Comparing the wake behind circular and elliptical cylinders in a uniform current

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Abstract

In this study, an experimental investigation has been conducted on the characteristics of the wake behind the elliptical cylinder with the axis ratio of $AR = 2$. In order to provide a comparison, the same experimental investigation has also been conducted for a circular cylinder with the vertical height equal to the height of the elliptical cylinder. For studying the wake behind these two cylinders, hot-wire anemometry measurement has been performed. All of the tests are carried out in Reynolds number of 3.0×10^4 (which is defined according to the mean velocity of air and the cylinder diameter). Results are presented in terms of time-averaged velocity, standard deviation, higher-order central moments of the hot-wire signals (skewness and kurtosis factors), Strouhal number and drag coefficient. For the regions near the wake, the velocity deficit of the elliptical cylinder is greater compared to the circular cylinder; however, this is completely reversed in the regions far from the wake. As the axis ratio of the cylinder decreases, the dimensionless standard deviation becomes higher, and its maximum value is located at an earlier stream-wise location. In addition, it is observed that by increasing the axis ratio of the cylinder, drag coefficient decreases, while the Strouhal number is increasing.

Keywords Wind tunnel · Hot-wire anemometry · Circular and elliptical cylinders · Drag coefficient · Standard deviation

1 Introduction

The cross flow through the cylinders have extensive applications in engineering field such as cooling towers, heat exchangers, chimney stacks, nuclear reactors, and offshore structures. Consequently, a large number of researches have been conducted on this kind of flow, mainly through circular cylinders. Here, we mention some of them, in which the experimental and numerical methods are applied. This geometrical configuration was firstly investigated experimentally by famous physicists like Strouhal, Von Kármán and Prandtl in late nineteenth and early twentieth century (Refer to Williamson [1] and its references).

The transition between laminar and turbulent flow occurs in special flow conditions and geometries, in

other words, it occurs mainly according to the interactions between unstable regions of flow and not due to the geometry of body itself. As can be seen in Fig. 1, the passing flow through cylinders is very complicated. The area around the cylinder can be divided into three main regions, including boundary layer region of the cylinder, the region comprising two shear layers on the top and bottom of the cylinder and finally, the region representing the wake. Several studies such as Bloor [2], Gerrard [3] and Roshko [4, 5] have been conducted for investigating the different regimes the corresponding phenomena in these regions.

In many researches the flow around a circular cylinder has been considered experimentally and numerically in moderate and high values of Reynolds numbers (Re).

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Fig. 1 Schematic diagram of the flow and coordinate system

Norberg [6] investigated the statistical moments of a circular cylinder (mean, *rms* and skewness) experimentally in the near wake region by the laser doppler velocimetry (LDV) method. He showed that for $Re < 8 \times 10^3$ the *rms* distribution of stream-wise velocity in centerline of the wake has two local maxima; however, in higher *Re*, only one maximum was identified. In an experimental investigation on a circular cylinder at $Re = 3900$, Ong and Wallace [7] studied the velocity profile in the turbulent regime of near wake region using hot-wire anemometry (HWA) method and measured the stream-wise velocity by single-sensor probes. Also, Olsen and Rajagopalan [8], Ezadi Yazdi et al. [9] and Norberg [10] investigated the flow around a circular cylinder using hot-wire anemometry experimentally. Parnaudeanu et al. [11] used both the particle image velocimetry (PIV) and large eddy simulation (LES) method in the subcritical flow regime.

Some researchers investigated the 3-D flow around a circular cylinder through the use of LES and direct numerical simulation (DNS) in the subcritical flow regime [12–19].

For the Reynolds number in the range of 0.5×10^6 and 4×10^6 , which is considered as high Reynolds numbers, Catalano et al. [20] applied two methods of 3-D LES with modeling wall and Unsteady Reynolds-Averaged Navier-Stokes (URANS) with $k - \epsilon$ model of Launder and Spalding [21] and compared the obtained results. Singh and Mittal [22] conducted a 2-D LES method for $100 < Re < 1 \times 10^7$. Catalano et al. [20] focused their numerical research on the Reynolds numbers of 0.5×10^6 and 1×10^6 , after the recovery of the drag coefficient from its sudden loss at $Re = 2 \times 10^5$ (which is known as drag crisis). In this case, the separation of the boundary layer was delayed and drag coefficients were reduced after drag crisis. The relation between the drag crisis and instability of the separated shear layer was investigated by Singh and Mittal [22]; they determined the drag crisis, which was close to the critical *Re*. They first studied the

subcritical regime ($300 < Re < 3 \times 10^5$); however, some of their results were reported in supercritical regime of $Re > 10^6$. Numerical evaluation of Contwell and Coles [23] experiments were conducted by Franke et al. [24] and Tutar and Holdø [25] in the subcritical flow regime ($Re = 1.4 \times 10^5$) at the beginning of drag crisis. Franke et al. [24] applied URANS along with the Launder and Spalding [21] standard $k - \epsilon$ model for high Reynolds number; while Tutar and Holdø [25] utilized both non-linear $k - \epsilon$ models and standard $k - \epsilon$ model for high Reynolds number.

In contrast, only a small number of researches studied the flow over an elliptical cylinder. Researchers investigating the turbulent transition and coherent eddies are particularly interested in the subcritical Reynolds number regime due to the fact that laminar to turbulent transition takes place in the separated shear layers. Nevertheless, laboratorial data of sub-critical range of Reynolds number for near wake region is fairly scarce in the literature.

Modi et al. [26–29] conducted a series of experimental investigations on elliptical cylinders for steady and unsteady flow. They determined the average and fluctuating distribution of pressure, Strouhal number, the moment, drag, and lift coefficients as well as near wake properties in the subcritical Reynolds number. Also, they pointed out that the aerodynamic properties showed no dependency on the Reynolds number; however, intense changes with variation of Reynolds number was observed in lift and moment coefficients of an elliptical cylinder with an axis ratio of 2:5, particularly for the attack angles smaller than 20° [27]. Furthermore, they declared that in an elliptical cylinder with an axis ratio of 1:6, the lift and moment coefficients are strongly dependent on Reynolds number [30].

Several experimental and theoretical investigations have been conducted on the flow at low and moderate

Reynolds number. Delany and Sorensen [31] carried out a study of drag coefficient and Strouhal number in an elliptical cylinder with an axis ratio of 1:2 at $\alpha = 0^\circ$ and 90° . They reported that the Strouhal number has a discontinuous changes at the critical Reynolds number; however, they had not provided detailed description of the flow around the cylinder.

In the study conducted by Ota et al. [32] on wind tunnel, hot wire anemometry data were collected for the flow through an elliptical cylinder with the axis ratio of 1:3, and at Reynolds numbers in the range of 85,000 to 312,000. According to their obtained results, the drag, moment, and lift coefficients alter irregularly at the critical Reynolds number, and the maximum value of Strouhal number is in the range of 1.0 to 1.5 and depends on α , upstream uniform flow velocity and the major axis of the elliptic.

Paul et al. [33, 34] investigated the laminar detachment and vortex shedding in the flow through the unrestrained elliptical cylinders. Their results revealed that as the axis ratio (AR) increases, dependency of Re_s , Re_{cr} , and St_{cr} on the angle of attack increases. Also, they conducted a numerical study on the properties of the near and far non-canonical wakes of 2-D elliptical cylinders. They obtained the solution of governing equations by the use of immersed boundary method (IBM) according to the projection scheme. They reported the occurrence of a phenomenon, which is called “dual near-wall instantaneous recirculation” as a function of the primary von Kármán vortex street length.

Kim et al. [35, 36] examined the applied drag and lift forces on elliptical cylinders. A 2-D incompressible flow was solved numerically for the Reynolds numbers in the range of 100 to 600 and the attack angles of 10° , 20° and 30° . They observed that the lift and drag forces are strongly dependent on the thickness of the elliptical cylinder, the angle of attack and the Reynolds number. Recently, Naik et al. [37] investigated the linear shear flow through an elliptical rotating cylinder, numerically. The IBM method was used to determine the flow characteristics and aerodynamic forces variation.

Newly, Ezadi Yazdi and Bak Khoshnevis [38, 39] studied the characteristics of the wake and the interference of the wake with the boundary layer for an elliptical cylinder at different Reynolds numbers. The HWA was used to measure the flow parameters. They found that increasing the gap ratio would result in an increase in the Strouhal number.

To measure the properties of the flow in turbulent flow regimes, Ardekani and Farhani [40] utilized a HWA as flow meter for medical and industrial applications. More recently, Ardekani et al. [41] used a single probe in the wake-survey method to calculate the drag coefficient of multistrand wires experimentally.

Afreen et al. [42] conducted an experimental investigation on flow around an elliptical cylinder with an axis ratio of 2:1. For determining the pressure distribution, they used 8 pressure sensors in 12 angles of attack between 0° and 90° . Their results revealed that the mean drag coefficients increases by increasing the angle of attack.

Also, Naik et al. [43] conducted a numerical study on the 2-D flow through a rotating elliptical cylinder and obtained the solution of incompressible Navier-Stokes equations by using IBM on a non-body conforming Cartesian grid for $Re = 100$ at various rotation rates, $\alpha = 0.5, 1.0$ and 1.5 and $AR = 0.1, 0.3, 0.5, 0.7$, and 1.0 . They concluded that drag forces reduce as AR increases up to 0.7, while, after that, they start to increase.

Hence, the present paper follows four objectives: (1) to conduct a comprehensive study on the characteristics of very near wake region ($x/B \leq 5$) of circular and elliptical cylinders through the method recognized as the best method. The statistical feature of velocity field (including time-averaged stream-wise velocity, standard deviation, skewness, and kurtosis) and vortex shedding frequency at Reynolds number of $Re_B = 3.0 \times 10^4$, in which B denotes the minor cylinder diameter, will be measured; (2) Compare the characteristics of the flow around the circular and elliptical cylinders; (3) To collect the turbulence characteristics in the wake region of an circular or elliptical cylinders, specially higher-order moments in the velocity signals of hot-wire, which contain useful information, including the related coherent structures, unfortunately, there is a little information in the literature. (4) To measure the drag coefficient of the circular and elliptical cylinders at the Reynolds number of interest and comparing the obtained results. The drag coefficient is computed for measurement data obtained via a single HWA probe with a desirable level of precision by utilizing the flow velocity profile and standard deviation downstream of the model and calculating the non-dimensional distance, x/B .

2 Experimental set-up

The experimental work of the present study is conducted in the wind tunnel of the aerodynamics laboratory located at Hakim Sabzevari University (HSU). Figure 2 shows a schematic diagram of the wind tunnel with a length of 1.68 m and a test section of $40 \times 40 \text{ cm}^2$ and no extra device, such splitter plates, is used for changing the condition, for example to force parallel shedding. The used wind tunnel is an open blower type and a motor controller was employed to control the air velocity. In order to monitor the flow velocity, speed of a fan is modified in range of 2 to 28 m/s. In addition, to attain sufficient air flow, 4 mesh screens are utilized in the settling chamber of the wind

Fig. 2 Schematic of wind tunnel

tunnel; one of the four screens is positioned before the honeycomb and the rest are positioned after. Figure 3 illustrates the schematic of wind tunnel test section along with the complete form of the model placed in the test section. The free-stream turbulence in the test section is less than 0.1%. The constant-temperature HWA [constructed by Fara Sanjesh Saba Co. (FSS)] is used to determine the velocity profiles of the flow and the turbulence intensity in downstream of the model. The single normal (SN) hot-wire probe is utilized with a 5 μm tungsten wire as its sensor. For data acquisition from hot-wire signal, the values of sampling frequency and time duration are chosen to be 5 kHz and 70 s respectively. For further analysis and processing using a dedicated application (Flow Ware), the voltage output from HWA is transmitted to a computer via a 12 bit A/D data acquisition card. A traverse mechanism of 0.01 mm precision is employed for moving the probe in the wind tunnel. Furthermore, a pitot tube has been utilized for measuring the total and static pressure in the

wake of cylinder in order to determine the mean velocity references. A spectrum analyzer and the FFT program on a PC is used to compute the vortex shedding frequencies.

The properties of the equipment employed in the present study is as follows:

- HONEYWELL DC005NDC4 Pressure transducer with the pressure range of ± 5.0 in H_2O and temperature compensation over 0°C to 50°C .
- The hot-wire filament probe with a length of 1.25 mm.
- The ratio of $((R_w - R_a)/R_a)$: 0.8, this ratio is known as overheat ratio. In which, R_a and R_w are the resistance of the hot wire sensor at ambient temperature and operating temperature, respectively.

The tested cylinders are made of Aluminum with alloy No. 7023. An elliptical cylinder with a minor cylinder diameter (B) of 21.2 mm, major cylinder diameter (A) of 42.4 mm, and an axis ratio of $AR = 2$ is employed. The cylinders are

Fig. 3 Schematic of test section set-up

placed in the test-section of the wind tunnel. The l/B ratio of the cylinder is 18.40, where l is the length. Profiles of mean velocity and standard deviation are obtained by moving the traverse mechanism in the wake of the cylinder at $x/B = 0.25, 1, 2.5, 5, 10, 15, 20$ and 25 at downstream. The Reynolds number is kept approximately at the constant value of 3.0×10^4 for all of the tests.

3 Data processing and uncertainly analysis

For determining the time-averaged velocity ($\bar{u}$) at desired points, instantaneous velocity data is collected. Therefore, the time-averaged velocity can be calculated from:

$$\bar{u} = \frac{1}{N} \sum_{i=1}^N U_i, \quad (1)$$

in which, N denotes the sample size. In order to calculate the instantaneous fluctuating velocity (u_i), we have:

$$u_i = U_i - \bar{u}. \quad (2)$$

For expressing the standard deviation, we usually utilize root mean square velocity (u_{rms}), which can be calculated from:

$$u_{rms} = \sqrt{\sum_{i=1}^N \frac{u_i^2}{N-1}}. \quad (3)$$

Both of the above quantities, ($\bar{u}$) and (u_{rms}) were non-dimensionalized with the free stream velocity.

In a random distribution, for evaluating the degree of asymmetry, the skewness factor (S_u) is used; which is equal to zero in a Gaussian distribution. Also, for checking the degree of peakedness, the kurtosis factor (F_u) is utilized, which is equal to 3 for a Gaussian distribution. For calculating these factors in an instantaneous flow velocity, we have [44]:

$$S_u = \frac{\sum_{i=1}^N \frac{(U_i - \bar{u})^3}{N-1}}{u_{rms}^{3/2}}, \quad (4)$$

$$F_u = \frac{\sum_{i=1}^N \frac{(U_i - \bar{u})^4}{N-1}}{u_{rms}^2}. \quad (5)$$

Both the S_u and F_u were non-dimensionalized with the third and fourth powers of the standard deviation.

Several uncertainties are involved in measurement and calculation of each parameter; these uncertainties can be classified in two main groups of bias and precision

uncertainties. The uncertainties originated from calibration, data acquisition and data reduction are considered as bias uncertainty (B), and can be estimated referring to the work of Jørgensen [45]. Furthermore, repeatability of the measurement is considered as precision uncertainty (P) and can be obtained from the method of Student's t -distribution with a 95% confidence interval [46]. So, the overall uncertainty can be obtained as:

$$E = \sqrt{B^2 + P^2}. \quad (6)$$

As a result, by using the above relations, the overall uncertainties of $\bar{u}$ and u_{rms} can be determined. For deriving the uncertainty of other parameters, error propagation [46] is used:

$$\frac{E_{\bar{u}}}{\bar{u}} = \sqrt{2 \left(\frac{E_{\bar{u}}}{\bar{u}} \right)^2}, \quad (7)$$

$$\frac{E_{u_{rms}}}{u_{rms}} = \sqrt{\left(\frac{E_{\bar{u}}}{\bar{u}} \right)^2 + \left(\frac{E_{u_{rms}}}{u_{rms}} \right)^2}, \quad (8)$$

$$\frac{E_{S_u}}{S_u} = \sqrt{\left(\frac{3 E_{\bar{u}}}{2 \bar{u}} \right)^2 + \left(\frac{3 E_{u_{rms}}}{2 u_{rms}} \right)^2}, \quad (9)$$

$$\frac{E_{F_u}}{F_u} = \sqrt{\left(\frac{2 E_{\bar{u}}}{\bar{u}} \right)^2 + \left(\frac{2 E_{u_{rms}}}{u_{rms}} \right)^2}. \quad (10)$$

Accordingly, maximum uncertainties of mean velocity, the dimensionless standard deviation of velocity, skewness factor, and kurtosis factor were obtained to be 3%, 6.5%, 5% and 6.5%, respectively. To see details of the calculation procedures, please refer to [44].

4 Results and discussion

4.1 Velocity measurements

In order to determine the characteristics of the near-wake and far-wake regions, data obtained from hot-wire measurements at different positions are used. An example of the obtained result in horizontal plane of elliptical cylinder downstream can be seen in Fig. 4a–h. The mean velocity, the dimensionless standard deviation, and the skewness profiles at $x/B = 2.5$ are shown in Fig. 4b–d, respectively, also, in Fig. 4f–h, these diagrams are provided for $x/B = 5$.

Fig. 4 **a** Hot-wire measurement at $x/B = 2.5$; **b** dimensionless mean velocity at $x/B = 2.5$; **c** dimensionless standard deviation at $x/B = 2.5$; **d** skewness at $x/B = 2.5$; **e** hot-wire measurement at

$x/B = 5$; **f** dimensionless mean velocity at $x/B = 5$; **g** dimensionless standard deviation at $x/B = 5$; **h** skewness at $x/B = 5$

In the dimensionless standard deviation profile, the peaks correspond to the positions in the wake, in which the maximum fluctuations occur, also, in the skewness profile, the negative peaks correspond to the wake edges; these positions are shown in Fig. 4a, e for the planes at $x/B = 2.5$ and $x/B = 5$, respectively. Skewness and the kurtosis, are the third and fourth orders of central moments, which become dimensionless by the third and fourth powers of the standard deviation. In both of these diagrams, a peak in the wake edges can be observed; which was also reported in [47] for a general two-dimensional mixing region, in [48] for wakes, in [49] for jets and in [50] for boundary layers. For determining the edge of the boundary layer, wake or jet, a conventional method is determining the thickness of a boundary layer or of the edge of a wake or jet, which is obtained by detecting a point in which the velocity is 99% (or in some references 95%) of the “outer velocity”. In this

procedure, several measurement uncertainties arise due to the fact that in the presence of stream-wise pressure gradients, the outer velocity is not known in advance and cannot be considered the same as the asymptotic velocity; also, normal derivative of the velocity is approximately zero in the region that belongs the shear layer edge. As a results, in this paper, we aim to use the negative peak of the skewness for determining the edge of the wakes, appropriately.

In Fig. 4a, e, the *black symbol* is the representative of the skewness peak, and the *white symbol* corresponds to the standard deviation peak.

Figure 5 depicts the mean velocity profiles at two different downstream positions ($x/B = 5$ and 10) for $Re = 3.0 \times 10^4$. The maximal mean velocity deficit $U_{wake,max}$ and the wake half-width b , which stands for the distance between two positions where the mean

Fig. 5 The wake deficit for the flow through a circular cylinder at Reynolds number of 3.0×10^4

Fig. 6 Profiles of the stream-wise mean velocity for circular and elliptical cylinders at four positions of the wake at $Re = 3.0 \times 10^4$

velocity deficit is the half of $U_{wake,max}$, are used for normalization of the velocities and lengths, respectively. The normalized mean velocity profiles in two different positions reach to a single curve approximately, which is compatible with data correlations and experimental measurements reported by Schlichting et al. [51], that is $U_{wake}/U_{wake,max} = [1 - (y_{wake}/1.1338b)^{1.5}]^2$. This suggests that the measured wake has attained a self-similar mode.

Figure 6 shows profiles of mean stream-wise velocities (u/u_∞) at downstream of the cylinders in comparison with the dimensionless distance that is perpendicular to wake region (y/B) at four x -positions ($x/B = 1, 2.5, 5$, and 10) and $Re = 3 \times 10^4$. These stream-wise locations have been selected to include important regions of interest, namely the near-wake region ($x/B = 1, 2.5$, and 5) and far-wake region ($x/B = 10$). Velocity and x – y coordinates become dimensionless by incoming stream and minor cylinder diameter, respectively. At $x/B = 1$, where the centerline velocity is higher than free-stream velocity,

velocity deficiency and velocity overflow occur. By further increasing the downstream distance, the centerline velocity reaches the free-stream velocity and due to the fact that the fluid is entrained in the wake, the wake becomes broader. Moreover, in far-wake region, viscous dissipation and diffusion weakens the eddies, which cause that the wake size reaches a limiting value. As can be seen in Fig. 6, the profile of u -velocity is severely dependent on the axial ratio. In higher axial ratios and for the stream-wise velocity, recovery is much faster compared to the lower axial ratios due to the size of the recirculation bubble in the stream-wise direction, as was expected, in recirculation region, a strong velocity deficit is occurred in (u)-profile, the profile is V -shaped near the cylinder it is U -shaped far from the cylinder in the downstream. For example, as can be seen, at $AR = 2$, the recovery is faster than $AR = 1$, also, as it is obvious, the velocity distributions are symmetric in both axis ratios except some deviation originated from uncertainties. The velocity defect is defined in terms of difference in the velocities of the free-stream u_∞ and the measured local mean velocity u , and it is maximal for $AR = 2$. The velocity defect and wake region decrease by increasing the AR . By comparing the wake region in circular and elliptical cylinders, it is observed that the wake of the circular cylinder is wider due to the smaller base pressure and larger drag coefficient C_D , as well as a lower Strouhal number (see Sect. 4.2).

Using the third-order moment (skewness), the information about the statistical distribution of the turbulent fluctuations around the mean velocity can be obtained. As mentioned before, $S_u = 0$ for a Gaussian distribution, so, none-zero values represents the asymmetry in the fluctuation distribution.

The stream-wise skewness factor in $AR = 1$ and 2 at four positions of $x/B = 10, 15, 20$, and 25 is presented in Fig. 7. As can be seen in this figure, due to the separation of skewness factor peaks and the rms profiles, the wake is extended. According to this figure, excellent symmetry can be observed about the wake centerline, so the skewness factor is approximately zero at the centerline of the wake. Also, at $AR = 1$, the velocity fluctuations about the wake centerline are positive, representative of positive skewness; however, they are negative at the outer edges of the wake, which indicates the negative skewness.

As shown in Fig. 7, axial ratio has a significant effect on the S_u . This is probably due to the presence of the separated shear layers and vortex structures in the upper and lower parts of the cylinders, which have different flow regimes. The obtained pattern of the skewness factor in this study is analogous to the results of Petrovic et al. [52] for an infinite circular cylinder. Just like the results of this study, Fabris [48] reported that at the centerline of the wake, the skewness is positive due to the movement of

Fig. 7 Variations of the stream-wise skewness factor at selected x/B positions

eddies toward the wake centerline. He also explained that due to the alternating flow near the edge of the wake, and the subsequent entrainment of the irrotational fluid in the turbulent wake, larger skewness and kurtosis values would be obtained [53]. The skewness peak was used for identifying the width of the wake by [39, 54, 55].

The wake edges (in which the skewness has the maximum negative peak) in the horizontal plane in both near and far-wake regions are shown in Fig. 8 for two axial ratios. According to this figure, first a decrease in the width of the wake up to a minimum value is observed, which is probably related to end of the mean recirculation region. In addition to the wake widening, by decreasing the AR, an increase is observed in the length of recirculation region, which would lead to a slight decrease in the curvature of the near-wake boundary that is convex at first and then it becomes concave.

Therefore, it can be concluded that the widening of the wake and the increasing of the recirculation region length, have a direct relation with the value of drag coefficient. As AR increases, the length of the recirculation area and the width of the wake decreases, which results about 50% reduction in drag coefficient (see Table 2). It can also be

concluded that the increase in AR leads to a delay in the separation of the boundary layer on the cylinder, which reduced the drag coefficient.

The variance of the stream-wise velocity fluctuations at four downstream positions of the wake are plotted in Fig. 9. As can be seen in this figure, the shear layer grows in the stream-wise direction, while in transverse direction, high turbulent zones becomes broader. Also, for the higher AR, the dimensionless standard deviation is smaller. As observed, the dimensionless standard deviation is high in the vicinity of the cylinders, indicating an increase up to 25% for AR = 2 and 38% for AR = 1. At a short distance (i.e., at $x/B = 1$), the dimensionless standard deviation is not maximum at the centerline of the wake. These profiles also are symmetric with respect to the centerline of the wake. In Fig. 9, a dual-peak pattern in the profile of *rms* velocity in stream-wise direction can be observed in the near-wake region. The observed peaks in std/u_∞ diagram are due to the formation of strong shear layers on both sides of the cylinder in the presence of the large mean velocity gradients ($\partial U_i/\partial x_j$). Therefore, TKE (turbulence kinetic energy) production rate increases on both sides of the cylinder, which would lead to the formation of dual-peak patterns,

Fig. 8 Effect of the variation of the axial ratio on the flow characteristics of near and far-wake regions: the white symbols shows skewness peaks in AR = 1, and black represents the skewness peaks at AR = 2

Fig. 9 Velocity standard deviation profiles at four downstream positions ($x/B = 1, 2.5, 5$ and 10) for $AR = 1$ and 2

as in Fig. 9. It should be noted that, the turbulence production is higher for smaller axial ratios according to the larger recirculation bubbles and the subsequent higher velocity gradient in the shear layer. In the far-wake regions, small mean velocity gradients would lead lower *rms* velocities for $|y/B| > 2$, at this region, predicted *rms* velocities at $AR = 2$ is less than $AR = 1$.

By careful evaluating the *rms* variation of the velocity fluctuations in the curves for near-wake and far-wake edges, according to Fig. 10, some interesting information about the velocity field can be achieved. It can be seen that variation of the fluctuations is related to the instability of the shear layer along these curves – and can be identified through the dynamics of the vorticity concentrations. Also, it can be observed that the increasing trend of fluctuations begins at a specific position ($x/B = 2.5$), which is consistent with the instability initiation along the recirculation boundary. By increasing the axial ratio, increase in the fluctuations is delayed and shear layer instability moves downstream, in addition, the length of recirculation

Fig. 10 Variation in the velocity standard deviation for near-wake and far-wake edges

region would be increased. Furthermore, it is evident that the fluctuations are higher at $AR = 1$ than the $AR = 2$. These differences are consistent with the different vortex structure due to the increase of AR .

The above analysis for the configuration of the wake in the downstream can also be obtained by studying kurtosis parameter, which is shown in Fig. 11. In fact, the kurtosis value may help us to understand the origin of high-level fluctuations. As mentioned before, high kurtosis values (higher than 3, which represents the Gaussian distribution) indicate the sharper and higher peaks values; however, for low values of kurtosis (lower than 3), the fluctuations have fewer distinct peaks and are likely sinusoidal (with the kurtosis value of 1.5). This usually represents that the identified fluctuations by the resulting velocity through the regular passage of concentrated vortices are dominant, as can be seen for the case of alternate vortex shedding for the wake of a 2-D body in cross-flow. In this study, most of the kurtosis values are slightly lower than 3, with the exception of kurtosis values are in the order of 2 for maximum *–rms* curve at the $x/B \leq 2.5$ (Fig. 11), due to the increase in the fluctuations, which are indicated in Fig. 10. The generation and passage of vorticity regions are corresponded to extension of upper and lower parts of the wake. Moreover, the effect of increasing the axial ratio on the generation of these structures is authenticated by the delayed variation of the kurtosis.

4.2 Strouhal number and drag coefficient

4.2.1 Vortex-shedding frequency

As mentioned before, HWA can be used to measure the maximum frequency of vortex shedding wake of the cylinder. The Strouhal number (St) is calculated using the peak in the velocity trace that is collected from the data obtained by HWA. St is obtained as follows:

Fig. 11 The kurtosis values for the peaks of the velocity standard deviation

Table 1 Comparison of the Strouhal number with other numerical and experimental results

Writers	Nature of study	Axis ratio	Re	St
Blevins [56]	Experimental	2	1.0×10^5	≈ 0.2
Modi and Dikshii [26]	Experimental	2	6.8×10^4	≈ 0.22
Hover et al. [57]	Experimental	1	3.0×10^4	≈ 0.2
Present	Experimental	2	3.0×10^4	≈ 0.22
Present	Experimental	1	3.0×10^4	0.2

$$St = \frac{f \times B}{u_{\infty}}, \quad (11)$$

where f represents the frequency of vortex shedding.

Table 1 provides an easy way to compare our experimental results for the Strouhal number with other experimental and numerical results in the literature. These results are compatible with those of other studies carried out on uniform flow. The width of two free shear layers and the velocity of the free stream affects the frequency of vortex shedding. The decrease in the axial ratio leads to an increase in the distance between the two free shear layers. Therefore, a decrease in the axial ratio would lead to a reduction in the interaction of these two shear layers, and consequently, the Strouhal number would also be reduced (see Table 1). Also, it is sometimes probable that separation of shear layer on one side stretches the adjacent vortex in the opposite shear layer towards it, which increases the Strouhal number.

4.2.2 Calculation of drag coefficient using wake-survey method

By applying the momentum balance principle for a control volume around a cylinder and using the wake-survey method, the time-averaged drag coefficient can be obtained. Here, for determining the drag coefficient, the effect of momentum deficit and time-averaged turbulent stress at the outflow plane of the wake are considered. The drag coefficient is obtained from the velocity and its fluctuations profiles along the test cell at a stream-wise position of $x/B = 25$. In this research, to calculate the drag coefficient, an extended equation is used, in which shear stress is minimum and intensity of turbulence can be regarded to be nearly homogenous ($u' = v' = w'$) [41, 58, 59]:

$$C_D = 2 \int_{-\infty}^{+\infty} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}} \right) d\left(\frac{y}{l}\right) + \int_{-\infty}^{+\infty} \frac{\overline{u'^2}}{u_{\infty}^2} \cdot d\left(\frac{y}{l}\right). \quad (12)$$

In the above relation, the first and second terms represent the momentum deficit of the time-averaged flow field and the contribution of the turbulent fluctuations, respectively. In all of the conducted experiments, it is found that the second term is about 10–15% of the total drag. Assuming a homogenous intensity for turbulence in the wake region (i.e., $u' = v' = w'$), we can simplify Eq. (12), and therefore, a single sensor normal HWA probe can be used for measuring the data with an acceptable accuracy (see Ardekani et al. [41] for details). Thus, in order to calculate the momentum difference (C_{Dm}/C_D) by a single sensor normal HWA probe, the proper distance (x/B) from the model should be determined, in which, the homogenous turbulence assumption is valid.

Figure 12 presents, C_{Dm}/C_D versus x/B for different axial ratio. As shown, at $x/B < 2.5$, the share of momentum difference term in drag coefficient in Eq. (12) is low (less than 80% of the overall drag coefficient). By increasing x/B , the drag coefficient also increases, at $x/B = 10$, its share becomes approximately 82%. By further increase of x/B , drag coefficient continues to increase and the slope of the momentum difference term increases; however, it becomes gentler. At $x/B = 25$, the share of momentum difference term become about 93% of the overall drag coefficient. The dependency of C_{Dm}/C_D on axial ratio is depicted in Fig. 12, in which, for $x/B = 25$, the ratio is about 97% at $AR = 1$, and 95% at $AR = 2$. In Fig. 13, C_{DT}/C_D , which represents the share of Reynolds stress term in the overall drag coefficient is shown for different axial ratio. At low x/B values, Reynolds stress is high, which decreases by increasing up to $x/B = 10$, after that, the decreasing trend also continues but its rate decreases. Moreover, by increasing the AR , the share of Reynolds stress term in overall drag coefficient is decreased. It should be noted that the homogenous assumption for flow turbulence is not valid at low x/B values [60], while it is valid for higher x/B values.

Fig. 12 Variations of momentum difference term (C_{Dm}/C_D) on the basis of distance downstream of cylinders (x/B) at different axial ratios

Fig. 13 Variations of Reynolds stress term (C_{DT}/C_D) on the basis of distance downstream of cylinders (x/B) at different axial ratios

Table 2 Comparison of the drag coefficient with other numerical and experimental results

Writers	Nature of study	Axis ratio	Re	C_D
Lindsey [61]	Experimental	2	3.0×10^4	≈ 0.62
Hover et al. [57]	Experimental	1	3.0×10^4	1.20
Present	Experimental	2	3.0×10^4	0.61
Present	Experimental	1	3.0×10^4	1.21

In Table 2, the values of drag coefficient are given. The results are then compared with the experimental results of Lindsey [61] for $AR = 2$ and Hover et al. [57] for $AR = 1$. This comparison is summarized in Table 2; we can see that the results of the current work are in a very good agreement with the experimental results. It is evident that with increasing axial ratio (AR), the drag coefficient decreased. The results provided for magnitude of drag coefficient show a higher value at the lower axial ratios (i.e. $AR = 1$) compared to the higher axial ratios (i.e. $AR = 2$), the opposite trend is seen for Strouhal number (Table 1), where it is higher at higher axial ratios and vice versa. Strouhal number demonstrates the interaction between shear layers; the positive values shows a positive interaction, which leads to the growth of vortex and initiation of early shedding as well as wake weakening through mixing and the consequent reduction in the drag coefficient.

5 Conclusions

In this paper, the 2-D flow around circular and elliptical cylinders was studied using HWA in the sub-critical Reynolds number regime, which was considered to be 3.0×10^4 . Time-averaged velocity, standard deviation, skewness and kurtosis factors, Strouhal number, and drag coefficient were reported. For calculation of the drag coefficient, the

wake-survey method was employed. The following conclusions could also be drawn from the present study:

- It was found that in the near and far-wake regions, the characteristics of the wake are strongly dependent on the axial ratio.
- It was observed that by increasing the axial ratio, width of the wake decreases.
- Profile of the dimensionless standard deviation in the wake becomes broader by moving downstream, and the peaks occur at positions with high velocity gradients in the velocity profile.
- The *rms* value of the stream-wise velocity component as well as the velocity deficit region decrease as the dimensionless distance (x/B) increases. The dimensionless standard deviation becomes smaller for the larger axial ratio, while mean recirculation region becomes smaller.
- In the near-wake region, a dual-peak was observed in the stream-wise *rms* velocity (u_{rms} component) profile, which is caused by formation and separation of strong shear layers on both sides of the cylinders. In addition, by increasing the distance from the cylinder in the far downstream region, the dual peaks are fall apart from each other. Also, it was observed that the stream-wise *rms* velocity (u_{rms}) profile decreases with increasing axial ratio (AR). In the far downstream region and in both circular and elliptical cylinders, the magnitude of *rms* velocities increases in the central region and decreases in side regions. It was shown that for the same position ($x/B = 2.5$), the maximum stream-wise *rms* velocity at $AR = 1$ is about 6% higher than $AR = 2$.
- At the higher axial ratio, mean recirculation region is smaller and consequently, drag coefficient is also smaller.
- Outside the wake center region, skewness and kurtosis factors were deviated from the standard values of Gaussian distribution due to the presence of the separated shear layers and vortex.
- In all axial ratios, high values of the skewness and kurtosis factors were observed, which is due to the presence of vortex structures in lower and upper surfaces of the cylinders; vortex are weaken as the AR increases.
- Application of a single-sensor normal HWA probe for computation of drag is valid only under homogenous turbulence assumption. Furthermore, at lower turbulence intensities, the homogenous turbulence assumption is of greater precision, and thus, measurements with single-sensor normal HWA probe would lead to a greater level of precision.
- The drag coefficient decreases with increasing AR , while the Strouhal number increases with increasing AR .

- The results of this research can be used in validating the computational studies, moreover, it provides a better understanding about the wake structure of circular or elliptical cylinders.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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