31	System (1)	2.652	15

3 Dynamical analysis of the new system

3.1 Dissipativity

The divergence of the system (1) is calculated by the following formula:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -1 - 1 - d + cxyz \quad (5)$$

Hence,

$$\nabla V = -2 - d + cxyz \quad (6)$$

Since, $cxyz < d + 2$ so, $\nabla V < 0$. Hence, the new 4D system is dissipative and all system (1) orbits are ultimately

confined to a specific subset of zero volume, and the asymptotic motion settles onto an attractor.

3.2 Equilibrium points and stability

The equilibrium points of the new system (1) are founded by solving the following set of equations:

$$\begin{cases} a(y - x - zw) = 0 \\ bx - xz^3 - w^3 = 0 \\ x^2 - z + w = 0 \\ cxyzw - dw = 0 \end{cases} \quad (7)$$

System (1) has a unique equilibrium point S:

$$S = [0, 0, 0, 0] \quad (8)$$

The stability of system (1) around this equilibrium point is analysed by using the below Jacobian matrix:

$$J_{x,y,z,w} = \begin{bmatrix} -a & a & -aw & -az \\ b - z^3 & 0 & -3xz^2 & -3w^2 \\ 2x & 0 & -1 & 1 \\ cyzw & cxzw & cxyw & -d + cxyz \end{bmatrix} \quad (9)$$

For the equilibrium point S, the Jacobian matrix become as follows:

$$J_S = \begin{bmatrix} -a & a & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -d \end{bmatrix} \quad (10)$$

In order to find the eigenvalues of the Jacobian matrix (10), and for the parameters values as in (2), we are solving the corresponding characteristic equation:

$$\lambda^4 + 5.5\lambda^3 - 12.7\lambda^2 - 26.8\lambda - 9.6 = 0 \quad (11)$$

The eigenvalues are found as:

$$\lambda_1 = -6.82, \lambda_2 = -1, \lambda_3 = -0.5, \lambda_4 = 2.82 \quad (12)$$

Since $\lambda_{1,2,3} < 0$ and $\lambda_4 > 0$ so, the equilibrium point S is unstable.

3.3 Bifurcation analysis

3.3.1 Fix $b=4.8, c=0.5, d=0.5$, and vary a

The Bifurcation diagram of the new system (1) with respect to parameter a is depicted in Fig. 5. Also, the Lyapunov exponents spectrum is shown in Fig. 6. We can see that

Fig. 5 Bifurcation diagram of the system (1) first state versus parameter a

Fig. 6 Lyapunov exponents of the system (1) versus parameter a

Fig. 7 Phase portraits of the periodic orbits

the Bifurcation diagram and the spectrum of Lyapunov exponents are completely compatible.

From Figs. 5 and 6, we note that the new system can exhibit different dynamical behaviors with the variation of parameter a , as it is summarized below:

- If $a = 11 \in [3.15, 3.25[\cup]3.40, 3.55[\cup]5.60, 5.80[\cup]6.20, 6.30[\cup]6.55, 6.80[\cup]7.00, 7.60[\cup]8.20, 8.40[\cup]8.80, 9.20[\cup]10.50, 12]$
The corresponding Lyapunov exponents are:
 $LE_1 = 0.00, LE_2 = -0.16, LE_3 = -0.75, LE_4 = -11.09$
system (1) generates periodic orbits such as depicted in Fig. 7.
- If $a = 9.5 \in [9.2, 10.5]$, the corresponding Lyapunov exponents are:
 $LE_1 = 0.00, LE_2 = 0.00, LE_3 = -0.75, LE_4 = -9.67$

So, system (1) generates quasi-periodic orbits such as depicted in Fig. 8.

- If $a = 4 \in [3.00, 3.15[\cup]3.25, 3.40[\cup]3.55, 5.6[\cup]5.8, 6.2[\cup]6.3, 6.55[\cup]6.8, 7[\cup]7.6, 8.2[\cup]8.4, 8.8]$
The corresponding Lyapunov exponents are:
 $LE_1 = 0.52, LE_2 = 0.00, LE_3 = -0.79, LE_4 = -4.72$

So, system (1) generates chaotic attractor such as depicted in Fig. 9.

3.3.2 Fix $a=4, c=0.5, d=0.5$, and vary b

The Bifurcation diagram of the new system (1) with respect to parameter b is depicted in Fig. 10. Also, the Lyapunov exponents spectrum is shown in Fig. 11. We can see that the Bifurcation diagram and the spectrum of Lyapunov exponents are completely compatible.

From Figs. 10 and 11, we note that the new system can exhibit periodic and chaotic behaviors with the variation of parameter b , as it is summarized below:

Fig. 8 Phase portraits of the quasi-periodic orbits **Fig. 9** Phase portraits of the snail-shaped chaotic attractor **Fig. 10** Bifurcation diagram of the system (1) first state versus parameter b

- If $b = 2.5 \in [2.00, 2.90[\cup]3.43, 3.46[\cup]4.60, 4.66]$ the corresponding Lyapunov exponents are:

$$LE_1 = 0.00, LE_2 = -0.48, LE_3 = -0.50, LE_4 = -4.02$$

Fig. 11 Lyapunov exponents of the system (1) versus parameter b

So, system (1) generates periodic orbits such as depicted in Fig. 12.

- If $b = 4 \in [2.90, 3.43[\cup]3.46, 4.60[\cup]4.66, 5.00]$, the corresponding Lyapunov exponents are:

$$LE_1 = 0.45, LE_2 = 0.00, LE_3 = -0.69, LE_4 = -4.75$$

So, system (1) generates chaotic attractor such as depicted in Fig. 13

Fig. 12 Phase portraits of the periodic orbits

Fig. 13 Phase portraits of the chaotic attractor

4 Electronic circuit implementation

In order to verify the feasibility of the proposed chaotic attractor, an electronic circuit is designed using Multisim software.

According to the dynamical equations of the new 4D system (1), the corresponding equations of the electronic circuit are given by:

$$\begin{cases} \dot{x} = \frac{1}{R_1 C_1} y - \frac{1}{R_2 C_1} x - \frac{1}{R_3 C_1} z w \\ \dot{y} = \frac{1}{R_4 C_2} x - \frac{1}{R_5 C_2} x z^3 - \frac{1}{R_6 C_2} w^3 \\ \dot{z} = \frac{1}{R_7 C_3} x^2 - \frac{1}{R_8 C_3} z + \frac{1}{R_9 C_3} w \\ \dot{w} = \frac{1}{R_{10} C_4} x y z w - \frac{1}{R_{11} C_4} w \end{cases} \quad (13)$$

The schematic of the circuit is shown in Fig. 14.

The values of the electronic circuit elements are chosen as follows:

$$\begin{cases} R_1 = R_2 = R_3 = 42.105 k\Omega \text{ (Quasi - periodic)} \\ R_1 = R_2 = R_3 = 36.363 k\Omega \text{ (Periodic)} \\ R_1 = R_2 = R_3 = 100 k\Omega \text{ (Chaotic)} \\ R_4 = 83.334 k\Omega \\ R_5 = R_6 = R_7 = R_8 = R_9 = 400 k\Omega \\ R_{10} = R_{11} = 800 k\Omega \\ R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = 100 k\Omega \\ C_1 = C_2 = C_3 = C_4 = 1 nF \end{cases} \quad (14)$$

Figures 15, 16 and 17 show respectively the snail-shaped attractor, the periodic orbit and the quasi-periodic orbit of the new system (1) via Multisim software which is consistent with the Matlab simulations.

All this results proved the real feasibility of the proposed snail-shaped chaotic system (1).

5 Synchronization of the new chaotic system with Rossler hyperchaotic system

In this section, synchronization between system (1) and the famous Rossler hyperchaotic system (15) is studied and achieved via active control. We are proved that the proposed chaotic system can be synchronized not only with chaotic systems but also with hyperchaotic systems, which make it

Fig. 14 The circuit schematic **Fig. 15** Multisim result of the snail-shaped chaotic attractor **Fig. 16** Multisim result of the periodic orbit

Fig. 17 Multisim result of the quasi-periodic orbit

very useful in different fields of science especially in secure transmission.

The classical Rossler hyperchaotic system is described as follows:

$$\begin{cases} \dot{x} = fy + gx \\ \dot{y} = 3 + yz \\ \dot{z} = -y - w \\ \dot{w} = x + z + hw \end{cases} \quad (15)$$

Where,

$$f = -0.5, g = 0.05, h = 0.25 \quad (16)$$

With these values of parameters, Rossler system has two positive Lyapunov exponents:

$$LE_1 = 0.109 \text{ and } LE_2 = 0.024$$

So, system (15) exhibits a hyperchaotic behavior as depicted in Fig. 18.

5.1 Control functions design

The Rossler system (15) is considered as a master system and it is described as follows:

$$\begin{cases} \dot{x}_m = fy_m + gx_m \\ \dot{y}_m = 3 + y_m z_m \\ \dot{z}_m = -y_m - w_m \\ \dot{w}_m = x_m + z_m + hw_m \end{cases} \quad (17)$$

The new system (1) is considered as a slave system and it is described as follows:

$$\begin{cases} \dot{x}_s = a(y_s - x_s - z_s w_s) + u_x \\ \dot{y}_s = bx_s - x_s z_s^3 - w_s^3 + u_y \\ \dot{z}_s = x_s^2 - z_s + w_s + u_z \\ \dot{w}_s = cx_s y_s z_s w_s - dw_s + u_w \end{cases} \quad (18)$$

Where u_x , u_y , u_z and u_w are the active control functions to be found. The state errors between the master and the slave systems are defined as:

$$e_x = x_s - x_m, e_y = y_s - y_m, e_z = z_s - z_m, e_w = w_s - w_m \quad (19)$$

Hence, the dynamical of state errors is described by the following equations:

$$\begin{cases} \dot{e}_x = ay_s - ax_s - az_s w_s - fy_m - gx_m + u_x \\ \dot{e}_y = bx_s - x_s z_s - w_s - 3 - y_m z_m + u_y \\ \dot{e}_z = x_s^2 - z_s + w_s + y_m + w_m + u_z \\ \dot{e}_w = cx_s y_s z_s w_s - dw_s - x_m - z_m - hw_m + u_w \end{cases} \quad (20)$$

Fig. 18 Rossler hyperchaotic attractor

Theorem If we choose the active control functions as the following:

$$\begin{cases} u_x = (a+g)x_m - (a-f)y_m + az_s w_s - ae_y + (a-1)e_x \\ u_y = x_s z_s^3 + w_s^3 + 3 + y_m z_m - bx_m - be_x - e_y \\ u_z = -x_s^2 + z_m - y_m - 2w_m - e_w \\ u_w = -cx_s y_s z_s w_s + (d+h)w_m + x_m + z_m + (d-1)e_w \end{cases} \quad (21)$$

The dynamical of state errors will be converge asymptotically to zero. Hence, the new chaotic system (1) and the Rossler hyperchaotic system (15) will be synchronized.

Proof Considering the active control functions defined in (21), the dynamical of state errors (20) can be rewritten as:

$$\begin{cases} \dot{e}_x = -e_x \\ \dot{e}_y = -e_y \\ \dot{e}_z = -e_z \\ \dot{e}_w = -e_w \end{cases} \quad (22)$$

Therefore,

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \\ \dot{e}_w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix} \quad (23)$$

All eigenvalues of the state matrix are negative, so, according to Routh–Hurwitz criterion; the errors dynamics are stable which ensure synchronization between the master system (15) and the slave system (1).□

5.2 Numerical simulation results

Matlab/Simulink software is used to implement the control laws in order to synchronize the two systems.

The parameters values as in (2) and the initial conditions $[20, -7, 0.2, 0.2]$ are chosen for the proposed system (1) and the parameters values as in (16) and the initial conditions $[14, 2, -17, 7]$ are chosen for the Rossler system (15).

Figures 19 and 20 show that the two systems are synchronized and the state errors converge to zero exponentially, which proved the effectiveness of the designed control functions to synchronize the proposed snail-shaped chaotic system (1) and the hyperchaotic system of Rossler (15).

Fig. 19 Coordinates times series of the master and the slave systems

Fig. 20 Errors time series between the master and the slave systems

6 Secure transmission scheme based on the proposed chaotic system

In this section, a new secure communication scheme is developed for the new snail-shaped chaotic system (1) using the drive response synchronization which has the advantage of strong self-synchronization and the use of only one coordinate to achieve synchronization.

The new scheme is tested using Matlab/Simulink software, simulation results showing the success of the proposed system to complete the encryption/decryption process.

Matlab/Simulink block diagram of the new secure communication scheme is depicted in Fig. 21.

6.1 Drive response synchronization

Consider the new snail-shaped chaotic system (1) as master system; the slave system is described by the following equations:

$$\begin{cases} \dot{x}_s = a(y_s - x_s - z_s w_s) \\ \dot{y}_s = b x_s - x_s z_s^3 - w_s^3 \\ \dot{z}_s = x_s^2 - z_s + w_s \\ \dot{w}_s = c x_s y_s z_s w_s - d w_s \end{cases} \quad (24)$$

By choosing the second state variable of the master system y_m as driving variable in order to lead the slave

system (24) to achieve synchronization with master system (1), the slave system can be rewritten as follows:

$$\begin{cases} \dot{x}_s = a(y_m - x_s - z_s w_s) \\ \dot{z}_s = x_s^2 - z_s + w_s \\ \dot{w}_s = c x_s y_m z_s w_s - d w_s \end{cases} \quad (25)$$

Simulation results depicted in Figs. 22, 23 and 24 showing the synchronization errors between the master coordinates and the slave coordinates, it can be seen that all synchronization errors converge to zero after few second (5 s), which confirm the success of the drive response method to synchronize the two identical master and slave snail-shaped chaotic systems starting from different initial values.

6.2 Encryption/decryption process

Using the drive response synchronization, a new encryption/decryption process is developed for the new snail-shaped system as depicted in Fig. 21b, c.

The encrypted message EM that will be transmitted is obtained by switching between two different encryption processes according to the value of the master system first state x_m as the following:

If $x_m < 0$: the encrypted message is obtained using the following equation:

$$\begin{cases} EM = F(m, x_m, z_m) \\ EM = p_1 m + p_2 x_m + p_3 z_m \end{cases} \quad (26)$$

where m is the clear message to be encrypted, p_1 , p_2 and p_3 are constant parameters considered as a secret key in order to enhance complexity and confidentiality of the encryption process.

If $x_m \geq 0$: the encrypted message is obtained using the following equation:

$$\begin{cases} EM = Q(m, x_m, w_m) \\ EM = k_1 m + k_2 x_m + k_3 w_m \end{cases} \quad (27)$$

where k_1 , k_2 and k_3 are constant parameters considered as a secret key in order to enhance complexity and confidentiality of the encryption process.

In the receiver, after achieving synchronization between the master system and the slave system, the decrypted message DM is obtained by switching between two decrypted processes according to the value of the slave system first state x_s using the following algorithm:

If $x_s < 0$: the decrypted message is obtained using the following equation:

(a) The new secure communication scheme

(b) Encryption process

(c) Decryption process

Fig. 21 Secure communication Matlab/Simulink block diagram based on the new snail-shaped chaotic system

Fig. 22 Synchronization error of the first coordinates

Fig. 25 Time evolution of the clear message

Fig. 23 Synchronization error of the third coordinates

Fig. 26 Time evolution of the encrypted message

Fig. 24 Synchronization error of the fourth coordinates

Fig. 27 Time evolution of the decrypted message

$$\begin{cases} DM = F'(m, x_s, z_s) \\ DM = \frac{1}{p_1}(EM - p_2 x_s - p_3 z_s) \end{cases} \quad (28)$$

where EM is the encrypted message to be decrypted, p_1 , p_2 and p_3 are the same constant parameters as in the transmitter.

If $x_s \geq 0$: the decrypted message is obtained using the following equation:

$$\begin{cases} DM = Q'(EM, x_s, w_s) \\ DM = \frac{1}{k_1}(EM - k_2 x_s - k_3 w_s) \end{cases} \quad (29)$$

where k_1 , k_2 and k_3 are the same constant parameters as in the transmitter.

- For numerical simulation

We chose the constant parameters of the encryption and the decryption functions as the following:

$$p_1 = p_2 = p_3 = k_1 = k_2 = k_3 = 1 \quad (30)$$

Fig. 28 Message reconstruction error

We choose the sine wave as the clear message to be transmitted as depicted in Fig. 25. Figure 26 shows the complex chaotic behavior of the encrypted message which confirm the effectiveness of the encrypting process to hide the clear message. The decrypted message is depicted in Fig. 27, we can see that after a few time (after achieving synchronization between master and slave systems) the obtained message is the same as the original message. Figure 28 shows the convergence of the message reconstruction error to zero after achieving synchronization, which confirms the success of the decryption process to reconstruct the original message.

7 Conclusion

In this work, a new snail-shaped chaotic system having eleven terms and four parameters is first presented. The new system can exhibit periodic, quasi-periodic and complex chaotic behaviors which are studied and proved theoretically by an exhaustive dynamical analysis and physically by implementing an equivalent electronic circuit via Multisim software. The new system is more complex and has a larger bandwidth than at least 30 recently reported chaotic systems, which is make it very useful in many fields of science such as: secure transmission and cryptography.

Also, synchronization between the novel system and the famous Rossler hyperchaotic system is achieved in order to illustrate the possibility of using the new system in applications, not only with chaotic systems but also with hyperchaotic systems. Finally, in order to test the applicability of the snail-shaped chaotic system a new secure communication scheme is developed for it by using drive response synchronization; simulation results proved the success of the proposed system to complete the secure transmission/reception process.

We strongly believe that the proposed chaotic system with its large bandwidth and complex behavior is desirable to use in a real secure transmission schemes in the near future.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interests.

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